The Impact of Manufacturer Rebates on Supply Chain Profits

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Abstract: Manufacturer rebates are commonly used as price discount tools for attracting end customers. In this study, we consider a two-stage supply chain with a manufacturer and a retailer, where a single seasonal product faces uncertain and price-sensitive demand. We characterize the impact of a manufacturer rebate on the expected profits of both the manufacturer and the retailer. We show that unless all of the customers claim the rebate, the rebate always benefits the manufacturer. Our results thus imply that "mail-in rebates," where some customers end up not claiming the rebate, particularly when the size of the rebate is relatively small, always benefit the manufacturer. On the other hand, an "instant rebate," such as the one offered in the automotive industry where every customer redeems the rebate on the spot when he/she purchases a car, does not necessarily benefit the manufacturer. © 2007 Wiley Periodicals, Inc. Naval Research Logistics 54: 667–680, 2007

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1. INTRODUCTION

Price promotions, that is, temporary price reductions, have been employed by many industries. This is nicely reflected in the marketing literature, where a large number of studies have focused on price promotions and their impact on retailers and, in some cases, on manufacturers. For a review, see [1].

Our focus in this paper is on price promotions provided by manufacturers through manufacturer rebates. Consider the following examples with products of short life-cycles:

- Nikon Coolpix Digital Camera is sold either online or in stores for about \$600. The manufacturer provides a rebate of \$100 independently of where the camera is purchased.
- Sharp VL-WD255U Digital Camcorder is sold for about \$500 at retail or virtual stores. Sharp provides a \$100 rebate to the customer independently of where the product was purchased.

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Both examples are part of a growing practice, where manufacturers use rebates, a form of price promotion, to improve their operations and, ultimately, their bottom line (see [38]). As mentioned by Jolson et al. [24], one of the major reasons for offering rebates is the increase in income due to "slippage," which refers to "the proportion of consumers who are enticed to purchase as a result of the rebate offer but fail to request refunds to which they are entitled." Jolson et al. have reported an overall slippage rate of 70% in their survey study. In fact, a significant slippage rate is expected when the manufacturer offers a "mail-in rebate," which requires the consumers to spend some effort to redeem it. However, slippage should not occur if the rebate is an "instant rebate," where the consumer can cash in the rebate with virtually no effort. Thus, in this paper we incorporate slippage into our model, and we analyze the case where slippage exists, as well as the case in which there is no slippage.

When manufacturer rebates are utilized in a two-echelon environment (i.e., a supply chain with a manufacturer and a retailer), they can be classified into two different types. The first type is the channel rebate, which is a payment from a

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manufacturer to a retailer based on the amount of sales that the retailer generates. The second type is the consumer rebate, where the manufacturer pays the end customer via a coupon. A number of studies have been conducted on channel rebates, including the work of Crafton and Hoffer [11], Wetzel and Hoffer [47], Taylor [41], and Beltramini and Chapman [5], among others.

Consumer rebates, which are the focus of this paper, have been investigated in the marketing literature. For example, Ali et al. [2] have developed a simple model to study the optimal size of a rebate when the consumers are categorized into loyal customers and brand switchers. Zhang et al. [51] have analyzed the choice between immediate value promotions (e.g., peel-off coupons, free-standing inserts, and direct-mail coupons) and delayed value promotions (e.g., in-pack coupons, on-pack coupons, and contests).

In the economics literature, a number of studies have been conducted to explain why rebates are used. Some studies have been performed on the use of manufacturer rebates as a means of price discrimination (see for example, [17]). On the other hand, Gerstner and Hess [14–16] have developed a model to demonstrate that a manufacturer may find it profitable to offer rebates even when price discrimination does not occur (i.e., when the redemption rate of the rebate is 100%). In their model, they assume the existence of two groups of consumers with different reservation prices and different redemption costs, where all cost parameters are deterministic. Ault et al. [3] have developed a deterministic multi-period inventory model and used it to demonstrate that manufacturer rebates can be used to increase the profits of manufacturers by mitigating arbitrage by retailers across temporally separated markets. Our model differs from these models in that we assume the retailer is facing a single-period stochastic demand, and we identify conditions under which the manufacturer and the retailer will benefit from the manufacturer rebate. One of our major findings is that for seasonal products (or products with short life-cycles), a manufacturer rebate can always increase the manufacturer's profit, as long as slippage exists.

In the operations management literature, models have been developed that integrates inventory and pricing decisions; see Khouja [26] for a recent survey of those models. In addition, Khouja [26] has developed and analyzed a deterministic lot-sizing model that incorporates pricing and rebate-offering decisions. Our work focuses on seasonal products, and therefore, our model has a newsvendor setting with pricing decisions. In the literature, various newsvendor models with price effects have been studied. Whitin [48] was the first to develop such a model in which the unit price of the product is a decision variable rather than a given parameter. Mills [31] has developed a newsvendor model that explicitly specifies an additive demand function. Karlin and Carr [25] have considered a newsvendor model with a multiplicative demand function. For reviews on various extensions of the newsvendor problem with price effects, see Petruzzi and Dada [35] and Yano and Gilbert [50]. More recently, newsvendor models with pricing decisions have appeared in various supply chain management applications; see, for example, Chen et al. [9], Granot and Yin [19], Chen et al. [10], and Song et al. [40].

Another area of related research involves manufacturer-retailer contractual relationships. In particular, the use of quantity discounts [46], return policies [12], channel rebates [41], consignment contracts [44], and so forth, to achieve channel coordination has been studied extensively in the supply chain contracting literature. To the best of our knowledge, none of these supply chain management publications has explicitly analyzed the impact of a consumer rebate on different parties in a supply chain; see Lariviere [27], Tsay et al. [42], and Cachon [7] for recent surveys of analyses on supply contracts.

In the next section, we formally describe our model. In Section 3, we analyze the impact of rebates on the manufacturer and prove that it is always beneficial for the manufacturer to offer some rebates to end customers, as long as the rebate claiming process is designed in such a way that some customers end up not claiming the rebate (e.g., a mail-in rebate). In Section 4 we identify conditions under which the rebate is also beneficial for the retailer. Thus, in Sections 3 and 4, conditions under which mail-in rebates increase supply chain profit are analyzed. In Section 5, an analysis is presented for the special case in which all customers claim the rebate. We show that, in this case, rebates are not necessarily beneficial for the manufacturer. Finally, numerical studies are reported in Section 6, followed by some concluding remarks in Section 7.

2. THE MODEL

We consider a two-stage supply chain with a manufacturer and its exclusive retailer, and assume a single seasonal product facing uncertain and price-sensitive demand. Each unit of the product incurs production cost $c > 0$. The manufacturer, which acts as a Stackelberg leader, first determines unit wholesale price $w > 0$ of the product and cash rebate x to be offered to the end customers to maximize its expected profit, where $0 \leq x \leq w$. Given the manufacturer's decisions on w and x, the retailer, which acts as a Stackelberg follower, determines order quantity $q \geq 0$ and unit retail price $p > 0$ to maximize its own expected profit (see Fig. 1). Hence, the manufacturer anticipates the retailer's reactions in making decisions on w and x.

We assume two customer segments in modeling the relationship among retail price p, rebate x, and demand $Q_b(p, x)$. The first segment represents customers who perceive price

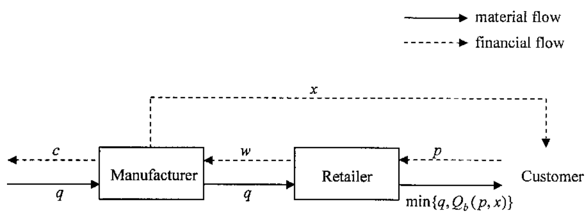


Figure 1. The model.

reduction by the amount of rebate x and respond to the decreased price, $p - x$, in their purchase decisions. On the other hand, the second segment represents customers who do not perceive the price reduction and consider the charged price p only in making purchase decisions. We call the first group "rebate-sensitive" segment. The share of this segment is $\rho(x) \times 100\%$, where ρ is a continuous and differentiable function such that $0 < \rho(x) \leq 1$ for any $x \geq 0$. In practice, ρ would be nondecreasing as a larger rebate tends to induce more customers to respond to the rebate, whereas a smaller rebate can be easily ignored by customers [37]. However, our analysis does not require such a monotonic condition on ρ. We call the second group "rebate-insensitive" segment, to which $[1 - \rho(x)] \times 100\%$ of the population belong.

