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Joint pricing and inventory management with deterministic demand and costly price adjustment

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ABSTRACT

We analyze a joint inventory and pricing model of a single product over a finite planning horizon with deterministic demand. In this model, an ordering quantity and a price are decided simultaneously at the beginning of each period, demand of the period depends on the price, and a price adjustment cost is incurred if the price is changed from the previous period. We develop polynomial time algorithms to maximize the total profit and discuss their computational complexity.

1. Introduction

Thanks to the development of new technologies, there is growing literature in developing and analyzing sophisticated mathematical models that integrate pricing and inventory decisions (see [4] for an up-to-date review). A predominate assumption in the literature is that price adjustment is costless. Yet, as evidenced by various empirical studies in economics, changing prices requires a significant amount of resources and the associated costs cannot be ignored in many business settings. For example, Levy et al. [16] notice in their study of supermarket chains that the price adjustment cost takes up as much as 35% of the reported profits. Zbaracki et al. [24] study the pricing practices of a one-billion-dollar industrial firm and observe that "the price adjustment costs comprise 1.22% of the company's revenue and 20.03% of the company's net margin". Similar observations are also made by Slade [18], Aguirregabiria [2] and Kano [14].

The purpose of this paper is to develop and analyze a joint pricing and inventory model with price adjustment costs and deterministic demands. Specifically, we consider a single product periodic review model over a finite planning horizon. In this model, an ordering quantity and a price are decided simultaneously at the beginning of each period. The demand of a period depends on the price in the current period. Similar to the classical economic lot sizing model, the replenishment incurs a fixed ordering cost and a variable ordering cost. After the demand of a period is satisfied, left over inventory is carried over to the next period incurring an inventory holding cost. In contrast to the majority of the literature, we assume that a price adjustment cost is incurred if the current price is changed from the previous period. The objective is to determine a joint ordering and pricing plan so as to maximize the total profit over the planning horizon.

The paper belongs to the rapidly growing literature on joint inventory and pricing models. Recently, significant progress has been made on analyzing joint inventory and pricing models without price adjustment cost (see, e.g., [5,6] for stochastic models, and [8,11,10] for deterministic models). Our contribution is to introduce costs associated with price adjustment into joint inventory and pricing models in *deterministic* settings, which are predominately ignored in the literature. The only exceptions are Aguirregabiria [2] and Chen et al. [7], who analyze the stochastic counterpart of our model. Two other related papers are Netessine [17] and Celik et al. [3], where Netessine [17] recognizes the importance of the impact of price adjustment costs on inventory and pricing decisions and formulates a deterministic continuous-time model to optimize the timing of a fixed number of price changes, and Celik et al. [3] analyze a continuous-time stochastic revenue management problem with costly price changes, which however does not consider inventory replenishment decisions and thus does not capture the intricate interaction of ordering and pricing.

As we illustrate in the next section, our model includes several integrated inventory and pricing models in the literature as special

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cases. For example, if price can be changed freely without any associated cost, it reduces to the joint inventory and dynamic pricing model analyzed in [22,19]. If the price adjustment cost is so high that prohibits any price change, our model reduces to the joint inventory and static pricing model analyzed in [15,12,20]. As we will demonstrate later, our model with general price adjustment costs becomes much more difficult. Nevertheless, we manage to develop polynomial time algorithms to finding the optimal coordinated pricing and ordering plan under a variety of settings. The remainder of the paper is organized as follows. The problem statement and associated mathematical formulation are given in Section 2. In Section 3, we develop algorithms to determine the optimal ordering and pricing decisions. Finally, we conclude with several remarks in Section 4.

