For network E, one approach is to condition on whether or not edge 5 fails, then use total probability.

When edge 5 is absent, network E reduces to network C.

When edge 5 is present, network E is equivalent to network D.

So $P(F) = p_5 \, P(\text{outage in network } C) + (1-p_5) \, P(\text{outage in network } D)$.

working things through, this simplifies to the expression we have.

Another approach is to directly get the expression in notes

→ If same four links used in network D used along with link 5, then network E fails whenever network D does.

→ there are two further ways network E can fail when network D does not.

① links 1, 4, 5 are only ones to fail $\rightarrow p_1 q_2 q_3 p_4 p_5$

② links 2, 3, 5 are only ones to fail $\rightarrow q_1 p_2 p_3 q_4 p_5$.

The $p_1 p_2$ operation for parallel paths is pretty straightforward

The $p_1 + p_2 - p_1 p_2$ operation for series paths is a little complicated.

but we can use union bound to simplify without too much loss in accuracy:

$$p_1 + p_2 - p_1 p_2 \leq p_1 + p_2 .$$

For example if $p = 0.01$, then $p^2 = .0001$

so $p_1 + p_2 - p_1 p_2 = .0199$ vs. $p_1 + p_2 = .02$.

In general, union bound is useful for simplifying series path computations without losing too much accuracy.

For network E, one approach is to condition on whether or not edge 5 fails, then use total probability.

When edge 5 is absent, network E reduces to network C.

When edge 5 is present, network E is equivalent to network D.

So $\Pr(F_E) = p_5 \Pr(\text{network C fails}) + (1-p_5)\Pr(\text{network D fails})$.

Working things through, this simplifies to the expression we have.

Another approach is to directly get the expression in words

→ If some links fail in network C, it fails along with link 5, then network E fails when network D does.

⇒ there are two other ways network E can fail when network C does not:

- links 1, 4, 5 are only ones to fail → $p_1 q_2 q_3 p_4 p_5$
- links 2, 3, 5 are only ones to fail → $q_1 p_2 p_3 q_4 p_5$.

The $p_1 p_2$ equation for parallel paths is pretty straightforward.

The $p_1+p_2-p_1p_2$ equation for series paths is a little complicated.

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In general, union bound is useful for simplifying series path computations without losing too much accuracy.

Another approach to computing outage probabilities is to enumerate all possible ~~[illegible]~~ present/absent edges and add together probabilities.

Distribution of flow over network.

network F.

$P_X(0) = p_1 p_2$

$P_X(10) = q_1 p_2$

$P_X(20) = p_1 q_2$

$P_X(30) = q_1 q_2$

What about network G?