E-companion to
"Coordinating Inventory Control and Pricing Strategies for Perishable Products"

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This e-companion contains the technical proof of Theorem 3, two extensions, and four tables of the numerical study.

EC.1. Proof of Theorem 3

It suffices to show that $R(d-\zeta, y-\zeta)$ is supermodular in (d, y, ζ) for $\zeta \in \{\zeta : d-\zeta \in \mathcal{D}_t, 0 \leq \zeta \leq y\}$.

Note that $R(d-\zeta, y-\zeta) = P(d-\zeta)(d-\zeta) + P(d-\zeta)E[\min(\epsilon_t, y-d)]$. Taking second-order cross-partial derivatives with respect to (d, y) yields

$$\frac{\partial^2 R}{\partial d \partial y} = P'(d-\zeta)\bar{F}(y-d) + P(d-\zeta)F'(y-d) = -P'(d-\zeta)\bar{F}(y-d)[\varrho(d,y)-1] \geq 0,$$

where the inequality is by condition (C2).

Taking second-order cross-partial derivatives with respect to (y, ζ) yields

$$\frac{\partial^2 R}{\partial \zeta \partial y} = -P'(d-\zeta)\bar{F}(y-d) \geq 0,$$

where the inequality is due to $P' < 0$.

Taking second-order cross-partial derivatives with respect to (d,ζ) yields

$$\frac{\partial^2 R}{\partial d \partial \zeta} = -2P'(d-\zeta) - P^{''}(d-\zeta)(d-\zeta) - P^{''}(d-\zeta)E[\min(\epsilon_t, y-d)] + P^{'}(d-\zeta)\bar{F}(y-d).$$

If $P^{''}(d) \le 0$, then

$$\begin{aligned}\frac{\partial^2 R}{\partial d \partial \zeta} &= -2P'(d-\zeta) - P^{''}(d-\zeta)E[\min(d-\zeta+\epsilon_t, y-\zeta)] + P^{'}(d-\zeta)\bar{F}(y-d) \\ &\ge -2P'(d-\zeta) + P^{'}(d-\zeta)\bar{F}(y-d) \\ &\ge 0,\end{aligned}$$

where the first inequality is by the nonnegativity of $E[\min(d-\zeta+\epsilon_t, y-\zeta)]$ and the second by the fact that $-P'(d-\zeta)+P^{'}(d-\zeta)\bar{F}(y-d) = -P'(d-\zeta)F(y-d) \ge 0$ and $P'(d) \le 0$.

If, otherwise, $P^{''}(d) > 0$, then

$$\begin{aligned}\frac{\partial^2 R}{\partial d \partial \zeta} &= -2P'(d-\zeta) - P^{''}(d-\zeta)(d-\zeta) - P^{''}(d-\zeta)E[\min(\epsilon_t, y-d)] + P^{'}(d-\zeta)\bar{F}(y-d) \\ &\ge -2P'(d-\zeta) - P^{''}(d-\zeta)(d-\zeta) + P^{'}(d-\zeta)\bar{F}(y-d) \\ &\ge -[P'(d-\zeta) + P^{''}(d-\zeta)(d-\zeta)] \\ &\ge 0,\end{aligned}$$

where the first inequality is from $E[\min(\epsilon_t, y-d)] \le E[\epsilon_t] = 0$, the second from $P'(d-\zeta) \le 0$ and the third from (C1). This completes the proof.

EC.2. Extensions

EC.2.1. Lost-Sales Model with Positive Lead Time

We now address the lost-sales model with positive lead time. Recall that in the lost-sales case $R(d, s_{l-k}) = P(d)E[\min(d+\epsilon_t, s_{l-k})]$ and

$$\phi_t(\boldsymbol{s}, s_l, d, a|\epsilon_t) = -(1-\gamma)cs_l - \gamma ca - \theta(a-d_t) - h^+(s_{l-k}-a)^+ - (h^- - \gamma c)(a-s_{l-k})^+ + \gamma f_{t+1}(\widetilde{\boldsymbol{s}}),$$

where

$$\widetilde{\boldsymbol{s}} = [(s_2-a)^+, ..., (s_{l-k}-a)^+, s_{l-k+1} - a \wedge s_{l-k}, ..., s_l - a \wedge s_{l-k}].$$

Note that the analysis for the lost-sales case with zero lead time relies on the monotonicity of $f_t(\boldsymbol{s})$. However, when the lead time is positive ($k \ge 1$), the partial monotone structure of the profit function under the dynamics (3) is not sufficient to ensure the preservation of $L^\natural$-concavity. The application of Lemma 5 requires that the underlying function to be monotone in all its components. However, in the lost-sales case with positive lead time, the profit function f_t may not always be monotone in all its components. To show that $L^\natural$-concavity can be preserved, we need to impose the following sufficient condition.

(C3) $h^+ + h^- - \gamma c \geq \gamma(h^- - \gamma c + \bar{p})$.