2. Model and preliminaries

Consider a firm that makes joint ordering and pricing decisions to satisfy a sequence of demands of a single product over a planning horizon with T periods. At the beginning of each period, an ordering quantity $y \geq 0$ and a price $p \in \mathcal{P}$ are determined simultaneously, where $\mathcal{P}$ is a closed interval. The replenishment incurs a fixed ordering cost $K\delta(y)$ and a variable ordering cost cy, where $\delta(0) = 0$ and $\delta(y) = 1$ for $y > 0$. Suppose that orders are delivered instantaneously and no backlogging is allowed. Inventory left at period t, denoted by I_t, is carried over to the next period with a marginal holding cost h. The demand of period t is modeled as a deterministic function of p by $D_t(p) = a_t d(p)$, where $a_t \geq 0$ and $d(p)$ is decreasing in p.

In contrast to most papers in the literature on joint inventory and pricing models, we assume that a cost $f(\tilde{p} - p)$ incurs if p and $\tilde{p}$ are prices in two consecutive periods, where $f(0) = 0, f(x) = u^+$ if $x > 0$ and $f(x) = u^-$ if $x < 0$ for some $u^+, u^- \geq 0$. Similar price adjustment cost structures have been proposed and analyzed in the literature. For example, Aguirregabiria [2] and Kano [14] consider the symmetric case $u^+ = u^-$. These costs, referred to as the menu costs or physical costs, are associated with activities like "constructing new price lists, printing and distributing new list prices and monthly supplemental price sheets, and notifying suppliers [24]". Our model allows asymmetric price adjustment costs to reflect the fact that firms may take different actions in response to price markdown and price markup.

Suppose that the initial inventory level $I_0 = 0$. The objective of the firm is to decide ordering quantities y_t and prices p_t in all periods so as to maximize the total profit. Mathematically, the firm faces the problem:

$$\max_{p_t, y_t, I_t} \quad \sum_{t=1}^{T} a_t p_t d(p_t) - \sum_{t=2}^{T} f(p_t - p_{t-1}) - \sum_{t=1}^{T} [K\delta(y_t) + cy_t + hI_t] \tag{1a}$$

$$\text{s.t.} \quad I_t = I_{t-1} + y_t - a_t d(p_t), \quad \forall 1 \leq t \leq T, \tag{1b}$$

$$I_t \geq 0, \qquad y_t \geq 0, \qquad p_t \in \mathcal{P}, \quad \forall 1 \leq t \leq T, \tag{1c}$$

where the three terms in (1a) represent the total revenue, price adjustment costs and inventory-related costs, respectively. The inventory balance equation (1b) and $I_t \geq 0$ in (1c) ensure that no demand is backlogged. The feasible sets of variables are given in (1c). It should be mentioned that our analysis and results hold under more general settings (e.g. the cost parameters K, c, h are time-dependent); see the discussion in Section 4.

Several important inventory (and pricing) models can be cast as special cases of the above problem. First, when $\mathcal{P}$ is a singleton set, (1) reduces to the economic lot sizing problem first analyzed in [23]. The authors show that it can be solved in $O(T^2)$ by finding a shortest path in an appropriately constructed acyclic network. More efficient algorithms with a running time $O(T \log T)$ are proposed by Aggarwal and Park [1], Federgruen and Tzur [9] and Wagelmans et al. [21]. In addition, if there is no speculative motive on holding inventories, i.e. $c_t + h_t \geq c_{t+1}$ for all $t < T$, then a T-period economic lot sizing problem can be solved in an $O(T)$ time as proved in, e.g., [9]. In the literature, the so-called zero inventory ordering property plays a very key role. It says that in an optimal plan an order is placed only when the inventory level drops to zero. The property also implies that if t is a reorder period, then the optimal ordering plan over periods $\{1, 2, \ldots, t-1\}$ can be determined independently of that over $\{t, t+1, \ldots, T\}$.

Second, if no price adjustment cost is incurred, i.e., $f(x) = 0$, then (1) reduces to the joint inventory and dynamic pricing model studied by Wagner and Whitin [22] and Thomas [19]. The basic idea is to construct an equivalent longest path problem on some network in a similar way as Wagner and Whitin [23]. Such problem can be solved in an $O(T^2)$ time.

