Probability with Engineering Applications ECE 313 – Section C – Lecture 14

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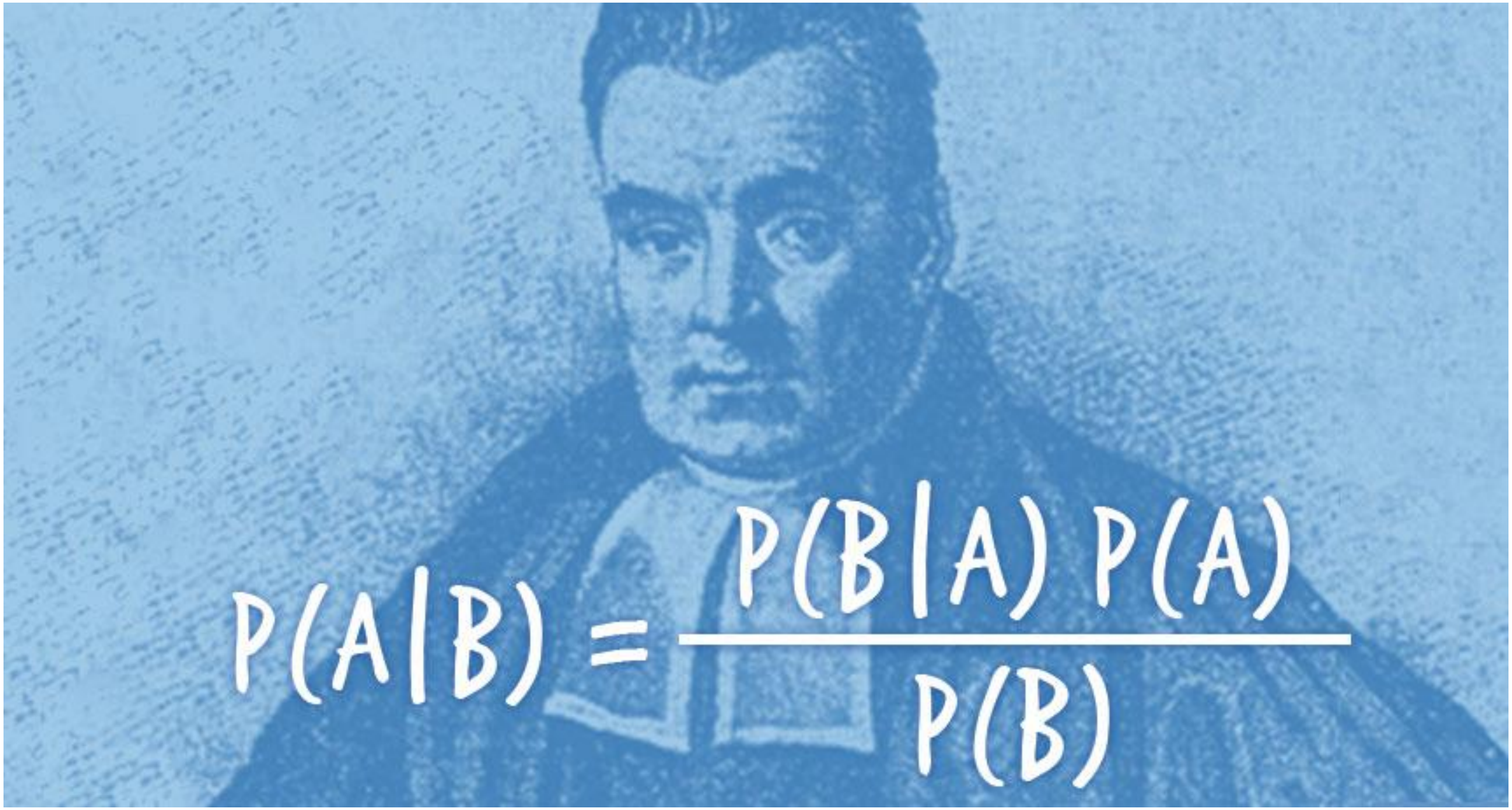
"Probability is common sense reduced to calculation"

Greatest Thing Ever!