Condition (C3) states that the inventory cost incurring in the current period is greater than the maximum potential benefit of carrying the inventory to the next period while holding on-hand inventory and facing unmet demand simultaneously. This condition implies that it is always more beneficial to meet the demand to the maximum extent. Note that the left-hand side of above inequality does not take into account of the revenue (price) impact of inventory whereas the right-hand side relaxes the price level to its upper bound, which implies that this condition can be potentially further relaxed.

The next theorem shows that under conditions (C1)-(C3) the desired structural properties hold.

Theorem EC.1 (Monotonicity Properties of Optimal Policies). *Suppose (C1)-(C3) hold and $k \geq 1$. For $t = 1, ..., T$, the functions $f_t(\boldsymbol{s})$, $g_t(\boldsymbol{s}, s_l, d)$ and $\phi_t(\boldsymbol{s}, s_l, d, a|\epsilon_t)$ are $L^\natural$-concave in $\boldsymbol{s}$, $(\boldsymbol{s}, s_l, d)$ and $(\boldsymbol{s}, s_l, d, a)$, respectively. The joint pricing, inventory replenishment and depletion policy has the same monotonicity properties as shown in Theorem 1.*

Proof. The proof is by induction as the proof of Theorem 1. It suffices to show that $\phi_t(\boldsymbol{s}, s_l, d, a|\epsilon_t)$ is $L^\natural$-concave in $(\boldsymbol{s}, s_l, d, a)$ if f_{t+1} is $L^\natural$-concave.

For any $\delta > 0, u > 0$ and $u + \delta \leq a \wedge s_{l-k}$, comparing the system starting with state $\tilde{\boldsymbol{s}} = ((s_2 - u)^+, ..., (s_{l-k} - u)^+, s_{l-k+1} - u, ..., s_l - u)$ and the system with state $\tilde{\boldsymbol{s}}' = ((s_2 - u - \delta)^+, ..., (s_{l-k} - u - \delta)^+, s_{l-k+1} - u - \delta, ..., s_l - u - \delta)$ at the beginning of period $t+1$, the latter has δ units less on-hand inventory. Then, compared to the first system, the second system has at most δ units of shortage in the following $l - k - 1$ periods, incurring at most δ units of lost sales, which implies

$$f_{t+1}(\boldsymbol{s}') - f_{t+1}(\boldsymbol{s}) \geq -(h^- - \gamma c + \bar{p})\delta.$$

Define $\psi_t(\boldsymbol{s}, s_l, u) = (h^+ + h^- - \gamma c)u + \gamma f_{t+1}(\tilde{\boldsymbol{s}})$. We have

$$\psi_t(\boldsymbol{s}, s_l, u + \delta) - \psi_t(\boldsymbol{s}, s_l, u) \geq \delta[(h^+ + h^- - \gamma c) - \gamma(h^- - \gamma c + \bar{p})] \geq 0,$$

where the first inequality is from above analysis and the second is from (C3). That is, $\psi_t(\boldsymbol{s}, s_l, u)$ is monotone increasing in u, which implies that

$$\psi_t(\boldsymbol{s}, s_l, a \wedge s_{l-k}) = \max_{u \leq a \wedge s_{l-k}} \psi_t(\boldsymbol{s}, s_l, u).$$

By Lemma 6, we know that $f_{t+1}(\boldsymbol{s})$ is nonincreasing in $(s_1, ..., s_{l-k-1})$. Then, by Lemma 4, we know that $f_{t+1}(\tilde{\boldsymbol{s}})$ is $L^\natural$-concave in $(\boldsymbol{s}, s_l, u)$. Clearly, $\phi_t(\boldsymbol{s}, s_l, u)$ is also $L^\natural$-concave in $(\boldsymbol{s}, s_l, u)$.

Note that the constraint set $\{u : 0 \le u \le a \wedge s_{l-k}\}$ forms a lattice. By Lemma 2, we know that $\phi_t(\boldsymbol{s}, a \wedge s_{l-k})$ is $L^\natural$-concave in $(\boldsymbol{s}, s_l, a)$.

Note that $(s_i - a)^+ = (s_i - s_{l-k} \wedge a)^+$ for all $i < l-k$. The dynamics of the system state can be expressed as

$$\widetilde{\boldsymbol{s}} = [(s_2 - s_{l-k} \wedge a)^+, ..., (s_{l-k} - s_{l-k} \wedge a)^+, s_{l-k+1} - s_{l-k} \wedge a, ..., s_l - s_{l-k} \wedge a],$$

Then, for any $a \le s_{l-k} \wedge d_t$,

$$\begin{aligned} &\phi_t(\boldsymbol{s}, s_l, d, a|\epsilon_t) \\ &= -(1-\gamma)cs_l - \gamma ca - \theta(a-d_t)^+ - h^+(s_{l-k} - a)^+ - (h^- - \gamma c)(a - s_{l-k})^+ + \gamma f_{t+1}(\widetilde{\boldsymbol{s}}) \\ &= -(1-\gamma)cs_l - \gamma ca - \theta(a-d_t)^+ - h^+[s_{l-k} - a \wedge s_{l-k}] - (h^- - \gamma c)[a - a \wedge s_{l-k}] + \gamma f_{t+1}(\widetilde{\boldsymbol{s}}) \\ &= -(1-\gamma)cs_l - \gamma ca - \theta(a-d_t)^+ - h^+ s_{l-k} - (h^- - \gamma c)a + \psi_t(\boldsymbol{s}, a \wedge s_{l-k}). \end{aligned}$$

