Operational Hedging through Dual-Sourcing under Capacity Uncertainty

Suggested Citation: Xin Chen (2017), "Operational Hedging through Dual-Sourcing under Capacity Uncertainty", : Vol. xx, No. xx, pp 1–16. DOI: 10.1561/XXXXXXXXX.

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Contents

Operational Hedging through Dual-Sourcing under Capacity Uncertainty

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ABSTRACT

This article studies a periodic-review dual-sourcing inventory system with demand and supply capacity uncertainty. Compared with dual-sourcing models with deterministic supply capacities, a challenge here is that the objective functions in the dynamic programming recursions are not convex in the ordering quantities. To address this challenge, a powerful transformation technique is developed which converts a non-convex minimization problem to an equivalent convex minimization problem. In addition, the transformation preserves $L^\natural$-convexity, allowing one to derive structural properties of the optimal policies for the dual-sourcing problem with random supply capacity. Dual-index policies are shown to be optimal under certain conditions. The effects of supply capacity uncertainties on the system performance are investigated.

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1.1 Motivation and description of the problem

Matching supply and demand is at the core of supply chain management. It becomes increasingly complicated and challenging as firms extend their supply chains at a global scale. One major concern for many multi-national firms with global supply networks is supply uncertainty, especially supply disruption. It may be caused by natural disasters (e.g., earthquake or Hurricane Sandy) or may be due to unreliability in firms' operational processes such as procurement, production and delivery, and lead to significant loss. For example, the 2011 Great Tohoku Earthquake and Tsunami caused a significant decline of global auto production in 2011 and many auto makers suffered severe financial losses (Canis, 2011, Robinet, 2011).

Such crisis has spurred the automobile firms to rethink their supply chain risk management strategies, in particular, the sourcing strategies, to build more resilient supply chains. Dual sourcing or multiple sourcing, as a natural operational hedging strategy to mitigate supply risks by diversifying supply sources, has become a common practice in supply chain management (cf. Canis, 2011, Kelleher, 2003, Van Mieghem, 2008, Rao *et al.*, 2000, Veeraraghavan and Scheller-Wolf, 2008).

Another important driving force for geographic dual sourcing is the tradeoff between cost and responsiveness. Van Mieghem (2008) provides a case study that introduces the dual-sourcing strategy of a $10 billion high-tech U.S. manufacturer of wireless transmission components with two assembly plants, one in China and the other in Mexico. Although the facility in China enjoyed lower costs, ocean transportation made its order lead times 5 to 10 times longer than those from Mexico. With highly volatile demand, the sole sourcing was unattractive: the plant in China was too unresponsive and the plant in Mexico was too expensive. The firm had to optimize the dual-sourcing strategy to better match supply and demand.

The above business practices indicate two distinctive characteristics of the operational environments of dual-sourcing firms: 1). supply uncertainty, 2). replenishment lead times. As far as we know, it remains an open question of how to optimize the dual-sourcing ordering decisions dynamically in such complex operational environments. Our research aims to address this issue and to provide insights on how the two factors drive the sourcing decisions.

More specifically, Chen and Pang (2014) consider a dual-sourcing problem in which a firm replenishes from two suppliers both with random capacities: an expedited supplier (or domestic source) with a higher ordering cost but shorter lead time and a regular supplier (or offshore plant) with a lower ordering cost but longer lead time. The firm's decision is to determine its inventory sourcing policy to satisfy volatile demand over time. The objective is to minimize the total expected discounted operating cost over a finite horizon. This article provides an overview of the model, methodology and results in Chen and Pang (2014).

1.2 Modeling approach and methodology

1.2.1 Dual-sourcing Model Setup

Consider a firm managing a T-period periodic-review inventory system in the presence of two capacitated suppliers in two geographically distant locations: a regular supplier (R) with a longer replenishment leadtime of l_r periods and a unit ordering cost c_r, and an expedited (emergency) supplier (E) with a shorter replenishment leadtime of l_e periods and a unit ordering cost c_e, where l_r and l_e are positive integers and $l_r > l_e$. There are no fixed ordering costs.