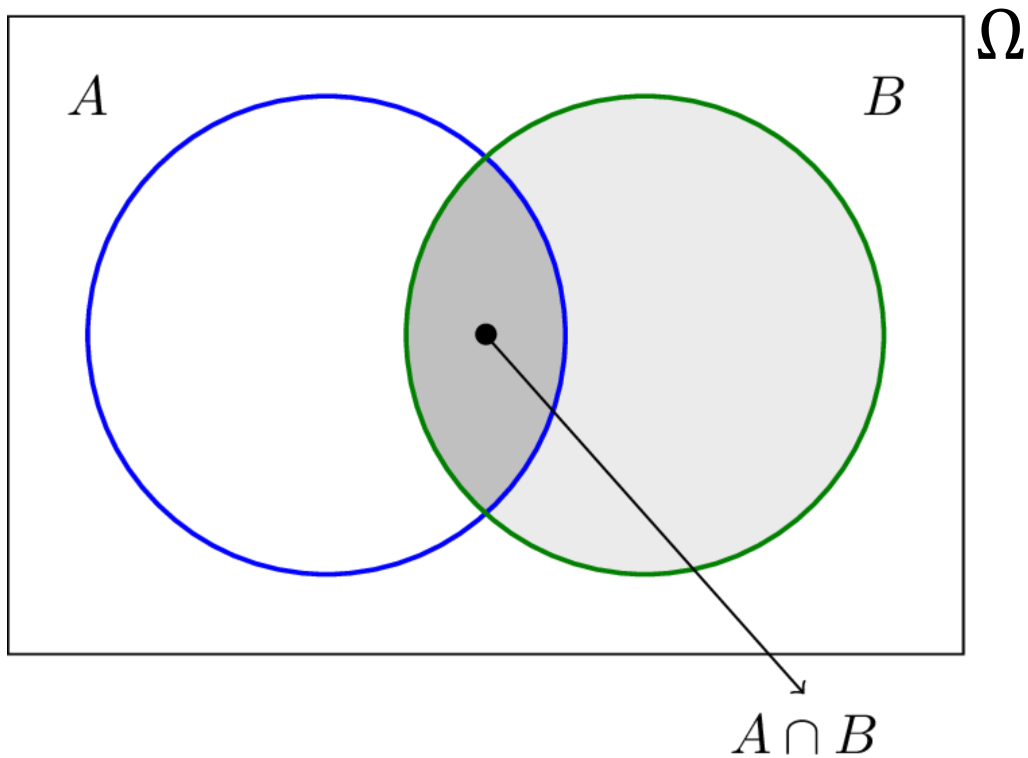


[https://www.sciencenews.org/article/bayesian-reasoning-implicated-some-mental-disorders]