Finally, if the price adjustment cost is very high, i.e., $f(x) = +\infty$ for any $x \neq 0$, then (1) reduces to the joint inventory and static pricing model analyzed by Kunreuther and Schrage [15] in which a constant price is determined at the beginning of the planning horizon. In this case, problem (1) becomes

$$\max_{p \in \mathcal{P}} [R(p, 1, T+1) - C(d(p))],$$

where $R(p, s, \tilde{s}) = \sum_{s \leq t < \tilde{s}} a_t p d(p)$ denotes the total revenue obtained from periods s to $\tilde{s} - 1$, and $C(d)$ defined below indicates the total inventory-related cost:

$$C(d) = \min_{y_t, I_t} \sum_{t=1}^{T} [K\delta(y_t) + cy_t + hI_t] \tag{2}$$

$$\text{s.t. } I_t = I_{t-1} + y_t - a_t d, \quad \forall 1 \leq t \leq T,$$

$$I_0 = 0, \qquad I_t \geq 0, \qquad y_t \geq 0, \quad \forall 1 \leq t \leq T.$$

Kunreuther and Schrage [15] show that $C(d)$ is concave and piecewise linear. Gilbert [12] illustrates that $C(d)$ consists of at most T linear pieces. Moreover, van den Heuvel and Wagelmans [20] prove the following result.

Lemma 1. *The function $C(d)$ defined by (2) consists of at most T linear pieces and its expression can be obtained in $O(T^2)$ time.*

3. Main results

In this section, we will derive polynomial time algorithms for problem (1) under various conditions, where the main idea is to construct an equivalent longest path problem. Recall that it is well known from the network flow literature that a longest path in an acyclic network with n links can be found in $O(n)$ time.

Throughout this paper, we make the following assumption on the function $d(p)$.

Assumption 1. For any given constants A_1 and A_2, the function $pd(p) + A_1 d(p) + A_2 p$ has $O(1)$ *local maximizers* in $\mathcal{P}$, and it takes $O(1)$ time to find all of them.

Observe that van den Heuvel and Wagelmans [20] implicitly use a weaker assumption that a global maximizer of the function $\varphi(p) = pd(p) + A_1 d(p) + A_2 p$ can be found in $O(1)$ time. As we will see later, it is not sufficient to consider only the global maximizer in our algorithm. Nevertheless, when $d(p)$ is linear and strictly decreasing, the associated $\varphi(p)$ is strictly concave and hence the two assumptions hold and are equivalent.

Before we proceed, several definitions are introduced as follows to simplify the discussion. First, observe that we can equivalently express the price adjustment cost $f(\tilde{P}-P)=u^{\tilde{\alpha}}$ whenever $P\neq\tilde{P}$, where $\tilde{\alpha}$, called the *price adjustment indicator* from P to $\tilde{P}$, satisfies that

$$\tilde{\alpha}\in\{+1,-1\}\quad\text{and}\quad\tilde{\alpha}(\tilde{P}-P)>0. \tag{3}$$

Moreover, for any given $\{p_t:1\le t\le T\}$, notice that it corresponds to a unique sequence $\{(s_n,\alpha_n,P_n):1\le n\le N\}$ such that the following conditions hold:

(a) Any two consecutive triples (s,α,P) and $(\tilde{s},\tilde{\alpha},\tilde{P})$ in the sequence satisfy (3) and

$$1\le s<\tilde{s}\le T+1\quad\text{and}\quad\alpha,\tilde{\alpha}\in\{-1,+1\}. \tag{4}$$

(b) $s_1=1$, $s_N=T+1$ and $\alpha_1=+1$;
(c) $p_t=P_n$ when $s_n\le t<s_{n+1}$.

Later we call such sequence a *pricing plan* and s_n are *price adjustment periods*, where 1 and $T+1$ are treated as artificial price adjustment periods for convenience.