Both suppliers offer limited and uncertain capacities, denoted by $K_{e,t}$ and $K_{r,t}$, $t \in \{1, ..., T\}$, for expedited and regular suppliers, respectively. The processes $\{K_{r,t}\}_{t=1}^{T}$ and $\{K_{e,t}\}_{t=1}^{T}$ are both independent over time and independent of each other. Demands of successive periods, denoted by d_t for period t, are stochastic, independent overtime, and independent of the supply capacities. For convenience, let $d_{[t,t+l]}$ be the total demand from period t to period $t+l$, i.e., $d_{[t,t+l]} = d_t + ... + d_{t+l}$.

The sequence of events is as follows. At the beginning of period t, orders from the regular supplier l_r periods ago and the expedited supplier l_e periods ago (if $l_e \geq 1$) are received. (Note that if $l_e = 0$, we assume that an order from the expedited supplier is received right away.) The firm then reviews the inventory level and the orders outstanding, and determines how much to order from the two suppliers before observing the suppliers' capacities $K_{r,t}$ and $K_{e,t}$. Let q_r and q_e be the (target) order quantities from the regular and expedited channels, respectively. After the orders are placed, the suppliers capacities $K_{r,t}$ and $K_{e,t}$ are realized and the amounts of inventories shipped from the regular and expedited suppliers are $q_r \wedge K_{r,t}$ and $q_e \wedge K_{e,t}$, respectively. Here we assume that the supply capacity uncertainties are resolved in the same period when the orders are placed. This is reasonable when the capacity uncertainties are mainly driven by the unreliability of the production process and the production time is no more than the period length while the shipping time is long. At the end of this period, the demand is realized and met with on-hand inventory (if any). Unmet demand is fully backlogged with a unit shortage cost h^-. Excess inventory is carried over to the next period with a unit holding cost h^+.

The objective of the firm is to find a dual-sourcing strategy so as to minimize the total expected discounted cost, including ordering cost, holding cost and backorder cost, over the planning horizon. To present the dynamic programming model for deriving the optimal strategy, one can describe the system state right before the firm places orders by a vector $z = (z_1, ..., z_k)$ with $k = l_r - l_e + 1$, where z_i is the amount of on-hand net inventory plus outstanding orders that will arrive within $i + l_e$ periods. Note that in a backlogging model, since the orders of each period will have an influence at least l_e periods later, and the on-hand net inventory level l_e period later solely depends on z_l, it suffices to use the now standard accounting technique to discount the future inventory cost to the current period and focus on the pipeline inventory levels l_e periods later. The state space is given by

$$\mathcal{S} = \{(z_1, \ldots, z_k) : z_1 \leq z_2 \leq \ldots \leq z_k\}.$$

Given the system state z, the system state of the next period is

$$\tilde{z} = (z_2 + q_e \wedge K_{e,t} - d_t, ..., z_k + q_e \wedge K_{e,t} - d_t, y \wedge (z_k + K_{r,t}) + q_e \wedge K_{e,t} - d_t),$$

where $y = z_k + q_r$ is the (target) order-up-to level from regular channel. To simplify our presentation, we denote $w = -q_e$ and $\tilde{K}_{e,t} = -K_{e,t}$. The dynamics of the system state can be rewritten as

$$\tilde{z} = [(z_2, ..., z_k, y \wedge (z_k + K_{r,t})) - (w \vee \tilde{K}_{e,t} + d_t)e],$$

where e is the k-dimensional all-ones vector, $x \wedge y = \min(x, y)$ and $x \vee y = \max(x, y)$.