Clearly, all the terms of the right-hand side of last equation are $L^\natural$-concave. Thus, $\phi_t(\boldsymbol{s}, s_l, d, a|\epsilon_t)$ is $L^\natural$-concave in $(\boldsymbol{s}, s_l, a)$. It is clear that f_t is $L^\natural$-concave under conditions (C1) and (C2). By induction, the desired structural results characterized in Theorem 1 hold. Q.E.D.

Theorem EC.1 shows that the desired structural properties could still hold in the lost-sales case with positive lead time. Its proof shows that (C3) supplies some monotonicity to a transformed profit-to-go functions, in addition to the partial monotonicity of the original profit-to-go function, which allows us to apply Lemma 4.

Nevertheless, one may also be interested in applying Lemma 5, given the fact that the profit-to-go function of the starting state of next period, $f_{t+1}(\widetilde{\boldsymbol{s}})$, has the same form as that of the function in Lemma 5. To this end, we need to impose new conditions under which some transformation of the profit-to-go function is monotone. Note that the key tradeoff is between the lost-sales cost and revenue in the future. It is possible to construct a transformed profit function with the lost-sales cost when the lost-sales cost is high. We now present the idea without providing the proof: Assume that $(h^- - \gamma c)\rho \ge \gamma\theta + \bar{p}$, where $\rho = \frac{1-\gamma}{\gamma(1-\gamma^k)}$. The transformed profit-go-to function $\hat{f}_t(\boldsymbol{s}) = f_t(\boldsymbol{s}) - \rho(h^- - \gamma c)[s_{l-k} + \gamma s_{l-k+1} + ... + \gamma^{k-1} s_{l-1}]$ is monotone decreasing in $\boldsymbol{s}$. By Lemma 5, we can show that $L^\natural$-concavity can be preserved.

Remark EC.1 (Lost-Sales Inventory Models with Exogenous Price). When the price p is exogenously given and constant over time, the model can be reduced to the standard lost-sales perishable inventory model with the equivalent lost-sales cost $h^- + p$. Then, condition (C3) is replaced by $(h^+ + h^- + p - \gamma c) \ge \gamma(h^- + p - \gamma c)$, which holds automatically. Thus, our structural analysis applies directly to the lost-sales perishable inventory model with positive leatimes, which implies that our model generalizes Nahmias' (2011) model to the case with positive lead time.

Remark EC.2 (Non-Stationary Lost-Sales Systems). Similar to the non-stationary backlogging systems, our analysis can be easily extended to the non-stationary case under the conditions $\theta_t \ge \gamma\theta_{t+1}$ and $h_t^+ + h_t^- - \gamma c_{t+1} \ge \gamma(h_{t+1}^- - \gamma c_{t+2} + \bar{p}_{t+1})$.

EC.2.2. Random Lifetime

We next extend our analysis to the case with random lifetime. For notational convenience, we restrict our attention to the backlogging case with zero lead time.

As summarized by Nahmias (1977), the useful lifetime of many products (e.g., fresh produce, meat, fowl, and fish) cannot be predicted in advance. Following Nahmias (1977), we assume that the inventories outdate in the same order in which they enter the system. For each period t, let K_t be a nonnegative integer random variable defined on the set $\{1,2,...,l\}$. Assume that $K_1,...,K_T$ are independent and identeically distributed. For a realization of K_t in period t, all on-hand inventory that is at least K_t periods old at the end of period t will expire. Let $\pi_i = Pr(K_t = i)$. The dynamics of the system state depend on the realization of K_t, denoted by $\tilde{\boldsymbol{s}}^{(i)}$. Let $a^{(i)}$ be the inventory depletion level when $K_t = i$ such that $a^{(i)} \in [s_{l-i+1} \vee d_t, s_l \vee d_t]$. Then,

$$\begin{aligned}
\tilde{\boldsymbol{s}}^{(l)} &= (s_2 - a^{(l)}, s_3 - a^{(l)}, ..., s_{l-1} - a^{(l)}, s_l - a^{(l)}),\\
\tilde{\boldsymbol{s}}^{(l-1)} &= (0, s_3 - a^{(l-1)}, ..., s_{l-1} - a^{(l-1)}, s_l - a^{(l-1)}),\\
&\vdots\\
\tilde{\boldsymbol{s}}^{(1)} &= (0,0,...,0,0).
\end{aligned}$$