There is a vast literature in marketing and psychology discussing reference price in customer purchase decisions. Customers do not perceive a price change *per se*; rather, they compare the new price with their own reference prices. If the new price falls within a region around the reference price, called the "latitude of acceptance," customers perceive no change in price and do not respond to the actual price change at all [28, 49]. It has been frequently observed that charged retail price can be a reference price, while price promotion is a change from the reference price [29]. Recent empirical studies show significant heterogeneity in reference price as well as in the latitude of acceptance [13, 32]; also see Mazumdar et al. [30] for a detailed review on reference price. Therefore, given the rebate x, we have two groups of customers, namely rebate-sensitive and rebate-insensitive customers.

In each group, we consider the bivariate utility model represented by the following indirect utility function:

$$v(p_c, m) = \left(\frac{1}{b-1}\right) p_c^{1-b} - e^{-m},$$

where p_c is the perceived price in purchase decisions (i.e., $p_c = p - x$ and $p_c = p$ in the first and second segments, respectively), and m is the customer budget allocated to this product category. The associated ordinary demand function is $d_b(p_c) = \theta p_c^{-b}$, where $\theta = e^m$ and b measures the price elasticity of demand ($b > 1$). It is shown that this continuous demand function can also be derived from a representative customer's stochastic utility maximization under the assumptions of discrete choice and perfect substitution [21]. The multiplicative form of demand with constant elasticity provides mathematical tractability and is flexible in exhibiting diverse economic properties [8]. Furthermore, it can be easily converted into an additive form with the log transformation. Hence, iso-elastic demand functions have been widely employed in many empirical and analytical studies; see, for example, Grabowski [18], Murray and Ginman [33], Welam [45], Oum et al. [34], Hoch et al. [23], and Petruzzi and Dada [35]. Recently, Song et al. [40] have demonstrated various analytical benefits of demand functions with small and nondecreasing "curvature." Iso-elastic demand functions, in particular, have curvature equal to 1.

Given retail price p, rebate x, and the shares $\rho(x)$ and $1 - \rho(x)$ of the two segments, we obtain the following total product demand after normalizing the budget effect (i.e., setting $\theta = 1$):

$$D_b(p, x) = \rho(x)(p - x)^{-b} + [1 - \rho(x)]p^{-b}. \quad (1)$$

We further assume that, in addition to price and rebate, other unobservable factors affect the actual demand at the retail marketplace via stochasticity involved in $d_b(p_c)$ as well as $\rho(x)$. Subsequently, the revealed retail demand becomes

$$Q_b(p, x) = D_b(p, x)\varepsilon = \{\rho(x)(p - x)^{-b} + [1 - \rho(x)]p^{-b}\}\varepsilon,$$

where ε is a nonnegative random variable with a general continuous probability distribution. We let $f(y)$ and $F(y)$ denote the probability density and cumulative distribution functions, respectively, of ε.

There are two subgroups in the rebate-sensitive segment depending on whether customers actually redeem the rebate offer or not. We assume that $\tilde{\rho}(x)$ is the share of customers who view $p - x$ as the selling price in purchase decisions, but do not claim the rebate, where $0 \leq \tilde{\rho}(x) \leq \rho(x)$. Such "slippage" may arise from situational factors and other uncontrollable factors that the customers do not expect or consider at the time of purchase [4]. Soman [39] has shown that consumers systematically underweight the future effort in the context of delayed reward and proposed that the decision to purchase a product can be independent of the decision to redeem an incentive later. Hence, a rebate offer that appears attractive at the time of purchase may appear unattractive at the time of redemption. Furthermore, numerous studies in psychology have shown that consumers systematically exhibit over-confidence in personal forecast [22, 36, 43]. Such optimistic bias can make customers overestimate their likelihood of redeeming a rebate offer later. Slippage in redemption may also be due to an error in estimating the effort involved in the redemption of a rebate on the part of customers. Subsequently, a fraction of customers may not claim cash rebates even though they take the rebates into consideration at the time of purchase.

If $\rho(x) \neq 0$, then only $\frac{\rho(x)-\tilde{\rho}(x)}{\rho(x)} \times 100\%$ of the customers in the first segment eventually redeem the rebate. Remaining

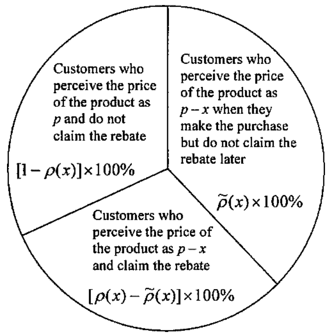


Figure 2. Functions ρ and $\tilde{\rho}$.

customers in the first segment, as well as the customers in the second segment, will never claim the rebate. Therefore, we can classify the population into three distinct groups depending on their responses to the rebate in purchase decisions and their redemption of the rebate offer (Fig. 2):

- $[\rho(x) - \tilde{\rho}(x)] \times 100\%$ of the customers perceive the selling price as $p - x$ and claim the rebate;
- $\tilde{\rho}(x) \times 100\%$ of the customers perceive the selling price as $p - x$ when they make purchase decisions, but do not claim the rebate;
- $[1 - \rho(x)] \times 100\%$ of the customers perceive the selling price as p and do not claim the rebate.

We assume that function $\tilde{\rho}$ is continuous and differentiable. We also assume that $\tilde{\rho}$ is nonincreasing because the larger a rebate, the more customers redeem the rebate [37]. In our model, we omit fixed costs and assume (but not without loss of generality) no salvage value or disposal cost of unsold items to isolate the effect of a manufacturer's rebate offer.

3. IMPACT OF REBATES ON THE MANUFACTURER

In this section, we analyze the impact of the rebates on the manufacturer. We first discuss how a rebate affects the manufacturer's expected total profit. We show that unless $\tilde{\rho} \equiv 0$, the optimal size of the rebate for the manufacturer is always nonzero. That is, we show that unless all customers claim the rebate, the rebate always benefits the manufacturer.

To simplify the notation, we denote

$$z = \frac{q}{D_{\mathrm{b}}(p, x)}.$$

We define

$$\Lambda(z) = \int_0^z F(y)dy = \int_0^z (z-y)f(y)dy$$

and

$$L(z) = 1 - \frac{\Lambda(z)}{z}.$$

It is easy to see that $0 < L(z) \leq 1$ for all $z > 0$. Note also that

$$E[\min\{z, \varepsilon\}] = z - \Lambda(z) = zL(z) \tag{2}$$

and

$$L'(z) = \frac{1}{z}[1 - F(z) - L(z)]. \tag{3}$$

We also denote $r = x/w$. Thus, $0 < r \leq 1$. We call r the "rebate factor." It is the fraction of the revenues that the manufacturer is prepared to return to the customers through the rebate program in case all customers claim the rebate. Given the values of w and r, the retailer's expected profit is

$$\begin{aligned} \Pi_{\mathrm{R}}(p, z, w, r) &= p \cdot E[\min\{q, D_{\mathrm{b}}(p, rw)\varepsilon\}] - wq \\ &= D_b(p, rw)\{p \cdot E[\min\{z, \varepsilon\}] - wz\} \\ &= D_b(p, rw)z[pL(z) - w], \end{aligned} \tag{4}$$

where the last equality follows from Eq. (2). Thus, by (3),

$$\frac{\partial \Pi_{\mathrm{R}}(p, z, w, r)}{\partial z} = D_{\mathrm{b}}(p, rw)\{p[1 - F(z)] - w\}.$$

Also,

$$\frac{\partial \Pi_{\mathrm{R}}(p, z, w, r)}{\partial p} = zD_{\mathrm{b}}(p, rw)L(z) - zb[pL(z) - w]D_{b+1}(p, rw).$$

From the first-order necessary conditions of optimality, we obtain, by setting these partial derivatives to zero, that

$$p^*(w, r) = \frac{w}{1 - F(z^*(w, r))} \tag{5}$$

and

$$D_b(p^*(w, r), rw)L(z^*(w, r)) - b[p^*(w, r)L(z^*(w, r)) - w]D_{b+1}(p^*(w, r), rw) = 0,$$

where $p^*(w, r)$ and $z^*(w, r)$ denote the optimal values of p and z, respectively, of the retailer for some given values of w and r. These two equations imply that

$$\begin{aligned} &D_b\left(\frac{w}{1 - F(z^*(w, r))}, rw\right) L(z^*(w, r)) \\ &-bw\left[\frac{L(z^*(w, r))}{1 - F(z^*(w, r))} - 1\right] D_{b+1}\left(\frac{w}{1 - F(z^*(w, r))}, rw\right) \\ &= 0. \end{aligned} \tag{6}$$

