Broken Paradigms
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Thank you Haruko and Tony for your tremendous hospitality and for keeping in touch year after year through your annual letters

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AJL@80: Challenges in Quantum Foundations, Condensed Matter Physics and Beyond, March 29,30 2018
• How to ask “good” questions
• How to make them sharper
Quantum Matter: Emergence

- Electron
- Electron = charge + spin
- Spin wave (magnon) S=1
- $\nabla \cdot B = 0$
  - No monopoles

Fractional Quantum Hall effect

Fractionalization: $S=1/2$ spinons:
- (neutral) spinon Fermi surface

Luttinger Liquids

Spin-charge separation

Fractionalization

Monopoles in spin liquids
One story.....
Disorder driven Superconductor-Insulator Transition SIT

Anderson Localization

BCS s-Superconductivity

Breakdown of both paradigms!
We now discuss our results in light of existing theories. We believe that MEM correctly captures the gap, coherence peaks, and even in the insulating state. Note that all the LDOS curves persist up to an even higher temperature. Hard gap and little evidence for coherence peaks, and the pseudogap is shown in Fig. 2e,f. Here the ground state is an insulator with a single-particle gap, what is the energy scale that vanishes on approaching the quantum critical point from the insulating side?

Phase Diagram
Disorder tuned SIT

\[ U = -2t \]

\[ E_{\text{gap}}/t \]

\[ \rho_s \]

\[ E_{\text{gap}} \]

\[ T_c = \min(E_{\text{gap}}, \rho_s) \]

\[ T^* \sim E_{\text{gap}} \]

\[ \omega_{\text{pair}} \]

\[ 2\omega_{\text{pair}} \]

A. Ghosal, M. Randeria, and N. Trivedi, PRL 81, 3940 (1998); PRB 65, 014501 (2001).
Prediction of robust Cooper pairs in insulator starting with a fermion model

Pairing of exact eigenstates; Bogoliubov de-Gennes inhomogenous mean field theory
Ghosal, Randeria & NT, PRL 81, 3940 (1998); PRB65, 014501 (2001)

Model: Attractive U Hubbard model with potential disorder
No coherence peaks are observed at any temperature–disorder plane. Figure 2 over 100 disorder realizations. Specific parameter values, we have run extensive simulations that average QMC. These results are obtained at fixed attraction in all the figures are dominated by disorder averaging and not from the finite-size scaling beyond the scope of this paper. The statistical error bars curves are guides to the eye; extracting critical exponents requires Fig. 2, is large and finite in all states. The two-particle energy scale $T$ decreases to zero at the critical disorder strength $V_c$. The coherence peaks (red) visible in the SC state, vanish for $0.1$.

Quantum Monte Carlo simulations and analytic continuation
Bouadim, Loh, Randeria & Trivedi Nature Phys. 7, 884 (2011)
No coherence peaks are observed at any temperature–disorder plane. Figure 2 over 100 disorder realizations. Between the insulator and the quantum critical region, and superconducting transition temperature, a hard gap (black region) persists for all finite systems provides an upper bound on the actual finite-size scaling beyond the scope of this paper. The statistical error bars in Fig. 3, is non-zero in the insulator but vanishes at the SIT. The dashed (red squares), obtained from the dynamical pair susceptibility shown in Fig. 2, is large and finite in all states. The single-particle energy scale decreases to zero at the critical disorder strength. The coherence peaks (red) visible in the SC state, vanish for 0.1, but the coherence peaks (red) exist only in the SC state and not in the insulator. The single-particle DOS.

