Introduction to Optimization (ECE 490) Final exam, 5.9.2016, 8:00-10:30am

Nine problems; 10 pages. No cell phones or internet usage. Show reasonings; box your answers.

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Problem 1

Which of the following subsets is closed? compact?

(1) $A = \{x : \sin^2(1/x) = 0\}$;
(2) $B = \{x : \sin^2(1/x) \geq .00001x^2\}$;
(3) $C = \{x : \sin^2(1/x) = x^2\}$.

Problem 2

Solve the optimization problem

$$5x^4 + 8x^2y^2 + 5y^4 - 18x^2 - 18y^2 \to \min$$

Problem 3

Consider the set of feasible points X given by the constraints

$$X = \begin{cases} e^x + e^y + e^z & = 3e; \\ x^2 + y^2 - z^3 & = 1. \end{cases}$$

(1) We know that

$$f = (x-a)^2 + y^2 + z^2$$

has a critical point on X at $(1,1,1)$. Find a.

(2) Is this critical point a local maximum? Minimum?

Problem 4

Consider the problem

$$\begin{aligned} -3x - 3y &\to \min \\ e^{-x} + e^{-y} + e^{x+y} &= 3 \end{aligned}$$

Is it convex? Solve it.

Problem 5

Consider the LP problem

$$\begin{array}{cccc} -5x_1 & -5x_2 & -6x_3 & \to \min, \text{subject to} \\ 2x_1 & +x_2 & +x_3 & \leq 8 \\ x_1 & +2x_2 & +x_3 & \leq 8 \\ x_1 & +x_2 & +2x_3 & \leq 8 \\ x_i & \geq 0 & i = 1 \ldots 3, & \end{array}$$

Starting at $x_1 = x_2 = x_3 = 0$, find the solution.

Problem 6

(1) Starting with $x_0 = 2$, find 4 iterations of Newton method to solve

$$f(x) = x^2 - x = 0.$$

(2) Sketch $|\log(f(x_k))|, k = 0, \ldots, 4$.
(3) If the iterations were for the function

$$f(x) = x^3 + x,$$

starting with $x_0 = 1$, what would the plot look like?

Problem 7

For the problem

$$\begin{aligned} ax + by &\to \min, \text{subject to} \\ x &\geq -1 \\ x &\leq 1 \\ y &\geq -1 \\ y &\leq 1 \end{aligned}$$

find and sketch the central paths for

- $a = 1, b = 1$;
- $a = 1, b = -1$

Problem 8

(1) Consider the 4-dimensional space spanned by the polynomials

$$p(x) = ax^3 + bx^2 + cx + d$$

of degree at most 3.

Run the Gram-Schmidt orthogonalization procedure on the vectors $\{1 + x, x + x^2, x^2 + x^3, x^3 + 1\}$, if the scalar product is given by

$$p_1' Q p_2 = \int_0^\infty e^{-x} p_1(x) p_2(x) dx.$$

(2) Consider quadratic form in $\mathbb{R}^{100}$ given by

$$Q = \begin{pmatrix} 2 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 1 & \cdots & 1 \\ 1 & 1 & 2 & \cdots & 1 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 1 & 1 & \cdots & 2 \end{pmatrix}$$

(i.e. 2 on the diagonal, 1 elsewhere). Run two iterations of the conjugate gradient method for the system

$$Qx = b,$$

where $b = (1, 0, \ldots, 0)$.

Problem 9

Items valued as

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14 18 12 15 2 10 6 12 16 2 19 2 16 13 12 19 9 7 14 15
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are lined up in a row. In the game, two players take, in turns, first or the last of the remaining items. What is the best total value the starting player can collect, under the best possible strategy (assuming that the opponent also does her best)?