Anticipating the reaction from the retailer to its wholesale price and rebate factor, the manufacturer's expected profit can be written as

$$\Pi_{\mathrm{M}}(w,r) = (w-c)q^*(w,r) - \frac{[\rho(rw)-\tilde{\rho}(rw)]\cdot[p^*(w,r)-rw]^{-b}}{D_b(p^*(w,r),rw)} \cdot rw \cdot E[\min\{q^*(w,r), D_b(p^*(w,r),rw)\epsilon\}],$$

where $p^*(w,r)$ is given by Eq. (5) and $q^*(w,r)$ denotes the optimal value of q for the retailer. The second term on the right-hand side of this equation corresponds to the expected reduction in revenues because of the redemption of the rebate by the end customers. Thus,

$$\Pi_{\mathrm{M}}(w,r) = z^*(w,r)\{(w-c)D_b(p^*(w,r),rw) - [\rho(rw)-\tilde{\rho}(rw)]\cdot[p^*(w,r)-rw]^{-b}\cdot rw \cdot L(z^*(w,r))\}. \tag{7}$$

We now analyze the model when the rebate factor, r, approaches zero. Note that when $r=0$, Eq. (6) reduces to

$$\frac{L(z^*(w,0))}{1-F(z^*(w,0))} = \frac{b}{b-1}. \tag{8}$$

This implies that when $r = 0$, the quantity $z^*(w,r)$ is independent of the wholesale price w.

Let $w^*(r)$ denote the value of w that maximizes $\Pi_{\mathrm{M}}(w,r)$. Again, from the first-order necessary conditions of optimality, we have

$$\left.\frac{\partial \Pi_{\mathrm{M}}(w,r)}{\partial w}\right|_{w=w^*(r)} = 0.$$

Hence, upon denoting $z^*(r)=z^*(w^*(r),r)$ and $z_r = \frac{\partial z^*(w,r)}{\partial r}|_{r=0,w=w^*(0)}$, we have

$$\begin{aligned}
\left.\frac{d\Pi_{\mathrm{M}}(w^*(r),r)}{dr}\right|_{r=0}
&= \left.\frac{\partial \Pi_{\mathrm{M}}(w,r)}{\partial r}\right|_{w=w^*(r),r=0}\\
&= \Pi_M(w^*(0),0)\left[\frac{1}{z^*(0)} - \frac{bf(z^*(0))}{1-F(z^*(0))}\right]\cdot z_r\\
&\quad + z^*(0)w^*(0)\left\{b[w^*(0)-c]\rho(0)\left[\frac{w^*(0)}{1-F(z^*(0))}\right]^{-b-1}\right.\\
&\quad \left. - [\rho(0)-\tilde{\rho}(0)]\left[\frac{w^*(0)}{1-F(z^*(0))}\right]^{-b} L(z^*(0))\right\},
\end{aligned} \tag{9}$$

where the last equality follows from the fact that

$$\begin{aligned}
\left.\frac{\partial}{\partial r} D_b\left(\frac{w}{1-F(z^*(w,r))}, rw\right)\right|_{r=0,w=w^*(0)}
&= -\left[\frac{w}{1-F(z^*(0))}\right]^{-b} \frac{bf(z^*(0))}{1-F(z^*(0))}\cdot z_r\\
&\quad + bw\rho(0)\left[\frac{w}{1-F(z^*(0))}\right]^{-b-1}.
\end{aligned}$$

We have the following lemma.

LEMMA 1: $\frac{\partial z^*(w,r)}{\partial r}|_{r=0,w=w^*(0)} = \rho(0)L(z^*(0))/[\frac{1}{z^*(0)} - \frac{bf(z^*(0))}{1-F(z^*(0))}]$.

PROOF: See Appendix. □

Note that at $r = 0$, the manufacturer's profit function becomes

$$\Pi_{\mathrm{M}}(w,0) = z^*(w,0)(w-c)\left[\frac{w}{1-F(z^*(w,0))}\right]^{-b}.$$

Recall that $z^*(w,r)$ is independent of w when $r=0$. Hence, by setting the first derivative of $\Pi_{\mathrm{M}}(w,0)$ to zero, we obtain

$$w^*(0) = \frac{bc}{b-1}. \tag{10}$$

Thus,

$$\Pi_{\mathrm{M}}(w^*(0),0) = \frac{z^*(0)}{b}[w^*(0)]^{-b+1}[1-F(z^*(0))]^b. \tag{11}$$

Using (8), (10), (11), and Lemma 1, upon simplification, Eq. (9) becomes

$$\left.\frac{d\Pi_{\mathrm{M}}(w^*(r),r)}{dr}\right|_{r=0} = z^*(0)L(z^*(0))[w^*(0)]^{-b+1}[1-F(z^*(0))]^b\tilde{\rho}(0). \tag{12}$$

This implies the following result:

THEOREM 1: If $\tilde{\rho}(0) > 0$, then $\frac{d\Pi_M(w^*(r),r)}{dr}|_{r=0} > 0$.

Note that $\tilde{\rho}$ is a non-increasing function. Thus, Theorem 1 implies that unless $\tilde{\rho} \equiv 0$, there exists $r > 0$ such that $\Pi_{\mathrm{M}}(w^*(r),r) > \Pi_M(w^*(0),0)$. This in turn implies that unless $\tilde{\rho} \equiv 0$, the optimal size of the manufacturer rebate is nonzero. This suggests that it is always beneficial for the manufacturer to offer some rebates to end customers, as long as the rebate claiming process is designed in such a way that some customers, who initially are attracted by the rebate, will nevertheless forgo the rebate (that is, slippage is nonzero). Such a condition is satisfied when the manufacturer rebate is a mail-in rebate, where some customers end up not claiming the rebate, particularly when the size of the rebate is very small. Therefore, a reason for manufacturers to offer mail-in rebates is to increase their own profits by introducing lower perceived selling prices for the products.

The above analysis focuses on the case of $r \to 0$, and it leads to a conclusion that the optimal size of the rebate must be strictly positive (unless $\tilde{\rho} \equiv 0$). However, it does not indicate the optimal size of the rebate and the magnitude of the benefit contributed by the rebate. The numerical studies in Section 6 will provide such quantified results.

REMARK: Theorem 1 remains valid when $\tilde{\rho}$ is a function of both x and p. In other words, for a customer who perceives the rebate as a direct reduction in the selling price, if his/her action of claiming the rebate depends not only on the size of the rebate but is also influenced by the retail price of the product (for example, the rebate claiming behavior of some customers may depend on the "percentage" reduction in price), then the result stated in Theorem 1 remains valid.

Finally, we comment on the differentiability of functions $z^*(w,r)$ and $w^*(r)$. As long as we assume that the optimal solution (p,z) that maximizes the retailer's expected profit $\Pi_R(p,z,w,0)$ is unique for any given w, the function $z^*(w,r)$ is differentiable over r for any w when r is sufficiently close to 0. If, in addition, the optimal solution that maximizes the manufacturer's expected profit $\Pi_M(w,r)$ is unique when $r=0$, then $w^*(r)$ is also differentiable for small values of r. It is also appropriate to point out that the uniqueness of the optimal solutions at $r=0$ is usually satisfied in practice. Thus, in this case, our analysis is valid.

4. IMPACT OF REBATES ON THE RETAILER

In this section, we analyze the impact of manufacturer rebates on the retailer. We first discuss how the rebate affects the retailer's expected profit. To simplify the analysis, we assume that ρ is independent of the rebate amount x. In this case, the solution $z^*(w,r)$ of Eq. (6) is independent of w. Therefore, we drop the parameter w and denote $z^*(r) = z^*(w,r)$. We have $\frac{dz^*(r)}{dr}|_{r=0} = \frac{\partial z^*(w,r)}{\partial r}|_{r=0,w=w^*(0)}$. Taking the partial derivative of $\Pi_M(w,r)$ with respect to w and setting it to zero, we obtain

$$(b-1)w^*(r)A(r) + bB(r) + \{(b-1)w^*(r)\tilde{\rho}(rw) - [w^*(r)]^2 r\tilde{\rho}'(rw)\}C(r) = 0, \quad (13)$$

where

$$A(r) = D_b\left(\frac{1}{1-F(z^*(r))}, r\right) - \rho\left[\frac{1}{1-F(z^*(r))} - r\right]^{-b} rL(z^*(r)),$$

$$B(r) = -cD_b\left(\frac{1}{1-F(z^*(r))}, r\right),$$

and

$$C(r) = \left[\frac{1}{1-F(z^*(r))} - r\right]^{-b} rL(z^*(r)).$$

Differentiating (13) with respect to r and setting r to zero, we get

$$(b-1)A'(0)w^*(0) + (b-1)A(0)\frac{dw^*(r)}{dr}\bigg|_{r=0} + bB'(0) + (b-1)w^*(0)\tilde{\rho}(0)C'(0) = 0.$$