3.1. Zero fixed ordering cost case

To illustrate the idea more clearly, we first consider the special case that no fixed ordering cost is charged (i.e., $K=0$), then move to the general case. Observe that it is optimal to order in every period when $K=0$, and the minimal total inventory-related cost is $\sum_{t=1}^{T}cD_t$ for any demand sequence $\{D_t\}$. It remains to determine the optimal pricing plan $\{(s_n,\alpha_n,P_n):1\le n\le N\}$. For this purpose, we partition the planning horizon according to price adjustment periods s_n and rewrite the associated total profit as

$$\sum_{n=1}^{N}G(P_n,s_n,\alpha_n,s_{n+1},\alpha_{n+1}),$$

where $G(p,s,\alpha,\tilde{s},\tilde{\alpha})$ defined below represents the total profit obtained in periods $s,s+1,\ldots,\tilde{s}-1$:

$$R(p,s,\tilde{s})-u^{\alpha}-\sum_{s\le t<\tilde{s}}c[a_td(p)].$$

Let $\mathcal{P}(s,\alpha,\tilde{s},\tilde{\alpha})$ be the set of local maximizers of $G(p,s,\alpha,\tilde{s},\tilde{\alpha})$ in term of $p\in\mathcal{P}$. Since that $P_{n-1}\neq P_n$ and $P_n\neq P_{n+1}$ by (3), slightly modifying P_n does not change the associated sequence $\{(s_n,\alpha_n):1\le n\le N\}$. It implies that $P_n\in\mathcal{P}(s_n,\alpha_n,s_{n+1},\alpha_{n+1})$ by the optimality of the pricing plan. This observation motivates us to convert (1) to an equivalent longest path problem.

Specifically, construct a directed network $(\mathcal{V},\mathcal{E})$ with the node set $\mathcal{V}$ and the link set $\mathcal{E}$ respectively given by

$$\begin{aligned}\mathcal{V}&=\{\mathbf{v}=(P,s,\alpha,\tilde{s},\tilde{\alpha}):P\in\mathcal{P}(s,\alpha,\tilde{s},\tilde{\alpha})\\&\qquad\text{and (4) holds}\}\cup\{\mathbf{v}^0,\mathbf{v}^e\},\\ \mathcal{E}&=\{\langle\mathbf{v},\tilde{\mathbf{v}}\rangle:\mathbf{v}=(P,s,\alpha,\tilde{s},\tilde{\alpha})\in\mathcal{V}\text{ and}\\&\qquad\tilde{\mathbf{v}}=(\tilde{P},\tilde{s},\tilde{\alpha},s',\alpha')\in\mathcal{V}\text{ satisfy (3)}\},\end{aligned}$$

where $\mathbf{v}^0=(p_0,1,+1,1,+1)$ and $\mathbf{v}^e=(p_0,T+1,-1,T+1,-1)$ for some $p_0<\min\mathcal{P}$. For each $\langle\mathbf{v},\tilde{\mathbf{v}}\rangle\in\mathcal{E}$, we assign the length $\ell\left(\mathbf{v},\tilde{\mathbf{v}}\right)=0$ if $\mathbf{v}=\mathbf{v}^0$ and $G(P,s,\alpha,\tilde{s},\tilde{\alpha})$ if $\mathbf{v}=(P,s,\alpha,\tilde{s},\tilde{\alpha})\neq\mathbf{v}^0$.

