How Downstream Retailer Merger Affects Upstream Manufacture and Downstream Retail Markets? A Generalized Theorem and Applications To Different Market Types

Ziyi Qiu
University of Chicago *
University of Illinois Urbana Champaign †

July 28, 2018

1 Abstract

This paper studies how the merger of downstream retail firms shall affect the market structures in both the upstream manufacture and downstream retail markets. By allowing downstream retail firms to merge, this paper studies the upstream manufacture firms' endogenous choices of product qualities and downstream retail firms' endogenous choices of retail prices pre- and post-merger. The paper finds that the post-merger retail prices shall increase for both the merged and non-merging products. The market shares shall decrease for the merged products and shall increase for the non-merging products. The adjustments of post-merger product qualities shall depend on the pre-merger market shares, consumer's preference over product quality and upstream manufacture firms' marginal and fixed costs of production. A generalized theorem is provided to characterize conditions to predict all possible post-merger changes of market shares, product qualities and retail prices for both the merged and non-merging products, under the cases when retail firms sell either single or multiple products. The paper also applies the general theorem to study particular market types: information goods and service goods markets.

*Department of Economics, the University of Chicago. 1126 E. 59th Street, Chicago, IL 60637. Email: ziyiqiu@uchicago.edu.

†Department of Economics, University of Illinois Urbana Champaign. 214 David Kinley Hall, 1407 W. Gregory, Urbana, IL, 61801. Email: ziyiqiu@illinois.edu.

2 Introduction

As the economy develops and product variety grows, it is widely observed that consumers go to retail stores to purchase products. Retail stores, by contracting wholesale prices with upstream manufacturers, distribute products and sell them directly to consumers. Instead of buying from each individual producer directly, buying from large retail stores has became a trend these days and brings much more convenience and accessibility to consumers. In the United States, for almost all daily-life related industries, there are major retail stores which lead consumers' purchases. Those stores include the large general merchandise retail stores such as Walmart and Target; online shopping retailers such as Amazon and Ebay; clothes retail stores such as Macy's, Newman Marcus and Saks Fifth Avenue; baby product retailers such as Toysrus and Buybuybaby; and grocery retail stores such as Wholefood, Trader Joe's and Costco. There are many benefits associated with shopping directly from retail stores. Firstly, large retail stores have big bargaining power compared to individual consumers, which can decrease wholesale prices significantly to much lower levels and may also provide lower retail prices to consumers. Secondly, larger retail stores carry a large variety of products and bring consumers much convenience by shopping everything at one place. Thirdly, large retail stores provide better shopping environment and better customer services. Last but not the least, it brings consumers more accessibility by shopping at large retail stores as those big-chain retail stores are almost everywhere. While on the other side, it is not even possible to have a handful of manufacturers located in the same place, especially in those small town areas. Besides, upstream manufacture firms can also benefit significantly from this three tier supply-distribution-consumption structure. By providing lower wholesale prices to retail firms, manufacturers can outsource all customer service, product distribution and display, and advertisement related costs, which allows manufacturers to concentrate on production without worrying too much about doing advertisement and finding customers.

As there are many retail firms existing in almost every industry, mergers between retail firms are frequently observed. The real data in the recent years show a clear rising trend in concentration of retail markets. We take U.K. grocery retail stores as an example. The number of grocery stores fell from over 140000 in 1960 to below 40000 in 1997, which is about 300 supermarket mergers and acquisitions each year in the U.K. In Italy, the number of food stores were about 340000 in 1983 but only about 290000 in 1993, which suggests about 5000 mergers happened each year. In addition, other famous examples of retailer mergers include the recent mergers of OfficeMax with Office Depot, and Walmart with Jet.com, which have received a great amount of attention from the public and could potentially influence consumers' purchase decisions a lot. Those increasing numbers of retailer mergers make understanding how retail mergers affect firms' competition behavior and consumers' purchase choices an important and timely research topic.

Since 2000, there has been a rising body of literature trying to understand how downstream retailers make profit maximization decisions, especially in the merger scenarios. The existing literature focuses on studying how the mergers between downstream retail firms affect the retail firms' pricing decisions in the downstream market and the consumers' welfare in a variety of dimensions. Among them, Fumagalli and Motta 2001 [7] claims that the downstream merger leads to higher market prices and lower consumer welfare. Hosken, Olson and Smith 2012 [12] finds that the post-merger prices frequently increase after the retailer merger. Montez 2006 [23] states that the downstream retailer merger induces a lower upstream capacity if the cost of capacity is high; a higher upstream capacity if the cost is low. Symeonidis 2010 [29] argues that the downstream retailer merger may raise or decrease consumer surplus and overall welfare, depending on different bargaining forms in the intermediate market. Mazzarotto 2004 [19] finds that for given contract terms in the intermediate market, a merger between contiguous retail stores lowers the quantity sold and raises final prices and profits per store at all neighbor locations. He also claims that with two-part tariff contracts a merger between contiguous retail stores leaves the wholesale price unchanged, lowers the quantity sold, raises final prices and profits per store as well as the value of the fixed fee per store. Inderst and Shaffer 2007 [13] finds that with the merger in the retail industry, suppliers will strategically choose to produce less differentiated products, which further reduces product variety.

However, there are some limitations of the existing work. The most important limitation of the existing literature is that upstream suppliers are not allowed to choose product qualities before and after mergers. While there has been an increasing body of literature studying downstream retail firms' optimal choices of prices post-merger, upstream manufacturers are generally assumed to produce products with exogenous product qualities. The assumption of exogenous product qualities does bring concerns. Firstly, since product qualities affect consumers' purchase decisions and also costs of production, it is reasonable that upstream manufacture firms could gain higher profits by adjusting product qualities. Allowing both prices and product qualities to be endogenous could potentially improve both upstream manufacturers' and downstream retailers' profits and necessitate the reconsideration of the classical pricing theory with exogenous product qualities. Secondly, while downstream retailer mergers do affect downstream retailer prices, they could also affect upstream firms' incentives to adjust their product qualities in order to maximize upstream profits. By allowing upstream manufacture firms to adjust their product qualities post-merger, it would affect consumers' utility and thus influence consumers' purchase decisions. The adjustments of post-merger retail prices could either be amplified or lessened given the adjustments of post-merger product qualities, depending on the directions of changes for post-merger product qualities. Thirdly, constant and exogenous product qualities are unduly demanding assumptions and are generally not supported by real data. Firms' adjustments over their product qualities are widely observed in real applications, in both non-merger

scenarios [20, 27, 4, 10, 28, 31, 9, 5, 3] and merger scenarios [1, 6, 22, 16, 21, 24]. All the concerns suggest that we should consider endogeneity of both prices and product qualities as we study firms' competition behavior, especially in a merger context. Therefore, a re-evaluation of both downstream retail firms' pricing and upstream manufacture firms' product repositioning incentives is an important and timely research topic.

In addition to the limitations of exogenous product quality assumption, the choices of firms' competition models, marginal and fixed cost assumptions, and consumers' utility functions in the existing literature are not applicable once we allow product qualities to affect both firms' marginal and fixed costs of production and consumers' utility of consumption. The existing literature usually adopts oligopoly competition models such as Cournot and Bertrand as the baseline models, which assume that products are identical and firms compete over either quantities or prices. The adoption of those models limits the ability to consider changes in the product quality dimension. In those models consumers shall only care about prices when they make purchase decisions, while product qualities are generally assumed to not affect consumers' utility and purchase choices. In addition, given that product qualities are constant, the existing literature usually assumes that marginal costs are also constant, which are not affected by different product quality choices. And the existing literature usually ignores the fixed costs of production, which are widely observed in firms' production and should also depend on different product quality choices.

Knowing the importance of understanding downstream retailer merger's effect on both the upstream manufacture and downstream retail markets, this paper shall consider both endogenous product quality choices made by upstream manufacturers and endogenous retail price choices made by downstream retailers. This paper contributes to the existing literature in several aspects. Firstly, this paper adopts the discrete choice model by allowing both product qualities and prices to affect consumers' utility and consumption choices. Secondly, this paper allows product quality choices to affect both marginal costs and fixed costs of production. Moreover, this paper allows firms to have different marginal cost and fixed cost productivities and shall study how the differences in firms' marginal and fixed cost productivities could cause different post-merger market outcomes. Thirdly, this paper distinguishes from the existing literature by studying how downstream retailer merger affects both the downstream retail market and upstream manufacture market. In particular, this paper studies how the downstream retailer merger affects product qualities chosen by the upstream manufacture firms, prices chosen by the downstream retail firms and market shares of both the merged and non-merging products post-merger. Last but the most important, this paper shall generalize the theorem to predict all possible post-merger market outcomes under different conditions and this paper also extends the theorem to consider retail firms which carry multiple products. In addition, this paper applies the generalized theorem to study particular market types: information goods and service goods markets.

The rest of the paper is organized as follows. Section 3 proposes the model with endogenous choices of product qualities for upstream manufacture firms and endogenous choices of retail prices for downstream retail firms. The model characterizes the post-merger adjustments of prices, product qualities and market shares for both the merged and non-merging products under the downstream retailer merger. Section 4 characterizes the main theorem for the post-merger predictions of market shares, prices and product qualities for both the merged and non-merging products when each retail firm carries a single product. Section 5 revises the model considering the multi-product retailer merger and studies how the merger between multi-product retail firms shall affect both the upstream manufacture and downstream retail markets. Section 6 extends the generalized theorem to predict post-merger market outcomes under the multi-product retailer merger. Section 7 and 8 apply the generalized theorem to particular market types and study the merger implications for the information goods market and service goods market, respectively. The paper shall conclude in Section 9.

3 Single Product Retailer Merger

3.1 The Model

In this section, we shall propose a model to characterize the merger between two downstream retail firms and study how the merger affects the upstream manufacture and downstream retail markets. In particular, we shall study the endogenous post-merger changes of market shares, product qualities and prices for both the merged and non-merging products. We shall consider a market of J manufacturers and J retailers. In line with many of the existing literature which assume that there is a one-to-one relation between the upstream manufacturer and downstream retail firms [11, 8, 32, 17, 18, 14], we shall assume in this section that there is an exclusive relation between one upstream manufacture firm and one downstream retail firm. We shall in the later section extend the model by allowing each retail firm to carry multiple products from the upstream producers.

To avoid the double marginalization problem that is common in the two-tier market structure, we shall assume that each upstream manufacture firm shall sign a contract with one downstream retail firm, where the downstream retail firm shall pay a two part tariff to the upstream manufacture firm [2, 15, 26, 30]. To be specific, we shall denote manufacturer j as the producer of product j. We shall denote retailer j as the seller of product j. For each manufacturer j and retailer j, there exists a set of contracting price per unit and a lump sum price, i.e. $\bar{P}_j$ and T_j, which are transferred from downstream retailer j to upstream manufacturer j. We shall show that there exists the optimal values of $\bar{P}_j$ and T_j for each product such that both upstream manufacturer j and re-

tailer j obtain the same profits as they are in the vertically integrated market (Appendix Note 1). Such two part tariff contracting mechanism eliminates the double marginalization problem in the two tier market structure. Without loss of generality, we assume that retailer j purchases products from manufacturer j with paying a fixed contracting price $\bar{P}_j$ per unit and a lump sum price T_j. Given the contracting price per unit and the lump sum price, each retailer j shall choose the optimal retail price to maximize its profit. On the other hand, given the contracting price per unit and the lump sum price, each manufacturer j shall choose the optimal product quality to maximize its profit. Both the endogenous product qualities and retail prices shall be released after the two part tariff contracting mechanism is released. Consumers observe both the retail price and product quality of each product and shall purchase the product with the highest utility. Moreover, to produce a product with different quality level, it shall require different levels of marginal and fixed costs of production.

For the simplicity of this section, we shall assume that the contracting price per unit and the lump sum price of each product j are given and the assignments of the one-to-one matchings between one upstream manufacture firm and one downstream retail firm is also given. I shall in my later work, discuss the endogenous choices of those terms. To proceed, we shall first characterize the demand side of the industry pre-merger by forming consumers' utility function and the closed forms of market shares. We then characterize the supply side of the industry pre-merger by characterizing the optimal product quality decisions of upstream manufacture firms and optimal product price decisions of downstream retail firms. Last, we shall characterize the merger between two downstream retail firms and find how such merger affects the product repositioning and pricing incentives of the upstream manufacture and downstream retail firms, respectively.

