Policy Distortions, the Single Country General Equilibrium and the Two Country Trading Pattern

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July 28, 2018

Abstract

This paper evaluates the effects of policy distortions under the single country closed economy and the two country open economies. I first study the effect of policy distortions on a single country general equilibrium in which production is carried out by heterogeneous firms under different policy distortions, i.e. with zero, random, negatively and positively correlated policy distortion respectively. I find that different tax policies that distort firms' production can lead to sizeable changes in equilibrium aggregate output, price level and the economy concentration. The tax policy that is in favor of more productive firms, i.e. negatively correlated tax policy, could increase the country's aggregate production level substantially without causing an inflation or a monopoly power. I then study the effect of policy distortions on the two country general equilibrium by considering different policy distortions in both the home and the foreign countries. My results show that trading with a country which has the negatively correlated tax policy is beneficial to both countries and hence should be encouraged. By doing the counterfactual analyses, I also find that trading with a foreign country with a lower iceberg trading cost or a higher baseline production technology can benefit the home country and shall be encouraged.

1 Introduction

Understanding how policy distortions affect the economy in various perspectives is of fundamental importance for policy making. Remaining a hot topic in development economics, there is a growing literature surrounded. A fairly large portion of the existing literature focuses on the effects of policy distortions on total factor productivity (TFP) and output level. Banerjee and Duflo 2005 [3] emphasize the importance of resource misallocation in understanding aggregate TFP differences across countries, and present suggestive evidence that gaps in marginal products of capital in India could play a large role in India's low manufacturing TFP relative to that of the U.S. Bartelsmann et al 2006 [4] study the effects of idiosyncratic distortions in the context of a model similar to ours using cross-country data on firms. Hsieh and Klenow 2007 [16] provide quantitative evidence on the potential impact of resource misallocation on aggregate TFP. Their paper claims that when capital and labor are hypothetically reallocated to equalize marginal products to the extent observed in the U.S, the manufacturing TFP gains are of $30\% - 50\%$ in China and $40\% - 60\%$ in India. Guner et al 2008 [12] consider policies that directly tax on the size of the establishment and find a substantial reduction in aggregate output. Restuccia and Rogerson 2008 [21] consider a broader and more abstract set of policies with the objective of capturing the wide array of policies that effectively cause idiosyncratic distortions to establishment-level decisions across countries. Their paper claims that if a country subsidizes the capital accumulation of low productivity units, then capital accumu

lation will increase but measured TFP will decline. Other related literature includes Parente and Prescott 1999 [20], Young 2000 [24], Schmitz 2001 [22], Bergoeing et al 2002 [5], Chu 2002 [7], Herrendorf and Texeira 2003 [15], Lagos 2006 [18], Alfaro et al 2008 [1], and Foster et al 2008 [11].

The existing literature on policy distortions, however, has limitations. First of all, the previous work focuses on the effects of policy distortions on the TFP and the aggregate output level. This paper improves on this by looking at the influence of policy distortions in more perspectives, including the effects on the equilibrium aggregate output, the price level and the economy concentration. Secondly, as the previous work focuses on particular distortions and lacks of a general representation of these types of policies, this paper contributes to the existing literature by analyzing the implications of policy distortions under different policy distortion scenarios: a) no policy distortion, i.e. $\tau_{ij} = 0$ for all firms, b) random productivity and policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) = 0$, c) negatively correlated policy distortion with productivity, i.e. $cov(s_{ij}, \tau_{ij}) < 0$, and d) positively correlated policy distortion with productivity, i.e. $cov(s_{ij}, \tau_{ij}) > 0$. The paper achieves these by characterizing the full set of general equilibrium.

Besides comparing different tax policies and adding more perspectives, this paper contributes to the existing literature by looking at the effects of policy distortions on both the closed and the open economies. Although some recent work provides cross-country analysis by using cross-sectional data, they fail to take into account the effects of policy distortions on the equilibrium trading pattern. As in the recent decades, international trade becomes ubiquitous and almost all countries participate in importing and exporting activities to some extent, it may be better to look at the effects of policy distortions in the open economies.

There are various versions of trading models that have been developed in the past few decades. The one that is widely used in recent work assumes constant elasticity of substitution demand for consumers and imperfect competition for firms. The major feature of this trading model is that firms do not fully pass through changes in their marginal costs to their prices because their optimal markups depend on their market shares (Atkerson and Burstein 2008 [2]). This model has been previously studied by Helpman and Krugman 1985 [14], Dornbusch 1987 [9], Krugman 1987 [17], Feenstra, Gahnon and Knetter 1996 [10], Yang 1997 [23], Bodnar, Dumas and Marston 2002 [6], Melitz 2003 [19], Corsetti and Dedola 2005 [8], Hellerstein 2006 [13], etc. This paper shall develop the two country trading equilibrium based on the constant elasticity of substitution demand for consumers and imperfect competition for firms. I then study the effects of policy distortions on the general equilibrium trading pattern by considering different policy distortion scenarios in either the home or the foreign country: a) zero policy distortion for both countries, b) random policy distortion for either the home or the foreign country, c) negatively correlated policy distortion for either the home or the foreign country, and d) positively correlated policy distortion for either the home or the foreign country. I also conduct the counterfactual studies by increasing the iceberg trading cost, the baseline production technology level and the fixed labor supply for the foreign country and test how they shall affect the general equilibrium in both countries.

The calibration results imply that different policy distortions can lead to sizeable changes in the equilibrium aggregate outputs, the price levels and the economy concentrations in both the home and the foreign countries. The negatively correlated tax policy is encouraged as it increases the aggregate production substantially without causing an inflation or a monopoly power. The calibration results for the two country trading model imply that trading with a foreign country with a) the negatively correlated tax policy, b) a lower iceberg trading cost and c) a higher baseline technology level can benefit the home country and shall be encouraged.

The rest of the paper proceeds as follows. Section 2 characterizes the single country general equilibrium model. The general equilibrium and the calibration algorithm are explained in Section 3. Section 4 presents the two country equilibrium trading model with Section 5 explains the calibration algorithm for the two country general equilibrium. Section 6 shows the calibration results of both the closed and the open economies. The paper concludes in Section 7.

2 The Model

This section presents a general equilibrium model to study the effects of policy distortions in the differentiated goods market. The model begins with a closed economy in which I characterize the single country general equilibrium under different tax policies. I then develop the two country trading model to further investigate the effects of policy distortions on the trading equilibrium in the open economies.

The model is along the lines of Restuccia and Rogerson 2008 [21] and Atkeson and Burstein 2008 [2] which allow for heterogeneity in the firm-level production. To proceed, I shall first consider a closed economy with a continuum $[0, 1]$ of sectors. Within each sector, there are J firms which adopt constant return to scale Cobb-Douglas production technology. Firms produce differentiated goods and compete oligopolistically by creating the market power over their products. I assume that each firm can ascertain its idiosyncratic productivity and policy distortion draw (s, τ) at the beginning of production. On the other side of the market, there are L consumers who provide labor and capital for production and own profits of firms. For simplicity, I normalize $L = 1$ and assume that the representative consumer providing 1 unit of labor inelastically. Each consumer has a constant elasticity of substitution (CES) demand and desires to maximize the consumption utility given the budget constraint.

To proceed, I first solve the consumer's maximization problem. I then solve firms' maximization problem by imposing the market clearing condition in the consumption goods market. The general equilibrium is characterized by combining them together with the labor market clearing condition and the government budget balance condition.

2.1 Consumer's Problem

It is assumed in this paper that the consumer faces a nested constant elasticity of substitution (CES) demand system. The aggregate consumption Q consists of two levels, with the upper level adding up consumptions across sectors and the lower level adding up goods across firms within the same sector. The aggregate consumption Q is referred to as the "final consumption" and is given by

$$Q = \left[\int_0^1 Q_i^{\frac{\eta-1}{\eta}} \, di \right]^{\frac{\eta}{\eta-1}}. \tag{1}$$

Q_i is the "sector good" which sums over all the "term goods", i.e. q_{ij} from each firm j in sector i

$$Q_i = \left[\sum_{j=1}^{J} q_{ij}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}. \tag{2}$$

The parameter ρ measures the elasticity of substitution across firms within the same sector and η measures the elasticity of substitution across sectors. I assume that $\rho > \eta > 1$, that is, products are more substitutable within sectors than across sectors.

Given the set of prices $\{p_{ij}\}$ for each term good and the equlibrium wage rate and the capital rental rate $\{w, r\}$, the representative consumer shall maximize his utility from consuming Q units of final goods subject to his income constraint. I assume that the consumption utility is linear in the final good and the consumer's maximization problem shall be characterized as

$$\begin{aligned} &\max_{\{q_{ij}\}} Q, \\ &\text{s.t.} \quad \int_{i=0}^{1} \sum_{j=1}^{J} p_{ij} q_{ij} \, di = I, \end{aligned} \tag{3}$$

where the consumer's income comes from three sources: providing labor and capital to firms, and owning the profits of firms, i.e. $I = wL + rK + \pi$. For simplicity, I shall define two levels of price indice here. The aggregate price index for the final consumption is given by

$$P = \left[\int_{i=0}^{1} P_i^{1-\eta} \, di \right]^{\frac{1}{1-\eta}}, \tag{4}$$

and the sectoral price index for each sector i is given by

$$P_i = \left[\sum_{j=1}^{J} p_{ij}^{1-\rho} \right]^{\frac{1}{1-\rho}}. \tag{5}$$

In order to solve the consumer's problem, I first form the Lagrangian of equation (3), i.e. $\mathcal{L} = Q + \lambda(I - \int_0^1 \sum_{j=1}^{J} p_{ij} q_{ij} \, di)$, where the parameter λ is the Lagrangian multiplier associated with the consumer's problem. The first order condition (FOC) with respect to the disaggregated demands q_{ij} is thus (refer to Appendix Note 1)

$$[q_{ij}] : \; Q^{\frac{1}{\eta}} Q_i^{(\frac{\eta-1}{\eta} - \frac{\rho-1}{\rho})} q_{ij}^{-\frac{1}{\rho}} = \lambda \, p_{ij}. \tag{6}$$

Solving the first order condition above suggests that the disaggregated demands of any two different firms j and j' from the same sector i is (refer to