Then, letting $f_{T+1}(\boldsymbol{s}) = 0$, the optimal profit-go-to function f_t satisfies

$$f_t(\boldsymbol{s}) = \max_{s_l \geq s_{l-1}, d\in\mathcal{D}_t} R(d, s_l) + \mathbb{E}\left[g_t(\boldsymbol{s}, s_l, d|\epsilon_t)\right],$$

where

$$\begin{aligned}
g_t(\boldsymbol{s}, s_l, d|\epsilon_t) &= \pi_l \max_{s_1\vee d_t \leq a^{(l)} \leq s_l \vee d_t} \{-v_t(s_l, a^{(l)}, d_t) + \gamma f_{t+1}(\tilde{\boldsymbol{s}}^{(l)})\}\\
&+ \pi_{l-1} \max_{s_2\vee d_t \leq a^{(l-1)} \leq s_l \vee d_t} \{-v_t(s_l, a^{(l-1)}, d_t) + \gamma f_{t+1}(\tilde{\boldsymbol{s}}^{(l-1)})\}\\
&\vdots\\
&+ \pi_1[-v_t(s_l, s_l \vee d_t, d_t) + \gamma f_{t+1}(0,0,...,0,0)],
\end{aligned}$$

and

$$v_t(s_l, a, d_t) = (1-\gamma)cs_l + \gamma ca + \theta(a - d_t) + h^+(s_l - a)^+ + h^-(a - s_l)^+.$$

Analogous to the analysis in the proof of Theorem 1, one can show that f_t is nondecreasing for $a^{(i)} \geq s_l \vee d_t$, the constraints $a^{(i)} \leq s_l \vee d_t$ are redundant, and the $L^\natural$-concavity is preserved.

EC.2.3. Inventory Rationing with Multiple Demand Classes

Parallel to the pricing management, inventory rationing in the presence of multiple demand classes is also an important strategy to leverage the supply and demand. Consider a lost-sales inventory system with N classes of customers with different unit payments and lost-sales penalty costs.

Assume that the unit payments (prices) are fixed throughout the planning horizon. At the beginning of each period, an order is placed and at the end of each period, after observing the realized demands the system operator decides which customers' demands to fulfill. In addition to satisfying demands, we assume that the system operator can intentionally dispose of some aged inventory to reduce the holding cost. The replenishment lead time of each order is of k periods.

Let n index the demand class. Let p_n be the unit payment, h_n^- be the unit penalty cost for class n, and d_{nt} be the demand of class n in period t. Let $\boldsymbol{d}_t = (d_{1t}, ..., d_{Nt})$. These demands could be correlated in each period but they are independent across periods. Without loss of generality, we assume that $p_1 + h_1^- > p_2 + h_2^- > ... > p_n + h_n^-$, which implies that the class-1 demand has the highest priority, then the class-2, and so on. Denote by a_n the amount of inventory allocated to class n's demands, a_0 the amount of additional inventory to be disposed of and $a = \sum_{n=0}^{N} a_n$. Note that $0 \le a_n \le d_{nt}$ and $s_1 \le a \le s_{l-k}$, the dynamics of the inventory are expressed as

$$\widetilde{\boldsymbol{s}} = ((s_2 - a)^+, ..., (s_{l-k} - a)^+, s_{l-k+1} - a, ..., s_l - a).$$

Letting $f_{T+1}(\boldsymbol{s}) = 0$, the optimality equation can be expressed as:

$$f_t(\boldsymbol{s}) = \max_{s_l \ge s_{l-1}} E[g_t(\boldsymbol{s}, s_l | \boldsymbol{d}_t)], \tag{EC.1}$$

where

$$g_t(\boldsymbol{s}, s_l | \boldsymbol{d}_t)$$
$$= \max_{0 \le a_n \le d_{nt}, s_1 \le \sum_{n=0}^{N} a_n \le s_{l-k}} \left\{ \tilde{R}(a_0, a_1, ..., a_N | \boldsymbol{d}_t) - (1-\gamma) c s_l - \gamma c \sum_{n=0}^{N} a_n - h^+ (s_{l-k} - \sum_{n=0}^{N} a_n) + \gamma f_{t+1}(\widetilde{\boldsymbol{s}}) \right\},$$

where

$$\tilde{R}(a_0, a_1, ..., a_N | \boldsymbol{d}_t) = \sum_{n=1}^{N} p_n a_n - \sum_{n=1}^{N} h_n^- (d_{nt} - a_n) - \theta a_0.$$

Clearly, for any given total amount allocation a, it is always optimal to meet the demands to the maximum extent from the highest priority to the lowest priority and the payoff function $\tilde{R}(a_1, ..., a_N | \boldsymbol{d}_t)$ is entirely determined by a (see, e.g., Zipkin (2008) for a similar treatment). Let $R(a | \boldsymbol{d}_t) = \max\{\tilde{R}(a_0, a_1, ..., a_N | \boldsymbol{d}_t) | 0 \le a_n \le d_{nt}, \sum_{n=0}^{N} a_n = a\}$. It is clear that $R(a | \boldsymbol{d}_t)$ is concave in a. Then, we can represent g_t as

$$g_t(\boldsymbol{s}, s_l | \boldsymbol{d}_t) = \max_{s_1 \le a \le s_{l-k}} \{ R(a | \boldsymbol{d}_t) - (1-\gamma) c s_l - \gamma c a - h^+ (s_{l-k} - a) + \gamma f_{t+1}(\widetilde{\boldsymbol{s}}) \}.$$

Applying the previous analysis, we can show by induction that f_t is nonincreasing in $(s_1, ..., s_{l-k-1})$. By Lemma 4, $f_{t+1}(\widetilde{\mathbf{s}})$ is $L^\natural$-concave. Similar to preceding analysis, we can show that $f_t(\mathbf{s})$ is also $L^\natural$-concave. Hence the optimal inventory replenishment, rationing and disposal policy has a similar structure to that characterized in Theorem 1.