3.2 Demand Side Pre-merger

We shall first characterize the demand side of the industry pre-merger. We shall define product j as the product that is supplied by manufacturer j and sold by retailer j. Consumers consider prices and product qualities to make their purchase decisions and both prices and product qualities are determined endogenously by the upstream manufacture and downstream retail firms. We shall apply the discrete choice model and the utility of consumer i choosing product j is characterized as

$$U_{ij} = \beta Z_j - \alpha P_j + \xi_j + \epsilon_{ij}, \tag{1}$$

for $j = \{1, ..., J\}$. P_j is the retail price of product j and Z_j is the quality of product j. ξ_j is the brand fixed effect of product j. ϵ_{ij} is the idiosyncratic shock term and follows the type one extreme value distribution. α is consumer i's preference over price and β is consumer i's preference over product quality. To simplify the model, we assume that the individual's variation in the utility comes

from the idiosyncratic shock term and we here do not consider the interaction of individual demographics with product qualities. By the property of type one extreme value distribution, the market share of product j is characterized as

$$S_j = \frac{\exp\left(\beta Z_j - \alpha P_j + \xi_j\right)}{1 + \sum_{j'=1}^{J} \exp\left(\beta Z_{j'} - \alpha P_{j'} + \xi_{j'}\right)}, \tag{2}$$

for $j = \{1, ..., J\}$. We assume that consumer i could also choose not to purchase any of the J products and instead purchase from the outside option. In this case, consumer i's utility of purchasing the outside option is

$$U_{i0} = \epsilon_{i0}, \tag{3}$$

and the market share of the outside option is

$$S_0 = \frac{1}{1 + \sum_{j'=1}^{J} \exp\left(\beta Z_{j'} - \alpha P_{j'} + \xi_{j'}\right)}. \tag{4}$$

3.3 Upstream Manufacture Firm Pre-merger

We then characterize the supply side of the industry. The supply side of the industry consists of two tiers: the upstream manufacture firms and downstream retail firms, where the upstream manufacture firms are product producers and downstream retail firms are product sellers. As the product producers, the upstream manufacture firms choose the endogenous product qualities to maximize profits. As the product sellers, the downstream retail firms choose the endogenous retail prices to maximize profits. Downstream retail firm j purchases products from upstream manufacturer j by paying a fixed contracting price per unit $\bar{P}_j$ and a lump sum price T_j.

We shall first characterize the profit maximization decisions of the upstream manufacture firms in the pre-merger period. Given the fixed contracting price per unit $\bar{P}_j$ and the lump sum price T_j between manufacturer j and retailer j, upstream manufacturer j tries to maximize its profit by choosing the optimal product quality

$$\max_{Z_j} N S_j(\bar{P}_j - mc_j) - Fixed_j + T_j, \tag{5}$$

for $j = \{1, ..., J\}$, where N is the total mass of consumers, S_j is the market share of product j, Z_j is the quality of product j that the upstream manufacture firm decides to produce, mc_j is the marginal cost of production and $Fixed_j$ is the fixed cost of production. It is sensible to assume that a manufacture firm's choice of product quality shall affect both its marginal and fixed costs of production, as a higher level of product quality should require higher amounts of marginal and fixed costs of production. In line with the existing literature [16, 6], we assume that the marginal cost is linear in product quality, and fixed

cost is quadratic in product quality. Thus the marginal cost and fixed cost of product j are characterized as

$$mc_j = \gamma_j Z_j + \omega_j, \tag{6}$$

and

$$\frac{dFixed_j}{dZ_j} = \delta_j Z_j + \theta_j, \tag{7}$$

for $j = \{1, ..., J\}$, where ω_j and θ_j are the marginal and fixed cost unobserved term respectively. To use consistent notations throughout this paper, we shall denote the derivative of product j's fixed cost with respect to its product quality, i.e. $\frac{dFixed_j}{dZ_j}$, the marginal fixed cost term. The necessary condition of the upstream manufacture firm j with respect to its product quality choice is

$$[Z_j] : NS_j(-\frac{dmc_j}{dZ_j}) + N\frac{dS_j}{dZ_j}(\bar{P}_j - mc_j) - \frac{dFixed_j}{dZ_j} = 0, \tag{8}$$

for $j = \{1, ..., J\}$. Simplifying the necessary condition with respect to the product quality of each upstream manufacture firm suggests that (Appendix Note 2)

$$[Z_j] : NS_j(-\gamma_j) + NS_j\beta(1 - S_j)(\bar{P}_j - \gamma_j Z_j - \omega_j) - \delta_j Z_j - \theta_j = 0, \tag{9}$$

for $j = \{1, ..., J\}$. We shall then simplify the above equation further to get that

$$Z_j = \frac{NS_j\beta(1 - S_j)(\bar{P}_j - \omega_j) - NS_j\gamma_j - \theta_j}{NS_j\beta(1 - S_j)\gamma_j + \delta_j}, \tag{10}$$

for $j = \{1, ..., J\}$. Based on equation (10), the pre-merger quality of product j can be characterized as a function of the product's pre-merger market share. Thus solving the equilibrium pre-merger market share guarantees the solution of the equilibrium pre-merger product quality of each product, which we shall discuss in the later section.

3.4 Downstream Retail Firm Pre-merger

We then characterize the profit maximization decisions of downstream retail firms. Given the contracting price per unit $\bar{P}_j$ and the lump sump price T_j of each product, the downstream retail firms sell products directly to consumers and each retail firm shall maximize its profit by choosing the optimal retail price

$$\max_{P_j} NS_j(P_j - \bar{P}_j) - T_j, \tag{11}$$

for $j = \{1, ..., J\}$. The necessary condition of each downstream retail firm j with respect to its retail price suggests that

$$[P_j] : NS_j + N\frac{dS_j}{dP_j}(P_j - \bar{P}_j) = 0, \tag{12}$$

fo $j = \{1, ..., J\}$. We shall simplify the above equation further to get (Appendix Note 3)

$$[P_j] : 1 - \alpha(1 - S_j)(P_j - \bar{P}_j) = 0, \tag{13}$$

and hence

$$P_j = \bar{P}_j + \frac{1}{\alpha(1 - S_j)}, \tag{14}$$

for $j = \{1, ..., J\}$. Based on equation (14), the pre-merger retail price of product j can be characterized as a function of the product's pre-merger market share. Thus solving the equilibrium pre-merger market share guarantees the solution of the equilibrium pre-merger retail price of each firm, which we shall discuss in the later section.

3.5 Merger between Downstream Retail Firms

We then characterize the merger in the downstream retailer market. We shall reconsider the demand and supply sides of the market in the post-merger period. We shall study how the merger of two downstream retail firms affects the post-merger market outcomes, in particular of market shares, product qualities and retail prices for both the merged and non-merging products. Without loss of generality, we assume that retail firm 1 and 2 decide to merge. The merged retail firm keeps both products post-merger and shall adjust the optimal retail prices of both products to maximize the added profit. For the case when the merged firm decides to keep only one product post-merger, please refer to my previous work on it [25]. The new merged retail firm shall maximize the sum of profits from selling products 1 and 2 by choosing the optimal retail prices of both products

$$\max_{P_1, P_2} NS_1(P_1 - \bar{P}_1) - T_1 + NS_2(P_2 - \bar{P}_2) - T_2. \tag{15}$$

Solving the necessary conditions of the merged retail firm with respect to P_1 and P_2 suggests that

$$[P_1] : N\frac{dS_1}{dP_1}(P_1 - \bar{P}_1) + NS_1 + N\frac{dS_2}{dP_1}(P_2 - \bar{P}_2) = 0, \tag{16}$$

and

$$[P_2] : N\frac{dS_2}{dP_2}(P_2 - \bar{P}_2) + NS_2 + N\frac{dS_1}{dP_2}(P_1 - \bar{P}_1) = 0. \tag{17}$$

Simplifying the equations above, we get that (Appendix Note 3)

$$[P_1] : 1 - \alpha(1 - S_1)(P_1 - \bar{P}_1) + S_2\alpha(P_2 - \bar{P}_2) = 0, \tag{18}$$

and

$$[P_2] : 1 - \alpha(1 - S_2)(P_2 - \bar{P}_2) + S_1\alpha(P_1 - \bar{P}_1) = 0. \tag{19}$$

Rewriting equations (18) and (19) above suggests the relations of the post-merger retail prices as a function of the post-merger market shares of the two merged products

$$P_1 = \bar{P}_1 + \frac{1}{\alpha(1 - S_1 - S_2)}, \tag{20}$$

and

$$P_2 = \bar{P}_2 + \frac{1}{\alpha(1 - S_1 - S_2)}. \tag{21}$$

Given that the merger happens in the downstream retail market, the profit maximization conditions would stay the same for the upstream manufacture firms. Thus the relations of the equilibrium product qualities with market shares shall stay the same post-merger for all upstream manufacture firms, as characterized in equation (10). For downstream non-merging retail firms, the relations of the equilibrium retail prices with market shares shall stay the same post-merger, as characterized in equation (14).

After we have fully characterized the relations of equilibrium prices and product qualities with market shares for both the upstream manufacture and downstream retail firms pre- and post-merger, we can then conduct the analysis to study how the downstream retailer merger shall affect the upstream manufacture and downstream retail markets. In particular, we want to study how the equilibrium market shares, prices, and product qualities of both the merged and non-merging products adjust post-merger. We shall conduct our analysis in the following orders. We shall first study how the retail prices change post-merger for the two merged products. We then study how the post-merger market shares change for both the merged and non-merging products. After that, we shall study how the post-merger prices change for the non-merging products. Last, we shall characterize how the post-merger product qualities adjust for both the merged and non-merging products. The main theorem which summarizes all the conditions required and all possible post-merger market outcomes shall be presented in the next section.

We shall first find how the post-merger retail prices change for the two merged products. Given that the merged retailer sells both product 1 and product 2 post-merger, we know that the merged retailer's profit should be the sum of profits from selling both products. Therefore, the post-merger profit of the merged retail firm should be strictly bigger than the pre-merger profit of either retailer 1 or retailer 2, i.e. $\Pi_{12}^{post} > \Pi_1^{pre}$ and $\Pi_{12}^{post} > \Pi_2^{pre}$, where we use Π_{12}^{post} to denote the post-merger profit of the merged retail firm. Applying proof by contradiction, we find that the sum of post-merger market shares of the two

merged products have to be bigger than the pre-merger market share of either product, i.e. $S_1^{post} + S_2^{post} > S_1^{pre}$ and $S_1^{post} + S_2^{post} > S_2^{pre}$ (Appendix Note 4). We then combine the above conditions with equations (14), (20) and (21) to find how the post-merger retail prices adjust for the two merged products. We find that the retail prices for the two merged products shall increase post-merger, i.e. $P_1^{post} > P_1^{pre}$ and $P_2^{post} > P_2^{pre}$ (Appendix Note 4).

We then characterize the post-merger changes of market shares for both the merged and non-merging products. In order to understand how the post-merger market shares adjust, we shall first take the ratio of each product's pre-merger market share with respect to the pre-merger market share of the outside option

$$\frac{S_j}{S_0} = \exp\left(\beta Z_j - \alpha P_j + \xi_j\right), \tag{22}$$

for $j = \{1, ..., J\}$. We then apply equation (10) and (14) to substitute each product's pre-merger retail price and product quality as a function of its pre-merger market share as

$$\frac{S_j}{S_0} = \exp\left(\beta \frac{NS_j\beta(1-S_j)(\bar{P}_j - \omega_j) - NS_j\gamma_j - \theta_j}{NS_j\beta(1-S_j)\gamma_j + \delta_j} - \alpha\bar{P}_j - \frac{1}{1-S_j} + \xi_j\right), \tag{23}$$

for $j = \{1, ..., J\}$. Solving equation (23) above for each product suggests a positive relation of each product's pre-merger market share with the market share of the outside option (Appendix Note 5). In other words, equation (23) suggests that there exists a strictly increasing function that characterizes the equilibrium pre-merger market share of each product j as a function of the equilibrium pre-merger market share of the outside option, i.e. $S_j = f_j(S_0)$, where $f_j'(S_0) > 0$ for $j = \{1, ..., J\}$.

In the post-merger period, we know from equations (20) and (21) above that for any given value of S_j, P_j^{post} are bigger for the two merged products. Thus the RHS of equation (22) shall move downward post-merger for the two merged products. Given the LHS of equation (22) stays the same as before and the LHS of equation (22) moves downward post-merger, we know that for any given S_0 value, the intersected S_j value would be smaller for the two merged products. In other words, it suggests that the function $f_j^{post}(S_0)$ shall move downward post-merger for the two merged products. Given that the function $f_j^{post}(S_0)$ moves downward for the two merged products and stays the same for the non-merging products post-merger, we know that for any given S_0, $S_0 + S_1 + ... + S_J$ shall become smaller post-merger. As all the market shares should always add up to 1 and $S_0 + S_1 + ... + S_J$ is a strictly increasing function of S_0, the market share of the outside option therefore must increase post-merger. Thus, the market shares of all the non-merging products shall also increase post-merger. On the opposite side, the market shares of the two merged products shall decrease post-merger. Please refer to Appendix Note 6 for details. Until now we

have fully characterized the post-merger changes of market shares for both the merged and non-merging products.