Appendix Note 2)

$$q_{ij'} = \left(\frac{p_{ij'}}{p_{ij}}\right)^{-\rho} q_{ij}. \tag{7}$$

From the equation above, the relative quantities of any two products within the same sector shall depend on the reciprocal of their relative prices. Intuitively, a higher price implies a lower quantity demanded of that product. The relationship of demand above allows us to rewrite all firms' disaggregated demands within the same sector as a function of the firm j's disaggregated demand q_{ij}. Recall that the sector consumption Q_i takes form as in the equation (2). After substituting all the disaggregated quantities into the equation (2), the demand function for the product j within the sector i is given as (refer to Appendix Note 3)

$$q_{ij} = \left(\frac{p_{ij}}{p_i}\right)^{-\rho} Q_i, \tag{8}$$

where Q_i is the demand of sector i. Moreover, by further investigating the disaggregated demand as a function of the aggregate demand Q, I get that (refer to Appendix Note 4)

$$q_{ij} = (\frac{p_{ij}}{p_i})^{-\rho}(\frac{p_i}{p})^{-\eta}Q. \tag{9}$$

The equation above shows that the disaggregated demand quantity for each firm q_{ij} shall depend on the relative prices, i.e. $\frac{p_{ij}}{p_i}$ and $\frac{p_i}{p}$. That is, an increase in the disaggregated demand q_{ij} can be either due to a decrease of the good price p_{ij} within the same sector, or due to a decrease of the sector price p_i, or both. Taking log on both sides of the equation (9) suggests that

$$\log q_{ij} = -\rho \log p_{ij} + \rho \log p_i - \eta \log p_i + \eta \log p + \log Q. \tag{10}$$

In order to characterize the price elasticity of demand, i.e. $\sigma_{ij} = -\frac{d \log q_{ij}}{d \log p_{ij}}$, I shall take the first order derivative of both the left and the right hand sides of the equation (10) with respect to $\log p_{ij}$. The price elasticity of demand is thus given by the following (refer to Appendix Note 5)

$$\sigma_{ij} \equiv -\frac{d \log q_{ij}}{d \log p_{ij}} = \rho(1 - sh_{ij}) + \eta sh_{ij}, \tag{11}$$

where sh_{ij} is the market share of the firm j and is defined as

$$sh_{ij} = \frac{p_{ij} q_{ij}}{\sum_{j'=1}^{J} p_{ij'} q_{ij'}}. \tag{12}$$

Simplifying the equation (12) above suggests that (refer to Appendix Note 6)

$$sh_{ij} = (\frac{p_{ij}}{p_i})^{1-\rho}. \tag{13}$$

Given the price for each firm p_{ij}, the equilibrium wage rate and the capital rental rate $\{w, r\}$, solving the consumer's maximization problem above shall return the demand for each good q_{ij} as a function of the relative prices and the aggregate consumption Q as described in the equation (9). Besides, the equation (11) characterizes the price elasticity of each firm σ_{ij} as a function of its market share sh_{ij}. The equation (13) shows that the market share sh_{ij} for each firm is a function of the relative price, i.e. $\frac{p_{ij}}{p_i}$. I will later on use these three equations to calibrate the general equilibrium of a single country closed economy.

2.2 Firms' Problem

My model is close to the setup in Restuccia and Rogerson 2008 in studying the firm's profit maximization problem. There is a continuum $[0, 1]$ of sectors and J firms in each sector in the market, i.e. $j = \{1, 2, ..., J\}$. Firms are assumed to have a Cobb-Douglas production function. Each firm can ascertain its idiosyncratic draw of productivity and policy distortion, i.e. (s_{ij}, τ_{ij}) at the beginning of production. The pair (s_{ij}, τ_{ij}) can be correlated for the same firm j but i.i.d. distributed across firms. The firm's production technology $f(s, k, l)$ is as following

$$f(s, k, l) = sk^{\alpha}l^{\beta},\ \alpha + \beta = 1. \tag{14}$$

where s measures the idiosyncratic productivity, k and l denote the capital and labor demand respectively. For simplicity, I assume that α and β values are the same across all firms and I assume the same distribution for s_{ij} across all firms. For each firm, the optimal labor and capital demand is solved by

$$\max_{\{l_{ij}, k_{ij}\}} s_{ij}{k_{ij}}^{\alpha}{l_{ij}}^{\beta} - wl_{ij} - rk_{ij}. \tag{15}$$

For a given pair of the wage rate and the capital rental rate $\{w, r\}$, solving the FOCs with respect to labor and capital shall return the demands for both inputs for each firm (refer to Appendix Note 7)

$$k_{ij} = \frac{q_{ij}}{s_{ij}(\frac{\beta}{\alpha})^{\beta}(\frac{r}{w})^{\beta}}, \tag{16}$$

and

$$l_{ij} = \left(\frac{\beta r}{\alpha w}\right)\frac{q_{ij}}{s_{ij}(\frac{\beta}{\alpha})^{\beta}(\frac{r}{w})^{\beta}}. \tag{17}$$

Given that the production function is constant return to scale, the marginal cost is constant and hence equals the average cost. Solving for the marginal cost suggests that (refer to Appendix Note 8)

$$mc_{ij} = \frac{(1+\frac{\beta}{\alpha})r}{s_{ij}(\frac{\beta}{\alpha})^{\beta}(\frac{r}{w})^{\beta}}. \tag{18}$$

From the equation above, the marginal cost for each firm is constant and depends on the wage rate and the capital rental rate $\{w, r\}$, and also the firm's idiosyncratic productivity draw s_{ij}.

Recall that firms produce differentiated products and compete oligopolistically in the market. Hence each firm can gain a market power by optimally setting its price above the marginal cost. Given the constant marginal cost for each firm, I shall rewrite the firm j's profit maximization problem with imposing the goods market clearing condition as

$$\begin{gathered} \max_{p_{ij}}\ p_{ij}(1-\tau_{ij})q_{ij} - mc_{ij}q_{ij}, \\ \text{s.t}\quad q_{ij} = (\tfrac{p_{ij}}{p_i})^{-\rho}(\tfrac{p_i}{p})^{-\eta}Q. \end{gathered} \tag{19}$$

The first order condition with respect to p_{ij} implies that

$$[p_{ij}] : (1-\tau_{ij})p_{ij}\frac{dq_{ij}}{dp_{ij}} + (1-\tau_{ij})q_{ij} - mc_{ij}\frac{dq_{ij}}{dp_{ij}} = 0. \tag{20}$$

Solving the first order condition returns the optimal pricing rule for each firm as (refer to Appendix Note 9)

$$p_{ij} = \frac{\sigma_{ij}}{\sigma_{ij}-1}\frac{1}{1-\tau_{ij}}mc_{ij}, \tag{21}$$

where I shall define $m_{ij} = \frac{\sigma_{ij}}{\sigma_{ij}-1}\frac{1}{1-\tau_{ij}}$ as the markup that the firm j charges over its marginal cost. We have that $m_{ij} > 1$ and $\frac{dm_{ij}}{ds_{ij}} > 0$. Intuitively, since firms produce differentiated products and compete oligopolistically in the market, firms have certain market power over their products and are able to set prices higher than marginal costs. The higher market share implies a higher market power and hence a higher markup of the firm.

Thus the total profit that each firm earns is

$$\pi_{ij} = \frac{1}{\sigma_{ij}-1}\frac{(1+\frac{\beta}{\alpha})r}{s_{ij}(\frac{\beta}{\alpha})^{\beta}(\frac{r}{w})^{\beta}}q_{ij}, \tag{22}$$

and the total profit owned by the representative consumer is

$$\pi = \int_{i=0}^{1}\sum_{j=1}^{J}\pi_{ij}di. \tag{23}$$

Given the wage rate and the capital rental rate $\{w, r\}$, each firm is able to set a price p_{ij} such that the goods market clears. I then combine it with the labor market clearing condition and the government budget balance condition to characterize the general equilibrium of a single country closed economy.

2.3 General Equilibrium

Section 2.1 solves the consumer's utility maximization problem given the price of each product $\{p_{ij}\}$ and the equilibrium wage rate and the capital rental rate $\{w, r\}$. Section 2.2 solves firms' profit maximization problem with imposing the goods market clearing condition. In particular, Section 2.2 characterizes firms' optimal choices of prices given the wage rate and the capital rental rate $\{w, r\}$. Taking the equilibrium wage rate and the capital rental rate as given, I then apply the labor market clearing condition to solve the equilibrium aggregate quantity Q

$$\int_{i=0}^{1} \sum_{j=1}^{J} l_{ij} di = L. \tag{24}$$

Applying the equations (9) and (17) for q_{ij} and l_{ij}, I get that

$$Q \int_{i=0}^{1} \sum_{j=1}^{J} \left(\frac{\beta r}{\alpha w} \right) \frac{(\frac{p_{ij}}{p_i})^{-\rho} (\frac{p_i}{p})^{-\eta}}{s_{ij} (\frac{\beta}{\alpha})^{\beta} (\frac{r}{w})^{\beta}} di = L. \tag{25}$$

Thus the aggregate quantity Q becomes

$$Q = \frac{L}{\int_{i=0}^{1} \sum_{j=1}^{J} \left(\frac{\beta r}{\alpha w} \right) \frac{(\frac{p_{ij}}{p_i})^{-\rho} (\frac{p_i}{p})^{-\eta}}{s_{ij} (\frac{\beta}{\alpha})^{\beta} (\frac{r}{w})^{\beta}} di}. \tag{26}$$

After solving the equilibrium aggregate quantity Q, the quantity, capital and labor demanded for each firm $\{q_{ij}, k_{ij}, l_{ij}\}$ can then be determined by the equations (9), (16) and (17). The equilibrium capital capacity can be characterize endogenously as

$$K(w, r) = \int_{i=0}^{1} \sum_{j=1}^{J} k_{ij} di. \tag{27}$$

In addition, I need to make sure the policy distortion draw for each firm satiesfies the government budget balance condition

$$\int_{i=0}^{1} \sum_{j=1}^{J} \tau_{ij} p_{ij} q_{ij} di = 0. \tag{28}$$

The general equilibrium can then be fully characterized for a single country closed economy.

