SUPPLEMENTARY MATERIALS FOR THE PAPER AOS1304-021: "COVARIANCE AND PRECISION MATRIX ESTIMATION FOR HIGH-DIMENSIONAL TIME SERIES"

BY XIAOHUI CHEN, MENGYU XU AND WEI BIAO WU

Proof of Relation (64), Spectral norm convergence rate for precision matrix

Proof. We follow the argument in Rothman et al (2008). Let $\hat{\Delta} = \hat{K}_\lambda - K$, $\Xi = \hat{R} - R$, $\mathcal{S}_u = \{(j,k) : |\omega_{jk}| \geq u, j \neq k\}$, $\mathcal{S}_u^c = \{(j,k) : |\omega_{jk}| < u, j \neq k\}$ and $\mathcal{W}_u = \{(j,k) : |\xi_{jk}| \geq u, j \neq k\}$. Clearly, $\xi_{jj} = 0$. Since $\varepsilon_0 \leq \rho(\Sigma) = \rho(\Omega^{-1}) \leq \varepsilon_0^{-1}$, then for all j, $\varepsilon_0^{1/2} \leq v_{jj} \leq \varepsilon_0^{-1/2}$. Note that $K = V\Omega V$ and $K_{jk} = \omega_{jk} v_{jj} v_{kk}$, we have

$$\begin{aligned} |K^-_{\mathcal{S}_u^c}|_1 &= \sum_{j \neq k} |K_{jk}| \mathbb{I}(|\omega_{jk}| < u) \\ &\leq \varepsilon_0^{-1} \sum_{j \neq k} |\omega_{jk}| \mathbb{I}(|\omega_{jk}| < u) \\ &\leq \varepsilon_0^{-1} p^2 u^{-1} D^-(u). \end{aligned}$$

By the argument of proving Theorem 3.1, we have that

$$|\hat{\Delta}|_F^2 \lesssim |\Xi_{\mathcal{W}_u}|_F^2 + u^2 S_u + u |K^-_{\mathcal{S}_u^c}|_1, \qquad S_u = |\mathcal{S}_u|.$$

Hence we obtain

$$\rho(\hat{\Delta})^2 \lesssim |\Xi_{\mathcal{W}_u}|_F^2 + p^2 D^-(u).$$

Now, by the argument of proving [RBLZ08, Theorem 2],

$$\begin{aligned} \rho(\hat{\Omega}_\lambda - \Omega) &\leq \rho(\hat{\Delta})\rho(\hat{V}^{-1})\rho(V^{-1}) + \rho^2(\hat{V}^{-1} - V^{-1})\rho(\hat{\Delta}) \\ &\quad + \rho(\hat{V}^{-1} - V^{-1})[\rho(\hat{K}_\lambda)\rho(V^{-1}) + \rho(\hat{V}^{-1})\rho(K)]. \end{aligned} \tag{1}$$

Under $\max[p^{1/q} n^{-1+1/q}, (\log p/n)^{1/2}] \lesssim \lambda$, we have $\rho(\hat{V}^2 - V^2) = O_{\mathbb{P}}(\lambda)$. Since $\varepsilon_0 \leq v_{jj} \leq \varepsilon_0^{-1}$ holds for all j, we have $\rho(\hat{V}^{-1} - V^{-1}) = O_{\mathbb{P}}(\lambda)$. Then the first term on the RHS of (1) is the dominating term for the spectral norm rate of convergence and (64) [numbered in the paper] follows from (56) [numbered in the paper]. ☐

References

[RBLZ08] Adam J. Rothman, Peter J. Bickel, Elizaveta Levina, and Ji Zhu. Sparse Permutation Invariant Covariance Estimation. *Electronic Journal of Statistics*, 2:494–515, 2008.