Once we understand the changes of post-merger market shares for both the merged and non-merging products, we could then characterize the changes of post-merger retail prices for all the products. For the non-merging products, given that the post-merger market shares increase, we know from equation (14) that the post-merger retail prices shall also increase for all the non-merging products. Recall that previously we have found that the post-merger prices shall increase for the two merged products. We thus can conclude that the post-merger retail prices shall increase for all the products. This conclusion is consistent with what other researchers find when holding the product qualities exogenous. The intuition is that the downstream merger shall make the downstream retail market more concentrated and less competitive, which shall give each retail firm a higher market power to charge a higher price over its cost of production. Until now we have fully characterized the post-merger changes of retail prices for both the merged and non-merging products.

We then characterize the changes of post-merger product qualities for both the merged and non-merging products. The product quality choices are made by the upstream manufacture firms, which have the same profit maximization necessary conditions pre- and post-merger. We therefore understand that the relation of each product's equilibrium quality with its market share shall stay the same pre- and post-merger as characterized in equation (10). Based on equation (10), how the post-merger product qualities adjust shall depend on the adjustments of post-merger market shares. To proceed, we shall first take the derivative of the pre-merger quality of each product j with respect to its pre-merger market share as

$$\frac{dZ_j}{dS_j} = \frac{(1-2S_j)N\beta((\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j) - N^2S_j^2\gamma_j^2\beta - N\gamma_j\delta_j}{(NS_j\beta(1-S_j)\gamma_j + \delta_j)^2}, \tag{24}$$

for $j = \{1, ..., J\}$. Based on equation (24) above, we find that the numerator of the derivative term $\frac{dZ_j}{dS_j}$ is strictly decreasing in the value of S_j. The higher the value of S_j, the more likely for $\frac{dZ_j}{dS_j}$ to be negative. We then calculate the threshold level of market share such that $\frac{dZ_j}{dS_j} = 0$ as

$$(1-2S_j^c)N\beta((\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j) = N^2S_j^2\gamma_j^2\beta + N\gamma_j\delta_j, \tag{25}$$

where S_j^c is the threshold market share of product j. Rewriting equation (25) above suggests that the threshold market share S_j^c of each product is a functon of the parameters of the demand and supply sides

$$S_j^c = \frac{\sqrt{\beta^2((\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j)^2 + \gamma_j^2 N\beta^2((\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j) - \gamma_j^3\beta N\delta_j} - \beta((\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j)}{N\gamma_j^2\beta}, \tag{26}$$

for $j = \{1, ..., J\}$. When $S_j < S_j^c$, we know that $\frac{dZ_j}{dS_j} > 0$ and the post-merger quality and market share of product j shall move in the same direction. When $S_j > S_j^c$, we find that $\frac{dZ_j}{dS_j} < 0$ and the post-merger quality and market share of product j shall move in the opposite direction. Comparing the threshold market share calculated in equation (26) with the pre-merger observed market share of each product and combining such $\frac{dZ_j}{dS_j}$ condition with the predicted change of post-merger market share of each product, we can thereby predict the change of post-merger product quality of each product j.

We find that how the post-merger product qualities change for both the merged and non-merging firms shall depend on the firms' pre-merger levels of market shares, and also on the firms' threshold market shares. By taking a closer look at equation (26) of the threshold market share, we find that the magnitude of the threshold market share shall depend on the demand and supply side parameters, in particular, the term of $(\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j$. By calculation, we find that the threshold market share of product j shall increase in the value of $(\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j$ (Appendix Note 7). If the term of $(\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j$ is smaller, the threshold market share will also be a smaller value. In that case, it's more likely for the pre-merger level of market share to surpass the threshold market share, and $\frac{dZ_j}{dS_j}$ is more likely to be negative. For the two merged products, the firms are more likely to increase their product qualities post-merger given the post-merger market shares decrease for those products. For the non-merging products, they are more likely to decrease their post-merger product qualities as their post-merger market shares increase. The opposite conclusion holds when the term of $(\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j$ is a bigger value, and the threshold market share of product j is more likely to be a bigger value. In that case, it's more likely for the pre-merger level of market share to be smaller than the threshold market share, and $\frac{dZ_j}{dS_j}$ is more likely to be positive. For the two merged products, the firms are more likely to decrease their product qualities post-merger given the post-merger market shares decrease for those products. For the non-merging products, they are more likely to increase their post-merger product qualities as their post-merger market shares increase. We shall denote $(\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j$ as the measure of firm j's overall productivity term. For manufacturer j with efficient productivities in both its marginal cost and fixed cost of production, γ_j and δ_j are smaller and thus the term of $(\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j$ is more likely to be a smaller value. On the other hand, for manufacturer j with inefficient productivities in both its marginal and fixed costs of production, γ_j and δ_j are bigger and thus the term of $(\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j$ is more likely to be a greater value.

Up to now, we have fully characterized the predictions of post-merger market

shares, retail prices, and product qualities for both the merged and non-merging products. The generalized conditions and predictions of all possible post-merger market outcomes shall be characterized in the main theorem in the next section.

4 Main Theorem

In this section, we shall present the main theorem by generalizing conditions for post-merger predictions of market shares, product qualities and prices for both the merged and non-merging products.

Theorem 4.1. *Let P_j be the retail price of product j set by the downstream retail firm j and Z_j be the quality of product j set by the upstream manufacture firm j. Let S_j be the market share of product j and S_j^c be the threshold market share of product j such that $\frac{dP_j}{dS_j} = 0$ as defined in equation (26)*

For the merged products,

1. *If $S_j < S_j^c$, product j's market share shall decrease, retail price shall increase, and product quality shall decrease post-merger.*
2. *If $S_j > S_j^c$, product j's market share shall decrease, retail price shall increase, and product quality shall increase post-merger.*

For the non-merging products,

1. *If $S_j < S_j^c$, product j's market share shall increase, retail price shall increase, and product quality shall increase post-merger.*
2. *If $S_j > S_j^c$, product j's market share shall increase, retail price shall increase, and product quality shall decrease post-merger.*

Remark 1: The merger between downstream retail firms shall make the downstream retail market more concentrated, which shall give the merged firm a stronger market power to increase the post-merger retail prices of the merged products. The market shares of the merged products shall decrease post-merger due to higher retail prices. The post-merger profit of the merged firm shall be bigger than the sum of profits from selling both products separately before the merger. The conclusion regarding to the post-merger changes of prices and market shares for the merged products remains consistent with the findings from other literature which adopted traditional competition models such as Incomplete Monopoly, Cournot and Stackelberg competition models. In addition, the other non-merging products shall respond to the merger as well. Given that the merger increases prices and decreases quantities of the merged products, the non-merging products' prices shall also increase after the merger. The conclusion regarding to the increasing prices of other non-merging products is also consistent with what are generally suggested in firms' parallel pricing behavior.

The increasing prices of the merged products shall give other non-merging products more room to increase their prices without decreasing their market shares, which can increase the profits of those non-merging products. Therefore, the downstream non-merging retail firms would have an incentive to increase their post-merger prices parallel to the merged firm in order to gain higher post-merger profits. Although the other non-merging products also increase their prices due to the higher market concentration and parallel pricing behavior, they may not increase as much as those directly merged products. Therefore, the post-merger market shares would still increase for the non-merging products as their prices get relatively smaller post-merger.

Remark 2: Knowing how the market shares and retail prices change post-merger for both the merged and non-merging products, we can then predict the changes of post-merger product qualities for all the products. We shall form the derivative terms, i.e. $\frac{dZ_j}{dS_j}$ for $j = \{1, ..., J\}$. The signs of these terms shall suggest how the post-merger product qualities adjust with the post-merger market shares. We find that the post-merger product qualities for both the merged and non-merging products can either increase or decrease, depending on the pre-merger market shares of those products and also the parameters which characterize the demand and supply sides of the market. When product j has a small pre-merger market share, i.e. $S_j < S_j^c$, $\frac{dZ_j}{dS_j} > 0$ and the post-merger product quality shall move in the same direction with the post-merger market share. When product j has a significant pre-merger market share, i.e. $S_j > S_j^c$, $\frac{dZ_j}{dS_j} < 0$ and the post-merger product quality shall move in the opposite direction with the post-merger market share. Given that the post-merger market shares decrease for the merged products and increase for the other non-merging products, we can then fully characterize the post-merger changes of product qualities. To be specific, when the merged product is with a small pre-merger market share, the product quality shall decrease post-merger. When the merged product is with a significant pre-merger market share, the product quality shall increase post-merger. To the opposite, when the non-merging product is with a small pre-merger market share, the product quality shall increase post-merger. When the non-merging product is with a significant pre-merger market share, the product quality shall decrease post-merger. Besides, we find that the products' overall productivities which are characterized in the terms of $(\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j$, for $j = \{1, ..., J\}$ shall also determine the changes of post-merger product qualities. For manufacturer j with better productivities in both its marginal cost and fixed cost of production, γ_j and δ_j are smaller and thus the term of $(\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j$ is more likely to be smaller. Given that $\frac{dS_j^c}{d\mu_j} > 0$ where $\mu_j = (\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j$ for $j = \{1, ..., J\}$, we can find how products' overall productivities shall affect the post-merger changes of product qualities. We find that when the merged products are relatively more productive, the post-merger product qualities shall increase for the merged products. When the merged products are relatively less productive, the post-merer product qualities shall decrease for the merged products. Moreover, when the non-merging products are relatively more productive,

the post-merger product qualities shall decrease for the non-merging products. When the non-merging products are relatively less productive, the post-merger product qualities shall increase for the non-merging products.

Remark 3: Moreover, we find that the products can either converge or diverge in their product qualities after the merger, depending on the consumer's preference over product quality, marginal and fixed costs and overall productivities of upstream manufacture firms. The product qualities would increase for all the products under the condition that the merged products have significant pre-merger market shares, i.e. $S_j^{pre} > S_j^c$ for $j = \{1, 2\}$ and the non-merging products have small pre-merger market shares, i.e. $S_j^{pre} < S_j^c$ for $j = \{3, ..., J\}$, or under the condition that the merged products are relatively more productive and the non-merging products are relatively less productive. The product qualities would decrease for all the products under the condition that the merged products have small pre-merger market shares, i.e. $S_j^{pre} < S_j^c$ for $j = \{1, 2\}$ and the non-merging products have significant pre-merger market shares, i.e. $S_j^{pre} > S_j^c$ for $j = \{3, ..., J\}$ or under the condition that the merged products are relatively less productive and the non-merging products are relatively more productive. When all the products have either small or significant pre-merger market shares, or being very efficient or very inefficient in their production technologies, we would expect the product qualities to move in the opposite directions for the merged and non-merging products. In particular, the post-merger product qualities shall increase for the merged products and decrease for the non-merging products if both the merged and non-merging products have significant pre-merger market shares, i.e. $S_j^{pre} > S_j^c$ for $j = \{1, ..., J\}$ or if all the upstream manufacture firms have high production efficiencies. On the other hand, the post-merger product qualities shall decrease for both the merged and non-merging products if all the products have small pre-merger market shares, i.e. $S_j^{pre} < S_j^c$ for $j = \{1, ..., J\}$ or if all the upstream manufacture firms have low production efficiencies.

Applying Theorem 4.1 above, we can predict the post-merger changes of market shares, product qualities and prices for both the merged and non-merging products. We shall list the conditions required for all possible post-merger market outcomes in both the upstream manufacture and downstream retail markets. To implement the theorem, we shall apply the theorem to consider the market with three major products existing pre-merger, with the quality of each product determined by the upstream manufacture firm and the retail price of each product determined by the downstream retail firm. Without loss of generality, we shall assume that retailer 1 and 2 decide to merge. We shall characterize the predictions of all possible post-merger market outcomes under different market conditions and the results are shown in Table 1. In particular, Table 1 characterizes the conditions required for all possible changes of post-merger market shares, prices and product qualities for both the merged and non-merging products.