Definition 2.1. Given the equilibrium wage rate and the capital rental rate $\{w, r\}$ and given the idiosyncratic productivity and policy distortion draw for each firm $\{s_{ij}, \tau_{ij}\}$, the general equilibrium of a single country closed economy consists of the equilibrium price and quantity of each firm $\{p_{ij}, q_{ij}\}$, the equilibrium capital and labor employment of each firm $\{k_{ij}, l_{ij}\}$, the equilibrium market share and elasticity of demand of each firm $\{sh_{ij}, \sigma_{ij}\}$ such that

1. the consumer maximizes utility;
2. firms maximize profits;
3. goods market clears;
4. labor market clears;
5. government budget balance clears.

I shall discuss the calibration of the single country general equilibrium in the next section.

3 Calibration of the Single Country General Equilibrium

I shall calibrate the general equilibrium of a single country closed economy considering four different government tax policy scenarios. Adopting from the existing literature, I assume firms' productivities in one country shall follow the Pareto distribution. I first consider the economy with no tax distortion, i.e. $\tau_{ij} = 0$ for all i, all j. In this case, the government will neither subsidize nor tax firms, regardless of their productivities. I then consider the economy with the random policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) = 0$, in which the government taxes or subsidizes firms randomly. I then conduct the calibration with the positively correlated tax distortion, i.e. $cov(s_{ij}, \tau_{ij}) > 0$. In this case, the government will tax the more productive firms and subsidize the less productive firms. Last, I shall consider the policy distortion which is negatively correlated with the productivity, i.e. $cov(s_{ij}, \tau_{ij}) < 0$. In this case, the government will subsidize the more productive firms while tax out the less productive firms. I shall compare the calibration results under the four different policy distortion scenarios. The calibration results shall give implications of the country's tax policy making.

To calibrate the general equilibrium, I shall apply the fixed point algorithm. I adopt the values of the wage rate and the capital rental rate $\{w, r\}$

and the values of parameters $\{\alpha, \beta\}$ from the existing literature. Given the distribution assumption for $\{s, \tau\}$, I shall first simulate the productivity and policy distortion pair $\{s_{ij}, \tau_{ij}\}$ for each firm. I then calculate the marginal cost mc_{ij} for each firm following the equation (18). I then apply the fixed point algorithm together with the equations $(5), (11), (13)$ and (21) to calculate the equilibrium solutions of each firm's market share sh_{ij}, the price index p_{ij}, and the elasticity of demand σ_{ij}. The calibration algorithm shall be described in the following

1. take an initial guess of each firm's market share sh_{ij},
2. apply the guess of sh_{ij} into the equation (11), i.e. $\sigma_{ij} = \rho(1-sh_{ij})+\eta sh_{ij}$, to calculate σ_{ij},
3. apply the value of σ_{ij} into the equation (21), i.e. $p_{ij} = \frac{\sigma_{ij}}{\sigma_{ij}-1}\frac{1}{1-\tau_{ij}}mc_{ij}$ to calculate p_{ij},
4. use p_{ij} to calculate p_i by the equation (5), i.e. $P_i = \left[\sum_{j=1}^{J} p_{ij}^{1-\rho}\right]^{\frac{1}{1-\rho}}$,
5. use p_{ij} and p_i to update sh_{ij} by the equation (13), i.e. $sh_{ij} = (\frac{p_{ij}}{p_i})^{1-\rho}$.

I shall repeat the above steps until sh_{ij} converges for each firm. Thus the fixed point algorithm returns the equilibrium market share, the price index and the price elasticity of demand for each firm $\{sh_{ij}, p_{ij}, \sigma_{ij}\}$, given the wage rate and the capital rental rate $\{w, r\}$, and given the productivity and policy distortion draw for each firm $\{s_{ij}, \tau_{ij}\}$. I then use the calibrated price level of each firm p_{ij} to solve the consumer's and firms' maximization problems to get $\{q_{ij}, l_{ij}, k_{ij}\}$ of each firm as a function of the aggregate output level Q. Given the fixed supply of labor in the market, i.e. $L = 1$, applying the labor market clearing conditon shall return the equilibrium aggregate output level Q by the equation (26).

After calculating the equilibrium aggregate quantity Q, I can then characterize the equilibrium quantity consumed and the capital and labor usage for each firm, i.e. $\{q_{ij}, k_{ij}, l_{ij}\}$ using the equations (9), (16) and (17). I can then fully characterize the general equilibrium outcomes of a single country closed economy. In the end, I shall apply the government budget balance conditon of the equation (28) to make sure my simulated policy distortion of each firm satisfies such condition.

3.1 Correlation between Productivity and Policy Distortion and the Tax Policy Implications

It is clearly seen that the general equilibrium of a single country closed economy shall depend on the distribution assumption of firms' productivity and policy distortion draws $\{s, \tau\}$. In this paper, I calibrate the single country general equilbirium under four different tax policies to study how the dependence of policy distortion and productivity shall affect the general equilibrium. The calibration results shall be helpful in making tax policy decisions.

I assume the productivities are drawn from a Pareto distribution and I shall consider four different policy distortion scenarios. I first calibrate the baseline model with no policy distortion, i.e. $\tau_{ij} = 0$ for all firms. In this case, the government will neither subsidize nor tax firms, regardless of their productivities. I then consider an independent and random policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) = 0$. In this case the government taxes or subsidizes firms randomly. I also consider a negatively correlated policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) < 0$ and a positively correlated policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) > 0$. In the former one the government subsidizes the more productive firms and taxes the less productive firms and in the later one the government taxes the more productive firms while subsidizes the less productive firms. In particular, I shall look at the effects of different tax policies on the aggregate production Q, the aggregate price index P and the economy concentration HHI.

In the later sections, instead of having a single country closed model, I shall modify the model by allowing two countries to trade with each other. There is an iceberg trading cost associated with exporting and I assume only the most productive firms from each sector can export to the other country. I then calibrate the general equilibrium trading pattern for both countries. I calibrate the model by assuming the home and the foreign countries with different tax policies. In addition, I conduct counterfactual analyses by increasing the iceberg trading cost, the baseline production technology level and the fixed labor supply for the foreign country and see how it will affect the general equilibrium trading pattern. The calibration results are helpful in making trading related policies. I will explain the two country general equilibrium model in the following section. And the calibration algorithm for the two country open economies are shown in Section 5.

4 Two Country Equilibrium Trading Pattern

Section 2 addresses the single country general equilibrium model. In this section, I shall introduce the foreign country and study the two country general equilibrium trading pattern. I assume there is an iceberg trading cost associated with each country for exporting. D denotes the iceberg trading cost for the home country and D^* denotes the iceberg trading cost for the foreign country. For further notation convenience, I shall denote all home country variables in the home country same as in Section 2 and I shall denote all foreign country variables in the foreign country with $*$. The prices of home firms in the home country are thus denoted as p_{ij} and of foreign firms in the foreign country are thus denoted as p^*_{ij}. The fixed labor supply for the home country is L^H and the fixed labor supply for the foreign country is L^F. Moreover, I denote the home firms that export to the foreign country with an F and the foreign firms that export to the home country with an H. Hence the price that a home firm in the foreign market charges is

$$P^F_{ij} = \frac{{\sigma_{ij}}^F}{{\sigma_{ij}}^F - 1} \frac{1}{1 - \tau_{ij}} D mc_{ij}, \tag{29}$$

and the price that a foreign firm in the home market charges is

$$P^{H*}_{ij} = \frac{\sigma^{*H}_{ij}}{\sigma^{*H}_{ij} - 1} \frac{1}{1 - \tau^*_{ij}} D^* mc^*_{ij}, \tag{30}$$

where mc_{ij} indicates the marginal cost of the home firm in the home market and mc^*_{ij} indicates the marginal cost of the foreign firm in the foreign market.

Recall that $\sigma_{ij} = \rho(1 - sh_{ij}) + \eta sh_{ij}$ is the elasticity of demand for the home firm in the home market, where sh_{ij} is the market share of the home firm in the home market. For the foreign country in the foreign market, $\sigma^*_{ij} = \rho(1 - sh^*_{ij}) + \eta sh^*_{ij}$ is the elasticity of demand for the foreign firm in the foreign market, where sh^*_{ij} is the market share of the foreign firm in the foreign market. In addition, $\sigma^{*H}_{ij} = \rho(1 - sh^{*H}_{ij}) + \eta sh^{*H}_{ij}$ is the elasticity of demand for the foreign country in the home market, where sh^{*H}_{ij} is the market share for the foreign firm in the home market. $\sigma^F_{ij} = \rho(1 - sh^F_{ij}) + \eta sh^F_{ij}$ is the elasticity of demand for the home country in the foreign market, where sh^F_{ij} is the market share for the home firm in the foreign market.

I assume there are J firms in each sector in both the home and the foreign countries. In the later calibration, I choose $J = 20$. I assume for both the home and the foreign countries, only the top 5 most productive firms in each sector can export. I rank the firms in each sector by their productivities. Hence $J = 1$ indicates the firm with the most productive technology and $J = 20$ indicates the firm with the least productive technology. After trading, there are 25 firms in each sector in both the home and the foreign countries. For the home country, all firms in each sector should consist of its own 20 firms and the 5 most productive firms from the foreign country. Similarly, for the foreign country, all firms in each sector should consist of its own 20 firms and the 5 most productive firms from the home country.

Given the wage rate and the capital rental rate $\{w, r\}$, the fixed point algorithm can solve the optimal prices, market shares and elasticities for firms in both the home and the foreign countries. I then use the labor market clearing conditions for both the home and the foreign countries to calculate the equilibrium aggregate quantities in both countries $\{Q^H, Q^F\}$, and thus solve the equilibrium labor and capital employment of each firm in both markets.

Different from Section 2, the labor market clearing condition for the home country now becomes

$$\int_{i=0}^{1} \sum_{j=1}^{J} l_{ij} di + \int_{i=o}^{1} \sum_{j'=1}^{5} l_{ij}^{*H} di = L^H. \tag{31}$$

Similarly, the labor market clearing condition for the foreign country now becomes

$$\int_{i=0}^{1} \sum_{j'=1}^{J} l_{ij}^{*} di + \int_{i=o}^{1} \sum_{j=1}^{5} l_{ij}^{F} di = L^F. \tag{32}$$

The calibration algorithm and results shall be shown in the following sections.

5 Calibration of the Two Country General Equilibrium Trading Pattern

In this section, I shall characterize the calibration algorithm for the two country trading model. Based on the calibration algorithm of a single country closed model, this section shall apply the similar fixed point algorithm as in Section 3. Note that the labor market clearing condition shall be different in the open economy.

