UIN	1	2	3	4	5	6
Name						

Show work; box answers; use attached sheets.

Problem 1 Consider the operator A on $\mathbb{R}^N = \{(x_0, \dots, x_{N-1})\}$ given by

$$(Ax)_n = x_{n-1} - 2x_n + x_{n+1},$$

where $n-1, n+1$ are understood $\mathtt{mod} N$.

1. Find the eigenvalues of A.
2. Find $\exp(tA)v$, where
$$v_k = \exp(2\pi i k l/N), 0 \leq k < N$$
(here l is an integer between 0 and $(n-1)$).

Problem 2

Consider vector field in $\mathbb{R}^2$ given by

$$\begin{aligned} \dot{x}_1 &= x_1^3 - 3x_1x_2^2 - x_1; \\ \dot{x}_2 &= 3x_1^2x_2 - x_2^3 - x_2. \end{aligned}$$

1. Find all stationary points of the vector field.
2. Linearize the vector field near the stationary points, and determine which of them are stable, asymptotically stable.

Problem 3

Find minimal realization of the system with transfer function

$$G(s) = \frac{1}{1-s^3}$$

Problem 4

For the operator A from Problem 1 (assume $N = 4$), set

$$\dot{x} = Ax + bu, b = (0, 0, 0, 1)^*.$$

1. Is the system controllable?
2. Find state feedback k such that the closed loop operator has eigenvalues $(-1), (-2)$ (both eigenvalues of multiplicity 2).

Problem 5 Consider the LPR problem $\dot{x} = Ax + bu, x \in \mathbb{R}^2, J = \int_0^T |cx|^2 + r|u|^2 dt$, where

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, c = (1 \quad 0), r = 1.$$

1. Solve algebraic Riccati equation.
2. Find the optimal state feedback for the infinite horizon, and determine the eigenvalues of the resulting closed loop operator.

Problem 6
Consider the travel time minimization problem

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= x_1 + u, \end{aligned}$$

where

$$|u| \leq 1,$$

and the final position is $(0, 0)$.
Find for which starting positions $x(0) = (x_1(0), x_2(0))$ the optimal time is finite.