Table 1: Prediction of post-merger Market Shares, Prices and Product Qualities

Pre-merger Conditions			Post-merger Predictions								
S_1^{pre}, S_1^c	S_2^{pre}, S_2^c	S_3^{pre}, S_3^c	S_1	S_2	S_3	P_1	P_2	P_3	Z_1	Z_2	Z_3
$S_1^{pre} > S_1^c$	$S_2^{pre} > S_2^c$	$S_3^{pre} > S_3^c$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
$S_1^{pre} > S_1^c$	$S_2^{pre} > S_2^c$	$S_3^{pre} < S_3^c$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
$S_1^{pre} > S_1^c$	$S_2^{pre} < S_2^c$	$S_3^{pre} > S_3^c$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
$S_1^{pre} < S_1^c$	$S_2^{pre} > S_2^c$	$S_3^{pre} > S_3^c$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
$S_1^{pre} > S_1^c$	$S_2^{pre} < S_2^c$	$S_3^{pre} < S_3^c$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$
$S_1^{pre} < S_1^c$	$S_2^{pre} > S_2^c$	$S_3^{pre} < S_3^c$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
$S_1^{pre} < S_1^c$	$S_2^{pre} < S_2^c$	$S_3^{pre} > S_3^c$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
$S_1^{pre} < S_1^c$	$S_2^{pre} < S_2^c$	$S_3^{pre} < S_3^c$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$

From Table 1, the post-merger market shares shall decrease for the two merged products and shall increase for the other non-merging product. The post-merger product prices shall increase for both the merged and non-merging products. The changes in post-merger product qualities, however, can go either way, depending on the threshold market shares which are determined by the upstream manufacture firms' marginal and fixed costs of production and the consumer's preference over product quality. When $S_j < S_j^c$, the post-merger product quality and market share shall move in the same direction. When $S_j > S_j^c$, the post-merger product quality and market share shall move in the opposite direction.

5 Multiple Product Retailer Merger

In Section 3 and 4, we have considered the market in which each retail firm has an exclusive relation with each upstream manufacture firm. In that case, each retail firm sells one product in the retailer market. Although the one-to-one matching between manufactures and retailers is a popular assumption in the existing literature, it is not widely observed in the real life applications. Most of the retail firms in the daily life related markets carry a variety of products. Although the merger in the single product retailer market has good implications for adjustments of retail prices, product qualities and market shares

post-merger, we shall ask ourselves whether the conclusions are consistent or not when we relax the exclusive matching assumption between retailers and manufacturers. In particular, we shall revise the model and extend the theorem from the previous sections by allowing each retail firm to carry multiple products. In this case, instead of the exclusive one-to-one relation between one upstream manufacturer and one downstream retailer, each downstream retailer can sign contracts with many upstream manufacturers which produce different products, where the number of contracts could vary among retail firms. Each manufacture firm, however, is still assumed to be in an exclusive contract with its downstream retailer. In other words, each manufacture firm cannot sell its product to different retailers at the same time. To proceed, we shall revise the model in this section and study how the merger between downstream retail firms which carry multiple products could affect both the upstream manufacture and downstream retail markets. We shall present the revised generalized theorem in the later section.

5.1 The Revised Model

In this section, We shall consider a market of J retail firms that carry multiple products. Without loss of generality, each downstream retail firm j can sign contracts with n_j number of upstream manufacturers, and each upstream manufacture firm can produce one product and sell to one unique retailer. There are a total of K different types of products in the market, where $K = \sum_{j=1}^{J} n_j$. There are K upstream manufacturers with each firm producing one product. There are J downstream retailers with each firm selling n_j different products. To avoid the double marginalization problem that is common in the two-tier market structure, we shall assume that each upstream manufacture firm shall sign one contract with one downstream retail firm, where the downstream retail firm shall pay a two part tariff to the upstream manufacture firm. To use consistent notation throughout this paper, I shall denote the product jk as the product which is produced by manufacturer jk and sold by retailer j. I shall denote the downstream retailer j as the one who buys products from all its n_j upstream suppliers. For each manufacturer jk, the retailer j shall pay a contracting price per unit $\bar{P_{jk}}$ and a lump sum price T_{jk} to the upstream manufacture firm. Such two part tariff contract eliminates the double marginalization problem in the two tier market structure. Given the contracting price per unit $\bar{P_{jk}}$ and the lump sum price T_{jk} for each product, each retailer j shall choose the optimal retail prices of all its products to maximize its total sum of profits. On the other hand, given the contracting price per unit $\bar{P_{jk}}$ and the lump sum price T_{jk}, each manufacturer chooses the optimal product quality to maximize its profit. For products with different qualities, it shall require different levels of marginal and fixed costs of production. Both the endogenous product qualities and retail prices shall be released after the two part tariff pricing mechanism is released. Consumers observe both the retailer prices and product qualities of all K products and shall purchase the product with the highest utility.

For the simplicity of this section, we shall assume that the contracting price per unit and the lump sum price of each product are given and the assignments of the matchings between downstream retail firms and upstream manufacture firms are also given. I shall in my later work, discuss the endogenous choices of those terms. To proceed, we shall first characterize the demand side of the market pre-merger by forming consumers' utility function and the closed forms of market shares. We then characterize the supply side of the market pre-merger by characterizing the optimal product quality decisions of upstream manufacture firms and optimal retail price decisions of downstream retail firms. Last, we shall characterize the merger between two downstream multi-product retail firms and study how such merger affects the product repositioning and pricing incentives of the upstream manufacture and downstream retail firms, respectively.

5.2 Demand Side Pre-merger

We shall first characterize the demand side of the market pre-merger. Same as in the previous section, we assume that consumers care both prices and product qualities to make their purchase decisions. We shall use the discrete choice model to characterize consumers' utility of purchasing each product. We shall define product jk as the product that is supplied by manufacturer jk and distributed by retailer j. The utility of consumer i choosing product jk is characterized as

$$U_{ijk} = \beta Z_{jk} - \alpha P_{jk} + \xi_{jk} + \epsilon_{ijk}, \tag{27}$$

for $j = \{1, ..., J\}$ and $k = \{1, ..., n_j\}$. P_{jk} is the price of product jk. Z_{jk} is the quality of product jk. ξ_{jk} is the brand fixed effect of product jk. ϵ_{ijk} is the idiosyncratic shock term and follows the type one extreme value distribution. α is consumer i's preference over price and β is consumer i's preference over product quality. Same as in the previous section, we assume that the individual's variation in the utility comes from the idiosyncratic shock term and we here do not consider the interaction of individual demographics with product qualities. By the property of type one extreme value distribution, the market share of product jk is characterized as

$$S_{jk} = \frac{\exp\left(\beta Z_{jk} - \alpha P_{jk} + \xi_{jk}\right)}{1 + \sum_{j'=1}^{J} \sum_{k'=1}^{n_{j'}} \exp\left(\beta Z_{j'k'} - \alpha P_{j'k'} + \xi_{j'k'}\right)}, \tag{28}$$

for $j = \{1, ..., J\}$ and $k = \{1, ..., n_j\}$. We assume that consumer i could also choose not to purchase any of the K products and instead purchase from the outside option. In this case, consumer i's utility of purchasing the outside option is

$$U_{i0} = \epsilon_{i0}, \tag{29}$$

and the market share of the outside option is

$$S_0 = \frac{1}{1 + \sum_{j'=1}^{J} \sum_{k'=1}^{n_{j'}} \exp\left(\beta Z_{j'k'} - \alpha P_{j'k'} + \xi_{j'k'}\right)}. \tag{30}$$

5.3 Upstream Manufacture Firm Pre-merger

We then characterize the supply side of the market. The supply side of the market consists of two tiers: upstream manufacture firms and downstream retail firms, where the upstream manufacture firms are product producers and downstream retail firms are product sellers. As the product producers, upstream manufacture firms shall choose the endogenous product qualities to maximize profits. As the product sellers which carry multiple products, downstream retail firms shall choose the endogenous retail prices of all the products they carry to maximize the sum of total profits. Downstream retail firm j purchases product jk from upstream manufacturer jk by paying a fixed contracting price per unit $\bar{P_{jk}}$ and a lump sum price T_{jk}. Each retail firm j shall carry n_j products by signing n_j two part tariff contracts with the upstream manufacturers.

We shall first characterize the profit maximization decisions of the upstream manufacture firms in the pre-merger period. Given the fixed contracting price per unit $\bar{P_{jk}}$ and the lump sum price T_{jk} between manufacturer jk and retailer j, the upstream manufacturer jk tries to maximize its profit by choosing the optimal product quality

$$\max_{Z_{jk}} NS_{jk}(\bar{P_{jk}} - mc_{jk}) - Fixed_{jk} + T_{jk}, \tag{31}$$

for $j = \{1, ..., J\}$ and $k = \{1, ..., n_j\}$. N is the total mass of consumers, S_{jk} is the market share of product jk which is sold by retailer j, Z_{jk} is the quality of product jk that the upstream manufacture firm decides to produce, mc_{jk} is the marginal cost of product jk and $Fixed_{jk}$ is the fixed cost of product jk. To be consistent with the previous model, we assume that the marginal cost is linear in product quality and fixed cost is quadratic in product quality. Thus the marginal and fixed costs of production of product jk are characterized as

$$mc_{jk} = \gamma_{jk} Z_{jk} + \omega_{jk}, \tag{32}$$

and

$$\frac{dFixed_{jk}}{dZ_{jk}} = \delta_{jk} Z_{jk} + \theta_{jk}, \tag{33}$$

for $j = \{1, ..., J\}$ and $k = \{1, ..., n_j\}$. ω_{jk} and θ_{jk} are the marginal and fixed cost unobserved term respectively. To use consistent notations throughout this paper, we shall denote the derivative of product jk's fixed cost with respect to its quality, i.e. $\frac{dFixed_{jk}}{dZ_{jk}}$, the marginal fixed cost term. The necessary condition of the upstream manufacture firm jk with respect to its product quality choice is thus

$$[Z_{jk}] : NS_{jk}(-\frac{dmc_{jk}}{dZ_{jk}}) + N\frac{dS_{jk}}{dZ_{jk}}(\bar{P_{jk}} - mc_{jk}) - \frac{dFixed_{jk}}{dZ_{jk}} = 0, \tag{34}$$

for $j = \{1, ..., J\}$ and $k = \{1, ..., n_j\}$. Simplifying the necessary condition above suggests that

$$[Z_{jk}] : NS_{jk}(-\gamma_{jk}) + NS_{jk}\beta(1-S_{jk})(\bar{P_{jk}} - \gamma_{jk}Z_{jk} - \omega_{jk}) - \delta_{jk}Z_{jk} - \theta_{jk} = 0, \quad (35)$$

for $j = \{1, ..., J\}$ and $k = \{1, ..., n_j\}$. We shall simplify the above equation further to get that

$$Z_{jk} = \frac{NS_{jk}\beta(1 - S_{jk})(\bar{P_{jk}} - \omega_{jk}) - NS_{jk}\gamma_{jk} - \theta_{jk}}{NS_{jk}\beta(1 - S_{jk})\gamma_{jk} + \delta_{jk}}, \quad (36)$$

for $j = \{1, ..., J\}$ and $k = \{1, ..., n_j\}$. Based on equation (36) above, the pre-merger quality of product jk can be characterized as a function of the product's pre-merger market share. Thus solving the equilibrium market share pre-merger guarantees the solution of the equilibrium product quality pre-merger of each product.

5.4 Downstream Multi-Product Retail Firm Pre-merger

We then characterize the profit maximization decisions of downstream retail firms. For each downstream retailer j, it sells n_j different products. For each product jk, downstream retail firm j shall sign the two part tariff contract with upstream manufacturer jk, in which the retailer shall pay the contracting price per unit $\bar{P_{jk}}$ and the lump sump price T_{jk} to the upstream manufacturer. The downstream retail firms sell products directly to consumers and each retail firm shall maximize its sum of total profits by choosing the optimal retail prices of all the products it carries

$$\max_{\{P_{jk}\}_{k=1}^{n_j}} \sum_{k=1}^{n_j} NS_{jk}(P_{jk} - \bar{P_{jk}}) - T_{jk}, \quad (37)$$

for $j = \{1, ..., J\}$ and $k = \{1, ..., n_j\}$, where P_{jk} is the retail price of product jk. The necessary condition of each downstream retail firm j with respect to the retail price of product jk suggests that

$$[P_{jk}] : NS_{jk} + N\frac{dS_{jk}}{dP_{jk}}(P_{jk} - \bar{P_{jk}}) + \sum_{k' \neq k} N\frac{dS_{jk'}}{dP_{jk}}(P_{jk'} - \bar{P_{jk'}}) = 0, \quad (38)$$

for $j = \{1, ..., J\}$ and $k = \{1, ..., n_j\}$. We shall simplify the above equation further to get

$$[P_{jk}] : 1 - \alpha(1 - S_{jk})(P_{jk} - \bar{P_{jk}}) + \sum_{k' \neq k} \alpha S_{jk'}(P_{jk'} - \bar{P_{jk'}}) = 0, \quad (39)$$

for $j = \{1, ..., J\}$ and $k = \{1, ..., n_j\}$. Solving the above necessary condition for each product suggests that

$$P_{jk} - \bar{P_{jk}} = P_{jk'} - \bar{P_{jk'}}, \tag{40}$$

for $k \neq k'$, $k = \{1, ..., n_j\}$ and $k' = \{1, ..., n_j\}$. Therefore, based on equation (40), the pre-merger retail price of product jk can be characterized as

$$P_{jk} = \bar{P_{jk}} + \frac{1}{\alpha(1 - \sum_{k'=1}^{n_j} S_{jk'})}, \tag{41}$$

for $j = \{1, ..., J\}$ and $k = \{1, ..., n_j\}$. Equation (41) suggests that the pre-merger retail price of each product carried by retailer j is a function of the pre-merger market shares of all the products that are carried by retailer j. Thus solving the equilibrium pre-merger market shares of all the products that each retail firm carries guarantees the solution of the equilibrium pre-merger retail price of each product, which we shall discuss in the later section.