I shall calibrate the two country general equilibrium under four policy distortion scenarios for either the home or the foreign country: a) both the home and the foreign countries with no policy distortion, i.e. $\tau_{ij} = 0$ and $\tau_{ij}^* = 0$; b) either the home or the foreign country with the random policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) = 0, \tau_{ij}^* = 0$ or $\tau_{ij} = 0, cov(s_{ij}^*, \tau_{ij}^*) = 0$; c) either the home or the foreign country with the negatively correlated policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) < 0, \tau_{ij}^* = 0$ or $\tau_{ij} = 0, cov(s_{ij}^*, \tau_{ij}^*) < 0$; and d) either the home or the foreign country with the positively correlated policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) > 0, \tau_{ij}^* = 0$ or $\tau_{ij} = 0, cov(s_{ij}^*, \tau_{ij}^*) > 0$. I shall compare the calibration results under different policy distortion scenarios described above. The calibration results shall give implications of the tax policy making under the open economies.

To calibrate the general equilibrium, I shall first apply the fixed point algorithm. I first simulate the firms' productivity and policy distortion pairs for both the home and the foreign countries, i.e. $\{\tau_{ij}, s_{ij}\}$ and $\{\tau_{ij}^*, s_{ij}^*\}$. I then rank the firms in each sector by their productivities from the highest to the lowest. Given the values of the wage rate and the capital rental rate $\{w, r\}$ and that the five most productive firms in each sector in both the home and the foriegn countries shall export, I then apply the fixed point algorithm to calculate the equilibrium marke shares, prices, and elasticities for firms in both the home and the foreign markets. The fixed point algorithm shall be described in the following

1. take an initial guess of market shares for firms in both the home and the foreign countries, i.e. $\{sh_{ij}\}_{j=1}^{20}$, $\{sh_{ij}^F\}_{j=1}^{5}$, $\{sh_{ij}^*\}_{j=1}^{20}$, and $\{sh_{ij}^{*H}\}_{j=1}^{5}$;

2. use the guess of market shares in both countries to calculate the elasticities of demand for firms in both the home and the foreign coun-

tries, i.e. $\{\sigma_{ij}\}_{j=1}^{20}$, $\{\sigma_{ij}^{*H}\}_{j=1}^{5}$, $\{\sigma_{ij}^{*}\}_{j=1}^{20}$, and $\{\sigma_{ij}^{F}\}_{j=1}^{5}$. The elasticity equations are $\sigma_{ij} = \rho(1 - sh_{ij}) + \eta sh_{ij}$, $\sigma_{ij}^{*H} = \rho(1 - sh_{ij}^{*H}) + \eta sh_{ij}^{*H}$, $\sigma_{ij}^{*} = \rho(1 - sh_{ij}^{*}) + \eta sh_{ij}^{*}$ and $\sigma_{ij}^{F} = \rho(1 - sh_{ij}^{F}) + \eta sh_{ij}^{F}$ respectively;

3. For $j = \{1, ..., 20\}$, update the price p_{ij} and p_{ij}^{*} by the equations $p_{ij} = \frac{\sigma_{ij}}{\sigma_{ij}-1}\frac{1}{1-\tau_{ij}}mc_{ij}$ and $p_{ij}^{*} = \frac{\sigma_{ij}^{*}}{\sigma_{ij}^{*}-1}\frac{1}{1-\tau_{ij}^{*}}mc_{ij}^{*}$ respectively. For $j = \{1, ..., 5\}$, update the price p_{ij}^{F} and p_{ij}^{*H} by the equations $p_{ij}^{F} = \frac{\sigma_{ij}^{*}}{\sigma_{ij}^{*}-1}\frac{1}{1-\tau_{ij}}Dmc_{ij}$ and $p_{ij}^{*H} = \frac{\sigma_{ij}}{\sigma_{ij}-1}\frac{1}{1-\tau_{ij}^{*}}D^{*}mc_{ij}^{*}$ respectively;

4. use the price of each individual firm to calculate p_i^H and p_i^F by the equations $p_i^H = [\sum_{j=1}^{20} p_{ij}^{1-\rho} + \sum_{j=1}^{5} p_{ij}^{*H\,1-\rho}]^{\frac{1}{1-\rho}}$ and $p_i^F = [\sum_{j=1}^{20} p_{ij}^{*\,1-\rho} + \sum_{j=1}^{5} p_{ij}^{F\,1-\rho}]^{\frac{1}{1-\rho}}$ respectively;

5. use the price of each individual firm, p_i^H and p_i^F to update market shares for firms in both the home and the foreign countries, i.e. $\{sh_{ij}\}_{j=1}^{20}$, $\{sh_{ij}^{F}\}_{j=1}^{5}$, $\{sh_{ij}^{*}\}_{j=1}^{20}$, and $\{sh_{ij}^{*H}\}_{j=1}^{5}$ by the equations $sh_{ij} = (\frac{p_{ij}}{p_i})^{1-\rho}$, $sh_{ij}^{F} = (\frac{p_{ij}^{F}}{p_i^{F}})^{1-\rho}$, $sh_{ij}^{*} = (\frac{p_{ij}^{*}}{p_i^{F}})^{1-\rho}$ and $sh_{ij}^{*H} = (\frac{p_{ij}^{*H}}{p_i^{H}})^{1-\rho}$ respectively.

I shall repeat the steps above until the market share of each firm converges in both the home and the foreign countries. Thus the fixed point algorithm returns the optimal market share, the elasticity of demand and the price of each firm in both the home and the foreign countries. I then apply the labor market clearing conditions for both the home and the foreign countries to calculate the equilibrium aggregate output levels of both countries, i.e. $\{Q^H, Q^F\}$. The labor market clearing conditons for the home and the foreign countries are characterized as

$$\int_{i=0}^{1}\sum_{j=1}^{J} l_{ij}di + \int_{i=0}^{1}\sum_{j=1}^{5} l_{ij}^{*H}di = L^H, \tag{33}$$

and

$$\int_{i=0}^{1}\sum_{j=1}^{J} l_{ij}^{*}di + \int_{i=0}^{1}\sum_{j=1}^{5} l_{ij}^{F}di = L^F. \tag{34}$$

Simplifying the two equations above, I get that

$$Q^H \int_{i=0}^{1}\sum_{j=1}^{J}(\frac{\beta}{\alpha}\frac{r}{w})\frac{(\frac{p_{ij}}{p_i^H})^{-\rho}(\frac{p_i^H}{p^H})^{-\eta}}{s_{ij}(\frac{\beta}{\alpha})^{\beta}(\frac{r}{w})^{\beta}}di + Q^H \int_{i=0}^{1}\sum_{j=1}^{5}(\frac{\beta}{\alpha}\frac{r}{w})\frac{(\frac{p_{ij}^{*H}}{p_i^H})^{-\rho}(\frac{p_i^H}{p^H})^{-\eta}}{s_{ij}^{*}(\frac{\beta}{\alpha})^{\beta}(\frac{r}{w})^{\beta}}di = L^H, \tag{35}$$

and

$$Q^F \int_{i=0}^{1}\sum_{j=1}^{J}(\frac{\beta}{\alpha}\frac{r}{w})\frac{(\frac{p_{ij}^{*}}{p_i^F})^{-\rho}(\frac{p_i^F}{p^F})^{-\eta}}{s_{ij}^{*}(\frac{\beta}{\alpha})^{\beta}(\frac{r}{w})^{\beta}}di + Q^F \int_{i=0}^{1}\sum_{j=1}^{5}(\frac{\beta}{\alpha}\frac{r}{w})\frac{(\frac{p_{ij}^{F}}{p_i^F})^{-\rho}(\frac{p_i^F}{p^F})^{-\eta}}{s_{ij}(\frac{\beta}{\alpha})^{\beta}(\frac{r}{w})^{\beta}}di = L^F. \tag{36}$$

After calculating the equilibrium aggregate quantities of both countries, i.e. $\{Q^H, Q^F\}$, I can then characterize the equilibrium quantity consumed and the labor and capital usage for each firm in both the home and the foreign countries. I therefore can fully characterize the general equilibrium trading pattern of the two country open economies.

6 The Calibration Results

This section consists of two subsections. In the first subsection, I shall calibrate the single country general equilibrium. I shall compare the four policy distortion scenarios for the home or the foreign country only: a) with no policy distortion, $\tau_{ij} = 0$ for each firm, b) with the random policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) = 0$, c) with the negatively correlated policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) < 0$ and d) with the positively correlated policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) > 0$. The calibration results are presented in TABLE 1 and TABLE 2 below, where I calibrate the single country general equilibrium for the home and the foreign country itself. I then introduce the international trade and calibrate the two country general equilibrium trading model. I shall calibrate the two country general equilibrium trading pattern under the four different policy distortion scenarios for either the home or the foreign country, with a) both countries having no policy distortion, i.e. $\tau_{ij} = 0, \tau^*_{ij} = 0$, b) with either the home or the foreign country having the random policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) = 0, \tau^*_{ij} = 0$ or $\tau_{ij} = 0, cov(s^*_{ij}, \tau^*_{ij}) = 0$, c) with either the home or the foreign country having the negatively correlated policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) < 0, \tau^*_{ij} = 0$ or $\tau_{ij} = 0, cov(s^*_{ij}, \tau^*_{ij}) < 0$ and d) with either the home or the foreign country having the positively correlated policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) > 0, \tau^*_{ij} = 0$ or $\tau_{ij} = 0, cov(s^*_{ij}, \tau^*_{ij}) > 0$. The calibration results are shown in TABLE 3 and TABLE 4 below.

In addition, I conduct three sensitivity analyses by a) increasing the iceberg trading cost for the foreign country and by b) increasing the baseline productivity level for the foreign country and by c) increasing the fixed labor supply for the foreign country. I shall study quantitatively the effects of the iceberg trading cost, the baseline production technology level and the fixed labor supply on the two country general equilibrium. The results of the counterfactual analyses shall be shown in TABLE 5 to TABLE 10 below. I shall discuss the policy implications based on the calibration results.

To proceed with the calibration, I consider both the home and the foreign countries with 100 sectors and 20 firms within each sector. I adopt the parameter values from the existing literature to have $\alpha = 0.3$, $\beta = 0.7$, $\rho = 5$, $\eta = 2$, $r = 1.1$ and $w = 1$. I assume that the productivity draw s_{ij} for each firm is from a Pareto distribution with both the location and the shape parameters equal to 1. I assume the fixed labor supply is 1 unit for both the home and the foreign countries, i.e. $L^H = 1, L^F = 1$. The iceberg trading costs is 1.2 for both countries, i.e. $D = 1.2, D^* = 1.2$.