In the network, $\mathbf{v}^0$ and $\mathbf{v}^e$ are artificial nodes introduced as the origin and the destination of the longest path to be constructed. There is neither an incoming link to $\mathbf{v}^0$ nor an outgoing link to $\mathbf{v}^e$ by (3); hence $(\mathcal{V},\mathcal{E})$ is an acyclic network. A typical node $\mathbf{v}\notin\{\mathbf{v}^0,\mathbf{v}^e\}$ specifies consecutive price adjustment periods $s<\tilde{s}$, associated price change indicators $\alpha,\tilde{\alpha}$, and a constant price $P\in\mathcal{P}(s,\alpha,\tilde{s},\tilde{\alpha})$ from period s to period $\tilde{s}-1$. Furthermore, a path from $\mathbf{v}^0$ to $\mathbf{v}^e$ specifies a feasible pricing plan $\{(s_n,\alpha_n,P_n):1\le n\le N\}$, and its length is equal to the total profit associated with the pricing plan. On the other hand, an optimal pricing plan $\{(s_n,\alpha_n,P_n):1\le n\le N\}$ of problem (1) corresponds to some path from $\mathbf{v}^0$ to $\mathbf{v}^e$. Thus determining an optimal pricing plan is equivalent to finding a longest path from $\mathbf{v}^0$ to $\mathbf{v}^e$ in the acyclic network $(\mathcal{V},\mathcal{E})$.

We next discuss the computational complexity. To obtain a link length, it suffices to maximize some function $G(p,s,\alpha,\tilde{s},\tilde{\alpha})$ of p over $\mathcal{P}$, where G has the form $A_0Pd(P)+A_1d(P)+A_2P+A_3$ for some $A_0\ge0$. By Assumption 1, each $\mathcal{P}(s,\alpha,\tilde{s},\tilde{\alpha})$ has $O(1)$ elements and can be determined in $O(1)$ time. Therefore it takes $O(T^2)$ time to prepare for all link lengths. In addition, the network has $O(T^3)$ links and the longest path can be found in $O(T^3)$ time. In summary, it follows the results below.

Theorem 1. *Problem (1) can be solved in $O(T^3)$ time if Assumption 1 holds and there is no fixed ordering cost.*

3.2. General case

Unlike the case with zero fixed ordering cost, the optimal ordering plan depends on realized demands and hence cannot be determined independent of pricing plans. It makes solving the general problem much more involved than the case with $K=0$. To circumstance the difficulty, we take into account the reorder period associated with each price adjustment period.

Specifically, consider two consecutive price adjustment periods $s<\tilde{s}$ with $p_t=P$ for $s\le t<\tilde{s}$. Let $d=d(P)$ and $\tau,\tilde{\tau}$ be reorder periods associated with $s,\tilde{s}$, respectively. There are two cases as follows:

$$\text{either }\tau=\tilde{\tau}\le s<\tilde{s}\quad\text{or}\quad\tau\le s<\tilde{\tau}\le\tilde{s}. \tag{5}$$

The first case in (5) indicates that all demands from periods s to $\tilde{s}-1$ are satisfied by an order at period τ. Hence the total inventory-related costs from s to $\tilde{s}-1$ is

$$C^0(d,\tau,s,\tilde{s})=K(\tau,s,\tilde{s})+\sum_{s\le t<\tilde{s}}c(\tau,t)(a_td+b_t),$$

where $K(\tau,s,\tilde{s})=K$ if $s=\tau<\tilde{s}$ and 0 otherwise, and

$$c(s,t)=c+(t-s-1)h$$

represents the marginal cost to satisfy the demand of period t by an order at period s. Note that the definition $K(\tau,s,\tilde{s})$ implies that the fixed cost incurred at period τ is included in the total inventory-related costs from s to $\tilde{s}-1$ only when $s=\tau$.