Both thermal and quantum phase fluctuations

$\rho_s \sim T_c < \omega_{\text{dos}}$

Scanning tunneling spectroscopy

Sacépé, Dubouchet, Chapelier, Sanquer,OVadia, Shahar, Feigel’man and Ioffe, Nat. Phys 7239 (2011)

Bouadim, Loh, Randearia & Trivedi Nature Phys. 7, 884 (2011)
No coherence peaks are observed at any SIT (temperature–disorder plane). These results are obtained at fixed attraction in all the figures are dominated by disorder averaging and not from the finite-size scaling beyond the scope of this paper. The statistical error bars, Fig. 3, is non-zero in the insulator but vanishes at the SIT. The dashed single-particle gap $T_c$ decreases to zero at the critical disorder strength $V$, the coherence peaks (red) visible in the SC state, vanish for $V > 1$. The superconducting transition temperature, from the DOS and the dynamical pair susceptibility discussed in the Supplementary Information. What gives us confidence is that our central results on the single- and two-particle energy scale $\omega_{\text{dos}}$ and $\omega_{\text{pair}}$ and goes to zero at the transition. These gap scales are extracted where the two-particle energy scale $\omega$ finally vanishes at the critical disorder $V$. A hard gap (black region) persists for all $V < V_c$. The superconducting amplitude inhomogeneity and phase fluctuations.

$T_c$ is the superfluid density $\rho_s$, calculated from the superfluid density $\rho_s$ and average $\langle U \rangle$ for the insulator ($V = 1$). $T_c$ is the Kosterlitz–Thouless universality class, we estimate the critical disorder $V_c$. $\rho_s$ remains non-zero across the SIT, whereas the two-particle energy scale $\omega_{\text{pair}}$ related to the pseudogap crossover scale described below. The superconducting transition temperature, $T_c$, calculated from the DOS shown in the Supplementary Information. These comparisons permit us to separate the effects of QMC results with self-consistent BdG calculations, which take into account only the spatial amplitude variations; see Supplementary Information. These comparisons permit us to separate the effects of $\rho_s$ and average $\langle U \rangle$ for the insulator ($V = 1$).

We have made extensive comparisons of the MEM correctly reproduces the low-energy structure of test spectra. We note that this procedure on $\rho_s$ and average $\langle U \rangle$ for the insulator ($V = 1$). We have verified that these results obey various sum rules to high precision, and that the MEM results are those obtained from the MEM.
Local gap and Local superfluid density

DOS gapped on both sides of transition

Scan-tunneling

Theory

Experiment

Disorder leads to “SC puddles”

Scan-squid

Field coil

SQUID pick-up loop

Bouadim et al., Nat. Phys. 7, 884 (2011)

With increasing disorder:

- DOS gapped on both sides of transition
- Superfluid density is vanishing at the transition

\[
\rho_s + 2 \int_{0^+}^{\infty} \frac{d\omega}{\pi} \operatorname{Re} \sigma(\omega) = \langle -k_x \rangle
\]

What processes contribute to the spectral weight at low freq?

Bosonic model

\[2\Delta \rightarrow \infty\]
Whatever happened to d-wave and disorder?
Why is the low energy V-shaped DOS unaffected by disorder?


\[ BSCCO = Bi_{2}Sr_{2}CaCu_{2}O_{8+\delta} \]

\( p=0.19 \)
"Strong correlations make high-temperature superconductors robust against disorder”
With attractive interactions

- **Wigner xtal**
- **Same-spin paired insulator**
- **BCS metal**
- **opposite spin-paired insulator**
- **Localized Insulator**
Magnetic field Puzzles:

Fluctuating vortices
Nernst

$vortex core$:

$H$
$H_{c2}$
$H_{c1}$

$T$

metallic core
insulating core

$\omega$

underlying state that goes SC: QH Insulator (with interactions)
Need measurements of the superfluid density (mutual inductance) as a function of magnetic field at low T.

Energy scales

BCS--BEC crossover in a field

Vortex cores?
• How to ask “good” questions
• How to make them sharper
Metal: Electron Waves

Instability of Fermi surface

\[ p = \frac{h}{\lambda} \rightarrow \text{wave length of electron wave} \]

\[ E_F = \frac{p_F^2}{2m} \rightarrow \text{Fermi energy} \]

\[ \Delta \sim E_m e^{-\frac{1}{g}} \]

\[ \Delta < E_m < E_F \]
How can the BCS paradigm break down?