6.1 Single Country General Equilibrium

I first calibrate the single country general equilibrium. I shall consider four policy distortion scenarios: a) zero policy distortion for all firms, i.e. $\tau_{ij} = 0$, b) random policy distortion draw with productivity, i.e. $cov(s_{ij}, \tau_{ij}) = 0$, c) negatively correlated policy distortion draw with productivity, i.e. $cov(s_{ij}, \tau_{ij}) < 0$, and d) positively correlated policy distortion draw with productivity, i.e. $cov(s_{ij}, \tau_{ij}) > 0$. The negatively correlated tax policy implies that more productive firms would receive subsidies while the less productive firms are taxed heavily. To the opposite, the positively correlated tax policy implies that the more productive firms are taxed heavily while the less productive firms are subsidized the most.

I shall calibrate the single country general equilibrium under those four policy distortion scenarios following the calibration algorithm described in Section 3 above. I shall compare the equilibrium aggregate production levels, the aggregate price indice and the economy concentrations HHI under the four cases. HHI here is defined as a measure of the economy concentration where

$$HHI = \sum_{i=1}^{100} \sum_{j=1}^{20} s_{ij}^2 * 10000; \tag{37}$$

Note that I average over the 100 sectors to calculate the average HHI index in each country. The calibration results of the home country are shown in TABLE 1 below.

Table 1: Single Country General Equilibrium under different Tax Policies - Home

Tax Policy	Aggregate Quantity	Aggregate Price	HHI
$\tau = 0$	6741.4	0.000381	6448.26
$cov(s, \tau) = 0$	6664.7	0.000386	6508.22
$cov(s, \tau) < 0$	6814.3	0.000378	6382.05
$cov(s, \tau) > 0$	5953.9	0.000411	5643.78

From TABLE 1, I can clearly see that the negatively correlated tax policy, which subsidizes the more productive firms while taxes heavily the less productive firms, can increase the country's aggregate output level significantly. The positively correlated tax policy, which taxes heavily the more productive firms while subsidizes the less productive firms, shall distort the economy the most by reducing the aggregate output level in the country. Having the random tax policy shall distort the economy compared to with no tax or with the negatively correlated tax policy.

To compare the aggregate price levels, I find that the negatively correlated tax policy, i.e. $cov(s_{ij}, \tau_{ij}) < 0$, shall generate the lowest equilibrium aggregate price level in the country. Having the positively correlated tax policy, on the other hand, shall generate the highest equilibrium aggregate price level in the country. Moreover, I find that having the randomly allocated policy distortion shall increase the aggregate price level in the country compared to with no policy distortion at all.

TABLE 1 also compares the economy concentrations under the four policy distortion scenarios. From the definition of HHI, a higher HHI value implies a more concentrated economy while a lower HHI value implies more competition and less market power of the economy. From the table, having the negatively correlated tax policy, instead of creating a higher monopoly power, indeed makes the market more competitive and less concentrated. Having the positively correlated tax distortion shall also decrease the HHI value, with all firms getting more similar after-tax and -subsidy and the economy getting less concentrated and more competitive. Having the random policy distortion makes the economy less competitive and more concentrated compared to the one with no policy distortion at all.

Overall, the negatively correlated tax policy should be encouraged as it increases the aggregate production dramatically for the home country while it does not create a monopoly power or an inflation for the economy. The positively correlated tax policy distorts the economy the most by dramatically decreasing the aggregate production level and increasing the price level for the home country and thus should be discouraged. Moreover, the results also imply that having no tax policy is better than having a random tax policy, as the random tax policy shall decrease the equilibrium aggregate output level and increase the price level as well as the market concentration for the country. I hence claim that the home country should choose the tax policy that is in favor of those more productive firms.

I then generate the single country general equilibrium for the foreign country only and the calibration results are shown in TABLE 2 below. Note that the difference between TABLE 2 and TABLE 1 is the random simulated firms' productivity draws. The reason I calibrate the general equilibrium for the foreign country is to compare the general equilibrium I get with the two country trading model that I shall present in the later section. From TABLE 2, I get consistent conclusions with what I find in TABLE 1 above.

Table 2: Single Country General Equilibrium under different Tax Policies - Foreign

Tax Policy	Aggregate Quantity	Aggregate Price	HHI
$\tau = 0$	5566.6	0.000451	6978.37
$cov(s, \tau) = 0$	5558.9	0.000452	6996.28
$cov(s, \tau) < 0$	5614.1	0.000448	6926.42
$cov(s, \tau) > 0$	5017.7	0.000481	6379.27

6.2 Two Country General Equilibrium Trading Pattern

In this section I shall modify the model by adding the foreign country and calibrate the two country general equilibrium trading pattern. I assume there are 100 sectors in both the home and the foreign markets, and 20 firms within each sector. I assume that for both the home and the foreign countries, the most productive 5 firms from each sector can export. Thus the general equilbrium consists of 25 firms for each sector in both countries, with 20 local firms and 5 international firms. I first assume the labor supplies and the iceberg trading costs are the same in both the home and the foreign countries, i.e. $L = L^* = 1$ and $D = D^* = 1.2$. I also assume that firms in the foreign country have the same productivity distribution as the ones in the home country. Recall that productivities are drawn from a Pareto distribution, with the location parameter $\mu = 1$ and the shape parameter $\theta = 1$.

To proceed with the calibration, I first consider the cases when the home country has four different policy distortion and the foreign country has no policy distortion, i.e. with a) no policy distortion for both countries, i.e. $\tau_{ij} = 0, \tau_{ij}^* = 0$, b) the random policy distortion for the home country and no policy distortion for the foreign country, i.e. $cov(s_{ij}, \tau_{ij}) = 0, \tau_{ij}^* = 0$, c) the negatively correlated policy distortion for the home country and no policy distortion for the foreign country, i.e. $cov(s_{ij}, \tau_{ij}) < 0, \tau_{ij}^* = 0$ and d) the positively correlated policy distortion for the home country and no policy distortion for the foreign country, i.e. $cov(s_{ij}, \tau_{ij}) > 0, \tau_{ij}^* = 0$. I shall compare the equilibrium aggregate production levels, the aggregate price indice and the economy concentrations HHI in both the home and the foreign countries under the four policy distortion scenarios above. The calibration results are shown in TABLE 3 below.

From TABLE 3 above, I can clearly see that allowing trading to happen between two countries shall significantly increase the aggregate production

Table 3: Two Country General Equilibrium $L = 1$, $L^* = 1$, $D = 1.2, D^* = 1.2$

Tax Policy	Aggregate Output	Aggregate Price Index	HHI
$\tau = 0$	10209.99	0.000261	6844.05
$\tau^* = 0$	10145.70	0.000266	6950.64
$cov(s, \tau) = 0$	10152.59	0.000263	6883.84
$\tau^* = 0$	10081.22	0.000268	6980.74
$cov(s, \tau) < 0$	10214.83	0.000260	6835.70
$\tau^* = 0$	10154.49	0.000265	6950.05
$cov(s, \tau) > 0$	10070.66	0.000275	6737.67
$\tau^* = 0$	10016.62	0.000278	6955.96

levels in both countries, regardless of the tax policy environments in those countries. Moreover, if one country has a different tax policy environment, it shall affect the general equilibrium in both countries. Trading with the home country which has the negatively correlated tax policy shall give both the home and the foreign countries the highest aggregate production levels. On the other hand, trading with the home country which has the positively correlated tax policy shall give both the home and the foreign countries the lowest aggregate production levels. Trading with the home country which has the random tax policy shall give less aggregate production levels in both the home and the foreign countries compared to the ones when the home country has no tax policy distortion or the negatively correlated policy distortion. Overall, the relative rankings of the aggregate production levels stay consistent with the single country general equilibrium in both the home and the foreign countries.

In addition, I also find that after trading, the aggregate market prices shall decrease in both countries, regardless of the tax policy environments in those countries. The decreases in price levels are simply caused by importing the top five most productive firms from the other country, which makes the firms on average more productive and hence decreases the aggregate price levels. I find that trading with the home country which has the negatively correlated tax policy shall give both the home and the foreign firms the lowest aggregate price indice. Trading with the home country which has the positively correlated tax policy shall give both countries the highest aggregate price indice. Trading with the home country which has the random tax policy shall increase the aggregate price indice slightly above the levels under no tax policy distortion. Overall, the relative rankings of the aggregate price indice stay consistent with the single country general equilibrium in both the home and the foreign countries. Besides that, the economy concentrations may either go up or down after trading and I do not observe a universal change pattern of economy concentrations in both countries. The HHI levels are relatively the same for both countries. Recall that in this calibration, I assume both countries have the same population size $L = L^* = 1$ and the same iceberg trading cost $D = D^* = 1.2$.

I then conduct a parallel analysis with considering the changes of tax policies in the foreign country. I shall calibrate the general equilibrium in both countries under the four different policy distortion scenarios for the foreign country, with a) no policy distortion for both countries, i.e. $\tau_{ij} = 0, \tau^*_{ij} = 0$, b) no policy distortion for the home country and the random policy distortion for the foreign country, i.e $\tau_{ij} = 0, cov(s^*_{ij}, \tau^*_{ij}) = 0$, c) no policy distortion for the home country and the negatively correlated policy distortion for the foreign country, i.e. $\tau_{ij} = 0, cov(s^*_{ij}, \tau^*_{ij}) < 0$ and d) no policy distortion for the home country and the positively correlated policy distortion for the foreign country, i.e. $\tau_{ij} = 0, cov(s^*_{ij}, \tau^*_{ij}) > 0$. I then compare the equilibrium aggregate production levels, the aggregate price indice and the economy concentrations HHI for both the home and the foreign countries under the four tax policies above. The calibration results are shown in TABLE 4 below.