We are now ready to present the dynamic program to derive the firm's optimal strategy. Let $\alpha \in (0, 1]$ be the discount factor. The optimality equations can be written as follows. Denote $\phi_t(z)$ the cost-to-go function at period t with state z. For $t = 1, ..., T$,

$$\phi_t(z) = \min_{y \geq z_k, w \leq 0} \left\{ E[g_t(z, y \wedge (z_k + K_{r,t}), w \vee \tilde{K}_{e,t})] \right\} \ \forall \ z \in \mathcal{S}, \quad (1.1)$$

where

$$g_t(z, y, w) = c_r(y - z_k) - c_e w + B_t(z_1 - w) + \alpha E[\phi_{t+1}[(z_2, ..., z_k, y) - (d_t + w)e]], \quad (1.2)$$

and

$$B_t(x) = \alpha^{l_e} E[h^+(x - d_{[t,t+l_e]})^+ + h^-(d_{[t,t+l_e]} - x)^+].$$

Note that the expectation of the right hand side of equation (1.1) is taken over the random capacities. The function g_t represents the expected total discounted cost after the capacities are realized but before the demand is realized. The first term of the right hand side of equation (1.2) is the ordering cost from the regular supplier, the second term is the ordering cost from the expedited supplier, the third term is the expected discounted holding and shortage cost of period $t + l_e$, and the last term is the expected total discounted future costs. For simplicity, we assume the terminal value function $\phi_{T+1}(z) = 0$ for any z, which implies that there is no salvage value for left inventory and no backlogging cost for unfilled demand after period $T + l_e$.

Problem (1.1) admits optimal solutions under rather general and standard conditions. Nevertheless, it is a challenging problem. First, the state space is a multi-dimensional space whose dimension is specified by the difference between the lead times of the two suppliers plus one. Second, the objective function of problem (1.1) is not convex. In fact, in general it is not convex even for the last period problem in which $g_T(z, y, w)$ is actually convex. Thus, it is far from clear whether the cost-to-go functions v_t are convex, and even if they are, the objective function of problem (1.1) is not, which makes it very challenging to characterize the structure of the optimal policy and design efficient algorithms to compute the optimal policy.

1.2.2 A Transformation Technique

To address the challenge of non-convexity identified above, Chen *et al.* (2017) develop a novel transformation technique . Specifically, given a function $f : \mathcal{F}^n \to \bar{\Re}$ and a random vector $\Xi \in \mathcal{F}^n$, consider the following optimization problem

$$\tau^* = \inf_{u \in \mathcal{F}^n} E[f(u \wedge \Xi)]. \tag{1.3}$$

Here $\mathcal{F}$ is either the real space $\Re$ or the space with integers $\mathcal{Z}$ and $\bar{\Re} = \Re \cup \{+\infty\}$.

In general, the above problem is not a convex minimization problem even if the function f is convex. Interestingly, we show that under certain conditions, our transformation technique converts it into an equivalent convex minimization problem.

Proposition 1.1 (Equivalent Transformation)**.** Suppose that (a) the function $f : \mathcal{F}^n \to \bar{\Re}$ is lower semi-continuous with $f(x) \to +\infty$ for

$|x| \to \infty$; (b) f is componentwise convex (componentwise discrete convex if $\mathcal{F} = \mathcal{Z}$); (c) the random vector $\Xi \in \mathcal{F}^n$ has independent components. Then, τ^* is also the optimal objective value of the following optimization problem.

$$\begin{array}{ll} \inf & E[f(v(\Xi))] \\ \text{s.t.} & v(\xi) \le \xi \ \forall \xi \in \mathcal{X} \\ & v(\xi) = (v_1(\xi_1), ..., v_n(\xi_n)) \in \mathcal{F}^n, \end{array} \tag{1.4}$$

where $\mathcal{X}$ is support of Ξ.

If Ξ has finite number of scenarios and f is convex, clearly problem (1.9) is a convex minimization problem, for which efficient algorithms usually are available. The transformation technique allows us to convert a non-convex minimization problem to an equivalent convex minimization problem. In addition, it allows the preservation of several commonly used structural properties: convexity, submodularity, and $L^\natural$-convexity, a fundamental concept in discrete convex analysis which finds applications in a variety of inventory models in recent years.

We first present the concept of $L^\natural$-*convexity*, which was defined by Murota (1998) as a fundamental concept to extend convex analysis from real space to spaces with integers (see Murota, 2009 for a survey of the recent developments in discrete convex analysis). It was found powerful to establish the structures of optimal policies in various inventory models (see Zipkin, 2008 Huh and Janakiraman, 2010, Pang *et al.*, 2012, Chen *et al.*, 2014, etc, and a recent survey by Chen, 2017). Following Simchi-Levi *et al.* (2014), we define $L^\natural$-convexity as follows.