5.5 Merger between Downstream Multi-Product Retail Firms

We then characterize the merger in the downstream retailer market where each retail firm carries multiple products. We shall reconsider the demand and supply sides of the market in the post-merger period, and we shall study how the merger of two downstream multi-product retail firms shall affect the post-merger market outcomes, in particular of market shares, product qualities and prices for both the merged and non-merging products. Without loss of generality, we assume that retail firm 1 and 2 decide to merge. The merged retail firm keeps all the products from retailer 1 and 2 post-merger and shall adjust the optimal retail prices of all the products to maximize the sum of profits. The new merged retail firm shall maximize the sum of profits from selling all the products as

$$\max_{\{P_{1k}\}_{k=1}^{n_1}, \{P_{2k'}\}_{k'=1}^{n_2}} \sum_{m=1}^{n_1} \{NS_{1m}(P_{1m} - \bar{P_{1m}}) - T_{1m}\} + \sum_{n=1}^{n_2} \{NS_{2n}(P_{2n} - \bar{P_{2n}}) - T_{2n}\}. \tag{42}$$

The necessary condition of the new merged retail firm with respect to the retail price of each product it carries post-merger suggests that

$$[P_{1k}] : N\frac{dS_{1k}}{dP_{1k}}(P_{1k} - \bar{P_{1k}}) + NS_{1k} + \sum_{m \neq k} N\frac{dS_{1m}}{dP_{1k}}(P_{1m} - \bar{P_{1m}}) + \sum_{n=1}^{n_2} N\frac{dS_{2n}}{dP_{1k}}(P_{2n} - \bar{P_{2n}}) = 0, \tag{43}$$

and

$$[P_{2k'}] : N\frac{dS_{2k'}}{dP_{2k'}}(P_{2k'} - \bar{P_{2k'}}) + NS_{2k'} + \sum_{n \neq k'} N\frac{dS_{2n}}{dP_{2k'}}(P_{2n} - \bar{P_{2n}}) + \sum_{m=1}^{n_1} N\frac{dS_{1m}}{dP_{2k'}}(P_{1m} - \bar{P_{1m}}) = 0, \tag{44}$$

for $k = \{1, ..., n_1\}$ and $k' = \{1, ..., n_2\}$. Simplifying the above equations further, we get that

$$[P_{1k}] : 1-\alpha(1-S_{1k})(P_{1k}-\bar{P_{1k}})+\sum_{m\neq k} \alpha S_{1m}(P_{1m}-\bar{P_{1m}})+\sum_{n=1}^{n_2} \alpha S_{2n}(P_{2n}-\bar{P_{2n}}) = 0, \tag{45}$$

and

$$[P_{2k'}] : 1-\alpha(1-S_{2k'})(P_{2k'}-\bar{P_{2k'}})+\sum_{n\neq k'} \alpha S_{2n}(P_{2n}-\bar{P_{2n}})+\sum_{m=1}^{n_1} \alpha S_{1m}(P_{1m}-\bar{P_{1m}}) = 0, \tag{46}$$

for $k = \{1, ..., n_1\}$ and $k' = \{1, ..., n_2\}$. Solving the above equations for each merged product suggests that

$$P_{1k} - \bar{P_{1k}} = P_{2k'} - \bar{P_{2k'}}, \tag{47}$$

for $k = \{1, ..., n_1\}$ and $k' = \{1, ..., n_2\}$. Based on equation (47), the post-merger retail price of each merged product can be characterized as

$$P_{1k} = \bar{P_{1k}} + \frac{1}{\alpha(1 - \sum_{m=1}^{n_1} S_{1m} - \sum_{n=1}^{n_2} S_{2n})}, \tag{48}$$

and

$$P_{2k'} = \bar{P_{2k'}} + \frac{1}{\alpha(1 - \sum_{m=1}^{n_1} S_{1m} - \sum_{n=1}^{n_2} S_{2n})}, \tag{49}$$

for $k = \{1, ..., n_1\}$ and $k' = \{1, ..., n_2\}$. Given that the merger happens in the downstream retail market, the profit maximization conditions would stay the same for upstream manufacture firms. Thus the relations of the equilibrium product qualities with market shares shall stay the same post-merger for all upstream manufacture firms, as characterized in equation (36). For the downstream non-merging retail firms, the relations of the equilibrium retail prices with market shares shall stay the same post-merger, as characterized in equation (41).

After we have fully characterized the relations of equilibrium retail prices and product qualities with market shares for both the upstream manufacture and downstream retail firms pre- and post-merger, we can then conduct the analysis to study how the downstream multi-product retail merger shall affect the upstream manufacture and downstream retail markets. In particular, we want to study how the equilibrium market shares, prices, and product qualities of both the merged and non-merging products adjust post-merger. We shall conduct our analysis in the following orders. We shall first study how the post-merger retail prices change for the merged products. We then study how the post-merger market shares change for both the merged and non-merging

products. After that, we shall study how the post-merger retail prices change for the non-merging products. Last, we shall characterize how the post-merger product qualities adjust for both the merged and non-merging products. The revised generalized theorem which summarizes all the conditions required and all possible post-merger market outcomes shall be presented in the later section.

We shall first figure out how the post-merger retail prices change for all the merged products. Given that the merged retailer's profit is the sum of profits from selling all the products which are carried by retailer 1 and 2 originally, we know that the post-merger profit of the merged firm should be strictly bigger than the pre-merger profit of either retailer 1 or 2, i.e. $\Pi_{12}^{post} > \Pi_{1}^{pre}$ and $\Pi_{12}^{post} > \Pi_{2}^{pre}$. Note that we use Π_{12}^{post} to denote the post-merger profit of the merged retail firm. Applying proof by contradiction (Appendix Note 8) , we can show that the sum of market shares of all the merged products post-merger is bigger than the sum of market shares of the products that are carried by either retailer 1 or 2 pre-merger, i.e. $\{S_{1k}\}_{k=1}^{n_1}{}^{post} + \{S_{2k'}\}_{k=1}^{n_2}{}^{post} > \{S_{1k}\}_{k=1}^{n_1}{}^{pre}$ and $\{S_{1k}\}_{k=1}^{n_1}{}^{post} + \{S_{2k'}\}_{k=1}^{n_2}{}^{post} > \{S_{2k'}\}_{k=1}^{n_2}{}^{pre}$. We then combine the above conditions with equations (41), (48) and (49) to find how the post-merger retail prices adjust for the merged products. We find that the post-merger retail prices for all the merged products shall increase, i.e. $P_{1k}^{post} > P_{1k}^{pre}$ for $k = \{1, ..., n_1\}$ and $P_{2k'}^{post} > P_{2k'}^{pre}$ for $k' = \{1, ..., n_2\}$ (Appendix Note 8).

We then characterize the post-merger changes of market shares for both the merged and non-merging products. In order to understand how the post-merger market shares change, we shall first take the ratio of each product's pre-merger market share with respect to the pre-merger market share of the outside option to get

$$\frac{S_{jk}}{S_0} = \exp\left(\beta Z_{jk} - \alpha P_{jk} + \xi_{jk}\right), \tag{50}$$

for $j = \{1, ..., J\}$ and $k = \{1, ..., n_j\}$. Substituting each product's pre-merger retail price and product quality as a function of its pre-merger market share in equations (36) and (41), we can get that

$$\frac{S_{jk}}{S_0} = \exp\left(\beta \frac{NS_{jk}\beta(1 - S_{jk})(\bar{P_{jk}} - \omega_{jk}) - NS_{jk}\gamma_{jk} - \theta_{jk}}{NS_{jk}\beta(1 - S_{jk})\gamma_{jk} + \delta_{jk}} - \alpha\bar{P_{jk}} - \frac{1}{1 - \sum_{n=1}^{n_j} S_{jn}} + \xi_{jk}\right), \tag{51}$$

for $j = \{1, ..., J\}$ and $k = \{1, ..., n_j\}$. Solving equation (51) above for each product suggests a positive relation of each product's pre-merger market share with the market share of the outside option. In other words, equation (51) suggests that there exists a strictly increasing function that characterizes the equilibrium pre-merger market share of each product as a function of the pre-merger market share of the outside option, i.e. $S_{jk} = g_{jk}(S_0)$, where $g'_{jk}(S_0) > 0$ for $j = \{1, ..., J\}$ and $k = \{1, ..., n_j\}$.

In the post-merger period, we know that for any given value of S_{jk}, P_{jk}^{post} is bigger for the merged products, i.e. for $j = \{1, 2\}$ and $k = \{1, ..., n_j\}$. Thus the RHS of equation (50) shall move downward post-merger as P_{jk}^{post} increases for any S_{jk} value for the merged products. Given that the LHS of equation (50) stays the same as before and the RHS of equation (50) moves downward post-merger, we know that for any given S_0 value, the intersected S_{jk} value would be smaller for the merged products, i.e. $j = \{1, 2\}$ and $k = \{1, ..., n_j\}$. In other words, it suggests that the function $g_{jk}^{post}(S_0)$ shall move downward post-merger for the merged products, i.e. $j = \{1, 2\}$ and $k = \{1, ..., n_j\}$. Given that the function $g_{jk}^{post}(S_0)$ moves downward for the merged products and stays the same for the non-merging products post-merger, we know that for any given S_0, $S_0 + \sum_{j=1}^{J} \sum_{k=1}^{n_j} S_{jk}$ shall become smaller post-merger. As all the market shares should always add up to 1 and $S_0 + \sum_{j=1}^{J} \sum_{k=1}^{n_j} S_{jk}$ is a strictly increasing function of S_0, the market share of the outside option shall increase post-merger. Therefore, the market shares of all the non-merging products shall also increase post-merger, i.e. $j = \{3, ..., J\}$ and $k = \{1, ..., n_j\}$. On the opposite side, the post-merger market shares for the merged products shall decrease, i.e. $j = \{1, 2\}$ and $k = \{1, ..., n_j\}$. Please refer to Appendix Note 5 and 6 for the detailed proof. Until now we have fully characterized the post-merger changes of market shares for both the merged and non-merging products.

Once we understand the changes of post-merger market shares for both the merged and non-merging products, we could then characterize the changes of post-merger retail prices for all the products. For the non-merging products, given that the post-merger market shares increase, we know from equation (41) that the post-merger retail prices shall also increase for all the non-merging products. Recall that previously we have found that the post-merger retail prices shall increase for the merged products. We thus can conclude that the post-merger retail prices shall increase for all the products. This conclusion is consistent with what we find under the single product retailer merger and is also consistent with what other researchers find when holding the product qualities exogenous. The downstream retailer merger shall make the downstream retail market more concentrated and less competitive, which shall give each retail firm a higher market power to charge a higher price over its cost of production. Until now we have fully characterized the post-merger changes of retail prices for both the merged and non-merging products.

We shall then characterize the changes of post-merger product qualities for both the merged and non-merging products. The product quality choices are made by the upstream manufacture firms, which have the same profit maximization necessary conditions pre- and post-merger. We therefore understand that the relation of each product's equilibrium quality with market share shall stay the same pre- and post-merger as characterized in equation (36). Based on equation (36), how the post-merger product qualities adjust shall depend on the changes of post-merger market shares. We shall first take the derivative of

the pre-merger quality of each product with respect to its pre-merger market share to get

$$\frac{dZ_{jk}}{dS_{jk}} = \frac{(1-2S_{jk})N\beta((\bar{P_{jk}}-\omega_{jk})\delta_{jk}+\theta_{jk}\gamma_{jk}) - N^2S_{jk}^2\gamma_{jk}^2\beta - N\gamma_{jk}\delta_{jk}}{(NS_{jk}\beta(1-S_{jk})\gamma_{jk}+\delta_{jk})^2}, \tag{52}$$

for $j = \{1, ..., K\}$ and $k = \{1, ..., n_j\}$. Based on equation (52) above, we find that the numerator of the deriviative term $\frac{dZ_{jk}}{dS_{jk}}$ is strictly decreasing in the value of S_{jk}. The higher the value of S_{jk}, the more likely for $\frac{dZ_{jk}}{dS_{jk}}$ to be negative. We then calculate the threshold market share of each product such that $\frac{dZ_{jk}}{dS_{jk}} = 0$ as

$$(1-2S_{jk}^c)N\beta((\bar{P_{jk}}-\omega_{jk})\delta_{jk}+\theta_{jk}\gamma_{jk}) = N^2S_{jk}^2\gamma_{jk}^2\beta + N\gamma_{jk}\delta_{jk}, \tag{53}$$

for $j = \{1, ..., J\}$ and $k = \{1, ..., n_j\}$. Rewriting equation (53) above suggests that the threshold market share S_{jk}^c for each product is a functon of the parameters of the demand and supply sides

$$S_{jk}^c = \frac{\sqrt{\beta^2\mu_{jk}^2 + \gamma_{jk}^2N\beta^2\mu_{jk} - \gamma_{jk}^3\beta N\delta_{jk}} - \beta\mu_{jk}}{N\gamma_{jk}^2\beta}, \tag{54}$$

where

$$\mu_{jk} = (\bar{P_{jk}}-\omega_{jk})\delta_{jk}+\theta_{jk}\gamma_{jk}, \tag{55}$$

for $j = \{1, ..., J\}$ and $k = \{1, ..., n_j\}$. When $S_{jk} < S_{jk}^c$, we know that $\frac{dZ_{jk}}{dS_{jk}} > 0$ and the post-merger quality and market share of product jk shall move in the same direction. When $S_{jk} > S_{jk}^c$, we find that $\frac{dZ_{jk}}{dS_{jk}} < 0$ and the post-merger quality and market share of product jk shall move in the opposite direction. Comparing the threshold market share with the pre-merger observed market share of each product and combining such $\frac{dZ_{jk}}{dS_{jk}}$ condition with the predicted change of post-merger market share of each product, we can thereby predict the changes of post-merger product qualities for both the merged and non-merging products.