Simplifying, we obtain

$$\frac{dw^*(r)}{dr}\bigg|_{r=0} = \frac{bc}{b-1}[\rho - \tilde{\rho}(0)]L(z^*(0)). \quad (14)$$

Recall from Eq. (4) that the expected profit of the retailer is

$$\Pi_R(p,z,w,r) = D_b(p,rw)z[pL(z) - w].$$

Notice that when $p = p^*(r)$, $z = z^*(r)$, and $w = w^*(r)$,

$$\frac{\partial \Pi_R(p,z,w,r)}{\partial p} = \frac{\partial \Pi_R(p,z,w,r)}{\partial z} = 0.$$

Thus, when $p = p^*(r)$, $z = z^*(r)$, and $w = w^*(r)$,

$$\begin{aligned}
&\frac{d\Pi_R(p^*(r),z^*(r),w^*(r),r)}{dr} \\
&= \frac{\partial \Pi_R(p,z,w,r)}{\partial w}\bigg|_{p=p^*(r),z=z^*(r),w=w^*(r)} \cdot \frac{dw^*(r)}{dr} + \frac{\partial \Pi_R(p,z,w,r)}{\partial r}\bigg|_{p=p^*(r),z=z^*(r),w=w^*(r)} \\
&= \bigg\{ rbz^*(r)[p^*(r)L(z^*(r)) - w^*(r)] \times \rho[p^*(r) - rw^*(r)]^{-b-1} \\
&\quad - z^*(r)D_b\left(\frac{w^*(r)}{1-F(z^*(r))}, rw^*(r)\right)\bigg\} \cdot \frac{dw^*(r)}{dr} \\
&\quad + b\rho\left[\frac{1}{1-F(z^*(r))} - r\right]^{-b-1} z^*(r)[w^*(r)]^{-b+1} \times \left[\frac{L(z^*(r))}{1-F(z^*(r))} - 1\right].
\end{aligned} \quad (15)$$

In particular, when $r = 0$,

$$\begin{aligned}
&\frac{d\Pi_R(p^*(r),z^*(r),w^*(r),r)}{dr}\bigg|_{r=0} \\
&= -z^* \cdot (w^*)^{-b}\left[\frac{1}{1-F(z^*)}\right]^{-b} \cdot \frac{dw^*(r)}{dr}\bigg|_{r=0} \\
&\quad + b\rho\left[\frac{1}{1-F(z^*)}\right]^{-b-1} z^* \cdot (w^*)^{-b+1}\left[\frac{L(z^*)}{1-F(z^*)} - 1\right]
\end{aligned}$$

$$= -z^* \cdot (w^*)^{-b}\left[\frac{1}{1-F(z^*)}\right]^{-b} \cdot \frac{bc}{b-1}[\rho - \tilde{\rho}(0)]L(z^*)$$
$$+ b\rho\left[\frac{1}{1-F(z^*)}\right]^{-b-1} z^* \cdot (w^*)^{-b+1}\left[\frac{L(z^*)}{1-F(z^*)} - 1\right]$$
(by (14))
$$= -z^* \cdot (w^*)^{-b}\left[\frac{1}{1-F(z^*)}\right]^{-b-1}$$
$$\times \frac{bc}{b-1}[\rho - \tilde{\rho}(0)]\frac{b}{b-1} + \rho\left[\frac{1}{1-F(z^*)}\right]^{-b-1} z^*$$
$$\times (w^*)^{-b+1}\frac{b}{b-1} \quad \text{(by (8))}$$
$$= \tilde{\rho}(0)\left[\frac{1}{1-F(z^*)}\right]^{-b-1} z^* \cdot (w^*)^{-b+1}\frac{b}{b-1},$$

where $z^* = z^*(0)$ and $w^* = w^*(0)$. Hence, we have the following result.

THEOREM 2: If ρ is constant and $\tilde{\rho}(0) > 0$, then $\frac{d\Pi_R(p^*(r),z^*(r),w^*(r),r)}{dr}|_{r=0} > 0$.

Theorem 2 implies that if ρ is constant and $\tilde{\rho}(0) > 0$, then there exists a manufacturer rebate that can benefit the retailer. Let

$$\Omega_M = \{r > 0 \mid \Pi_M(w^*(r), r) > \Pi_M(w^*(0), 0)\}$$

and

$$\Omega_R = \big\{r > 0 \mid \Pi_R(p^*(r), z^*(r), w^*(r), r) > \Pi_R(p^*(0), z^*(0), w^*(0), 0)\big\}.$$

Thus, $r \in \Omega_M$ if and only if it is beneficial for the manufacturer to offer a rebate of amount rw. Similarly, $r \in \Omega_R$ if and only if a manufacturer rebate of amount rw is beneficial to the retailer. Theorems 1 and 2 imply that if ρ is constant and $\tilde{\rho}(0) > 0$, then $\Omega_M \cap \Omega_R \neq \emptyset$. Hence, in such a case, there must exist a rebate factor $r > 0$ that can benefit both parties. The manufacturer may either choose a rebate factor $r \in \Omega_M$ that maximizes its own benefit, or choose a rebate factor $r \in \Omega_M \cap \Omega_R$ that benefits both parties. Note that if the manufacturer chooses a rebate factor $r \in \Omega_M$ that maximizes its own benefit, then the rebate may not benefit the retailer.

The condition "$\tilde{\rho}(0) > 0$" is satisfied as long as some slippage exists. As mentioned in Section 3, this condition is satisfied when the rebate is a mail-in rebate. The condition "ρ = constant" requires that the size of the group of customers who completely ignore the rebate is insensitive to the rebate size x. When ρ is a function of x, the analysis becomes highly complex, but we conjecture that Theorem 2 remains valid.

We now investigate the impact of a mail-in rebate on the retailer's order quantity and selling price. Notice that the retailer's order quantity is given as

$$q^*(r) = z^*(r)[w^*(r)]^{-b} D_b\left(\frac{1}{1-F(z^*(r))}, r\right).$$

Thus, when $r = 0$,

$$\left.\frac{dq^*(r)}{dr}\right|_{r=0} = q^*(0)\left\{\left.\frac{dz^*(r)}{dr}\right|_{r=0} \cdot \left[\frac{1}{z^*} - \frac{bf(z^*)}{1-F(z^*)}\right] - \left.\frac{dw^*(r)}{dr}\right|_{r=0} \times \frac{b}{w^*} + \rho b[1-F(z^*)]\right\}$$
$$= q^*(0)\{\rho L(z^*) - b[\rho - \tilde{\rho}(0)]L(z^*) + \rho b[1-F(z^*)]\}$$
[by (10), (14), and Lemma 1]
$$= q^*(0)L(z^*)b\tilde{\rho}(0) \quad \text{[by (8)]}.$$

Hence, if $\tilde{\rho}(0) > 0$, then $\frac{dq^*(r)}{dr}|_{r=0} > 0$. In other words, if not all the rebate-sensitive customers claim the rebate, then the retailer tends to order more than the case with no rebate being offered. Of course, this is intuitive since the retailer is expecting more customers.

With regard to the retailer's selling price, one legitimate guess is that the retailer will increase its selling price if a manufacturer rebate is provided. However, our analysis shows that this is not necessarily the case. Recall that $p^*(r) = \frac{w^*(r)}{1-F(z^*(r))}$.

$$\left.\frac{dp^*(r)}{dr}\right|_{r=0} = \frac{[1-F(z^*)] \cdot \frac{dw^*(r)}{dr}|_{r=0} + w^* f(z^*) \cdot \frac{dz^*(r)}{dr}|_{r=0}}{[1-F(z^*)]^2}$$
$$= c\left(\frac{b}{b-1}\right)^2\left[\rho - \tilde{\rho}(0) + \frac{\rho z^* f(z^*)}{1-F(z^*) - bz^* f(z^*)}\right]$$
(by (8), (10), (14), and Lemma 1).

