Probability with Engineering Applications
ECE 313 – Section C – Lecture 17

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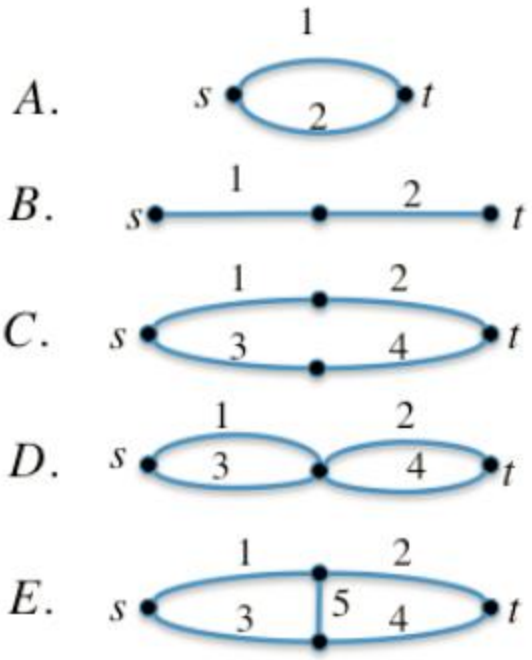
Network Reliability

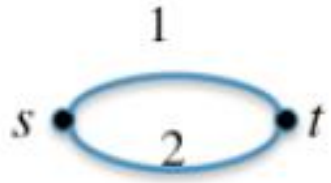
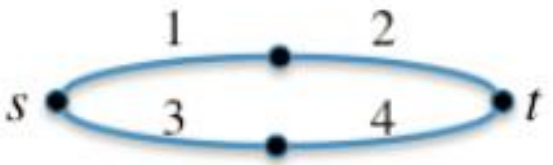
- An s–t network consists of a source node s, a terminal node t, possibly some additional nodes, and edges that connect pairs of nodes

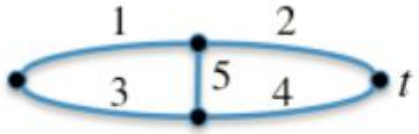


Various combinations of series/parallel connections

Network Reliability

- Let F_i be the event that edge i fails, with probability p_i, independently for each edge
- Network outage is said to occur if at least one link fails along every s–t path, which is the event F
- Want to find $P(F)$ either exactly or approximately

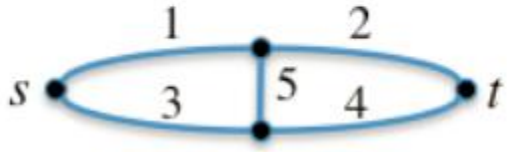
		$P(F)$
A.		$p_1 p_2$
B.		$p_1 + p_2 - p_1 p_2$
C.		$(p_1 + p_2 - p_1 p_2)(p_3 + p_4 - p_3 p_4)$
D.		$p_1 p_3 + p_2 p_4 - p_1 p_2 p_3 p_4$

E. 

$$p_1p_3 + p_2p_4 - p_1p_2p_3p_4 + p_1q_2q_3p_4p_5 + q_1p_2p_3q_4p_5$$

D.

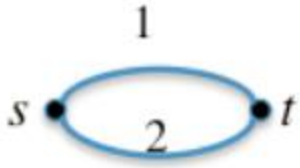
E.



Boole's Inequality (Union Bound)

- For a countable set of events $A_1, A_2, \ldots$, we have:

$$P\left(\bigcup_i A_i\right) \leq \sum_i P(A_i)$$

A. 

$p_1 p_2$

B. 