Definition 1.1 ($L^\natural$-Convexity). A function $f : \mathcal{F}^n \to \bar{\Re}$ is $L^\natural$-convex if and only if $f(x - \xi e)$ is submodular on $(x, \xi) \in \mathcal{F}^n \times \mathcal{S}$, where $\mathcal{S}$ is the intersection of $\mathcal{F}$ and any unbounded interval in $\Re$. A set $\mathcal{V} \subseteq \mathcal{F}^n$ is called $L^\natural$-convex if its indicator function $\delta_{\mathcal{V}}$, defined as $\delta_{\mathcal{V}}(x) = 0$ for $x \in \mathcal{V}$ and $+\infty$ otherwise, is $L^\natural$-convex.

In an attempt to deal with inventory models with random capacity or revenue management problems using booking limit controls, Chen *et al.* (2017) derive useful properties of convexity, submodularity and $L^\natural$-convexity for the following problem.

$$f(x, z) = \inf_{u:(x,z,u) \in \mathcal{A}} E_\Xi[g(x, u \Diamond_k (z + \Xi))], \tag{1.5}$$

where $g(\cdot,\cdot) : \mathcal{F}^m \times \mathcal{F}^n \to \bar{\Re}$, $x \in \mathcal{F}^m$, $z \in \mathcal{F}^n$, the set $\mathcal{A} \subseteq \mathcal{F}^m \times \mathcal{F}^n \times \mathcal{F}^n$ is non-empty, and $\Diamond_k$ is defined as $u \Diamond_k \xi \triangleq (u_1 \wedge \xi_1, ..., u_k \wedge \xi_k, u_{k+1} \vee \xi_{k+1}, ..., u_n \vee \xi_n)$.

Assumption 1. For any given x, (a) $g(x,\cdot)$ is lower semi-continuous with $g(x,u) \to +\infty$ for $\|u\|_2 \to \infty$; (b) $g(x,\cdot)$ is component-wise convex (component-wise discrete convex if $\mathcal{F} = \mathcal{Z}$.

Assumption 2. The random vector Ξ has independent components. Its support is denoted by $\mathcal{X}$ and the support of the j-th component is denoted by $\mathcal{X}_j$.

Assumption 3. The set $\mathcal{A} = \{(x,z,u) | Au \leq b, u_1 \geq \underline{u}_1, ..., u_k \geq \underline{u}_k, u_{k+1} \leq \bar{u}_{k+1}, ..., u_n \leq \bar{u}_n\}$, where $b, \underline{u}_1, ..., \underline{u}_k, \bar{u}_{k+1}, ..., \bar{u}_n$ are parameters that may depend on x and z, $A = (a_{ij})$ with entries $a_{ij} \geq 0$ for any i and $j = 1, ..., k$, and $a_{ij} \leq 0$ for any i and $j = k+1, ..., n$. In addition, $\mathcal{X}_j$ is contained in $[\underline{u}_j - z_j, +\infty)$ for $j = 1, ..., k$, and $\mathcal{X}_j$ is contained in $(-\infty, \overline{u}_j - z_j]$ for $j = k+1, ..., n$.

Theorem 1.1. (Chen *et al.*, 2017) Consider the optimization problem (1.5). Under Assumptions 1-3, problem (1.5) and the following optimization problem have the same optimal objective value.

$$\begin{array}{ll} \inf & E[g(x, v_1(\Xi_1), \ldots, v_n(\Xi_n))] \\ \text{s.t.} & v_j(\xi_j) \leq z_j + \xi_j \quad \forall \xi_j \in \mathcal{X}_j, \ \forall\, j = 1, \ldots, k \\ & v_j(\xi_j) \geq z_j + \xi_j \quad \forall \xi_j \in \mathcal{X}_j, \ \forall\, j = k+1, \ldots, n \\ & (x, z, v_1(\xi_1), \ldots, v_n(\xi_n)) \in \mathcal{A}^{\Xi} \quad \forall \xi \in \mathcal{X}, \end{array} \tag{1.6}$$

where $\mathcal{A}^{\Xi} = \{(x,z,w) | w = u \Diamond_k (z + \xi), (x,z,u) \in \mathcal{A}, \xi \in \mathcal{X}\}$. Furthermore,

(a) If g and $\mathcal{A}^{\Xi}$ are convex, then f is also convex.

