Six problems. No cell phones or internet usage. Show reasonings; box your answers.

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1. (a) Does there exist a function on real line that is bounded, has local minima and maxima, but does not have global maxima or minima?
   
   (b) Does there exist a continuous function on real line with infinitely many local maxima and minima, such that the values at these local maxima are all equal (say, to \(a\)); the values at the local minima are all equal (say, to \(b, b < a\)) yet the function does not attain its global maximum?

2. Describe the sets of points where the following functions on \(\mathbb{R}^2\) are not (Frechet) differentiable:
   
   (a) \(|x_1| - |x_2|\)
   
   (b) \(\max(\min(x_1, x_2), \min(x_2, -x_1 - x_2), \min(-x_1 - x_2, x_1))\)
   
   (c) \(\max(x_1^2 - x_2^2 - 1, 0)^2\).

3. How many critical points does the function
   
   \[\sin x + \frac{x^2}{101\pi}\]

   have on real line?

4. For a given \(m \times n\) matrix \(A\), solve the optimization problem:
   
   \[\text{tr}XX^* \rightarrow \min, \text{subject to } \text{tr}XA^* = 1.\]

   Here \(X\) is an \(m \times n\) matrix, \(^*\) means matrix conjugation (transposition).

5. Among all probability distributions on nonnegative integers \(X = \{0, 1, 2, \ldots\}\) with mean \(a > 0\), find one maximizing entropy. In other words, solve
   
   \[-\sum_k x_k \log x_k \rightarrow \max, \text{subject to } x_k > 0; \sum_k x_k = 1, \sum_k kx_k = a.\]

6. Formulate the dual problem to
   
   \[
   \begin{align*}
   x_1 + x_2 + x_3 & \rightarrow \text{max} \\
   3x_1 + x_2 + x_3 & \leq 5 \\
   x_1 + 4x_2 + x_3 & \leq 6 \\
   x_1 + x_2 + 5x_3 & \leq 7 \\
   x_1, x_2, x_3 & \geq 0.
   \end{align*}
   
   Solve the dual problem.