Remark EC.3 (Backlogging Model with Multiple Demand Classes). Our analysis can also be easily extended to the backlogging case with N demand classes. Let b_n denote the number of class-n backorders and h_n^- denote the unit backlogging cost per period. Assume that $h_1^- > .. > h_N^-$. Define $\mathbf{z} = (z_1, ..., z_N)$ where $z_n = b_1 + ... + b_n$ represents the partial sum of backorders from class 1 to class n, $n = 1, ..., N$. Then the system state can be represented by $(\mathbf{z}, \mathbf{s})$. Again, let a be the total amount of inventory to be depleted such that $s_1 \le a \le s_{l-k}$. Let $\hat{d}_{nt} = d_{1t} + ... + d_{nt}$ be the partial sum of demands from class 1 to class n. The dynamics of the system state can be expressed as $(\tilde{\mathbf{z}}, \tilde{\mathbf{s}}) = ((z_1 + \hat{d}_{1t} - a)^+, ..., (z_N + \hat{d}_{Nt} - a)^+, s_2 - a, ..., s_l - a)$. Let $f_{T+1}(\tilde{\mathbf{z}}, \tilde{\mathbf{s}}) = 0$. The optimal value functions satisfy the following optimality equations.

$$f_t(\mathbf{z}, \mathbf{s}) = \max_{s_l \ge s_{l-1}} E[g_t(\mathbf{z}, \mathbf{s}, s_l | \mathbf{d_t})],$$

where

$$g_t(\mathbf{z}, \mathbf{s}, s_l | \mathbf{d}_t) = \max_{s_1 \le a \le s_{l-k}} \{R(a | \mathbf{d}_t, \mathbf{z}_t) - (1-\gamma) c s_l - \gamma c a - h^+ (s_{l-k} - a)^+ + \gamma f_{t+1}(\tilde{\mathbf{z}}, \tilde{\mathbf{s}})\},$$

with $R(a|\mathbf{d}_t, \mathbf{z}_t) = \sum_{n=1}^N p_n d_{nt} - \sum_{n=1}^N (h_n^- - h_{n+1}^-)(z_n + \hat{d}_{nt} - a)^+ - \theta(a - z_N - \hat{d}_{Nt})^+$ and $h_{N+1}^- = 0$. Clearly, $R(a, \mathbf{z}_t | \mathbf{d}_t)$ is $L^\natural$-concave in $(a, \mathbf{z}_t)$, and f_t is decreasing in $z_n, n = 1, ..., N$. By Lemma 4, $f_{t+1}(\tilde{\mathbf{z}}, \tilde{\mathbf{s}})$ is $L^\natural$-concave. Similar to preceding analysis, we can obtain the desired results.

EC.3. Tables of Numerical Study

References

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Table EC.1 Long-Run Average Profits Per Period