Table 4: Two Country General Equilibrium $L = L^* = 1$, $D = D^* = 1.2$

Tax Policy	Aggregate Output	Aggregate Price Index	HHI
$\tau = 0$	10209.99	0.000261	6844.05
$\tau^* = 0$	10145.70	0.000266	6950.64
$\tau = 0$	10202.37	0.000261	6841.31
$cov(s^*, \tau^*) = 0$	10138.31	0.000267	6948.28
$\tau = 0$	10214.86	0.000260	6847.22
$cov(s^*, \tau^*) < 0$	10151.20	0.000266	6945.39
$\tau = 0$	10133.15	0.000269	6803.30
$cov(s^*, \tau^*) > 0$	10007.31	0.000276	6779.38

From TABLE 4, I get consistent conclusions with what I find in TABLE 3 above. Trading shall increase the aggregate production levels in both countries significantly, regardless of the tax policy environment in those countries. In particular, trading with the foreign country which has the negatively correlated tax policy shall give both the home and the foreign countries the highest aggregate production levels, while trading with the foreign country which has the positively correlated tax policy shall give both countries the lowerest aggregate production levels. Trading with the foreign country which has the random tax policy shall give less aggregate production levels in both the home and the foreign countries compared to the ones when the foreign country has no tax policy distortion or with the negatively correlated policy distortion. Overall, the relative rankings of the aggregate production levels stay consistent with the single country general equilibrium in both the home and the foreign countries.

I also find that trading shall decrease the aggregate price levels in both countries regardless of the tax policy environment in each country, while the changes of economy concentrations can be either positive or negative. Trading with the foreign country which has the negatively correlated tax policy shall give both the home and the foreign firms the lowest aggregate price indice. Trading with the foreign country which has the positively correlated tax policy shall give both countries the highest aggregate price indice. Trading with the foreign country which has the random tax policy shall increase the aggregate price indice slightly above the levels under no tax policy distortion. Overall, the relative rankings of the aggregate price indice stay consistent with the single country general equilibrium in both the home and the foreign countries.

I then conduct the counterfactual analysis by modifying the iceberg trading cost for the foreign country. I increase $D^* = 1.5$ and see how the increase of the iceberg trading cost shall affect the general equilibrium trading pattern. I recalibrate the general equilibrium under the four policy distortion scenarios for either the home or the foreign country as before and the calibration results are shown in TABLE 5 and TABLE 6 below.

Comparing TABLE 5 with TABLE 3 implies that a higher iceberg trading cost in the foreign country shall lower the aggregate production levels in the home country under all four policy distortion scenarios. The home country faces relatively higher imported good prices now and the aggregate price index shall increase in the home country, regardless of the tax policy environment. Moreover, the home country now faces a lower economy concentration due to

Table 5: Two Country General Equilibrium $L = L^* = 1$, $D = 1.2$, $D^* = 1.5$

Tax Policy	Aggregate Output	Aggregate Price Index	HHI
$\tau = 0$	9925.87	0.000283	6758.98
$\tau^* = 0$	10145.70	0.000266	6950.64
$cov(s, \tau) = 0$	9875.23	0.000285	6800.34
$\tau^* = 0$	10081.22	0.000268	6980.74
$cov(s, \tau) < 0$	9929.54	0.000281	6747.42
$\tau^* = 0$	10154.49	0.000265	6950.05
$cov(s, \tau) > 0$	9777.25	0.000299	6603.19
$\tau^* = 0$	10016.62	0.000278	6955.96

the more costly importing from the foreign country. I also find that although the absolute values change for all the three variables in the home country, the relative rankings of the aggregate output levels and the aggregate price indice remain consistent with what I have before in the home country. Besides, increasing the iceberg trading cost in the foreign country does not affect the general equilibrium in the foreign country.

The results above imply that increasing the iceberg trading cost in the foreign country shall distort the aggregate production and cause a higher aggregate price level in the home country. Home country loses more as the imported goods become more expensive. Therefore, for policy implication purpose, trading with the foreign country with less exporting tariff is favored and should be encouraged. I then conduct the calibration of the general equilibrium when the foreign country has four tax distortion policies and the home country has no policy distortion. The estimation results are shown in TABLE 6 below. Overall, I find that the results are consistent with what I find in TABLE 5.

In the previous two cases, I assume both the home and the foreign countries

Table 6: Two Country General Equilibrium $L = L^* = 1$, $D = 1.2$, $D^* = 1.5$

Tax Policy	Aggregate Output	Aggregate Price Index	HHI
$\tau = 0$	9925.87	0.000283	6758.98
$\tau^* = 0$	10145.70	0.000266	6950.64
$\tau = 0$	9916.79	0.000283	6757.72
$cov(s^*, \tau^*) = 0$	10138.31	0.000267	6948.28
$\tau = 0$	9935.97	0.000282	6762.04
$cov(s^*, \tau^*) < 0$	10151.20	0.000266	6945.39
$\tau = 0$	9787.92	0.000290	6720.22
$cov(s^*, \tau^*) > 0$	10007.31	0.000276	6779.38

have the same baseline production technology level - i.e. the same distribution of productivities. I now allow the foreign country to have a more advanced baseline production technology level and study how the increase of the production technology shall affect the general equilibrium. Recall that in the previous case, both the home and the foreign countries have productivities drawn from the Pareto distribution with the location parameter $\mu = 1$ and the shape parameter $\theta = 1$. In the counterfactual analysis below, I shall change the location parameter to be $\mu^* = 5$ for the foreign country. I then recalibrate the general equilibrium trading pattern under the policy distortion scenarios as described before. The calibration results are shown in TABLE 7 and TABLE 8 below.

Comparing TABLE 7 with TABLE 3, I find that the better baseline technology level in the foreign country shall increase the aggregate production levels in both countries, regardless of the tax policy environment in those countries. When the foreign country has more productive firms than before, the lower marginal costs of all firms shall bring down the aggregate price levels in both countries, regardless of the tax policy environment in each country. Moreover, as the foreign firms get more productive, the competition level in-

Table 7: Two Country General Equilibrium $L = L^* = 1$, $D = 1.2$, $D^* = 1.2$, $\mu^* = 5$

Tax Policy	Aggregate Output	Aggregate Price Index	HHI
$\tau = 0$	10471.93	0.000254	6563.29
$\tau^* = 0$	10358.79	0.000255	6328.21
$cov(s, \tau) = 0$	10413.67	0.000255	6594.34
$\tau^* = 0$	10293.34	0.000256	6340.94
$cov(s, \tau) < 0$	10472.49	0.000252	6561.26
$\tau^* = 0$	10369.00	0.000254	6331.15
$cov(s, \tau) > 0$	10400.34	0.000267	6502.70
$\tau^* = 0$	10212.95	0.000265	6292.11

creases in each country and hence the market concentrations shall decrease in both countries, regardless of the tax policy environment in each country. Besides, I also find that although the absolute values change for all the three variables in the home and the foreign countries, the relative rankings of the aggregate output levels and the aggregate price indice in both countries remain consistent with what I have before.

Overall, the results imply that if the foreign country has a better technology level, then both countries will benefit from the improved technology. The foreign country benefits directly from the better technology by lowering the costs of production of all foreign firms. The home country can also benefit from the better technology by importing less expensive firms from the foreign country. Hence, trading with the foreign country with a better baseline production technology shall be encouraged. I then conduct the parallel analysis to test the effect of the increased baseline production technology in the foreign country on the two country trading pattern when the foreign country has four different tax policy scenarios. The estimation results are shown in TABLE 8 below.

Table 8: Two Country General Equilibrium $L = L^* = 1$, $D = 1.2$, $D^* = 1.2$, $\mu^* = 5$

Tax Policy	Aggregate Output	Aggregate Price Index	HHI
$\tau = 0$	10471.93	0.000254	6563.29
$\tau^* = 0$	10358.79	0.000255	6328.21
$\tau = 0$	10462.60	0.000254	6554.37
$cov(s^*, \tau^*) = 0$	10346.13	0.000255	6316.19
$\tau = 0$	10476.54	0.000253	6565.41
$cov(s^*, \tau^*) < 0$	13686.52	0.000249	5762.06
$\tau = 0$	10397.00	0.000262	6534.84
$cov(s^*, \tau^*) > 0$	5294.42	0.000234	4372.73

The implication of TABLE 8 is consistent with what I find in TABLE 7 above except that when the foreign country has the positively correlated policy distortion, the aggregate production in the foreign country decreases as compared with the one when $\mu^* = 1$. Overall, both TABLE 7 and TABLE 8 suggest that trading with the foreign country which has a better baseline production technology shall benefit the home country in terms of increasing the aggregate production levels, decreasing the aggregate price indice, and making the market more competitive and less concentrated, regardless of the policy distortion environments in both countries. Thus trading with the foreign country which has a better production technology should be encourage for the home country.

I then conduct the counterfactual analysis by modifying the fixed labor supply in the foreign country. I shall increase $L^* = 1.2$ and study how the increase of the fixed labor supply shall affect the general equilibrium trading pattern. I recalibrate the general equilibrium under the four policy distortion scenarios for both the home and the foreign countries as before and the calibration results are shown in TABLE 9 and TABLE 10 below.

Table 9: Two Country General Equilibrium $L = 1$, $L^* = 1.2$, $D = D^* = 1.2$, $\mu^* = 1$

Tax Policy	Aggregate Output	Aggregate Price Index	HHI
$\tau = 0$	10209.99	0.000261	6844.05
$\tau^* = 0$	12174.84	0.000266	6950.64
$cov(s, \tau) = 0$	10152.59	0.000263	6883.85
$\tau^* = 0$	12097.47	0.000268	6980.74
$cov(s, \tau) < 0$	10214.83	0.000260	6835.70
$\tau^* = 0$	12185.39	0.000265	6950.05
$cov(s, \tau) > 0$	10070.66	0.000275	6737.67
$\tau^* = 0$	12019.94	0.000278	6955.96

Comparing TABLE 9 with TABLE 3 implies that a higher fixed labor supply in the foreign country shall increase the aggregate production levels in the foreign country under all four policy distortion scenarios. The home country shall not be influenced by it and thus the aggregate production levels stay the same as before. Since I take the wage rate and the capital rental rate as given, the increase in the foreign country's labor supply shall not influence the marginal costs of all firms and thus the aggregate price indice and economy concentrations stay the same as before for both countries. Moreover, the relative rankings of the aggregate output levels remain consistent with what I have before in both the home and the foreign countries. I then conduct the calibration of the general equilibrium when the foreign country has four tax policies and the home country has no policy distortion. The estimation results are shown in TABLE 10 below. Overall, I find that the results are consistent with what I find in TABLE 9.