If ε is uniformly distributed in $[0,1]$, then $z^* = \frac{2}{b+1}$ and $\frac{dp^*(r)}{dr}|_{r=0} = c(\frac{b}{b-1})^2[\frac{b-1}{b+1}\rho - \tilde{\rho}(0)]$. Therefore, if $\tilde{\rho}(0) < \frac{b-1}{b+1}\rho$, then $\frac{dp^*(r)}{dr}|_{r=0} > 0$. On the other hand, if $\tilde{\rho}(0) > \frac{b-1}{b+1}\rho$, then $\frac{dp^*(r)}{dr}|_{r=0} < 0$. This demonstrates that the retailer may not increase its selling price when the manufacturer offers a rebate to the end customers. In other words, if the rebate benefits the retailer, then the benefit is contributed by the increase in demand and not necessarily by the increase in the selling price.

Theorems 1 and 2 show results fundamentally consistent with the findings in Bruce et al. [6], even though their structure of market demand is quite different from ours. They examine a manufacturer's cash rebate on durable goods, of which used ones are traded in the second-hand market, such as automobiles. They assume perfect redemption of cash rebate. The rebate may increase the demand of new products because of increasing demand from the customers who would not purchase the product without the rebate. On the other hand, the supply of used products from customers who replace old ones with new ones with the rebate lowers the price of used products and may decrease the demand of new products. They derive market conditions for the existence of a positive rebate and show that, under those conditions, the rebate not only increases the manufacturer's profit, but also the retailer's profit because of the increase in demand even though their margins decrease.

5. WHEN ALL CUSTOMERS CLAIM THE REBATE

Next, we investigate the special case in which $\rho(x) = 1$ and $\tilde{\rho}(x) = 0$ for all $x \geq 0$. Note that in this case, $D_b\big(p, x\big) = (p - x)^{-b}$, and Eq. (6) reduces to

$$r = \frac{b}{L(z^*(r))} - \frac{b-1}{1 - F(z^*(r))}. \tag{16}$$

From our previous analysis, we know that the optimal wholesale price can be obtained by solving Eq. (13). When $\rho \equiv 1$ and $\tilde{\rho} \equiv 0$, the optimal wholesale price is

$$w^*(r) = \frac{bc}{b-1} \cdot \frac{1}{1 - rL(z^*(r))}$$

and the manufacturer's profit becomes

$$\Pi_{\mathrm{M}}(w^*(r), r) = z^*(r)\left[\frac{bc}{b-1} \cdot \frac{1}{1 - rL(z^*(r))}\right]^{-b} \times \left[\frac{1}{1 - F(z^*(r))} - r\right]^{-b} \cdot \frac{c}{b-1}.$$

By Eq. (16), and upon simplification, we have

$$\Pi_{\mathrm{M}}(w^*(r), r) = b^{-2b}(b-1)^{2b-1}c^{-b+1}z^*(r)[L(z^*(r))]^b.$$

We now investigate the manufacturer's profit for the special case in which ε is uniformly distributed in [0, 1]. In this case, $f(y) = 1$, $F(y) = y$, and $L(z) = 1 - \frac{z}{2}$, where $0 \leq z < 2$. Hence,

$$\Pi_{\mathrm{M}}(w^*(r), r) = b^{-2b}(b-1)^{2b-1}c^{-b+1}z^*(r)\left[1 - \frac{z^*(r)}{2}\right]^b. \tag{17}$$

Furthermore, Eq. (16) becomes

$$r = \frac{b}{1 - [z^*(r)/2]} - \frac{b-1}{1 - z^*(r)}.$$

This implies that

$$z^*(r) = \begin{cases} \frac{2}{b+1}, & \text{if } r = 0; \\ \frac{1}{2r}\left[\sqrt{(b+1-3r)^2 - 8r(r-1)} \right. \\ \qquad \left. -(b+1-3r)\right], & \text{if } r > 0. \end{cases}$$

It is easy to show that $z^*(r) < \frac{2}{b+1}$ for any $r \in (0, 1]$. Thus, $z^*(r)$ is maximized when $r = 0$. Define $\phi(z) = z(1 - \frac{z}{2})^b$. Then, $\phi'(z) = (1 - \frac{z}{2})^{b-1}(1 - \frac{b+1}{2}z) \geq 0$ whenever $0 \leq z < \frac{2}{b+1}$. Hence, from (17), we conclude that in this special case $\Pi_{\mathrm{M}}(w^*(r), r)$ increases as $z^*(r)$ increases. This implies that $\Pi_{\mathrm{M}}(w^*(r), r)$ is maximized when $r = 0$. Therefore, in this special case, the optimal decision of the manufacturer is not to offer a rebate.

In summary, in the case with "$\rho \equiv 1$ and $\tilde{\rho} \equiv 0$," it is not necessarily beneficial for the manufacturer to provide rebates. Note that the conditions "$\rho \equiv 1$ and $\tilde{\rho} \equiv 0$" imply that the customers will always claim the rebate, regardless of the size of the rebate. Such a case occurs when the rebate offered by the manufacturer is an instant rebate, where every customer will redeem the rebate on the spot when they make the purchase. An instant rebate of x can easily cause every potential customer to perceive the selling price of the product as $p - x$, and the rebate will have a redemption rate of 100%. However, the analysis in this section implies that, in such a case, a manufacturer rebate does not necessarily help the manufacturer to improve its profit. Note that this result seems contradictory to that of Bruce et al. [6], who demonstrated that a cash rebate with perfect redemption can actually increase the manufacturer's profit. However, in Bruce et al.'s model, they consider the durable goods industry in which used products can be traded. They show that a rebate increases both the manufacturer's and retailer's profits when the fraction of customers who would not purchase the new product without the rebate is sufficiently large and the rebate can effectively solve the negative equity problem of their used product.

Note that in practice there are other scenarios in which an instant rebate is beneficial to the manufacturer. An instant rebate is actually a price reduction and has a long-term effect on the product demand. This long-term effect, which is not captured in our newsvendor model setting, may make the rebate attractive to the manufacturer. Furthermore, an instant rebate can be employed as a temporary price reduction mainly for brand switching [20]. An instant rebate can also be used to alleviate a retailer's arbitrage behavior across the temporally separated markets [3].

6. NUMERICAL STUDIES

Theorem 1 provides evidence that a mail-in rebate benefits the manufacturer unless all customers claim their rebate. However, it does not quantify the magnitude of the benefit and how the rebate impacts the retailer. In this section, we conduct numerical studies to determine the impact of the rebate on the manufacturer's profit, the retailer's profit, and the overall channel profit.

In these numerical studies, the manufacturer's expected profit, denoted as Π_{M}, is given by Eq. (7). That is, for any given rebate factor r,

$$\Pi_{\mathrm{M}}(r) = z^* \left\{ (w^* - c) D_b \left(\frac{w^*}{1 - F(z^*)}, r w^* \right) - [\rho(rw^*) - \tilde{\rho}(rw^*)] \left[\frac{w^*}{1 - F(z^*)} - r w^* \right]^{-b} r w^* L(z^*) \right\}.$$

Here, w^* is obtained numerically by searching for the value of w that maximizes the function, and z^* is obtained by solving Eq. (6) numerically. We assume that the manufacturer is maximizing its own expected profit without considering the impact of the rebate on the retailer's profit. Thus, the manufacturer's optimal expected profit is given as

$$\Pi_{\mathrm{M}}(r^*) = \max_{r \geq 0} \{\Pi_{\mathrm{M}}(r)\}.$$

By Eqs. (4) and (5), the retailer's expected profit is given as

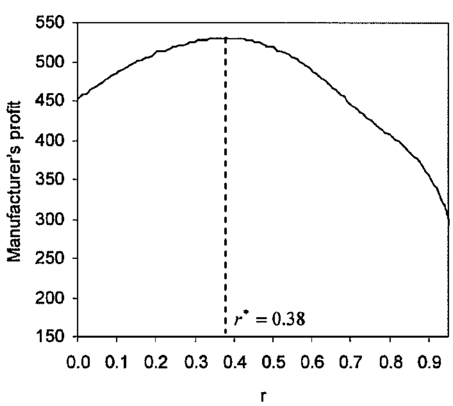
$$\Pi_{\mathrm{R}}(r^*) = D_b \left(\frac{w^*}{1 - F(z^*)}, r^* w^* \right) z^* \left[\frac{w^*}{1 - F(z^*)} \cdot L(z^*) - w^* \right].$$

