Probability with Engineering Applications ECE 313 – Section C – Lecture 22

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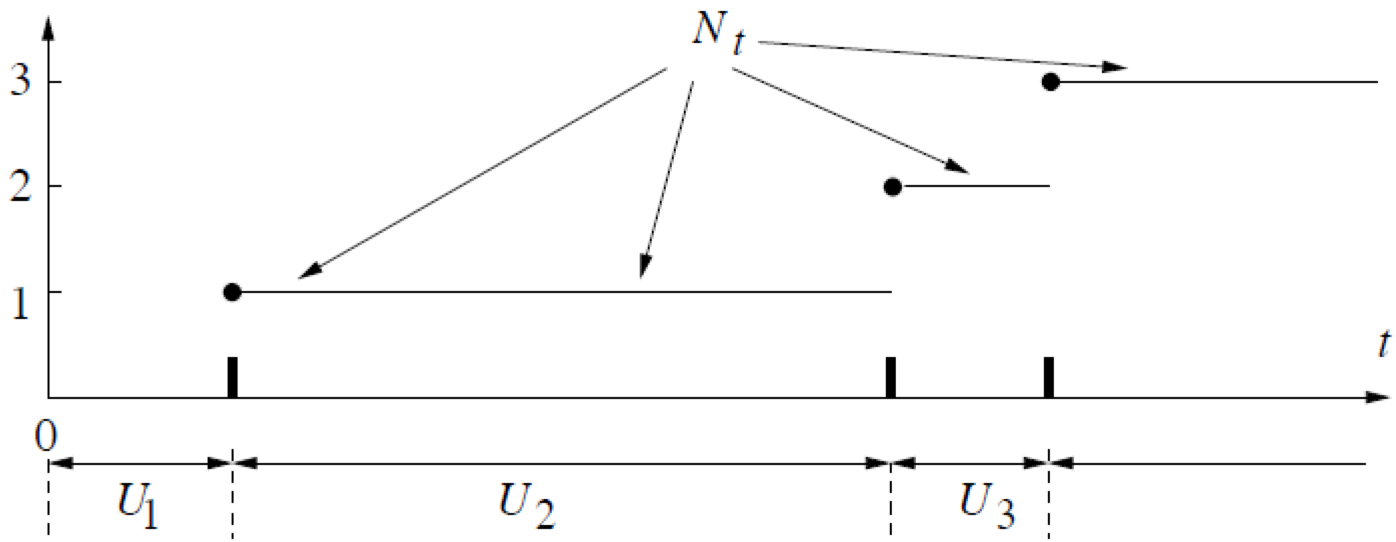
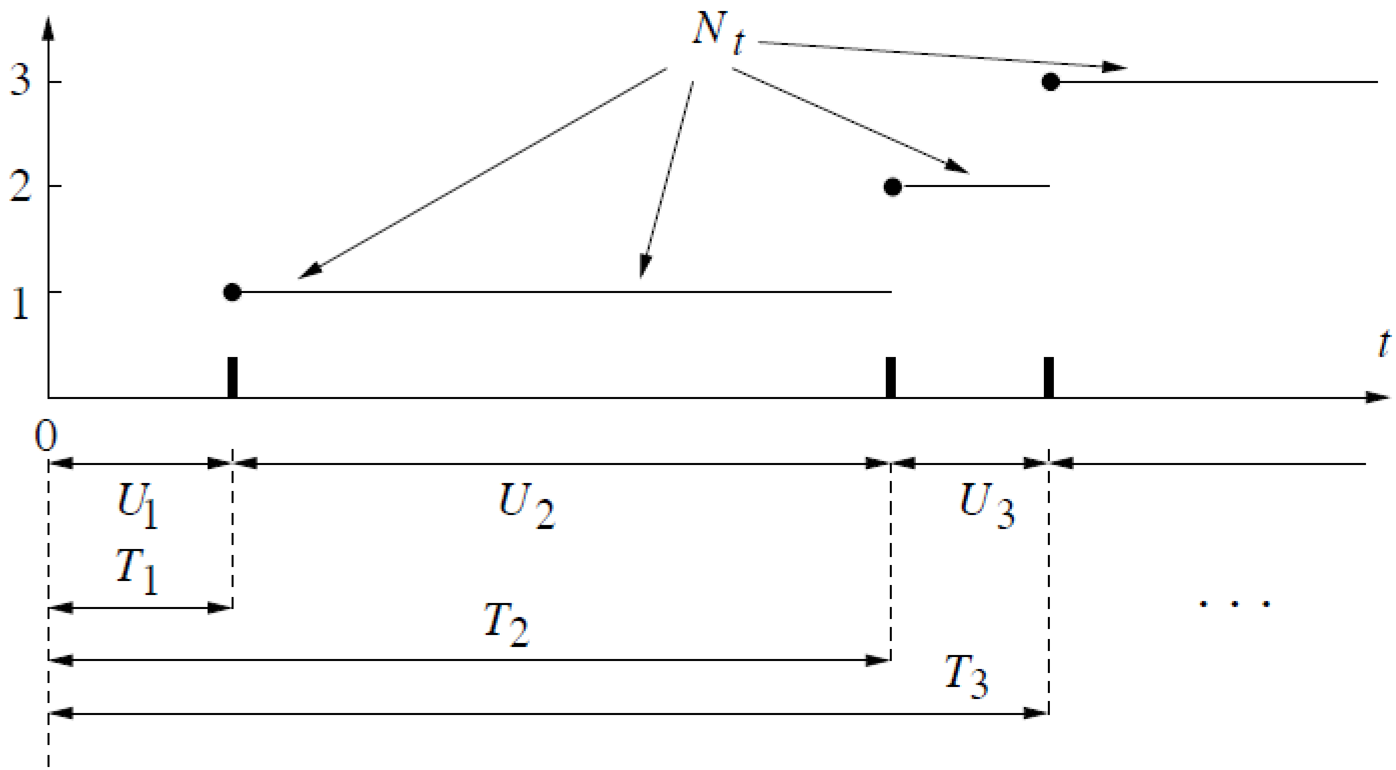
Poisson Process