For the second case in (5), demands from period τ to period s are satisfied by an order at period τ, and demands from periods $\tilde{\tau}$ to $\tilde{s}$ are satisfied by the order at period $\tilde{\tau}$. By the zero inventory ordering property, the minimal inventory-related cost incurred to satisfy the demands from periods s to $\tilde{\tau}-1$, denoted by $C^1(d,\tau,s,\tilde{\tau})$, can be expressed as

$$\begin{aligned}\min_{y_t,I_t}\quad&[K(\tau,s,\tilde{\tau})+c(\tau,s)y_s+h_s\tilde{I}_s]\\&+\sum_{s<t<\tilde{\tau}}[K\delta(y_t)+cy_t+h\tilde{I}_t]\\ \text{s.t.}\quad&\tilde{I}_t=\tilde{I}_{t-1}+y_t-a_td,\quad\forall s\le t<\tilde{s},\\&\tilde{I}_{s-1}=0,\qquad\tilde{I}_t\ge0,\qquad y_t\ge0,\quad\forall s\le t<\tilde{s}.\end{aligned}$$

Notice that here the fixed ordering cost incurred at period τ is assigned to the total inventory related cost from period s to period $\tilde{s}-1$ only when $s=\tau$. We also observe that the total inventory-related costs from period $\tilde{\tau}$ to period $\tilde{s}-1$ is $C^0(d,\tilde{\tau},\tilde{\tau},\tilde{s})$.

To summarize the two cases in (5), the minimal inventory-related cost from period s to period $\tilde{s}-1$, denoted by $C(d,\tau,s,\tilde{\tau},\tilde{s})$, can be expressed by

$$\begin{cases}C^0(d,\tau,s,\tilde{s}), & \text{if }\tau=\tilde{\tau},\\ C^1(d,\tau,s,\tilde{\tau})+C^0(d,\tilde{\tau},\tilde{\tau},\tilde{s}), & \text{if }\tau<\tilde{\tau}.\end{cases}$$

From the above discussion, we know that for an optimal pricing plan $\{(s_n, \alpha_n, P_n) : 1 \leq n \leq N\}$, if the demand of period s_n is satisfied by an order at period τ_n, then the maximal total profit can be expressed by

$$\sum_{n=1}^{N} G(P_n, \tau_n, s_n, \alpha_n, \tau_{n+1}, s_{n+1}, \alpha_{n+1}),$$

where $G(P, \tau, s, \alpha, \tilde{\tau}, \tilde{s}, \tilde{\alpha})$ defined below represents the maximal total profit from periods s to $\tilde{s} - 1$:

$$R(P, s, \tilde{s}) - u^{\alpha} - C(d(P), \tau, s, \tilde{\tau}, \tilde{s}).$$

We are ready to convert problem (1) to a longest path problem. Let $\mathcal{P}(\tau, s, \alpha, \tilde{\tau}, \tilde{s}, \tilde{\alpha})$ be the set of all local maximizers of the function G in term of $p \in \mathcal{P}$. Construct a network $(\mathcal{V}, \mathcal{E})$ with

$$\begin{aligned}\mathcal{V} &= \{(P, \tau, s, \alpha, \tilde{\tau}, \tilde{s}, \tilde{\alpha}) : P \in \mathcal{P}(\tau, s, \alpha, \tilde{\tau}, \tilde{s}, \tilde{\alpha}), \\ &\quad (4) \text{ and } (5) \text{ hold}\} \cup \{\mathbf{v}^0, \mathbf{v}^e\}, \\ \mathcal{E} &= \{\langle \mathbf{v}, \tilde{\mathbf{v}} \rangle : \mathbf{v} = (P, \tau, s, \alpha, \tilde{\tau}, \tilde{s}, \tilde{\alpha}) \in \mathcal{V} \text{ and} \\ &\quad \tilde{\mathbf{v}} = (\tilde{P}, \tilde{\tau}, \tilde{s}, \tilde{\alpha}, \tau', s', \alpha') \in \mathcal{V} \text{ satisfy } (3)\},\end{aligned}$$

where $\mathbf{v}^0 = (p_0, 1, 1, +1, 1, 1, +1)$ and $\mathbf{v}^e = (p_0, T+1, T+1, -1, T+1, T+1, -1)$ for some $p_0 < \min \mathcal{P}$. For each $\langle \mathbf{v}, \tilde{\mathbf{v}} \rangle \in \mathcal{E}$, assign the length $\ell(\mathbf{v}, \tilde{\mathbf{v}}) = 0$ if $\mathbf{v} = \mathbf{v}^0$ and $G(P, \tau, s, \alpha, \tilde{\tau}, \tilde{s}, \tilde{\alpha})$ if $\mathbf{v} = (P, \tau, s, \alpha, \tilde{\tau}, \tilde{s}, \tilde{\alpha}) \neq \mathbf{v}^0$.