We let

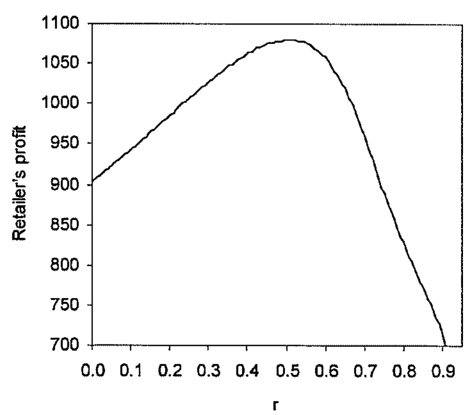
$$\Delta_M = \frac{\Pi_{\mathrm{M}}(r^*) - \Pi_{\mathrm{M}}(0)}{\Pi_{\mathrm{M}}(0)} \times 100\%,$$

which is the percentage increase in the manufacturer's expected profit when the manufacturer introduces a rebate that maximizes its own profit. We let

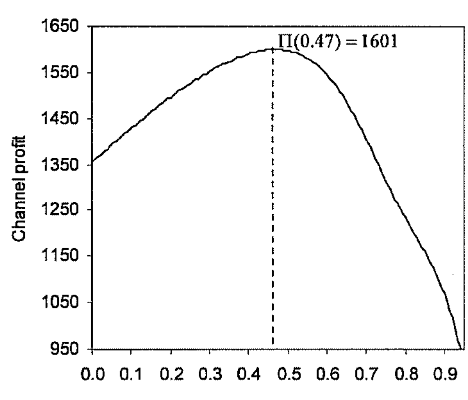
$$\Delta_R = \frac{\Pi_{\mathrm{R}}(r^*) - \Pi_{\mathrm{R}}(0)}{\Pi_{\mathrm{R}}(0)} \times 100\%,$$



(a) $\Pi_M(r)$ versus r



(b) $\Pi_R(r)$ versus r



(c) $\Pi(r)$ versus r

Figure 3. A numerical example with varying rebate factor (a) $\Pi_{\mathrm{M}}(r)$ versus r, (b) $\Pi_{\mathrm{R}}(r)$ versus r, and (c) $\Pi(r)$ versus r.

which is the percentage change in the retailer's expected profit when the manufacturer introduces such a rebate. Note that $\Delta_R < 0$ if the manufacturer rebate results in a decrease in retailer's expected profit. Denote

$$\gamma = \frac{\Pi_{\mathrm{M}}(r^*)}{\Pi_{\mathrm{R}}(r^*)},$$

which is the ratio of the manufacturer's expected profit to the retailer's expected profit. Denote $\Pi(r) = \Pi_{\mathrm{M}}(r) + \Pi_{\mathrm{R}}(r)$, which is the overall channel profit.

Consider an example with $\varepsilon \sim N(10000, 4000^2)$, $c=1$, $b=2$, $\rho(x)=1$ for $x \geq 0$, and $\tilde{\rho}(x) = \frac{1}{2}e^{-x}$ for $x \geq 0$. The values of $\Pi_M(r)$, $\Pi_R(r)$, and $\Pi(r)$ are plotted in Fig. 3. From these graphs, we observe that the manufacturer rebate can benefit both the manufacturer and the retailer. This is consistent with the results of Sections 3 and 4. In this numerical example, $r^* = 0.38$, $\Pi_{\mathrm{M}}(r^*) = 530$, $\Pi_{\mathrm{R}}(r^*) = 1056$, $\Pi_{\mathrm{M}}(0) = 452$, and $\Pi_{\mathrm{R}}(0) = 904$. This implies that $\Delta_M = 17.3\%$, $\Delta_R = 16.8\%$, and $\Pi(r^*) = 1586$. Note that this result is based on the assumption that the manufacturer chooses a rebate factor r^* that maximizes its own benefit. However, the expected channel profit is maximized when $r = 0.47$, where $\Pi(0.47) = 1601$.

Next, we perform a computational study to analyze the impact of the rebate on the expected channel profit. For simplicity, in this computational study, we set $\rho(x) = 1$ and $\tilde{\rho}(x) = \lambda \exp(-\beta x)$ for $x \geq 0$, where $\lambda \geq 0$ and $\beta > 0$. This is the case when all of the customers view the selling price of the product as $p - x$, and when the proportion of customers who forgo the rebate decreases exponentially as the size of the rebate increases. We let ε be a truncated normal random variable with a mean μ and standard deviation σ.

We perform the computational study with various parameter settings. Without loss of generality, we assume that $\mu = 10{,}000$ and $c = 1$ (for any problem instance, we can

Table 1. Numerical results for $\lambda = 1$.

	$b = 1.25$	$b = 1.5$	$b = 2$	$b = 3$
$\beta = 0.5, \sigma = 2000$	$\gamma = 0.198$	$\gamma = 0.334$	$\gamma = 0.509$	$\gamma = 0.658$
	$\Delta_M = 28.2\%$	$\Delta_M = 73.6\%$	$\Delta_M = 227.6\%$	$\Delta_M = 1104.4\%$
	$\Delta_R = 29.2\%$	$\Delta_R = 73.0\%$	$\Delta_R = 222.1\%$	$\Delta_R = 1119.8\%$
$\beta = 0.5, \sigma = 3000$	$\gamma = 0.199$	$\gamma = 0.333$	$\gamma = 0.513$	$\gamma = 0.675$
	$\Delta_M = 23.8\%$	$\Delta_M = 63.5\%$	$\Delta_M = 198.8\%$	$\Delta_M = 957.8\%$
	$\Delta_R = 24.6\%$	$\Delta_R = 63.5\%$	$\Delta_R = 191.2\%$	$\Delta_R = 945.0\%$
$\beta = 0.5, \sigma = 4000$	$\gamma = 0.199$	$\gamma = 0.333$	$\gamma = 0.506$	$\gamma = 0.693$
	$\Delta_M = 20.4\%$	$\Delta_M = 55.1\%$	$\Delta_M = 173.3\%$	$\Delta_M = 819.2\%$
	$\Delta_R = 20.9\%$	$\Delta_R = 55.5\%$	$\Delta_R = 170.0\%$	$\Delta_R = 784.8\%$
$\beta = 0.5, \sigma = 5000$	$\gamma = 0.199$	$\gamma = 0.332$	$\gamma = 0.510$	$\gamma = 0.705$
	$\Delta_M = 18.1\%$	$\Delta_M = 49.2\%$	$\Delta_M = 155.0\%$	$\Delta_M = 720.3\%$
	$\Delta_R = 18.7\%$	$\Delta_R = 49.8\%$	$\Delta_R = 149.8\%$	$\Delta_R = 675.6\%$
$\beta = 1, \sigma = 2000$	$\gamma = 0.200$	$\gamma = 0.334$	$\gamma = 0.502$	$\gamma = 0.672$
	$\Delta_M = 9.6\%$	$\Delta_M = 21.6\%$	$\Delta_M = 51.3\%$	$\Delta_M = 136.8\%$
	$\Delta_R = 9.5\%$	$\Delta_R = 21.4\%$	$\Delta_R = 50.7\%$	$\Delta_R = 134.8\%$
$\beta = 1, \sigma = 3000$	$\gamma = 0.200$	$\gamma = 0.334$	$\gamma = 0.503$	$\gamma = 0.674$
	$\Delta_M = 8.5\%$	$\Delta_M = 19.6\%$	$\Delta_M = 47.4\%$	$\Delta_M = 128.0\%$
	$\Delta_R = 8.5\%$	$\Delta_R = 19.4\%$	$\Delta_R = 46.7\%$	$\Delta_R = 125.5\%$
$\beta = 1, \sigma = 4000$	$\gamma = 0.200$	$\gamma = 0.334$	$\gamma = 0.503$	$\gamma = 0.669$
	$\Delta_M = 7.6\%$	$\Delta_M = 17.8\%$	$\Delta_M = 43.8\%$	$\Delta_M = 119.3\%$
	$\Delta_R = 7.7\%$	$\Delta_R = 17.7\%$	$\Delta_R = 42.9\%$	$\Delta_R = 118.5\%$
$\beta = 1, \sigma = 5000$	$\gamma = 0.200$	$\gamma = 0.334$	$\gamma = 0.500$	$\gamma = 0.671$
	$\Delta_M = 7.0\%$	$\Delta_M = 16.5\%$	$\Delta_M = 41.0\%$	$\Delta_M = 112.4\%$
	$\Delta_R = 7.1\%$	$\Delta_R = 16.4\%$	$\Delta_R = 40.9\%$	$\Delta_R = 111.1\%$
$\beta = 2, \sigma = 2000$	$\gamma = 0.200$	$\gamma = 0.333$	$\gamma = 0.501$	$\gamma = 0.667$
	$\Delta_M = 4.2\%$	$\Delta_M = 9.1\%$	$\Delta_M = 20.2\%$	$\Delta_M = 46.7\%$
	$\Delta_R = 4.3\%$	$\Delta_R = 9.2\%$	$\Delta_R = 19.9\%$	$\Delta_R = 46.5\%$
$\beta = 2, \sigma = 3000$	$\gamma = 0.200$	$\gamma = 0.333$	$\gamma = 0.499$	$\gamma = 0.668$
	$\Delta_M = 3.8\%$	$\Delta_M = 8.4\%$	$\Delta_M = 18.9\%$	$\Delta_M = 44.4\%$
	$\Delta_R = 3.9\%$	$\Delta_R = 8.4\%$	$\Delta_R = 19.1\%$	$\Delta_R = 44.2\%$
$\beta = 2, \sigma = 4000$	$\gamma = 0.200$	$\gamma = 0.333$	$\gamma = 0.499$	$\gamma = 0.668$
	$\Delta_M = 3.4\%$	$\Delta_M = 7.7\%$	$\Delta_M = 17.7\%$	$\Delta_M = 42.1\%$
	$\Delta_R = 3.5\%$	$\Delta_R = 7.8\%$	$\Delta_R = 17.9\%$	$\Delta_R = 41.8\%$
$\beta = 2, \sigma = 5000$	$\gamma = 0.200$	$\gamma = 0.333$	$\gamma = 0.499$	$\gamma = 0.668$
	$\Delta_M = 3.2\%$	$\Delta_M = 7.2\%$	$\Delta_M = 16.8\%$	$\Delta_M = 40.3\%$
	$\Delta_R = 3.3\%$	$\Delta_R = 7.3\%$	$\Delta_R = 16.9\%$	$\Delta_R = 39.9\%$