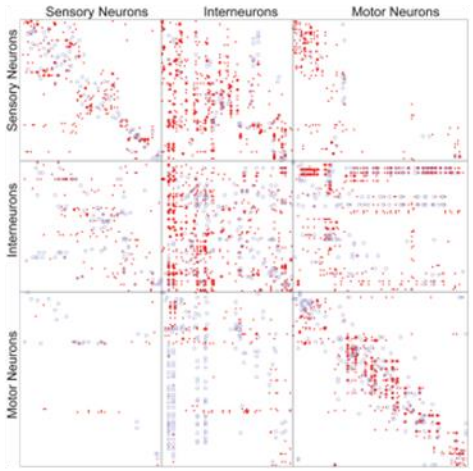
$p_1 + p_2 - p_1 p_2$

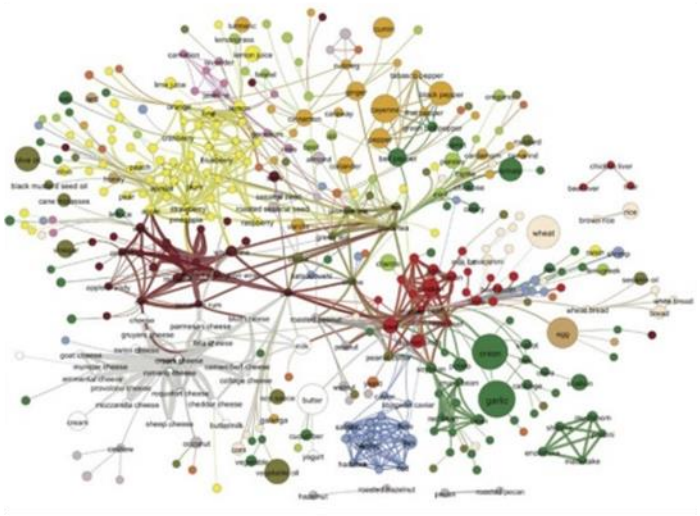
C.

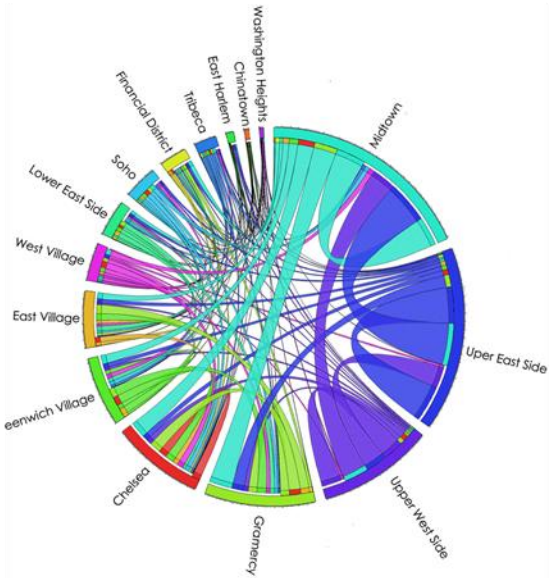
State	Network fails?	probability
0000	no	
0001	no	
0010	no	
0011	no	
0100	no	
0101	yes	$q_1p_2q_3p_4$
0110	yes	$q_1p_2p_3q_4$
0111	yes	$q_1p_2p_3p_4$
1000	no	
1001	yes	$p_1q_2q_3p_4$
1010	yes	$p_1q_2p_3q_4$
1011	yes	$p_1q_2p_3p_4$
1100	no	
1101	yes	$p_1p_2q_3p_4$
1110	yes	$p_1p_2p_3q_4$
1111	yes	$p_1p_2p_3p_4$

Network Flow

ECE 498LV — Network Science: Dynamics and Flow
Spring 2018





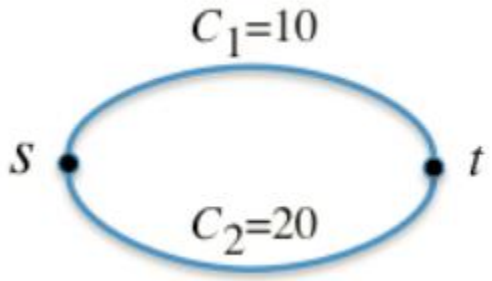


- Can we predict the behavior of the nematode *C. elegans* by looking at the connectivity pattern of its neurons?
- Can we characterize the pulse of life in New York City by looking at flows of taxi cabs?
- Can we design flavorful culinary recipes by understanding knowledge on shared flavor compounds in ingredients?

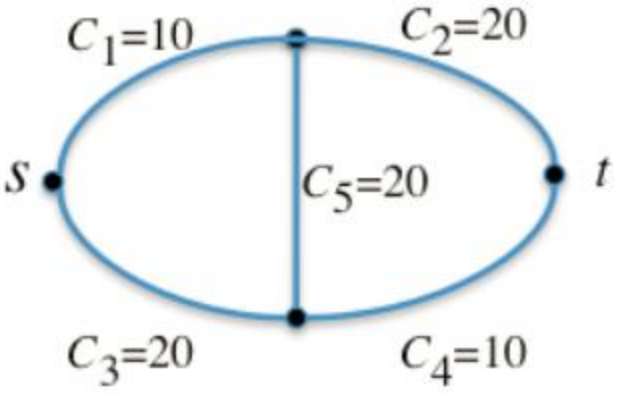
By taking an engineering perspective on network science, we can address these problems; more traditional problems in communications, computing, and power; and more!