By a similar argument as the previous subsection, one can see that determining an optimal pricing plan is equivalent to finding a longest path in $(\mathcal{V}, \mathcal{E})$. Furthermore, once the optimal pricing plan is specified, the optimal ordering plan can be determined by solving an economic lot sizing problem of the form (2).

We now consider the computational complexity. To construct the network, we need the expressions of linear functions $C^0(d, \tau, s, t)$ and piecewise linear functions $C^1(d, \tau, s, t)$ for all combinations (τ, s, t), where the former can be obtained in $O(T^3)$, and the latter in $O(T^5)$ time by Lemma 1. Therefore all $C(d, \tau, s, \tilde{\tau}, \tilde{s})$ can be determined in $O(T^5)$ time and each of them consists of at most T linear pieces. It implies that the function $G(P, \tau, s, \alpha, \tilde{\tau}, \tilde{s}, \tilde{\alpha})$ of P consists of $O(T)$ pieces of the form $A_0Pd(P) + A_1d(P) + A_2P + A_3$ for some $A_0 \geq 0$. By Assumption 1, the set $\mathcal{P}(\tau, s, \alpha, \tilde{\tau}, \tilde{s}, \tilde{\alpha})$ has $O(T)$ elements and can be obtained in $O(T)$ time. Therefore the network has $O(T^5)$ nodes and $O(T^8)$ links whose lengths can be obtained in $O(T^5)$ time. Thus, it takes $O(T^8)$ time to find the optimal pricing plan plus an additional $O(T)$ time for the optimal ordering plan. In summary, we have the following results.

Theorem 2. *Determining the optimal pricing plan of problem (1) is equivalent to finding a longest path from $\mathbf{v}^0$ to $\mathbf{v}^e$ in the network $(\mathcal{V}, \mathcal{E})$, where the network has $O(T^5)$ nodes and $O(T^8)$ links whose lengths can be determined in $O(T^5)$ time if Assumption 1 holds. Furthermore, it takes $O(T^8)$ time to determine the optimal pricing plan plus an additional $O(T)$ time to obtain the optimal ordering plan of the form (2).*

4. Concluding remarks

In this paper, we present a joint pricing and inventory management model with deterministic demand and price adjustment cost, and develop polynomial time algorithms to solve problem (1), where the basic idea is to construct an acyclic network such that determining the optimal pricing plan of problem (1) is equivalent to finding the longest path in the network. The same idea also works for the general problem with time-dependent cost parameters $K_t, c_t, h_t, u_t^{\pm}$ and the demand model $D_t(p) = a_t d(p) + b_t$. Note that under the general settings the function $C(d)$ previously defined by (2) becomes

$$\begin{aligned} C(d) = \min_{y_t, I_t} \quad & \sum_{t=1}^{T} [K_t \delta(y_t) + c_t y_t + h_t I_t] \\ \text{s.t.} \quad & I_t = I_{t-1} + y_t - (a_t d + b_t), \quad \forall 1 \leq t \leq T, \\ & I_0 = 0, \quad I_t \geq 0, \quad y_t \geq 0, \quad \forall 1 \leq t \leq T. \end{aligned}$$

The computational complexity to solve the general problem depends on how many linear pieces the above function $C(d)$ has. If $C(d)$ consists of S_T linear pieces, then its expressions can be determined by solving S_T economic lot sizing problems with the same parameters a_t, b_t, K_t, c_t and h_t by van den Heuvel and Wagelmans [20] (they in fact claim that $S_T = O(T^2)$; however, there is a flaw in their proof according to our private communication with van den Heuvel). Recall that solving a general economic lot sizing problem takes $O(T \log T)$ time. Similar to Theorem 2, if Assumption 1 holds then the acyclic network has $O(S_T T^4)$ nodes and $O(S_T^2 T^6)$ links whose lengths can be constructed in $O(S_T T^4 \log T)$ time. Furthermore, the optimal pricing and ordering plan can be determined in $O(S_T^2 T^6)$ time.