Table 2. Numerical results for $\lambda = 0.5$.

	$b = 1.25$	$b = 1.5$	$b = 2$	$b = 3$
$\beta = 0.5, \sigma = 2000$	$\gamma = 0.200$ $\Delta_M = 9.6\%$ $\Delta_R = 9.6\%$	$\gamma = 0.334$ $\Delta_M = 21.4\%$ $\Delta_R = 21.2\%$	$\gamma = 0.506$ $\Delta_M = 50.1\%$ $\Delta_R = 48.4\%$	$\gamma = 0.672$ $\Delta_M = 132.8\%$ $\Delta_R = 131.0\%$
$\beta = 0.5, \sigma = 3000$	$\gamma = 0.200$ $\Delta_M = 8.5\%$ $\Delta_R = 8.5\%$	$\gamma = 0.335$ $\Delta_M = 19.4\%$ $\Delta_R = 18.9\%$	$\gamma = 0.508$ $\Delta_M = 45.8\%$ $\Delta_R = 43.4\%$	$\gamma = 0.677$ $\Delta_M = 122.0\%$ $\Delta_R = 118.7\%$
$\beta = 0.5, \sigma = 4000$	$\gamma = 0.200$ $\Delta_M = 7.6\%$ $\Delta_R = 7.6\%$	$\gamma = 0.335$ $\Delta_M = 17.6\%$ $\Delta_R = 16.9\%$	$\gamma = 0.510$ $\Delta_M = 41.9\%$ $\Delta_R = 39.0\%$	$\gamma = 0.682$ $\Delta_M = 111.8\%$ $\Delta_R = 107.0\%$
$\beta = 0.5, \sigma = 5000$	$\gamma = 0.200$ $\Delta_M = 7.0\%$ $\Delta_R = 7.0\%$	$\gamma = 0.335$ $\Delta_M = 16.3\%$ $\Delta_R = 15.8\%$	$\gamma = 0.509$ $\Delta_M = 39.0\%$ $\Delta_R = 36.5\%$	$\gamma = 0.686$ $\Delta_M = 103.9\%$ $\Delta_R = 98.1\%$
$\beta = 1, \sigma = 2000$	$\gamma = 0.200$ $\Delta_M = 4.2\%$ $\Delta_R = 4.3\%$	$\gamma = 0.334$ $\Delta_M = 9.1\%$ $\Delta_R = 9.0\%$	$\gamma = 0.501$ $\Delta_M = 20.0\%$ $\Delta_R = 19.7\%$	$\gamma = 0.668$ $\Delta_M = 45.8\%$ $\Delta_R = 45.5\%$
$\beta = 1, \sigma = 3000$	$\gamma = 0.200$ $\Delta_M = 3.8\%$ $\Delta_R = 3.8\%$	$\gamma = 0.334$ $\Delta_M = 8.3\%$ $\Delta_R = 8.2\%$	$\gamma = 0.502$ $\Delta_M = 18.6\%$ $\Delta_R = 18.2\%$	$\gamma = 0.673$ $\Delta_M = 43.2\%$ $\Delta_R = 41.9\%$
$\beta = 1, \sigma = 4000$	$\gamma = 0.200$ $\Delta_M = 3.4\%$ $\Delta_R = 3.5\%$	$\gamma = 0.334$ $\Delta_M = 7.7\%$ $\Delta_R = 7.5\%$	$\gamma = 0.502$ $\Delta_M = 17.3\%$ $\Delta_R = 16.8\%$	$\gamma = 0.674$ $\Delta_M = 40.6\%$ $\Delta_R = 39.2\%$
$\beta = 1, \sigma = 5000$	$\gamma = 0.200$ $\Delta_M = 3.2\%$ $\Delta_R = 3.2\%$	$\gamma = 0.334$ $\Delta_M = 7.2\%$ $\Delta_R = 6.9\%$	$\gamma = 0.503$ $\Delta_M = 16.3\%$ $\Delta_R = 15.7\%$	$\gamma = 0.674$ $\Delta_M = 38.6\%$ $\Delta_R = 37.0\%$
$\beta = 2, \sigma = 2000$	$\gamma = 0.200$ $\Delta_M = 2.0\%$ $\Delta_R = 2.0\%$	$\gamma = 0.333$ $\Delta_M = 4.2\%$ $\Delta_R = 4.3\%$	$\gamma = 0.501$ $\Delta_M = 9.1\%$ $\Delta_R = 8.9\%$	$\gamma = 0.667$ $\Delta_M = 19.8\%$ $\Delta_R = 19.7\%$
$\beta = 2, \sigma = 3000$	$\gamma = 0.200$ $\Delta_M = 1.8\%$ $\Delta_R = 1.8\%$	$\gamma = 0.333$ $\Delta_M = 3.9\%$ $\Delta_R = 4.0\%$	$\gamma = 0.501$ $\Delta_M = 8.5\%$ $\Delta_R = 8.3\%$	$\gamma = 0.669$ $\Delta_M = 18.8\%$ $\Delta_R = 18.4\%$
$\beta = 2, \sigma = 4000$	$\gamma = 0.200$ $\Delta_M = 1.6\%$ $\Delta_R = 1.6\%$	$\gamma = 0.333$ $\Delta_M = 3.6\%$ $\Delta_R = 3.7\%$	$\gamma = 0.501$ $\Delta_M = 8.0\%$ $\Delta_R = 7.7\%$	$\gamma = 0.669$ $\Delta_M = 17.9\%$ $\Delta_R = 17.4\%$
$\beta = 2, \sigma = 5000$	$\gamma = 0.200$ $\Delta_M = 1.5\%$ $\Delta_R = 1.5\%$	$\gamma = 0.333$ $\Delta_M = 3.4\%$ $\Delta_R = 3.4\%$	$\gamma = 0.500$ $\Delta_M = 7.6\%$ $\Delta_R = 7.5\%$	$\gamma = 0.669$ $\Delta_M = 17.1\%$ $\Delta_R = 16.7\%$

always rescale the quantity unit to obtain $\mu = 10{,}000$ and rescale the monetary unit to obtain $c = 1$). The standard deviation of ε (i.e., σ) is set to 2000, 3000, 4000, and 5000. This corresponds to setting the coefficient of variation of the demand to 0.2, 0.3, 0.4, and 0.5, respectively. The price-elasticity index (i.e., b) is set to 1.25, 1.5, 2, and 3 to cover different scenarios of the price-demand relationship. Parameter β is set to 0.5, 1, and 2. Parameter λ is set to 0, 0.5, and 1. The case with $\lambda = 0$ is the case in which all of the customers claim the rebate.

Tables 1 and 2 summarize the results for the cases of λ = 1 and 0.5, respectively. From these computational results, we observe that γ is approximately $(b-1)/b$ and is nearly independent of σ. In other words, the ratio of the profits between the two parties depends mainly on demand elasticity and has little dependence on demand uncertainty. This observation is consistent with the findings of Song et al. [40], who showed in their buyback contract model that the profit ratio is "distribution-free" and depends only on the curvature of the deterministic demand part.