Poisson Process

	Poisson Process	Bernoulli Process
times of arrival	continuous	discrete
pmf of # arrivals	Poisson	binomial
interarrival cdf	exponential	geometric
arrival rate	λ per unit time	p per trial

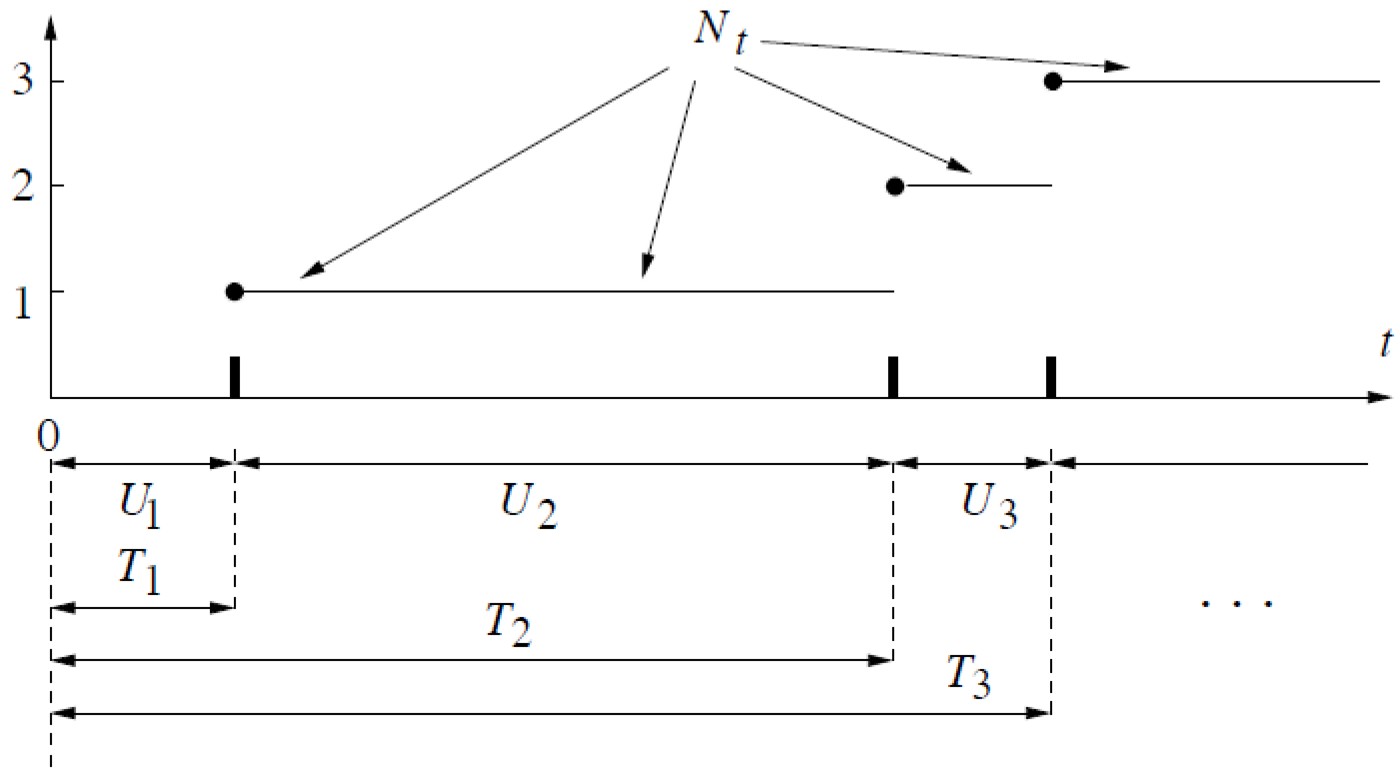
Poisson Process



Poisson Process

	Poisson Process	Bernoulli Process
times of arrival	continuous	discrete
pmf of # arrivals	Poisson	binomial
interarrival cdf	exponential	geometric
count cdf	Erlang	negative binomial
arrival rate	λ per unit time	p per trial

Poisson Process: Count Times



Let T_r be the time of the rth count of a Poisson process, so $T_r = U_1 + U_2 + \cdots + U_r$, where the U_i are i.i.d. exponentially distributed random variables (λ)

Poisson Process: Count Times

- Notice that for a fixed time t, the event $\{T_r > t\}$ can be written as $\{N_t \leq r - 1\}$ since the rth count happens after time t iff the number of counts that happened by time t is less than or equal to $r - 1$.
- Thus:

$$P\{T_r > t\} = \sum_{k=0}^{r-1} \frac{\exp(-\lambda t)(\lambda t)^k}{k!}$$

Poisson Process: Count Times

- To get the pdf, we take the negative of the derivative of the survival function:

$$f_{T_r}(t) = -\frac{d}{dt}P\{T_r > t\}$$

(rest on board)