We provide several additional remarks as below, where the detailed discussions can be found in [13].

Remark 1. In the model we assumed that the price can take all possible values within the interval $\mathcal{P}$. However, for practical purposes, one often focus on finite number of predetermined price levels. In the case that $\mathcal{P}$ has S_P elements, the problem can be handled in a similar way. The only difference is that when constructing the network, we can simply focus on all S_P price levels rather than finding local maximizers of the function G of p. Specifically, solving the problem is equivalent to finding a longest path in an acyclic network; moreover, it takes an $O(S_P T^4 \log T)$ time to construct the network and an $O(S_P^2 T^4)$ time to find the desirable longest path.

Remark 2. In general, S_T could be large and hence we may have an extremely large acyclic network. Interestingly, as demonstrate numerically in [13], we observe that the ordering plan is quite stable under different discretization levels of the prices. This suggests the following efficient heuristics: we first solve the joint inventory and pricing problem with a coarse discretization of the prices to identify an ordering plan; then fix the ordering plan and solve the joint inventory and pricing problem with zero ordering cost, which based on Theorem 1 can be solved very efficiently.

Remark 3. Celik et al. [3] argue that the price adjustment cost may also depend on inventory level I on hand. Specifically, they consider a price adjustment cost of the form $f(\tilde{p} - p) + f_0(I)$, where $f_0(I) = K_0 + c_0 I$ denotes the inventory-related cost. In fact, the main idea of our approach also works for price adjustment cost functions in the following form:

$$f(\tilde{p}, p, I) = \begin{cases} u^+(I) + v^+(\tilde{p}) + w^+(p) & \text{if } \tilde{p} > p, \\ u^-(I) + v^-(\tilde{p}) + w^-(p) & \text{if } \tilde{p} < p, \end{cases}$$

where $u^{\pm}(I)$, $v^{\pm}(p)$ and $w^{\pm}(p)$ can be general continuous functions such that $u^{\pm}(I) \geq 0$.

Remark 4. In the case $u^+ = u^-$, the price adjustment cost is independent of price adjustment directions and hence we do not need to verify condition (3) when construct the network. The problem in this case can be solved more efficiently. Specifically, when there is no fixed ordering cost, it is equivalent to finding a longest path in an acyclic network, and it takes an $O(T^2)$ time in total to solve the problem. In addition, solving the general problem is equivalent to finding a longest path in another network, and it takes an $O(S_T T^5 \log T)$ time in total to solve the problem.

There are several interesting extensions along different directions. First, it is interesting to see the algorithms can be improved.

Observe that when we construct the acyclic networks, a variety of optimization subproblems with small differences have to be solved. Thus, one direction is to eliminate possible redundant computations in solving these subproblems. Moreover, solving such a subproblem indeed corresponds to deciding the optimal price for some joint static pricing and inventory model. In the general case we assume that $C(d)$ defined by T-period problem (2) consists of S_T linear pieces. However, it is not clear if S_T is polynomial in T, which may constitute another direction. Finally, it remains a challenge to incorporate ordering capacity constraints into our model. In this case, breaking down the total profit to terms involving single constant prices becomes impossible because the zero inventory ordering property does not hold anymore. Even if this could be done, it is likely that we have to solve the joint static pricing and inventory model with capacity constraints as a subroutine, which itself is challenging.

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