We also observe that the manufacturer rebate has a larger impact on both the manufacturer's profit and the retailer's profit as b and λ increase and as β and σ decrease. As b increases, the demand is more price-sensitive and, therefore, a rebate attracts more demand. As a result, the rebate leads to a higher channel profit, since not all customers claim the rebate. The overall channel profit is higher as λ increases and as β decreases, because in such a case, more customers forgo the rebate.

As σ increases, we observe from the computational results that the manufacturer rebate has a smaller impact on the expected profit of the system. This can be explained as follows. In all the test instances, as σ increases, the value of $D_b(p^*, r^*w^*)\sigma$ increases; that is, the variability of demand increases. When demand variability gets higher, a higher portion of the cost is incurred in the safety stock. The rebate can help increase the profitability of the sales, but it does not help reduce the cost incurred in the safety stock. As a result, the percentage increase in channel profit due to the rebate is lower.

From Tables 1 and 2, we observe that $\Delta_R > 0$ in all the test instances with $\lambda = 1$ or 0.5. In fact, in most cases, the retailer's percentage increase in expected profit is very close to that of the manufacturer, despite the manufacturer's objective being to maximize its own expected profit. This is because $\Pi_M(0)/\Pi_R(0) = (b-1)/b$ and, as mentioned earlier, $\Pi_M(r^*)/\Pi_R(r^*) \approx (b-1)/b$, which implies that $\Delta_M \approx \Delta_R$. Hence, we conclude that a mail-in rebate not only benefits the manufacturer, but it also benefits the retailer. When $\lambda = 0$, we obtained $\Delta_M = \Delta_R = 0$ in all test instances, which is consistent with the findings in Section 5.

7. CONCLUSIONS

In this paper we analyzed the impact of a manufacturer rebate in a two-stage decentralized supply chain. We demonstrated that unless all of the customers claim the rebate, the rebate always benefits the manufacturer. On the other hand, an instant rebate, where every customer redeems the rebate on the spot when purchasing the product, does not necessarily benefit the manufacturer.

Thus, our results nicely explain the use of mail-in rebates by manufacturers. In this case, the manufacturer reduces the perceived selling price but adds a significant hurdle to the buying process, the need to complete and mail in the rebate. This may cause some customers, even some who value the rebate, not to send the coupon to the manufacturer.

However, our results suggests that instant rebates used by automotive manufacturing companies do not necessarily benefit the manufacturer when they are applied to seasonal products. Indeed, in this case, every customer receives the manufacturer rebate and, thus, the manufacturer's profit does not necessarily increase.

Finally, it is important to point out that this paper would be incomplete if we did not emphasize some important limitations of our model. First, we have omitted the manufacturer's processing costs of a redeemed rebate because most rebate redemptions are handled by rebate clearinghouses, such as Young America, that specialize in processing rebates. These specialized firms can minimize the incremental costs of processing rebates. However, if the processing costs are significant, these costs should be considered as an additional variable cost to serve the customers who claim the rebate. Second, we have assumed that customer demand is a function of their perceived price and random factors at the time of purchase, but not their redemption efforts at the time of requesting the rebate. It is because empirical studies have shown that the decision to purchase a product can be independent of the decision to redeem a rebate later [37, 39]. The efforts that customers realize at the time of redemption, however, will affect $\tilde{\rho}(x)$; that is, the proportion of customers who decide not to exercise the rebate offer. However, if customers take the redemption effort into account at the time of purchase, then this effort should affect the demand and, as a result, product purchase becomes related to the redemption behavior. This would be an interesting direction for future research.

Our analysis relies on the multiplicative form of our stochastic demand model with constant elasticity, and while we distinguish between rebate sensitive and non-sensitive customers, the functions ρ and $\tilde{\rho}$ are deterministic functions. Such a demand model setting significantly simplifies our analysis. In addition, we have ignored salvage value and disposal cost of excess inventory in our model. If a salvage value v or a disposal cost $-v$ is included in the model, then the retailer's expected profit becomes $\Pi_R(p, z, w, r) = p'E[\min\{q, D_b(p, rw)\varepsilon\}] - w'q$, where $p' = p - v$ and $w' = w - v$. Although this equation looks similar to (4), the demand function therein is $D_b(p, rw)$ and not $D_b(p', rw')$. Therefore, the analysis in Section 3 no longer holds. In fact, in such a case there exist scenarios in which the optimal size of the manufacturer rebate is zero even when $\tilde{\rho} \not\equiv 0$.

Our analysis is also limited to a single-period setting. A more general setting would have a multi-period framework in which customers who have forgone a rebate may "learn" from the experience and start ignoring the rebate in their next purchase. However, it is highly challenging to analyze such a generalized model. Of course, while these limitations are valid, our model, as well as similar models in the marketing literature, can serve as a valid approximation providing insights on the benefit of manufacturer rebates.

APPENDIX

PROOF OF LEMMA 1: Taking the derivative over Eq. (6) with respect to r, we obtain

$$G(z^*(w,r), w, r)\frac{\partial z^*(w,r)}{\partial r} + H(z^*(w,r), w, r) = 0,$$

where

$$\begin{aligned} G(z,w,r) = &-2bwD_{b+1}\left(\frac{w}{1-F(z)}, rw\right)\frac{f(z)L(z)}{[1-F(z)]^2} \\ &+ D_b\left(\frac{w}{1-F(z)}, rw\right)\frac{1-F(z)-L(z)}{z} \\ &+ b(b+1)w^2\left[\frac{L(z)}{1-F(z)} - 1\right]D_{b+2}\left(\frac{w}{1-F(z)}, rw\right)\frac{f(z)}{[1-F(z)]^2} \\ &- bwD_{b+1}\left(\frac{w}{1-F(z)}, rw\right)\frac{1-F(z)-L(z)}{z[1-F(z)]} \end{aligned}$$

and

$$H(z,w,r) = bw\rho(rw)\left[\frac{w}{1-F(z)} - rw\right]^{-b-1} L(z)$$
$$- b(b+1)w^2\rho(rw)\left[\frac{w}{1-F(z)} - rw\right]^{-b-2}\left[\frac{L(z)}{1-F(z)} - 1\right]$$
$$+ w\rho'(rw)\left\{\left[\frac{w}{1-F(z)} - rw\right]^{-b} - \left[\frac{w}{1-F(z)}\right]^{-b}\right\} L(z) - bw^2\rho'(rw)$$
$$\times\left\{\left[\frac{w}{1-F(z)} - rw\right]^{-b-1} - \left[\frac{w}{1-F(z)}\right]^{-b-1}\right\}\left[\frac{L(z)}{1-F(z)} - 1\right].$$

When $w = w^*(r)$ and $r = 0$,

$$G(z, w^*(0), 0) = -\left[\frac{w^*(0)}{1-F(z)}\right]^{-b}\left[b(1-b)\frac{f(z)L(z)}{1-F(z)}\right.$$
$$\left. - (1-b)\ \frac{1-F(z)-L(z)}{z} + b(b+1)f(z)\right].$$

By (8), this implies that

$$G(z^*(0), w^*(0), 0) = \left[\frac{w^*(0)}{1-F(z^*(0))}\right]^{-b}\left[\frac{1-F(z^*(0))}{z} - bf(z^*(0))\right].$$

Also,

$$H(z, w^*(0), 0)$$
$$= b\rho(0)\left[\frac{w^*(0)}{1-F(z)}\right]^{-b}\{(b+1)[1-F(z)]^2 - bL(z)[1-F(z)]\},$$

which implies that

$$H(z^*(0), w^*(0), 0) = -\rho(0)\left[\frac{w^*(0)}{1-F(z^*(0))}\right]^{-b}[1-F(z^*(0))]L(z^*(0)).$$

Thus,

$$\left[\frac{w^*(0)}{1-F(z^*(0))}\right]^{-b}\left[\frac{1-F(z^*(0))}{z} - bf(z^*(0))\right]\cdot\frac{\partial z^*(w,r)}{\partial r}\bigg|_{r=0,w=w^*(0)}$$
$$- \rho(0)\left[\frac{w^*(0)}{1-F(z^*(0))}\right]^{-b}[1-F(z^*(0))]L(z^*(0)) = 0,$$

or equivalently,

$$\frac{\partial z^*(w,r)}{\partial r}\bigg|_{r=0,w=w^*(0)} = \frac{\rho(0)L(z^*(0))}{\frac{1}{z^*(0)} - \frac{bf(z^*(0))}{1-F(z^*(0))}}. \qquad \square$$

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