(b) If g is submodular and $\mathcal{A}^{\Xi}$ is a lattice, then f is also submodular.

(c) If g and $\mathcal{A}^{\Xi}$ are $L^{\natural}$-convex, then f is also $L^{\natural}$-convex.

The following theorem characterizes the monotonicity properties of the solution set to the optimization problem (1.5). Let $\mathcal{U}^*(x,z)$ denote the the optimal solution set of (1.5).

Theorem 1.2. (Chen *et al.*, 2017) Consider the optimization problem (1.5). Under Assumptions 1-3, if $\mathcal{A}^{\Xi}$ is closed, and $u_j \le z_j + \bar{\xi}_j, j = 1, ..., k, u_j \ge z_j + \underline{\xi}_j, j = k+1, ..., n$, then we have the following results:

(a) If g is a submodular function, and $\mathcal{A}, \mathcal{A}^{\Xi}$ are lattices, then $\mathcal{U}^*(x, z)$ is increasing in (x, z). There exist a greatest element and a least element in $\mathcal{U}^*(x, z)$, which are increasing in (x, z).

(b) If g is an $L^\natural$-convex function, and $\mathcal{A}, \mathcal{A}^{\Xi}$ are $L^\natural$-convex sets, then $\mathcal{U}^*(x, z)$ is increasing in (x, z) and $\mathcal{U}^*((x, z) + \omega e) \sqsubseteq \mathcal{U}^*(x, z) + \omega e$ for any $\omega > 0$. Within $\mathcal{U}^*(x, z)$, there exist a greatest element and a least element, which have the above monotonicity properties with limited sensitivity.

We apply the methodologies presented here to analyze the dual-sourcing problem in the next section and refer to Chen *et al.* (2017), Chen *et al.* (2015) and Gao (2017) for many other applications including inventory models with random capacities and revenue management using booking limit controls.

1.3 Results and Insights

1.3.1 Optimal Structural Policy

The methodology developed in the previous section allows us to derive the key monotone comparative statics of the optimal solution of the dual-sourcing problem (1.1), which otherwise would have been extremely difficult to establish.

Theorem 1.3. For all t, $v_t(z)$ is $L^\natural$-convex in $z \in \mathcal{S}$. The optimal solution of problem (1.1), denoted by $(y_t(z), u_t(z))$, are increasing in z with limited sensitivity. That is, for any $\omega > 0$,

$$y_t(z) \le y_t(z + \omega e) \le y_t(z) + \omega, \quad u_t(z) \le u_t(z + \omega e) \le u_t(z) + \omega. \quad (1.7)$$

The above theorem implies that the regular order quantity decreases in on-hand inventory level and the sizes of the outstanding orders, and the sensitivity decreases in the age of the outstanding order, where the age refers to the number of periods from the period passed since the order was placed. On the other hand, the expedited order quantity decreases in the sizes of outstanding orders in the pipeline, but the sensitivity increases in the age of the outstanding order.

To gain sharper insights, we examine a special case with $k = l_r - l_e = 1$. It is known that when $l_r = l_e + 1$ the optimal inventory policy under dual sourcing with unlimited capacities can be characterized by a dual-index policy (see, e.g., Fukuda 1964). That is, in each period t, there exist two critical inventory positions for the regular and expedited ordering decisions, denoted by $S_{r,t}$ and $S_{e,t}$ respectively, with $S_{e,t} \leq S_{r,t}$. When the inventory position is below $S_{e,t}$, order up to $S_{e,t}$ from the expedited supplier and then place an order from the regular supplier; when the inventory position is above $S_{e,t}$ but below $S_{r,t}$, order up to $S_{r,t}$ from the regular supplier. In the presence of random capacities, such a policy may no longer be optimal. Nevertheless, the following theorem shows that a dual-index policy can be optimal when there is no capacity limit for the expedited supplier and there is a random capacity limit for the regular supplier.