Table 10: Two Country General Equilibrium $L = 1$, $L^* = 1.2$, $D = D^* = 1.2$, $\mu^* = 1$

Tax Policy	Aggregate Output	Aggregate Price Index	HHI
$\tau = 0$	10209.99	0.000261	6844.05
$\tau^* = 0$	12174.84	0.000266	6950.64
$\tau = 0$	10202.37	0.000261	6841.31
$cov(s^*, \tau^*) = 0$	12165.97	0.000267	6948.28
$\tau = 0$	10214.86	0.000260	6847.22
$cov(s^*, \tau^*) < 0$	12181.44	0.000266	6954.39
$\tau = 0$	10133.15	0.000269	6803.30
$cov(s^*, \tau^*) > 0$	12008.77	0.000276	6779.38

Overall, both TABLE 9 and TABLE 10 suggest that trading with the foreign country which has a higher fixed labor supply shall benefit the foreign country in terms of increasing the aggregate production levels regardless of the policy distortion environments in both countries. Trading with the foreign country which has a higher fixed labor supply shall not affect the general equilibrium in the home country given the constant wage rate and the capital rental rate and thus does not suggest any trading implication.

To summarize, the negatively correlated tax policy is encouraged in a closed economy given that it expands the aggregate production level in the single country without causing an inflation or a monoply power. The positively correlated tax policy is worse than no tax policy by distorting the aggregate production level in the closed economy and shall be discouraged. For the two country open economies, trading with the foreign country which has the negatively correlated tax policy shall benefit both the home and the foreign economies by increasing the aggregate production levels the most. The home country is thus encouraged to trade with the foreign country that has the tax policy in favor of highly productive firms (i.e. negatively correlated tax policy). The counterfactual analysis implies that trading with the foreign country that has a higher iceberg trading cost shall distort the aggregate output level in the home country. The home country is thus encouraged to trade with the foreign country that has a lower exporting tariff. Moreover, trading with the foreign country that has a higher baseline production technology shall be beneficial to the home country as it can import growth from importing less expensive products by increasing the aggregate production level in the home economy. Therefore, the home country is encouraged to trade with the foreign country with a better baseline technology. The higher fixed labor supply in the foreign country shall not afffect the general equilibrium in the home country and thus does not suggest any trading implication.

7 Conclusion

This paper studies the effects of tax policy distortions on the general equilibrium of both the single country closed economy and the two country open economies. The paper starts with a single country closed economy. Solving the consumer's and the firms' maximization problems together with goods and labor market clearing conditions characterize the single country general equilibrium. The paper then extends the analysis to the two country trading model. The paper solves the general equilibrium trading pattern in both countries by assuming the top five most productive firms in each sector of each country shall export to the other country while facing an iceberg trading cost.

The paper then calibrates the general equilibrium to solve the equilibrium output and price level for each firm, $\{p_{ij}, q_{ij}\}$, the labor and capital employment $\{l_{ij}, k_{ij}\}$ and the elasticity of demand and marke share $\{\sigma_{ij}, s_{ij}\}$ for each firm. The paper then summarizes the general equilibrium of the aggregate output levels, the aggregate price indice and the market concentrations in both the home and the foreign countries to get policy related and trading related implications.

The paper first calibrates the single country general equilibrium of the aggregate production levels, the aggregate price indice and the market concentrations in either the home or the foreign country under four policy distortion scenarios: a) no policy distortion i.e. $\tau_{ij} = 0$ or $\tau^*_{ij} = 0$, b) random policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) = 0$ or $cov(s^*_{ij}, \tau^*_{ij}) = 0$, c) negatively correlated policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) < 0$ or $cov(s^*_{ij}, \tau_{ij})^* < 0$ and d) positively correlated policy distortion, i.e. $cov(s_{ij}, \tau_{ij}) > 0$ or $cov(s^*_{ij}, \tau^*_{ij}) > 0$. The calibration results imply that the negatively correlated tax policy shall expand the aggregate production the most without causing an inflation or a monopoly power and thus should be encouraged. The positively correlated tax policy shall distort the economy the most by reducing the aggregate production level and thus should be discouraged.

The paper then calibrates the two country general equilibrium trading pattern of the aggregate production levels, the aggregate price indice and the market concentrations under four policy scenarios for the home country: a) no policy distortion for both countries, i.e. $\tau_{ij} = 0, \tau^*_{ij} = 0$, b) random policy distortion for the home country and no policy distortion for the foreign country, i.e. $cov(s_{ij}, \tau_{ij}) = 0, \tau^*_{ij} = 0$, c) negatively correlated policy distortion for the home country and no policy distortion for the foreign country, i.e. $cov(s_{ij}, \tau_{ij}) < 0, \tau^*_{ij} = 0$, and d) positively correlated policy distortion for the home country and no policy distortion for the foreign country, i.e. $cov(s_{ij}, \tau_{ij}) > 0, \tau^*_{ij} = 0$. Overall, the paper suggests that allowing trading to happen between two countries shall increase the aggregate production levels in both countries, regardless of the tax policy environment in each country. In particular, the paper finds that trading with the home country which has the negatively correlated tax policy shall increase the aggregate production levels in both countries the most, while trading with the home country which has the positively correlated tax policy shall increase the aggregate production levels in both countries the least. Hence trading with a country that has the negatively correlated tax policy should be encouraged. Moreover, I then conduct three counterfactual analyses by a) increasing the iceberg trading cost for the foreign country from $D^* = 1.2$ to $D^* = 1.5$ and b) increasing the baseline technology level for the foreign country by changing the location parameter $\mu^* = 1$ to $\mu^* = 5$ and c) increasing the fixed labor supply in the foreign country from $L^* = 1$ to $L^* = 1.2$. The counterfactual analyses suggest that trading with the foreign country with a lower iceberg trading cost or with a better baseline technology level is beneficial to the home country and hence should be encouraged. Trading with the foreign country with an increased fixed labor supply does not affect the general equilibrium in the home country and thus does not suggest any trading implication.

The paper also conducts the calibration analysis under the two country trading pattern of the aggregate production levels, the aggregate price indice and the market concentrations when the foreign country has four policy distortion scenarios: a) no policy distortion for both countries, i.e. $\tau_{ij} = 0, \tau_{ij}^* = 0$, b) no policy distortion for the home country and the random policy distortion for the foreign country, i.e. $\tau_{ij} = 0, cov(s_{ij}^*, \tau_{ij}^*) = 0$, c) no policy distortion for the home country and the negatively correlated policy distortion for the foreign country, i.e. $\tau_{ij} = 0, cov(s_{ij}^*, \tau_{ij}^*) < 0$ and d) no policy distortion for the home country and the positively correlated policy distortion for the foreign country, i.e. $\tau_{ij} = 0, cov(s_{ij}^*, \tau_{ij}^*) > 0$. The calibration results together with the three counterfactual analyses suggest consistency with what I find above. Generally speaking, trading with the foreign country that has the encouraging tax policy toward productive firms, has less exporting tariff, and has more advanced baseline production technology level shall benefit the home country and should be encouraged. Trading with the foreign country with an increased fixed labor supply shall not affect the general equilibrium in the home country and thus does not suggest any trading implication. An important objective for future work is to calibrate the models with the U.S. and China manufacturing and trading data. The results should suggest useful policy implications for both tax policies and trading related policies.

8 Appendix

8.1 Appendix Note 1

Forming the Lagrangian of the consumer's maximization problem in the equation (3) gives that

$$L = Q + \lambda(I - \int_{i=0}^{1} \sum_{j=1}^{J} p_{ij} q_{ij} di). \tag{38}$$

Simplying the equation (38) above suggests that

$$\frac{dQ}{dq_{ij}} = \lambda p_{ij}. \tag{39}$$

From the equations (1) and (2), I have that

$$\frac{dQ}{dq_{ij}} = \frac{\eta}{\eta - 1}[\int_{i=o}^{1} Q_i^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}-1} \frac{\eta - 1}{\eta} Q_i^{\frac{\eta-1}{\eta}-1} \frac{\rho}{\rho - 1}[\sum_{j=1}^{J} q_{ij}^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}-1} \frac{\rho - 1}{\rho} q_{ij}^{\frac{\rho-1}{\rho}-1}. \tag{40}$$

Simplifying the equation (40) above suggests that

$$\frac{dQ}{dq_{ij}} = [\int_{i=0}^{1} Q_i^{\frac{\eta-1}{\eta}} di]^{\frac{1}{\eta-1}} Q_i^{-\frac{1}{\eta}} [\sum_{j=1}^{J} q_{ij}^{\frac{\rho-1}{\rho}}]^{\frac{1}{\rho-1}} q_{ij}^{-\frac{1}{\rho}}. \tag{41}$$

Thus,

$$\frac{dQ}{dq_{ij}} = Q^{\frac{1}{\eta}} Q_i^{\frac{1}{\rho}-\frac{1}{\eta}} q_{ij}^{-\frac{1}{\rho}}. \tag{42}$$

So the equation (6) is obtained.

8.2 Appendix Note 2

I consider two firms j and j' in the same sector i. I take the ratio of the first order conditions for firm j and firm j' to have that

$$\frac{q_{ij'}^{-\frac{1}{\rho}}}{q_{ij}^{-\frac{1}{\rho}}} = \frac{p_{ij'}}{p_{ij}}. \tag{43}$$

Simplifying the equation (43) above suggests that

$$q_{ij'} = (\frac{p_{ij'}}{p_{ij}})^{-\rho} q_{ij}, \tag{44}$$

which yields that relationship of the disaggregated demands for two firms j and $j's$ as in the equation (7).

8.3 Appendix Note 3

From the equation (2), I have that

$$Q_i^{\frac{\rho-1}{\rho}} = \sum_{j'=1}^{J} q_{ij'}^{\frac{\rho-1}{\rho}}. \tag{45}$$

Given the equation (7), I have that

$$Q_i^{\frac{\rho-1}{\rho}} = \sum_{j'=1}^{J} [\left(\frac{p_{ij'}}{p_{ij}}\right)^{-\rho} q_{ij}]^{\frac{\rho-1}{\rho}}. \tag{46}$$

Simplifying the equation (46) above suggests that

$$Q_i^{\frac{\rho-1}{\rho}} = q_{ij}^{\frac{\rho-1}{\rho}} \sum_{j'=1}^{J} (\frac{p_{ij'}}{p_{ij}})^{1-\rho}. \tag{47}$$

Thus,

$$Q_i^{\frac{\rho-1}{\rho}} = q_{ij}^{\frac{\rho-1}{\rho}} \frac{1}{p_{ij}^{1-\rho}} \sum_{j'=1}^{J} p_{ij'}^{1-\rho}. \tag{48}$$

Given the equation (5), I can simplify the equation (48) above further to get that

$$Q_i^{\frac{\rho-1}{\rho}} = q_{ij}^{\frac{\rho-1}{\rho}} \frac{1}{p_{ij}{}^{1-\rho}} P_i^{1-\rho}. \tag{49}$$

I then simplify the equation (49) above to get the equation (8).