Consistent with the implications we get from the single product retailer merger, we find that how the post-merger product qualities change for both the merged and non-merging products shall depend on the pre-merger level of market share, and the threshold market share of each product. We also find that manufacture firms' overall productivities, which are captured in the term of $(\bar{P_{jk}}-\omega_{jk})\delta_{jk}+\theta_{jk}\gamma_{jk}$ shall also affect the post-merger market outcomes. If the merged product has a smaller overall productivity term, i.e. efficient production in both its marginal and fixed costs of production, the threshold market share of the merged product will be small and the post-merger quality is more likely to

increase when the post-merger market share decreases for the merged product. On the other hand, if the non-merging product has a smaller overall productivity term, the threshold market share of the non-merging product that product will be small and the post-merger product quality is more likely to decrease when the post-merger market share increases for the non-merging product. The opposite conclusions hold when products have high overall productivity terms, i.e. inefficient production in both its marginal and fixed costs of production. If the merged product has a high overall productivity term, the threshold market share of the merged product will be big and the post-merger quality is more likely to decrease when the post-merger market share decreases for the merged product. On the other hand, if the non-merging product has a bigger overall productivity term, the threshold market share of the non-merging product will be big and the post-merger product quality is more likely to increase when the post-merger market share increases for the non-merging product.

Up to now, we have fully characterized the predictions of post-merger market outcomes for both the merged and non-merging products. Overall, we find that the conclusions regarding to post-merger changes of market shares, retail prices and product qualities for both the merged and non-merging products shall stay consistent with what we find under the single product retailer merger. We shall present the revised generalized theorem and the predictions of all possible post-merger market outcomes in the next section.

6 Revised Generalized Theorem

We shall present the revised generalized theorem by characterizing the post-merger predictions of market outcomes under the multi-product retailer merger. To be specific, we shall generalize conditions for all possible post-merger changes of market shares, product qualities and retail prices for both the merged and non-merging products.

Theorem 6.1. *Each downstream retail firm j sells n_j different products which are produced by n_j upstream manufacturers. Let P_{jk} be the retail price of product jk set by the downstream retail firm j and Z_{jk} be the quality of product jk set by the upstream manufacture firm jk. Let S_{jk} be the market share of product jk and S^c_{jk} be the threshold market share of product jk such that $\frac{dP_{jk}}{dS_{jk}} = 0$ as defined in equation (54)*

For the merged products,

1. *If $S_{jk} < S^c_{jk}$, product jk's market share shall decrease, retail price shall increase, and product quality shall decrease post-merger.*

2. *If $S_{jk} > S^c_{jk}$, product jk's market share shall decrease, retail price shall increase, and product quality shall increase post-merger.*

For the non-merging products,

1. *If $S_{jk} < S^c_{jk}$, product jk's market share shall increase, retail price shall increase, and product quality shall increase post-merger.*

2. *If $S_{jk} > S^c_{jk}$, product jk's market share shall increase, retail price shall increase, and product quality shall decrease post-merger.*

The theorem implications are consistent with what we find in Theorem 4.1. Please refer to the remarks in Section 4 for details.

7 Information Goods Market

In this and the next sections, we shall apply the generalized theorem to study more specific market types. We shall study the information goods market in this section and the service goods market in the next section. As internet and computers become ubiquitous in everyday life, people's working, living and shopping behaviors have been influenced significantly. It becomes much easier for people nowadays to shop all around the world and to share pricy information and technologies through the internet. The information goods market becomes one of the trending market types nowadays. It is typical in the information goods market that production requires a large fixed cost to invent but a relatively negligible marginal cost to deliver online. Those examples could include technology goods such as softwares, electronic books, online music and movies.

We shall apply Theorem 4.1 to study the post-merger changes of prices, product qualities and market shares of both the merged and non-merging products in the information goods market. For upstream manufacture firms in the information goods market, the marginal costs of production are negligibly small compared to the fixed costs of production, where we generally assume that $\gamma_j = 0$ and $\omega_j = 0$ for $j = \{1, ..., J\}$. Therefore, based on equations (10) and (14), the pre-merger product quality and retail price of each product can be characterized as

$$Z_j = \frac{NS_j\beta(1 - S_j)\bar{P}_j - \theta_j}{\delta_j}, \tag{56}$$

and

$$P_j = \bar{P}_j + \frac{1}{\alpha(1 - S_j)}, \tag{57}$$

for $j = \{1, ..., J\}$. Given that the merger happens between the downstream retail firms, the upstream manufacture firms' profit maximization conditions shall remain the same post-merger for both the merged and non-merging products. Therefore the relation of each product j's equilibrium quality with market share characterized in equation (56) above shall remain the same in both the

pre- and post-merger periods. The relation of product j's equilibrium retail price with market share shall remain the same as in equation (57) for all non-merging products, i.e. $j = \{3, ..., J\}$. For the two merged products, the relation of equilibrium post-merger retail price with market share shall become

$$P_j = \bar{P}_j + \frac{1}{\alpha(1 - S_1 - S_2)}, \tag{58}$$

for $j = \{1, 2\}$. As we have fully characterized the relations of equilibrium retail prices and product qualities with market shares for both the merged and non-merging products pre- and post-merger, we can then conduct the analysis to study how the downstream retail merger shall affect the upstream manufacture and downstream retail markets. We know from Section 4 that the post-merger retail prices shall increase for both the merged and non-merging products. The post-merger market shares shall decrease for the merged products and increase for the non-merging products.

We shall then characterize the changes of post-merger product qualities for both the merged and non-merging products. Given that the product quality choices are made by the upstream manufacture firms, which have the same profit maximization necessary conditions pre- and post-merger, we therefore understand that the relation of each product's quality with market share shall stay the same pre- and post-merger. Based on equation (56), we find that how the post-merger quality of each product adjusts shall depend on the change of the post-merger market share. Taking the derivative of each product's pre-merger quality with its pre-merger market share suggests that

$$\frac{dZ_j}{dS_j} = \frac{N\beta\bar{P}_j(1 - 2S_j)}{\delta_j}, \tag{59}$$

for $j = \{1, ..., J\}$. Based on equation (59), we find that the threshold market share of each product j is the same regardless of the marginal and fixed costs of production and the consumer's preference over product quality terms. We claim that $S_j^c = \frac{1}{2}$ for $j = \{1, ..., J\}$. When the pre-merger market share of product j is less that $\frac{1}{2}$, the post-merger product quality shall increase as the post-merger market share increases, i.e. $\frac{dZ_j}{dS_j} > 0$. On the other hand, when the pre-merger market share of product j is bigger than $\frac{1}{2}$, the post-merger product quality shall decrease as the post-merger market share increases for that product, i.e. $\frac{dZ_j}{dS_j} < 0$. Combining the threshold market share condition above with the predicted change of post-merger market share for each product, we can fully characterize the post-merger changes of product qualities for both the merged and non-merging products. To be specific, if the merged products have the pre-merger market shares less than $\frac{1}{2}$, then the product qualities shall decrease for those merged products post-merger. If the non-merging products have the pre-merger market shares less than $\frac{1}{2}$, then the product qualities shall increase for those non-merging products post-merger. The opposite conclusions hold when

the pre-merger market shares are significant. We find that if the merged products have the pre-merger market shares greater than $\frac{1}{2}$, then the product qualities shall increase for those merged products post-merger. If the non-merging products have the pre-merger market shares greater than $\frac{1}{2}$, then the product qualities shall decrease for those non-merging products post-merger. Up to now, we have fully characterized the post-merger changes of market shares, retail prices and product qualities for both the merged and non-merging products.

Given that most of the recent retailer mergers happen in the information goods market where none of the retail firms are dominant, i.e. $S_j < \frac{1}{2}$, for $j = \{1, ..., J\}$, we can apply the analysis above to predict the post-merger changes of market shares, product qualities and retail prices for both the merged and non-merging products. We find that there are frequent mergers in software industry these days, with the famous ones include the takeover of Autonomy by Hewlett-Packard, Skype by Microsoft, and Cognos by IBM. A recent comparison among 49 industries discloses that the number of merger and acquisition transactions in the software industry exceeds all other industries in the U.S. and in Europe. Our theorem predicts that for the recent mergers in the software industry, the post-merger retail prices shall increase for all the products post-merger; the post-merger market shares shall decrease for the merged products and shall increase for the non-merging products; and the post-merger product qualities shall decrease for the merged products and shall increase for the non-merging products.

8 Service Goods Market

We shall then consider the application of Theorem 4.1 in the service goods market. These days, service goods become very popular and bring much convenience to people's daily life. Service goods could be in a variety of forms in today's economy, from education and tutoring service to lawn cutting service, from haircut service to beauty centers, and also in many types of consulting services. Opposite to information goods, service goods usually do not require much fixed costs but only marginal costs in order to produce. In the service goods market, the fixed cost of production is usually negligibly small compared to the marginal cost of production, where we generally assume that $\delta_j = 0$ and $\theta_j = 0$. In that case, the endogenous pre-merger product quality and retail price of each product can be characterized as a function of the pre-merger market share

$$Z_j = \frac{NS_j\beta(1 - S_j)(\bar{P}_j - \omega_j) - NS_j\gamma_j}{NS_j\beta(1 - S_j)\gamma_j}, \tag{60}$$

and

$$P_j = \bar{P}_j + \frac{1}{\alpha(1 - S_j)}, \tag{61}$$

for $j = \{1, ..., J\}$. Given that the merger happens between the downstream retail firms, the upstream manufacture firms' profit maximization conditions shall remain the same pre- and post-merger for both the merged and non-merging products. Therefore the relation of product j's pre-merger quality with market share as characterized in equation (60) shall remain the same in both the pre- and post-merger periods. The relation of product j's pre-merger retail price with market share shall remain the same pre- and post-merger for the non-merging products as in equation (61). For the two merged products, the relation of the product's post-merger retail price with market share shall become

$$P_j = \bar{P}_j + \frac{1}{\alpha(1 - S_1 - S_2)}, \tag{62}$$

for $j = \{1, 2\}$. As we have fully characterized the relations of equilibrium prices and product qualities with market shares for both the merged and non-merging products pre- and post-merger, we can then conduct the analysis to study how the downstream retailer merger shall affect the upstream manufacture and downstream retail markets. Same as what we find in Section 4, the post-merger market shares shall decrease for the merged products and shall increase for the non-merging products. The post-merger retail prices shall increase for both the merged and non-merging products in the service goods market.

We then characterize the changes of post-merger product qualities for both the merged and non-merging products. Given that the product quality choices are made by the upstream manufacture firms, which have the same profit maximization necessary conditions pre- and post-merger, we therefore understand that the relation of each product's quality with market share shall stay the same pre- and post-merger. Based on equation (60), we find that how the post-merger quality of each product adjusts shall depend on the change of post-merger market share of each product j. We shall first take the derivative of product j's pre-merger quality with respect to its market share as

$$\frac{dZ_j}{dS_j} = \frac{-1}{\beta(1 - S_j)^2}, \tag{63}$$

for $j = \{1, ..., J\}$. Based on equation (63), we find that the derivative of each product j's pre-merger quality with market share should always be negative regardless of the demand and supply sides' parameters and each product's pre-merger market share, i.e. $\frac{dZ_j}{dS_j} < 0$, for $j = \{1, ..., J\}$. Combining the above $\frac{dZ_j}{dS_j}$ condition with the prediction that the post-merger market shares shall decrease for the two merged products and shall increase for the other non-merging products, we can claim that the post-merger product qualities shall increase for the two merged products and shall decrease for the other non-merging products. Up to now, we have fully characterized the post-merger changes of market shares, retail prices and product qualities for both the merged and non-merging

products.