Theorem 1.4 (Dual-Index Policy)**.** Suppose that $K_{e,t} = \infty$. A dual-index policy is optimal.

Theorem 1.4 shows that when only the regular supplier has limited uncertain supply capacity a dual-index policy is optimal. However, when only the expedited supplier has limited uncertain capacity, a dual-index policy may not be optimal. Nevertheless, it is intuitive to expect that the dual-index policy is close to being optimal even when both suppliers have uncertain supply capacities. Theorem 1.4 serves as a building block for us to develop the dual-index heuristic.

Theorem 1.4 also shows that it is not necessarily always optimal to order from the regular supplier even when it has a lower unit cost. The following corollary shows that if the regular supply capacity has no capacity limit and the regular ordering cost is larger than the expedited ordering cost then a sole expedited sourcing strategy is optimal.

Corollary 1.5 (Sole Expedited Sourcing Policy)**.** If $c_r \geq \alpha c_e$ and the expedited supply capacity is unlimited (i.e., $K_{e,t} = \infty$), a sole expedited sourcing is optimal.

Note that c_r is the unit cost for the regular order that will arrive in the next period and αc_e is discounted unit cost for the expedited order that will be placed and arrive in the next period. The condition $c_r \geq \alpha c_e$ implies that it is always cheaper to order from the expedited supplier. Hence it follows immediately the sole expedited sourcing is optimal. On the other hand, if regular supply capacity is unlimited and $c_e > c_r + h^-$, i.e., it is always better off to use regular orders to meet backorders instead of using expedited orders, then a sole regular sourcing policy is optimal.

The structure properties identified here play an important role in reducing the computation of the optimal policies. To compute the optimal value function and the optimal policy, one can adopt the standard value iteration approach (See Bertsekas, 1995 for the detailed introduction to the value iteration approach). When computing the optimal ordering decisions for each state, we conduct the line search for the optimal regular order-up-to level and (negative) expedited order quantity. $L^\natural$-convexity property allows us to reduce the searching space. The advantage of $L^\natural$-convexity is to ensure the local optimum to be global optimum and the steepest accent method can be used (instead of searching the whole decision space). Moreover, the monotonicity of the optimal decisions with respect to the system state ensures that the optimal solutions of smaller states can serve as lower bounds of the solutions of larger states, which helps further reduce the computational effort. Note that a state vector is smaller (larger) than another state vector if all its components are smaller (larger).

1.3.2 Effects of Supply Capacity Uncertainty

To investigate the effects of supply capacity uncertainties on the system costs, Chen and Pang (2014) conduct a careful numerical study to compare an infinite horizon inventory system with random capacities to a corresponding one with deterministic capacities (we refer to Chen and Pang (2014) for the detailed model parameters). More specifically, they consider the following four profiles: (I) no supply capacity uncertainty; (II) only regular supply capacity is uncertain; (III) only expedited supply capacity is uncertain; and (IV) both supply capacities are uncertain. The corresponding expected capacities for the regular supply and expedited supply under different profiles are the same. The effect of supply capacity uncertainties under profile i, referred to as ρ^i ($i \in \{II, III, IV\}$), is measured by the percentage of cost increase over profile I.

Chen and Pang (2014) first show that the optimal regular order-up-to level with deterministic dual supply capacity (profile I) is lower than those under all other profiles, which implies that uncertainty induces the system manager to order more from the regular supply channel. On the other hand, the optimal expedited orders have a different pattern. When the inventory level is negative and the amount of backorders is relatively large, the optimal expedited order quantities under random expedited supply (profile III) and random dual supply (profile IV) are higher than the other two. When the inventory level is median, the optimal expedited order quantity under random expedited supply (profile III) is the lowest. When the inventory level is relatively high, the optimal expedited order quantities are similar. These observations imply that when the system faces many backorders, the system should use more expedited supply to meet the backorders in a shorter time and hence the system manager tends to place more expedited orders when the maximum possible level of expedited supply capacity is higher.