Network Flow

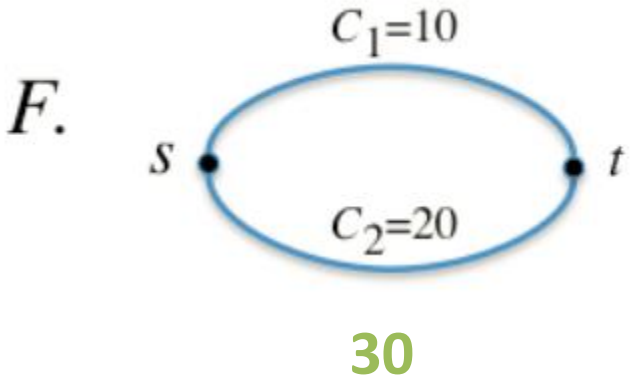
F.

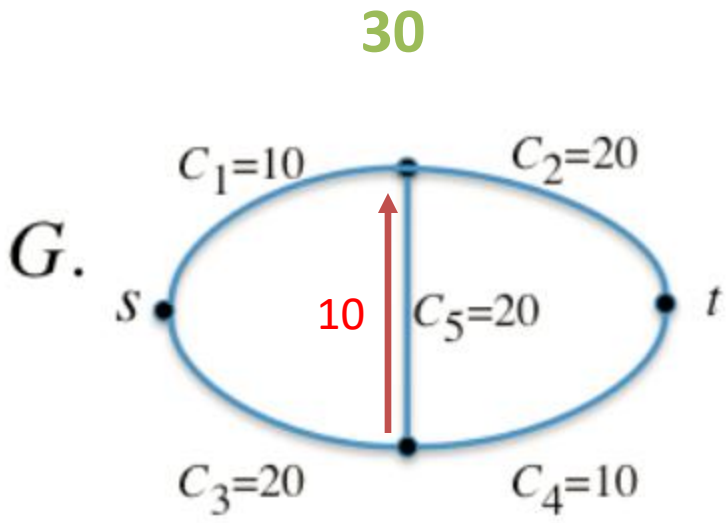


G.



Network Flow





Network capacity

- If a link fails, it cannot carry any flow. If a link does not fail, it can carry flow at a rate up to the link capacity. Link capacities and flows are given in units of some quantity per unit time. For example, the units could be gallons per minute (for an irrigation system), packets per second (for a communication network), or cars per hour (for a transportation system).

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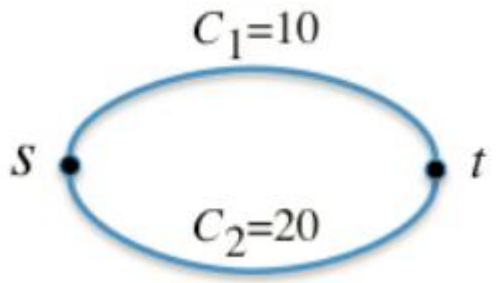

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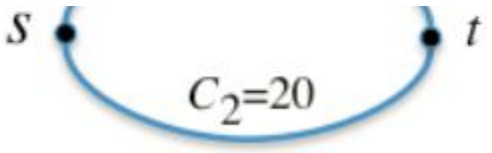

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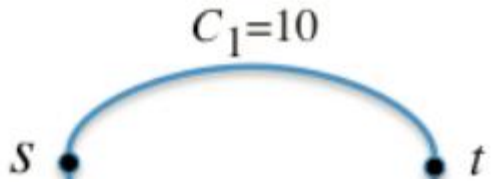

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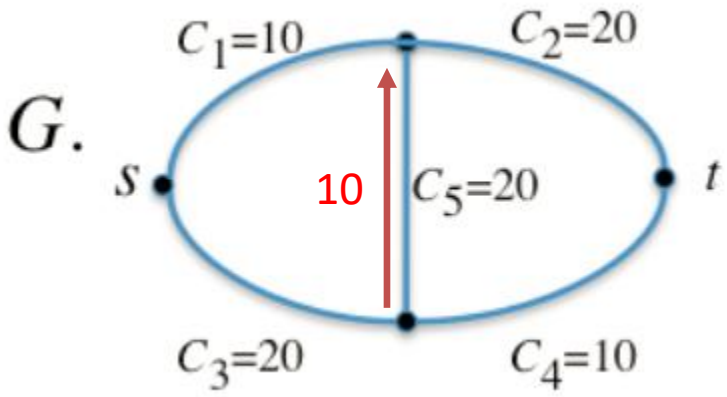

A table for calculating the distribution of capacity of network G.

State	capacity	probability
00000	30	$q_1q_2q_3q_4q_5$
00001	20	$q_1q_2q_3q_4p_5$
00010	20	$q_1q_2q_3p_4q_5$
00011	10	$q_1q_2q_3p_4p_5$
00100	10	$q_1q_2p_3q_4q_5$
00101	10	$q_1q_2p_3q_4p_5$
00110	10	$q_1q_2p_3p_4q_5$
00111	10	$q_1q_2p_3p_4p_5$
$\vdots$	$\vdots$	$\vdots$
11111	0	$p_1p_2p_3p_4p_5$