We shall then apply the analysis above to predict the post-merger changes of market outcomes in the recent retailer mergers in the service goods market. There have been frequent mergers and acquisitions happened in the service goods market in the past decade, with the most frequent and influential ones in consulting service industry. The analysis on the number of mergers and acquisitions in the consulting industry since 2006 shows that over the past 10 years there has been more than 2000 mergers and acquisitions in consulting service industry each year. Among them, the famous ones include the frequent mergers and acquisitions from the Big Four companies, Deloitte, EY, KPMG and PwC. Between the year of 2010 and 2014, the Big Four companies have closed a total of 185 merger and acquisition deals, with Deloitte 57, PwC 44, KPMG 43, and EY 41. Our analysis above suggests that for the recent mergers in the consulting industry, the merged products shall increase their qualities of services post-merger, while the other non-merging products shall decrease their qualities of services post-merger. Both the merged and non-merging products shall increase their service prices post-merger. The merged products shall decrease their market shares and the non-merging products shall increase their market shares post-merger.

9 Conclusion

Different from the existing literature in studying the downstream retail merger, this paper contributes to the current literature by considering the downstream retailer merger's effects on both the downstream retail firms' optimal price choices and the upstream manufacture firms' optimal product quality choices post-merger. The paper forms the merger analysis by providing the generalized theorem to predict post-merger changes of market shares, product qualities, and prices for both the merged and non-merging products. The paper also makes contributions by allowing firms to be differentiated in their production technologies, with both the marginal costs and fixed costs being heterogeneous and depending on the product qualities. The paper studies how differentiated production technologies of marginal and fixed costs of production, the pre-merger market share levels, and the consumer's preference over product quality shall affect the downstream retail firms' pricing incentives and the upstream manufacture firms' product repositioning incentives post-merger.

Overall, the paper finds that the downstream merger between two retail firms shall decrease the market shares of the two merged products and increase the market shares of the non-merging products. Moreover, the post-merger product prices shall increase for both the merged and non-merging products as the market becomes more concentrated. The post-merger product qualities can either increase or decrease, depending on the products' pre-merger levels of market shares and the demand and supply sides of the market, in particular the

consumer's preference over product quality and the firms' marginal and fixed costs of production. The paper finds that for the merged products, when the pre-merger market shares are small or when the firms have low production efficiencies, the post-merger product qualities shall decrease. When the pre-merger market shares are significant or when the firms have high production efficiencies, the post-merger product qualities shall increase. The conclusions hold oppositely for the non-merging products. When the pre-merger market shares are small or when the firms have low production efficiencies, the post-merger product qualities shall improve for the non-merging products. When the pre-merger market shares are significant or when the firms have high production efficiencies, the post-merger product qualities shall decrease for the non-merging products. Under different demand and supply sides' conditions, the product qualities of both the merged and non-merging products can either converge or diverge after the downstream retailer merger. The paper then extends the theorem to allow each retail firm to carry multiple products. The paper finds that the conclusions under multi-product retailer mergers remain consistent with what we find under single product retailer mergers.

The paper then applies Theorem 4.1 to consider two special market types: information goods market and service goods market. We find that for the information goods market, the post-merger retail prices shall increase for both the merged and non-merging products. The post-merger market shares shall decrease for the two merged products and increase for the other non-merging products. When product j's pre-merger market share is less than $\frac{1}{2}$, the post-merger product quality shall move in the same direction with the post-merger market share. When product j's pre-merger market share is bigger than $\frac{1}{2}$, the post-merger product quality shall move in the opposite direction with the post-merger market share.

For the service goods market, we find that the post-merger retail prices shall increase for both the merged and non-merging products. The post-merger market shares shall decrease for the two merged products and increase for the other non-merging products. The post-merger product qualities shall increase for the two merged products and decrease for the other non-merging products.

References

[1] Orley C. Ashenfelter, Daniel S. Hosken, and Matthew C. Weinberg. The price effects of a large merger of manufacturers: A case study of maytag-whirlpool. *NBER Working Paper*, 2011.

[2] J.M.A. Cheng. Breakthrough in transfer pricing. *Management Accounting Quarterly*, 3, 2002.

[3] Chenghuan Sean Chu. The effect of satellite entry on cable television prices and product quality. *RAND Journal of Economics*, 41, 2010.

[4] Gregory Crawford and Matthew Shum. Empirical modeling of endogenous quality choice: the case of cable television. *working Paper*, 2001.

[5] Michaela Draganska, Michael Mazzeo, and Katja Seim. Addressing endogenous product choice in an empirical analysis of merger effects. *Discussion paper, Northwestern University*, 2009.

[6] Ying Fan. Ownership consolidation and product characteristics: A study of the u.s. daily newspaper market. *American Economic Review*, 103, 2013.

[7] Chiara Fumagalli and Massimo Motta. Upstream mergers, downstream mergers, and secret vertical contracts. *Research in Economics*, 2001.

[8] Esther Gal-Or. Duopolistic vertical restraints. *European Economic Review*, 35.6, 1991.

[9] Amit Gandhi, Luke Froeb, Steven Tschantz, and Gregory Werden. Post-merger product repositioning. *Journal of Industrial Economics*, 56, 2008.

[10] Lisa George. What's fit to print: The effect of ownership concentration on product variety in daily newspaper markets. *Economics of the Media*, 19, 2007.

[11] Henrick Horn and Asher Wolinsky. Bilateral monopolies and incentives for merger. *The RAND Journal of Economics*, 1988.

[12] Daniel Hosken, Luke Olson, and Loren Smith. Do retail mergers affect competition? evidence from grocery retailing. *Bureau of Economics Federal Trade Commission*, 2012.

[13] Roman Inderst and Greg Shaffer. Retail mergers, buyer power and product variety. *The Economic Journal*, January, 2007.

[14] Francine Lafontaine and Margaret Slade. 10 exclusive contracts and vertical restraints: Empirical evidence and public policy. *Handbook of antitrust economics*, 2008.

[15] Bjorn Lantz. The double marginalization problem of transfer pricing: Theory and experiment. *European Journal of Operational Research*, 196, 2009.

[16] Jinkook Lee. Endogenous product characteristics in merger simulation: A study of the u.s. airline industry. *working paper*, 2013.

[17] Kjell Erik Lommerud, Odd Rune Straume, and Lars Sorgard. Downstream merger with upstream market power. *European Economic Review*, 49.3, 2005.

[18] Constantine Manasakis and Minas Vlassis. Downstream mode of competition with upstream market power. *Dusseldorf Institute for Competition Economics*, 2013.

[19] Nicola Mazzarotto. Retail mergers and buyer power. *Center for Competition and Regulation*, 2004.

[20] Michael Mazzeo. Product choice and oligopoly market structure. *Rand Journal of Economics*, 33, 2002.

[21] Michael Mazzeo. Competition and service quality in the u.s. airline industry. *Review of Industrial Organization*, 22, 2003.

[22] Michael Mazzeo, Katja Seim, and Mauricio Varela. The welfare consequences of mergers with product repositioning. *working paper*, 2013.

[23] Joao V Montez. Downstream mergers and producer's capacity choice: Why bake a large pie when getting a smaller slice? 2006.

[24] C. Peters. Evaluating the performance of merger simulation: Evidence from the u.s. airline industry. *Journal of Law and Economics*, 2006.

[25] Ziyi Qiu. Merger analysis with endogenous prices and product characteristics - generalized theorem and application to the u.s. airline industry. 2016.

[26] Patrick Rey. The economics of vertical restraints. *Economics for an imperfect world: essays in honor of Joseph E. Stiglitz*, 247, 2003.

[27] Katja Seim. An empirical model of firm entry with endogenous product-type choices. *Rand Journal of Economics*, 37, 2006.

[28] Andrew Sweeting. Dynamic product repositioning in differentiated product industries: the case of format switching in the commercial radio industry. *NBER working paper*, 2007.

[29] George Symeonidis. Downstream merger and welfre in a bilateral oligopoly. *International Journal of Industrial Organization*, 2010.

[30] Sofia Berto Villas-Boas. Vertical relationships between manufacturers and retailers: Inference with limited data. *The Review of Economic Studies*, 74, 2007.

[31] Randal Watson. Product variety and competition in the retail market for eyeglasses. *Journal of Industrial Economics*, 57, 2009.

[32] Steffen Ziss. Vertical separation and horizontal mergers. *The Journal of Industrial Economics*, 1995.

10 Appendix

10.1 Appendix Note 1

Assume that upstream manufacturer j supplies product j to downstream retailer j. Downstream retailer j shall pay a contracting price per unit $\bar{P}_j$ and a lump

sum price T_j to the upstream manufacture firm j. The pre-merger profit maximization decisions for both the upstream manufacturers and the downstream retailers are characterized as

$$Z_j = \frac{NS_j\beta(1-S_j)(\bar{P}_j - \omega_j) - NS_j\gamma_j - \theta_j}{NS_j\beta(1-S_j)\gamma_j + \delta_j}, \tag{64}$$

and

$$P_j = \bar{P}_j + \frac{1}{\alpha(1-S_j)}, \tag{65}$$

for $j = \{1, ..., J\}$. For any given set of $\bar{P}_j$ and T_j for $j = \{1, ..., J\}$, solving equation (64) and (65) for each firm suggests the equilibrium pre-merger market share of each firm. The profit of upstream manufacturer j is

$$\Pi_j^{preU}(\bar{P}_j, T_j) = NS_j^{pre}(\bar{P}_j - \gamma_j Z_j^{pre} - \omega_j) - Fixed_j^{pre}(Z_j^{pre}) + T_j, \tag{66}$$

for $j = \{1, ..., J\}$ and the profit of downstream retailer j is

$$\Pi_j^{preD}(\bar{P}_j, T_j) = NS_j^{pre}(\bar{P}_j - \gamma_j Z_j^{pre} - \omega_j) - T_j, \tag{67}$$

for $j = \{1, ..., J\}$. By allowing the upstream manufacturer j and downstream retailer j to vertically merge together, we shall have that the merged firm's profit maximization problem characterized as

$$\max_{Z_j, P_j} NS_j(P_j - mc_j) - Fixed_j, \tag{68}$$

for $j = \{1, ..., J\}$. Solving the profit maximization decision of the vertically integrated firm, we shall get that the optimal product quality and price of the vertically integrated firm are characterized as

$$Z_j = \frac{NS_j(\frac{\beta}{\alpha} - \gamma_j) - \theta_j}{\delta_j}, \tag{69}$$

and

$$P_j = \frac{NS_j\gamma_j(\frac{\beta}{\alpha} - \gamma_j) - \gamma_j\theta_j}{\delta_j} + \omega_j + \frac{1}{\alpha(1-S_j)}, \tag{70}$$

for $j = \{1, ..., J\}$. Solving the post-merger necessary conditions of all firms suggests the equilibrium post-merger market share of each firm. For the vertically integrated firm, the post-merger profit of the upstream manufacturer is

$${\Pi_j}^{postU} = NS_j^{post}(P_j^{post} - \gamma_j Z_j^{post} - \omega_j) - Fixed_j^{post}(Z_j^{post}), \tag{71}$$

for $j = \{1, ..., J\}$ and the post-merger profit of downstream retailer j is

$${\Pi_j}^{postD} = NS_j^{post}(P_j^{post} - \gamma_j Z_j^{post} - \omega_j), \tag{72}$$

for $j = \{1, ..., J\}$. We shall then solve $\bar{P}_j$ and T_j of each product j by combining equations (66), (67), and (71), (72) together as

$$\Pi_j{}^{preU}(\bar{P}_j, T_j) = \Pi_j{}^{postU}, \tag{73}$$

$$\Pi_j{}^{preD}(\bar{P}_j, T_j) = \Pi_j{}^{postD}, \tag{74}$$

for $j = \{1, ..., J\}$. Solving equations (73) and (74) for each product suggests the unique set of two part tariff contract for each product, i.e. $\{\bar{P}_j, T_j\}$ for $j = \{1, ..., J\}$, such that the two part tariff contracts guarantee the same profit of each firm as in the vertically integrated scenario. Therefore we prove that the two part tariff pricing mechanism provides a solution to the double marginalization problem in the two-tier market structure.