id	l	$c.v.$	$\frac{h^-}{h^++h^-}$	θ	V^*	ρ^{FP}	d^{FP}	ρ^{H1}	(d^{H1}, s_l^{H1})	ρ^{H2}	(d^{H2}, s_l^{H2})
1	2	1	0.98	10	846.13	1.06%	58	1.39%	(59, 69)	1.39%	(59, 69)
2	–	0.6	–	–	899.46	0.68%	57	0.85%	(58, 73)	0.85%	(58, 73)
3	–	0.8	–	–	868.43	0.92%	58	1.25%	(59, 71)	1.16%	(58, 70)
4	–	1.2	–	–	830.30	1.15%	58	1.52%	(59, 67)	1.52%	(59, 67)
5	–	1.5	–	–	814.31	1.21%	58	1.62%	(59, 65)	1.62%	(59, 65)
6	–	–	0.90	–	926.19	0.24%	55	0.28%	(55, 55)	0.28%	(55, 55)
7	–	–	0.95	–	899.38	0.51%	56	0.64%	(57, 55)	0.60%	(56, 54)
8	–	–	0.99	–	790.80	1.58%	60	2.41%	(61, 80)	2.23%	(61, 81)
9	–	–	–	5	851.49	0.85%	58	1.31%	(59, 70)	1.21%	(58, 69)
10	–	–	–	20	838.05	1.45%	59	1.76%	(59, 66)	1.76%	(59, 66)
11	–	1.5	0.99	20	717.00	2.78%	62	3.82%	(63, 77)	3.82%	(63, 77)
12	3	1	0.98	10	921.66	0.63%	57	0.74%	(57, 92)	0.74%	(57, 92)
13	–	0.6	–	–	945.22	0.20%	55	0.22%	(55, 90)	0.22%	(55, 90)
14	–	0.8	–	–	933.05	0.44%	56	0.56%	(57, 92)	0.53%	(57, 93)
15	–	1.2	–	–	912.09	0.76%	57	0.98%	(58, 92)	0.98%	(58, 92)
16	–	1.5	–	–	901.01	0.90%	58	1.12%	(58, 91)	1.12%	(58, 91)
17	–	–	0.90	–	945.60	0.15%	55	0.17%	(55, 71)	0.17%	(55, 71)
18	–	–	0.95	–	936.97	0.32%	56	0.37%	(56, 80)	0.37%	(56, 80)
19	–	–	0.99	–	906.71	0.94%	58	1.26%	(59, 102)	1.19%	(59, 103)
20	–	–	–	5	922.92	0.53%	57	0.65%	(57, 93)	0.65%	(57, 93)
21	–	–	–	20	919.77	0.83%	57	1.00%	(58, 91)	1.00%	(58, 91)
22	–	1.5	0.99	20	869.66	1.87%	60	2.32%	(61, 103)	2.32%	(61, 103)
23	4	1	0.98	10	940.93	0.21%	55	0.20%	(55, 107)	0.20%	(55, 109)
24	–	0.6	–	–	950.80	0.01%	54	0.01%	(54, 101)	0.01%	(54, 101)
25	–	0.8	–	–	946.04	0.10%	55	0.11%	(55, 107)	0.11%	(55, 107)
26	–	1.2	–	–	936.03	0.32%	56	0.38%	(56, 110)	0.38%	(56, 110)
27	–	1.5	–	–	929.69	0.43%	56	0.54%	(57, 113)	0.54%	(57, 113)
28	–	–	0.90	–	950.36	0.03%	54	0.03%	(54, 87)	0.03%	(54, 87)
29	–	–	0.95	–	946.64	0.09%	54	0.10%	(55, 98)	0.10%	(55, 98)
30	–	–	0.99	–	935.80	0.34%	56	0.40%	(56, 117)	0.38%	(56, 118)
31	–	–	–	5	941.18	0.17%	55	0.20%	(55, 109)	0.20%	(55, 109)
32	–	–	–	20	940.55	0.28%	59	0.33%	(56, 108)	0.32%	(56, 109)
33	–	1.5	0.99	20	917.86	0.93%	57	1.09%	(58, 121)	1.09%	(58, 121)

Table EC.2 Average Disposal Costs Per Period

id	l	$c.v.$	$\frac{h^-}{h^++h^-}$	θ	DC^*	δ^*	DC^{FP}	δ^{FP}	DC^{H1}	δ^{H1}	DC^{H2}	δ^{H2}
1	2	1	0.98	10	9.67	1.14%	13.24	1.58%	9.65	1.16%	9.65	1.16%
2	–	0.6	–	–	6.60	0.73%	8.85	0.99%	7.04	0.79%	7.04	0.79%
3	–	0.8	–	–	8.78	1.01%	11.66	1.36%	8.50	0.99%	9.23	1.08%
4	–	1.2	–	–	10.16	1.22%	13.71	1.67%	9.80	1.20%	9.80	1.20%
5	–	1.5	–	–	8.78	1.01%	11.66	1.36%	8.50	0.99%	9.23	1.08%
6	–	–	0.90	–	1.50	0.16%	2.23	0.24%	1.57	0.17%	1.57	0.17%
7	–	–	0.95	–	3.64	0.40%	5.33	0.60%	3.59	0.40%	4.04	0.45%
8	–	–	0.99	–	17.74	2.24%	22.95	2.95%	16.24	2.10%	17.37	2.24%
9	–	–	–	5	5.99	0.70%	7.68	0.91%	5.23	0.62%	5.66	1.67%
10	–	–	–	20	13.62	1.63%	19.39	2.35%	14.87	1.81%	14.87	1.81%
11	–	1.5	0.99	20	28.18	3.93%	39.64	5.69%	27.10	3.93%	27.10	3.93%
12	3	1	0.98	10	2.30	0.25%	4.14	0.45%	3.26	0.36%	3.26	0.36%
13	–	0.6	–	–	0.81	0.09%	1.53	0.16%	1.35	0.14%	1.35	0.14%
14	–	0.8	–	–	1.63	0.17%	3.14	0.34%	2.11	0.23%	2.31	0.25%
15	–	1.2	–	–	2.73	0.30%	5.04	0.56%	3.31	0.37%	3.31	0.37%
16	–	1.5	–	–	3.07	0.34%	5.53	0.62%	3.89	0.44%	3.89	0.44%
17	–	–	0.90	–	0.47	0.05%	0.86	0.09%	0.72	0.08%	0.72	0.08%
18	–	–	0.95	–	1.05	0.11%	1.86	0.20%	1.42	0.15%	1.42	0.15%
19	–	–	0.99	–	3.71	0.41%	6.80	0.76%	4.45	0.50%	4.79	0.53%
20	–	–	–	5	1.42	0.15%	2.38	0.26%	1.77	0.19%	1.77	0.19%
21	–	–	–	20	3.11	0.34%	6.97	0.76%	4.62	0.51%	4.62	0.51%
22	–	1.5	0.99	20	7.30	0.84%	14.87	1.74%	9.86	1.16%	9.86	1.16%
23	4	1	0.98	10	0.45	0.05%	1.14	0.12%	0.83	0.09%	0.91	0.10%
24	–	0.6	–	–	0.05	0.01%	0.11	0.01%	0.11	0.01%	0.11	0.01%
25	–	0.8	–	–	0.23	0.03%	0.52	0.05%	0.46	0.05%	0.46	0.05%
26	–	1.2	–	–	0.65	0.07%	1.50	0.16%	1.10	0.12%	1.10	0.12%
27	–	1.5	–	–	0.86	0.09%	2.11	0.23%	1.39	0.15%	1.39	0.15%
28	–	–	0.90	–	0.07	0.01%	0.17	0.02%	0.15	0.02%	0.15	0.02%
29	–	–	0.95	–	0.18	0.02%	0.49	0.05%	0.34	0.04%	0.34	0.04%
30	–	–	0.99	–	0.75	0.08%	1.74	0.19%	1.38	0.15%	1.49	0.16%
31	–	–	–	5	0.27	0.03%	0.62	0.07%	0.50	0.05%	0.50	0.05%
32	–	–	–	20	0.63	0.07%	1.94	0.21%	1.27	0.14%	1.39	0.15%
33	–	1.5	0.99	20	1.99	0.22%	6.04	0.66%	3.84	0.42%	3.84	0.42%