Chen and Pang (2014) observe that though both regular and expedited supply uncertainties are costly, when the backorder cost is low, the regular supply capacity uncertainty is more costly and thus has a bigger effect ($\rho^{II} > \rho^{III}$), while for higher backorder cost the expedited supply capacity uncertainty is more costly and thus has a bigger effect ($\rho^{II} < \rho^{III}$). This may be due to the fact that when the backorder cost is higher the system relies more on the expedited supplier, which leads to a larger effect of expedited supply capacity uncertainty. In addition, the joint effect of both regular and expedited supply uncertainties, i.e., ρ^{IV}, is greater than the sum of the separate effects of individual supply capacity uncertainties, i.e., $\rho^{II} + \rho^{III}$.

Chen and Pang (2014) also illustrate that when the leadtime difference of the two supplies increases, the effect of regular supply capacity uncertainty, ρ^{II}, decreases. For the expedited supplier, when the backorder cost is low, the effects of supply capacity uncertainties, ρ^{III} and

ρ^{IV}, decrease in the leadtime difference; but when the backorder cost is high, these effects increase in the leadtime difference. Again this may be due to the dependence on the expedited supplier increases as the backorder cost becomes larger.

1.4 Future research

There are several interesting extensions we plan to explore. First, our transformation technique, though powerful to derive monotone comparative statics, requires independently distributed random variables. In addition, it only allows charges on delivered quantities. Gao (2017) provides some preliminary results along these lines. Specifically, he considers the following optimization problem

$$\inf_{u \in \mathcal{F}^n} \ell(u) + E[f(u \wedge \Xi)], \tag{1.8}$$

where $\ell, f : \mathcal{F}^n \to \bar{\Re}$.

Proposition 1.2 (Equivalent Transformation)**.** Suppose that (a) the objective function of problem (1.8) is lower semi-continuous and goes to $+\infty$ when $|u| \to \infty$; (b) f is componentwise convex (componentwise discrete convex if $\mathcal{F} = \mathcal{Z}$) and supermodular, ℓ is increasing; (c) the random vector $\Xi = (\Xi_1, \ldots, \Xi_n) \in \mathcal{F}^n$ are positively dependent, i.e., $\{\Xi_1, \ldots, \Xi_{i-1}, \Xi_{i+1}, \ldots, \Xi_n | \Xi_i\}$ is stochastically increasing for all i. Then, problem (1.8) has the same optimal objective value as the following optimization problem.

$$\begin{aligned} \inf \quad & \ell(u) + E[f(v(\Xi))] \\ \text{s.t.} \quad & v(\xi) = (v_1(\xi_1), ..., v_n(\xi_n)) \in \mathcal{F}^n \\ & v(\xi) \le u \ \forall \xi \in \mathcal{X} \\ & v(\xi) \le \xi \ \forall \xi \in \mathcal{X} \\ & v_j(\xi_j) \text{ is increasing } \forall \xi_j \in \mathcal{X}_j, j = 1, \ldots, n. \end{aligned} \tag{1.9}$$

Gao (2017) also illustrates how decision makers' risk preference can be incorporated.

Second, although $L^\natural$-convexity property helps reducing the searching space in computing the optimal policy, the computation still suffers from the curse-of-dimensionality. Dual-index policies also become difficult to compute in nonstationary systems where the policy parameters are time-varying. Hence, more efficient heuristics are needed to address high-dimensional and nonstationary problems. Third, we assume that the supply capacity uncertainty is resolved in the same period when the order is placed. It is interesting to extend our approach to settings in which the random supply capacities are resolved only upon the receipts of the orders. Finally, by far our analysis has been restricted to dual sourcing problems. It will be interesting to further extend the analysis to the systems with multiple sources and various lead times.

Acknowledgements

This research is partly supported by National Science Foundation (NSF) Grants CMMI-1363261, CMMI-1538451, CMMI-1635160 and National Science Foundation of China (NSFC) Grant 71520107001.

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