(1) Strong coupling to glue:
Fermi sphere greatly perturbed

\[ T_c \propto \Delta \]

(2) Non-adiabatic limit:
electrons are slower than the mode

\[ \Delta \propto E_m e^{-\frac{1}{g}} \]
BCS paradigm \[ T_c \ll E_{\text{mode}} \ll E_F \]

non-adiabatic limit:
electrons are slower than the mode

\[ \tau_e > \tau_{\text{mode}} \]
\[ E_F < E_{\text{mode}} \]

Bismuth: \( E_F = 25 \text{ meV} \)
\( E_{\text{mode}} = 12 \text{ meV} \)
\( T_c = 0.5 \text{ mK} \sim 0.05 \text{ meV} \)

Can an insulator become a SC? \textit{at same doping}
How does SC arise when there is \textbf{no} Fermi surface?
Fixed attraction $U$

$U=0$

$E_{1p} = \Omega$

$E_{2p} = 2\Omega$

$U \neq 0$

$crossover$

$SC$- Insulator Transition

Increase hopping $t$ between wells
Insulator-Superconductor Transition:

- Fermi Insulator
- Bose Insulator
- BEC
- BCS

Crossover

SC-Insulator Transition

Lowest energy excitation is a charge 1 fermion

Lowest energy excitation has charge 2e -- "boson"

Minimum Gap locus is a Point

Minimum Gap locus is a Contour

* Topology of minimum gap locus
* Gap-edge singularity in DOS

Loh, Randeria, Trivedi, Chen, Scalettar, Phys. Rev. X 6, 021029 (2016)
Minimum gap locus in $k$-space

BEC regime ("strong pairing")

$E_k$ vs $\epsilon_k$

$\mu < 0$

$\Delta$

$E_{gap}$

$\bullet$

point

$\epsilon_k = 0$

or $k = 0$

BCS regime ("weak pairing")

$E_k$ vs $\epsilon_k$

$\mu$

$E_{gap} = \Delta$

$E_k = \sqrt{(\epsilon_k - \mu)^2 + \epsilon_k}$

contour

$\epsilon_k = \mu$

or $k = "k_F"$
Can a topological insulator become a SC? Is the SC topological? What is the role of the Berry phase?
Dirac points protected by inversion and time reversal symmetry

Break inversion
Sublattice potential

Break TR complex nn hopping

(Color: Berry curvature = Berry flux density)
Band Structure: Kane-Mele Model

\[ H_{KM} = -t \sum_{\langle i,j \rangle, \sigma} c_{i \sigma}^\dagger c_{j \sigma} + i \lambda \sum_{\langle i,j \rangle, \sigma} \nu_{ij}^\sigma c_{i \sigma}^\dagger c_{j \sigma} + m_{AB} \sum_{i \in \Lambda, \sigma} c_{i \sigma}^\dagger c_{i \sigma} - m_{AB} \sum_{i \in B, \sigma} c_{i \sigma}^\dagger c_{i \sigma} \]

trivial phase

\[ x = 0.25 \]

\[ m_{AB} > 3\sqrt{3}\lambda \]

\[ x = 0.5 \]

\[ m_{AB} < 3\sqrt{3}\lambda \]

topological phase transition

\[ x \equiv \frac{3\sqrt{3}\lambda}{3\sqrt{3}\lambda + m_{AB}} \]

topological phase

\[ x = 0.75 \]
Pairing Instabilities at Bulk and Edge of Topological Insulator
Phases

1. Time-reversal Invariant
   - Topological SC ($\tilde{\nu} = 1$)

2. Time-reversal breaking
   - Topological SC ($\tilde{C} = \pm 1$)

3. Time-reversal invariant
   - Trivial SC ($\tilde{\nu} = 0$)

4. Time-reversal breaking
   - Trivial SC ($C = 0$)
Tony: Happy Birthday