10.2 Appendix Note 2

Solving the derivatives of S_j with respect to Z_j and $Z_{j'}$ suggests that

$$\frac{dS_j}{dZ_j} = S_j\beta(1 - S_j), \tag{75}$$

$$\frac{dS_j}{dZ_{j'}} = -\beta S_j S_{j'}. \tag{76}$$

$$\tag{77}$$

10.3 Appendix Note 3

Solving the derivatives of S_j with respect to P_j and $P_{j'}$ suggests that

$$\frac{dS_j}{dP_j} = S_j(-\alpha)(1 - S_j), \tag{78}$$

$$\frac{dS_j}{dP_{j'}} = \alpha S_j S_{j'}. \tag{79}$$

$$\tag{80}$$

10.4 Appendix Note 4

Proof by contradition. If $S_1^{post} + S_2^{post} < S_1^{pre}$ or $S_1^{post} + S_2^{post} < S_2^{pre}$, we have that

$$\begin{aligned}
\Pi_{12}^{post} &= NS_1^{post}(P_1 - \bar{P}_1) + NS_2^{post}(P_2 - \bar{P}_2), && (81)\\
&= \frac{N(S_1^{post} + S_2^{post})}{\alpha(1 - S_1^{post} - S_2^{post})}, && (82)\\
&< \frac{NS_1^{pre}}{\alpha(1 - S_1^{pre})}, && (83)\\
&< \frac{NS_1^{pre}}{\alpha(1 - S_1^{pre})} + \frac{NS_2^{pre}}{\alpha(1 - S_2^{pre})}, && (84)\\
&= \Pi_1^{pre}, && (85)
\end{aligned}$$

or

$$\begin{aligned}
\Pi_{12}^{post} &= NS_1^{post}(P_1 - \bar{P}_1) + NS_2^{post}(P_2 - \bar{P}_2), && (86)\\
&= \frac{N(S_1^{post} + S_2^{post})}{\alpha(1 - S_1^{post} - S_2^{post})}, && (87)\\
&< \frac{NS_2^{pre}}{\alpha(1 - S_2^{pre})}, && (88)\\
&< \frac{NS_1^{pre}}{\alpha(1 - S_1^{pre})} + \frac{NS_2^{pre}}{\alpha(1 - S_2^{pre})}, && (89)\\
&= \Pi_2^{pre}. && (90)
\end{aligned}$$

Contradiction. Thus

$$S_1^{post} + S_2^{post} > S_1^{pre}, \tag{91}$$

and

$$S_1^{post} + S_2^{post} > S_2^{pre}. \tag{92}$$

Combing what we get above with the necessary conditions that $P_1^{post} = \bar{P}_1 + \frac{1}{\alpha(1-S_1^{post}-S_2^{post})}$ in equation (20) and $P_1^{pre} = \bar{P}_1 + \frac{1}{\alpha(1-S_1^{pre})}$ in equation (14), we know that P_1 shall increase post-merger. Similarly, P_2 shall also increase post-merger.

10.5 Appendix Note 5

For each product j, equation (23) characterizes the relation of the product's pre-merger market share with the pre-merger market share of the outside option. We find that for any given S_0, the left hand side of equation (23) is a linear function in S_j. As the market share of the outside option increases, the linear line becomes flatter but is always upward sloping. The right hand side of

equation (23) is a non-linear function in S_j. To understand how the right hand side of equation (23) evolves with S_j, we shall first denote the function in the exponential symbol as

$$L_j = \beta \frac{NS_j\beta(1-S_j)(\bar{P}_j - \omega_j) - NS_j\gamma_j - \theta_j}{NS_j\beta(1-S_j)\gamma_j + \delta_j} - \alpha\bar{P}_j - \frac{1}{1-S_j} + \xi_j, \qquad (93)$$

for $j = \{1, ..., J\}$. Taking the derivative of L_j with respect to S_j suggests that

$$\frac{dL_j}{dS_j} = \beta \frac{(1-2S_j)N\beta((\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j) - N^2S_j^2\gamma_j^2\beta - N\gamma_j\delta_j}{(NS_j\beta(1-S_j)\gamma_j + \delta_j)^2} - \frac{1}{(1-S_j)^2}, \qquad (94)$$

for $j = \{1, ..., J\}$. From the equation above, we can tell that the function $L_j(S_j)$ shall be upward sloping in S_j when S_j is sufficiently small and the function $L_j(S_j)$ shall be downward sloping in S_j when S_j is sufficiently big. As the value of S_j gets bigger, the slope becomes less positive and/or more negative. Based on the analysis above, we could draw the curve of the RHS of equation (23). Given its concave down shape, we know that the RHS shall intersect with the LHS at one point. By allowing the market share of the outside option to increase, the LHS would become flatter, and the RHS is not a function of S_0. Hence the intersected market share of product j shall become bigger as the market share of the outside option increases. Therefore we could characterize the market share of product j as a function of the market share of the outside option, i.e. $S_j = f_j(S_0)$, which we know would be strictly increasing.

10.6 Appendix Note 6

We first take product 1 as an example. Given that $S_2 > 0$, we know from equations (14) and (20) that for any value of S_1, the post-merger price of product 1 should always be greater than the pre-merger value. Therefore, $L_1 = \beta Z_1 - \alpha P_1 + \xi_1$ should decrease post-merger for any value of S_1. The RHS of equation (23) should be lower for each value of S_1. The new intersection of LHS and RHS suggests that for any given value of S_0, S_1 will be smaller in the post-merger period. Thus, the strictly positive function that characterizes the relation of S_1 with S_0 shall move downward. The same argument holds for product 2. Given that all market shares add up to 1, i.e. $S_0 + S_1 + ... + S_J = 1$, and the market share of each product has a positive relation with the market share of the outside option, we therefore find that S_0 should increase post-merger. Given that the strictly increasing function of S_j as a function of S_0 stays the same pre- and post-merger for all the non-merging products, $j = \{3, ..., J\}$, we know that the post-merger market shares of all the non-merging products shall increase as the post-merger market share of the outside option increases. Accordingly, we know that S_1 and S_2 shall decrease in the post-merger period.

10.7 Appendix Note 7

We first denote that $\mu_j = (\bar{P}_j - \omega_j)\delta_j + \theta_j\gamma_j$. The threshold market share S_j^c in equation (26) can be characterized as

$$S_j^c = \frac{\sqrt{\beta^2\mu_j^2 + \gamma_j^2 N\beta^2\mu_j - \gamma_j^3\beta N\delta_j} - \beta\mu_j}{N\gamma_j^2\beta}, \tag{95}$$

for $j = \{1, ..., J\}$. Taking the derivative of S_j^c with respect to μ_j suggests that

$$\frac{dS_j^c}{d\mu_j} = \frac{1}{N\gamma_j^2\beta}(\frac{1}{2}\frac{2\beta^2\mu_j + \gamma_j^2 N\beta^2}{\sqrt{\beta^2\mu_j^2 + \gamma_j^2 N\beta^2\mu_j - \gamma_j^3\beta N\delta_j}} - \beta), \tag{96}$$

$$= \frac{2\beta^2\mu_j + \gamma_j^2 N\beta^2 - 2\beta\sqrt{\beta^2\mu_j^2 + \gamma_j^2 N\beta^2\mu_j - \gamma_j^3\beta N\delta_j}}{2(\beta^2\mu_j^2 + \gamma_j^2 N\beta^2\mu_j - \gamma_j^3\beta N\delta_j)^{\frac{1}{2}} N\gamma_j^2\beta}, \tag{97}$$

for $j = \{1, ..., J\}$. To show the numerator is positive, we need to prove that

$$2\beta^2\mu_j + \gamma_j^2 N\beta^2 > 2\beta\sqrt{\beta^2\mu_j^2 + \gamma_j^2 N\beta^2\mu_j - \gamma_j^3\beta N\delta_j}, \tag{98}$$

for $j = \{1, ..., J\}$, which can be simplified as

$$\beta\mu_j + \frac{\gamma_j^2 N\beta}{2} > \sqrt{\beta^2\mu_j^2 + \gamma_j^2 N\beta^2\mu_j - \gamma_j^3\beta N\delta_j}, \tag{99}$$

for $j = \{1, ..., J\}$. Squaring on both sides of the equation above suggests that

$$\beta^2\mu_j^2 + \frac{\gamma_j^4 N^2\beta^2}{4} + \beta^2\mu_j\gamma_j^2 N > \beta^2\mu_j^2 + \gamma_j^2 N\beta^2\mu_j - \gamma_j^3\beta N\delta_j, \tag{100}$$

for $j = \{1, ..., J\}$, which is always true given that

$$\frac{\gamma_j^4 N^2\beta^2}{4} > -\gamma_j^3\beta N\delta_j, \tag{101}$$

for $j = \{1, ..., J\}$. We therefore prove that the numerator of equation (97) is always positive, thus $\frac{dS_j^c}{d\mu_j} > 0$ for $j = \{1, ..., J\}$.

10.8 Appendix Note 8

Proof by contradiction. If $\{S_{1k}^{post}\}_{k=1}^{n_1}+\{S_{2k'}^{post}\}_{k'=1}^{n_2} < \{S_{1k}^{pre}\}_{k=1}^{n_1}$ or $\{S_{1k}^{post}\}_{k=1}^{n_1}+\{S_{2k'}^{post}\}_{k'=1}^{n_2} < \{S_{2k'}^{pre}\}_{k'=1}^{n_2}$, we have that

$$\begin{aligned}
\Pi_{12}^{post} &= \sum_{k=1}^{n_1} NS_{1k}^{post}(P_{1k}-\bar{P_{1k}}) + \sum_{k'=1}^{n_2} NS_{2k}^{post}(P_{2k}-\bar{P_{2k}}), && (102)\\
&= \frac{\sum_{k=1}^{n_1} NS_{1k}^{post} + \sum_{k'=1}^{n_2} NS_{2k'}^{post}}{\alpha(1-\sum_{k=1}^{n_1} S_{1k}^{post} - \sum_{k'=1}^{n_2} S_{2k'}^{post})}, && (103)\\
&< \frac{N\sum_{k=1}^{n_1} S_{1k}^{pre}}{\alpha(1-\sum_{k=1}^{n_1} S_{1k}^{pre})}, && (104)\\
&< \frac{N\sum_{k=1}^{n_1} S_{1k}^{pre}}{\alpha(1-\sum_{k=1}^{n_1} S_{1k}^{pre})} + \frac{N\sum_{k'=1}^{n_2} S_{2k'}^{pre}}{\alpha(1-\sum_{k'=1}^{n_2} S_{2k'}^{pre})}, && (105)\\
&= \Pi_1^{pre}, && (106)
\end{aligned}$$

or

$$\begin{aligned}
\Pi_{12}^{post} &= \sum_{k=1}^{n_1} NS_{1k}^{post}(P_{1k}-\bar{P_{1k}}) + \sum_{k'=1}^{n_2} NS_{2k'}^{post}(P_{2k'}-\bar{P_{2k'}}), && (107)\\
&= \frac{N\sum_{k=1}^{n_1} S_{1k}^{post} + N\sum_{k'=1}^{n_2} S_{2k'}^{post}}{\alpha(1-\sum_{k=1}^{n_1} S_{1k}^{post} - \sum_{k'=1}^{n_2} S_{2k'}^{post})}, && (108)\\
&< \frac{N\sum_{k'=1}^{n_2} S_{2k'}^{pre}}{\alpha(1-\sum_{k'=1}^{n_2} S_{2k'}^{pre})}, && (109)\\
&< \frac{N\sum_{k=1}^{n_1} S_{1k}^{pre}}{\alpha(1-\sum_{k=1}^{n_1} S_{1k}^{pre})} + \frac{N\sum_{k'=1}^{n_2} S_{2k'}^{pre}}{\alpha(1-\sum_{k'=1}^{n_2} S_{2k'}^{pre})}, && (110)\\
&= \Pi_2^{pre}, && (111)
\end{aligned}$$

Contradiction. Thus

$$\sum_{k=1}^{n_1} S_{1k}^{post} + \sum_{k'=1}^{n_2} S_{2k'}^{post} > \sum_{k=1}^{n_1} S_1^{pre}, \qquad (112)$$

and

$$\sum_{k=1}^{n_1} S_{1k}^{post} + \sum_{k'=1}^{n_2} S_{2k'}^{post} > \sum_{k'=1}^{n_2} S_{2k'}^{pre}, \qquad (113)$$

Given that $P_{1k}^{post} = \bar{P_{1k}} + \frac{1}{\alpha(1-\sum_{k=1}^{n_1} S_{1k}^{post} - \sum_{k'=1}^{n_2} S_{2k'}^{post})}$ in equation (48) and $P_{1k}^{pre} = \bar{P_{1k}} + \frac{1}{\alpha(1-\sum_{k=1}^{n_1} S_{1k}^{pre})}$ in equation (41), we know that P_{1k} shall increase post-merger for $k = \{1, ..., n_1\}$. Similarly, $P_{2k'}$ shall also increase post-merger for $k' = \{1, ..., n_2\}$.