Table EC.3 Finite-Horizon Models with Non-Stationary Demand

id	l	$c.v.$	$\frac{h^-}{h^++h^-}$	θ	$\hat{f}_1(\mathbf{0})$	DC^*	ρ^{FP}	DC^{FP}	ρ^{H1}	DC^{H1}	ρ_I^{FP}	ρ_{II}^{FP}
1	2	1	0.98	10	4030.01	36.35	7.06%	52.83	5.25%	11.91	2.88%	1.14%
2	–	0.6	–	–	4254.17	26.84	6.32%	39.32	13.10%	2.29	2.37%	0.65%
3	–	0.8	–	–	4120.29	34.40	6.79%	48.46	10.21%	4.86	2.69%	0.96%
4	–	1.2	–	–	3968.62	37.51	7.24%	54.21	0.92%	34.57	2.99%	1.26%
5	–	1.5	–	–	3908.39	36.76	7.39%	53.83	0.35%	60.80	3.09%	1.35%
6	–	–	0.90	–	4580.56	1.13	4.77%	1.52	0.44%	1.41	1.34%	0.02%
7	–	–	0.95	–	4385.61	8.73	5.36%	13.07	1.81%	4.55	1.78%	0.26%
8	–	–	0.99	–	3671.74	77.92	9.26%	107.76	8.72%	28.19	4.29%	2.43%
9	–	–	–	5	4050.30	22.43	6.82%	30.31	4.75%	6.83	2.67%	0.94%
10	–	–	–	20	399.49	51.30	7.51%	81.66	6.27%	19.48	3.26%	1.54%
11	–	1.5	0.99	20	3352.01	122.58	11.40%	186.14	2.81%	276.49	5.84%	4.27%
12	3	1	0.98	10	4275.98	7.95	6.37%	19.03	8.10%	0.08	2.10%	0.26%
13	–	0.6	–	–	4419.89	3.71	5.49%	11.30	12.58%	0.01	1.55%	0.04%
14	–	0.8	–	–	4338.89	6.52	6.05%	18.67	11.56%	0.03	1.86%	0.15%
15	–	1.2	–	–	4228.68	8.91	6.57%	21.50	4.15%	0.39	2.27%	0.36%
16	–	1.5	–	–	4178.09	9.45	6.67%	23.45	2.65%	0.64	2.38%	0.45%
17	–	–	0.90	–	4591.46	0.13	4.76%	0.15	0.29%	0.00	1.32%	0.00%
18	–	–	0.95	–	4460.42	1.78	5.11%	4.58	2.33%	0.02	1.45%	0.03%
19	–	–	0.99	–	4112.85	16.89	7.70%	43.24	14.34%	0.32	2.75%	0.65%
20	–	–	–	5	4280.44	5.07	6.23%	10.51	7.47%	0.05	1.98%	0.21%
21	–	–	–	20	4269.36	10.96	6.62%	31.10	9.02%	0.15	2.30%	0.37%
22	–	1.5	0.99	20	3918.44	29.29	9.52%	78.71	3.73%	7.90	3.92%	1.57%
23	4	1	0.98	10	4340.39	1.22	5.76%	8.24	7.70%	0.00	1.54%	0.01%
24	–	0.6	–	–	4445.33	0.25	5.09%	2.21	9.54%	0.00	1.37%	0.00%
25	–	0.8	–	–	4387.17	0.77	5.46%	5.63	9.56%	0.00	1.45%	0.00%
26	–	1.2	–	–	4303.57	1.58	5.98%	9.87	5.29%	0.00	1.62%	0.02%
27	–	1.5	–	–	4262.57	1.90	6.21%	11.23	4.54%	0.00	1.73%	0.04%
28	–	–	0.90	–	4592.30	0.00	4.76%	0.00	0.18%	0.00	1.32%	0.00%
29	–	–	0.95	–	4475.38	0.14	4.96%	1.03	1.95%	0.00	1.36%	0.00%
30	–	–	0.99	–	4237.33	3.01	6.77%	15.33	14.02%	0.00	1.86%	0.05%
31	–	–	–	5	4341.07	0.77	5.67%	4.57	7.29%	0.00	1.51%	0.01%
32	–	–	–	20	4339.40	1.62	5.91%	13.87	8.36%	0.00	1.59%	0.02%
33	–	1.5	0.99	20	4107.71	6.06	8.07%	33.36	7.73%	0.00	2.56%	0.26%

