Coupling with the past

Tropp & Wilson ’98: a remarkable algorithm to generate exact sample from stationary dist. \(\mathbf{\Pi}\).

Algorithm: going backwards, simulate jumps from all states. This defines a (random) sequence of mappings \(\mathbf{f}^{-1}: S \rightarrow S\).

Random mapping \(\mathbf{f}^{-1}\).

Eventually, at some point the range of \(\mathbf{F} \circ \ldots \circ \mathbf{F}^{-t}\) is just \(1,2,4\).

This random element has distribution \(\mathbf{\Pi}\).
Immediate Q: why going backwards?

Can one ever stop at $a$?

Proof of the Thin
Important simplification happens for ordered $S$, if we can couple jumps from different states so that $s > s' \Rightarrow F_t(s) > F_t(s')$.

(If this case, trajectories don't cross).

Then, if there are largest & smallest states, one can just wait till they collide.

Example: Ising model.

Non-example: Blob.
\[ P(+) \to (-) \text{ on the left} \]
\[ > P(+) \to (-) \text{ on the right} \]

\[ \sum \sigma_k \sigma_e \]

\[ \Pi(\vec{\sigma}) \sim e^\rho \sum \sigma_k \sigma_e \]

\[ \rho > 0 \]

\[ \begin{array}{c}
+ \quad \nearrow \\
- \quad \searrow
\end{array} \]