8.4 Appendix Note 4

Substituting the equation (8) back into the equation (6) suggests that

$$Q^{\frac{1}{\eta}} Q_i^{-\frac{1}{\eta}} \frac{p_{ij}}{p_i} = \lambda p_{ij}, \tag{50}$$

which simplifies to get that

$$Q^{\frac{1}{\eta}} Q_i^{-\frac{1}{\eta}} = \lambda p_i. \tag{51}$$

I then take a ratio of Q_i and $Q_{i'}$ to get that

$$(\frac{Q_{i'}}{Q_i})^{-\frac{1}{\eta}} = \frac{p_{i'}}{p_i}. \tag{52}$$

Thus the sectoral quantity from any other sector i' can be written as a function of

$$Q_{i'} = (\frac{p_{i'}}{p_i})^{-\eta} Q_i. \tag{53}$$

Recall the formula for Q in the equation (1) and for P in the equation (4) above. I then substitute all other sectors' quantities $Q_{i'}$ as a function of Q_i. The aggregate quantity Q is now a function of Q_i. Simplify to get that

$$Q = p_i^{\eta} Q_i p^{-\eta}. \tag{54}$$

Simplifying the equation (54) above suggests that

$$Q_i = \frac{Q}{{p_i}^{\eta} p^{-\eta}} = (\frac{p_i}{p})^{-\eta} Q. \tag{55}$$

I then combine the equation (8) and (55) to get the equation (9).

8.5 Appendix Note 5

Taking the derivative with respect to $\log q_{ij}$ on both sides of the equation (10) implies the price elasticity of demand

$$\frac{d \log q_{ij}}{d \log p_{ij}} = -\rho + \rho \frac{d \log p_i}{d \log p_{ij}} - \eta \frac{d \log p_i}{d \log p_{ij}}. \tag{56}$$

To calculate $\frac{d \log p_i}{d \log p_{ij}}$, I recall the price index for sector i as defined earlier in the equation (5). Taking derivative with respect to p_{ij} suggests that

$$\frac{dp_i}{dp_{ij}} = [\sum_{j=1}^{J} p_{ij}^{1-\rho}]^{\frac{\rho}{1-\rho}} p_{ij}^{-\rho}. \tag{57}$$

Given that $\frac{d\log q_{ij}}{d\log p_{ij}} = \frac{dp_i}{dp_{ij}} \frac{p_{ij}}{p_i}$ and combining it with the equations (5) and (57) above, I thus get that

$$\frac{d\log q_{ij}}{d\log p_{ij}} = [\sum_{j=1}^{J} p_{ij}^{1-\rho}]^{\frac{\rho}{1-\rho}} p_{ij}^{-\rho} \frac{p_{ij}}{p_i}. \tag{58}$$

Simplifying the equation above suggests that

$$\frac{d\log q_{ij}}{d\log p_{ij}} = [\sum_{j=1}^{J} p_{ij}^{1-\rho}]^{-1} p_{ij}^{1-\rho}. \tag{59}$$

Given that $[\sum_{j=1}^{J} p_{ij}^{1-\rho}]^{-1} p_{ij}^{1-\rho} = \frac{p_{ij}(\frac{p_{ij}}{p_i})^{-\rho} Q_i}{\sum_{j'=1}^{J} p_{ij'}(\frac{p_{ij'}}{p_i})^{-\rho} Q_i}$ and combining it with the equation (8), I thus show that

$$\frac{d\log p_i}{d\log p_{ij}} = \frac{p_{ij} q_{ij}}{\sum_{j'=1}^{J} p_{ij'} q_{ij'}} = sh_{ij}. \tag{60}$$

Thus, I get the price elasticity of demand σ_{ij} as

$$\sigma_{ij} = -\frac{d\log q_{ij}}{d\log p_{ij}} = \rho(1 - s_{ij}) + \eta sh_{ij}. \tag{61}$$

8.6 Appendix Note 6

By definition,

$$sh_{ij} = \frac{p_{ij} q_{ij}}{\sum_{j'=1}^{J} p_{ij'} q_{ij'}}. \tag{62}$$

Applying the equation (8) suggests that

$$\frac{p_{ij}q_{ij}}{\sum_{j'=1}^{J} p_{ij'}q_{ij'}} = \frac{p_{ij}\frac{p_{ij}}{p_i}^{-\rho}Q_i}{\sum_{j'=1}^{J} p_{ij'}(\frac{p_{ij'}}{p_i})^{-\rho}Q_i}. \tag{63}$$

Simplifying the equation (63) above implies that

$$\frac{p_{ij}\frac{p_{ij}}{p_i}^{-\rho}Q_i}{\sum_{j'=1}^{J} p_{ij'}(\frac{p_{ij'}}{p_i})^{-\rho}Q_i} = \frac{p_{ij}^{1-\rho}}{\sum_{j'=1}^{J} p_{ij'}^{1-\rho}} = (\frac{p_{ij}}{p_i})^{1-\rho}. \tag{64}$$

Thus, I have that

$$sh_{ij} = (\frac{p_{ij}}{p_i})^{1-\rho}, \tag{65}$$

which is the equation (13).

8.7 Appendix Note 7

The FOCs are

$$[l_{ij}] : s_{ij}{k_{ij}}^{\alpha}\beta{l_{ij}}^{\beta-1} = w, \tag{66}$$

and

$$[k_{ij}] : s_{ij}\alpha{k_{ij}}^{\alpha-1}{l_{ij}}^{\beta} = r. \tag{67}$$

Take a ratio of the two FOCs to get that

$$l_{ij} = \frac{\beta}{\alpha}\frac{r}{w}k_{ij}. \tag{68}$$

I then apply the above equation into $q_{ij} = s_{ij}k_{ij}^{\alpha}l_{ij}^{\beta}$ to get that

$$q_{ij} = s_{ij}k_{ij}(\frac{\beta}{\alpha})^{\beta}(\frac{r}{w})^{\beta}, \tag{69}$$

which then implies the equations (16) and (17) above.

8.8 Appendix Note 8

Given the constant return to scale assumption, the marginal cost of each firm shall equal the average cost, i.e. $mc_{ij} = \frac{wl_{ij}+rk_{ij}}{q_{ij}}$. From the equations (16) and (17) above, I have that

$$mc_{ij} = \frac{w\frac{\beta r}{\alpha w}\frac{q_{ij}}{s_{ij}(\frac{\beta}{\alpha})^{\beta}(\frac{r}{w})^{\beta}} + r\frac{q_{ij}}{s_{ij}(\frac{\beta}{\alpha})^{\beta}(\frac{r}{w})^{\beta}}}{q_{ij}}. \tag{70}$$

Simplify the equation (70) above to get that

$$mc_{ij} = \frac{(1+\frac{\beta}{\alpha})r}{s_{ij}(\frac{\beta}{\alpha})^{\beta}(\frac{r}{w})^{\beta}}. \tag{71}$$

8.9 Appendix Note 9

From the FOC, I first calculate $\frac{dq_{ij}}{dp_{ij}}$ by taking derivative with respect to p_{ij} for the goods market clearing constraint, i.e. $q_{ij} = (\frac{p_{ij}}{p_i})^{-\rho}(\frac{p_i}{p})^{-\eta}Q$ to get that

$$\frac{dq_{ij}}{dp_{ij}} = (-\rho)p_{ij}^{-\rho-1}p_i^{\rho-\eta}p^{\eta}Q + p_{ij}^{-\rho}(\rho-\eta)p_i^{\rho-\eta-1}\frac{dp_i}{dp_{ij}}p^{\eta}Q. \tag{72}$$

Given the equation (5) above, I can then calculate $\frac{dp_i}{dp_{ij}}$ as

$$\frac{dp_i}{dp_{ij}} = [\sum_{j=1}^{J} p_{ij}^{1-\rho}]^{\frac{\rho}{1-\rho}}p_{ij}^{-\rho} = p_i^{\rho}p_{ij}^{-\rho}. \tag{73}$$

Combining the equation (72) together with the equation (73) implies that

$$\frac{dq_{ij}}{dp_{ij}} = (-\rho)\frac{q_{ij}}{p_{ij}} + (\rho - \eta){p_{ij}}^{-\rho}{p_i}^{\rho-1}q_{ij}. \tag{74}$$

I then apply the expression I derive for $\frac{dq_{ij}}{dp_{ij}}$ of the equation (74) above into the first order condition to get that

$$(p_{ij}(1-\tau_{ij}) - mc_{ij})q_{ij}(\frac{\rho}{p_{ij}} - (\rho-\eta){p_{ij}}^{-\rho}{p_i}^{\rho-1}) = (1-\tau_{ij})q_{ij}. \tag{75}$$

Given that $sh_{ij} = (\frac{p_{ij}}{p_i})^{1-\rho}$, the equation above becomes

$$(p_{ij}(1-\tau_{ij}) - mc_{ij})(\rho - (\rho-\eta)sh_{ij}) = (1-\tau_{ij})p_{ij}. \tag{76}$$

Simplifying the equation above suggests that

$$\frac{mc_{ij}}{p_{ij}} = (1-\tau_{ij})\frac{\rho-(\rho-\eta)sh_{ij}-1}{\rho-(\rho-\eta)sh_{ij}}. \tag{77}$$

Thus,

$$p_{ij} = \frac{\rho-(\rho-\eta)sh_{ij}}{\rho-(\rho-\eta)sh_{ij}-1}mc_{ij}\frac{1}{1-\tau_{ij}}. \tag{78}$$

I then use $\sigma_{ij} = \rho - (\rho-\eta)sh_{ij}$ to get that

$$p_{ij} = \frac{\sigma_{ij}}{\sigma_{ij}-1}\frac{1}{1-\tau_{ij}}mc_{ij}. \tag{79}$$

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