Table EC.4 Average Costs and Disposal Costs Per Period for Cost-Minimization Problems

id	l	$c.v.$	$\frac{h^-}{h^+ + h^-}$	θ	C^{FP}	DC^{FP}	ρ^{H1}	DC^{H1}	s_l^{H1}	ρ^{H2}	DC^{H2}	s_l^{H2}
1	2	1	0.98	10	1329.39	14.62	0.24%	11.32	61	0.24%	11.32	61
2	–	0.6	–	–	1270.30	9.79	0.12%	8.34	67	0.12%	8.34	67
3	–	0.8	–	–	1305.02	13.49	0.24%	10.00	63	0.17%	10.81	64
4	–	1.2	–	–	1346.42	15.74	0.34%	10.64	58	0.26%	11.53	59
5	–	1.5	–	–	1363.36	15.98	0.27%	11.58	57	0.27%	11.58	57
6	–	–	0.90	–	1236.55	2.25	0.02%	1.84	44	0.02%	1.84	44
7	–	–	0.95	–	1267.01	5.44	0.09%	4.04	50	0.09%	4.04	50
8	–	–	0.99	–	1395.38	27.52	0.61%	19.72	69	0.48%	20.97	70
9	–	–	–	5	1321.39	8.80	0.32%	6.11	62	0.25%	6.58	63
10	–	–	–	20	1342.43	22.94	0.22%	17.74	57	0.22%	17.74	57
11	–	1.5	0.99	20	1484.95	49.12	0.71%	33.67	62	0.57%	36.04	63
12	3	1	0.98	10	1247.11	5.20	0.13%	3.82	85	0.13%	3.82	85
13	–	0.6	–	–	1217.16	1.70	0.02%	1.48	88	0.02%	1.48	88
14	–	0.8	–	–	1232.76	3.70	0.06%	2.97	87	0.06%	2.97	87
15	–	1.2	–	–	1258.90	6.24	0.21%	4.22	83	0.16%	4.55	84
16	–	1.5	–	–	1272.20	7.27	0.22%	4.93	82	0.17%	5.31	83
17	–	–	0.90	–	1216.03	0.97	0.01%	0.81	69	0.01%	0.81	69
18	–	–	0.95	–	1226.78	2.25	0.06%	1.58	75	0.04%	1.74	76
19	–	–	0.99	–	1267.95	8.91	0.29%	6.32	92	0.22%	6.75	93
20	–	–	–	5	1244.36	2.97	0.15%	2.06	86	0.11%	2.23	87
21	–	–	–	20	1251.81	8.42	0.16%	6.00	82	0.12%	6.52	83
22	–	1.5	0.99	20	1320.79	21.60	0.51%	14.06	87	0.42%	15.02	88
23	4	1	0.98	10	1221.47	1.34	0.03%	0.99	105	0.03%	0.99	105
24	–	0.6	–	–	1209.33	0.11	0.00%	0.11	101	0.00%	0.11	101
25	–	0.8	–	–	1214.96	0.63	0.01%	0.50	104	0.01%	0.50	104
26	–	1.2	–	–	1227.78	1.92	0.06%	1.41	105	0.04%	1.52	106
27	–	1.5	–	–	1235.90	2.63	0.08%	1.92	105	0.08%	1.92	105
28	–	–	0.90	–	1209.91	0.17	0.00%	0.17	88	0.00%	0.17	88
29	–	–	0.95	–	1214.22	0.49	0.01%	0.38	95	0.01%	0.43	96
30	–	–	0.99	–	1228.51	2.30	0.05%	1.87	113	0.05%	1.87	113
31	–	–	–	5	1220.78	0.72	0.03%	0.54	106	0.02%	0.59	107
32	–	–	–	20	1222.67	2.28	0.03%	1.82	104	0.03%	1.82	104
33	–	1.5	0.99	20	1255.48	8.03	0.20%	5.51	110	0.16%	5.90	111