Making inferences of causes from observations of effects

Conditional Probability

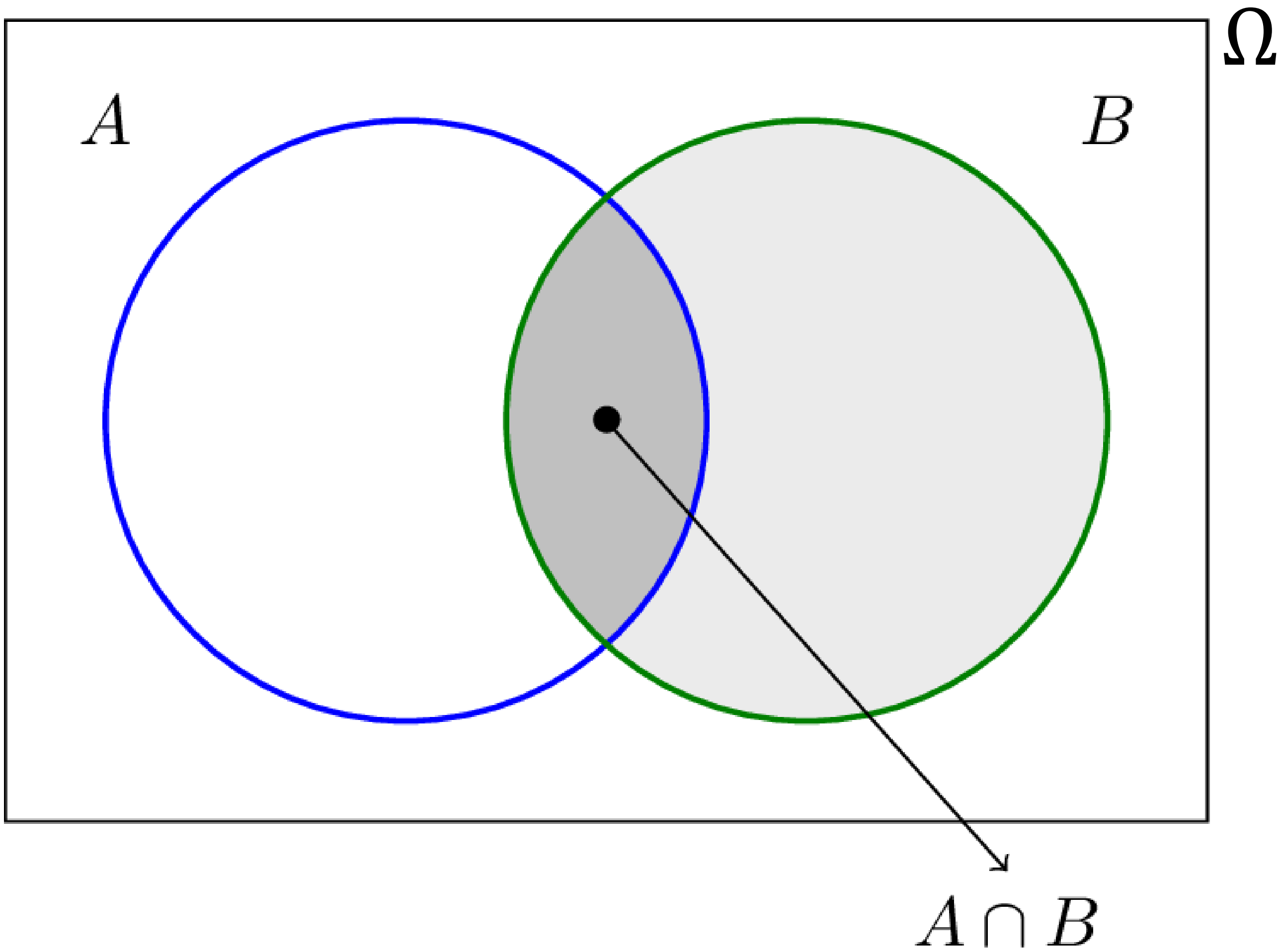
- Probabilities based on information/knowledge
  - Revising the knowledge base should lead to revisions of probabilities



Classical Conditional Probability

- Consider the probability of an event A, $P(A)$
- If we are now informed that event B has occurred, how should we revise $P(A)$ so that it is the *conditional probability* $P(A|B)$?

- $P(A) = \frac{|A|}{|\Omega|}$ gets revised to $P(A|B) = \frac{|AB|}{|B|}$



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

[https://www.probabilitycourse.com/chapter1/1_4_0_conditional_probability.php]

Conditional Mean

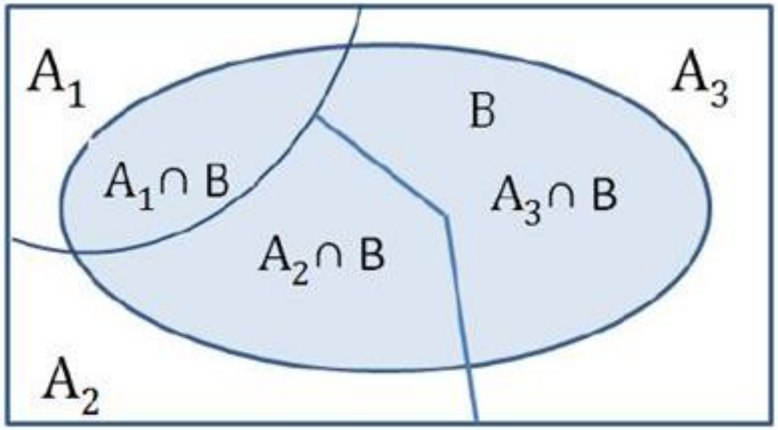
- Define conditional mean as:

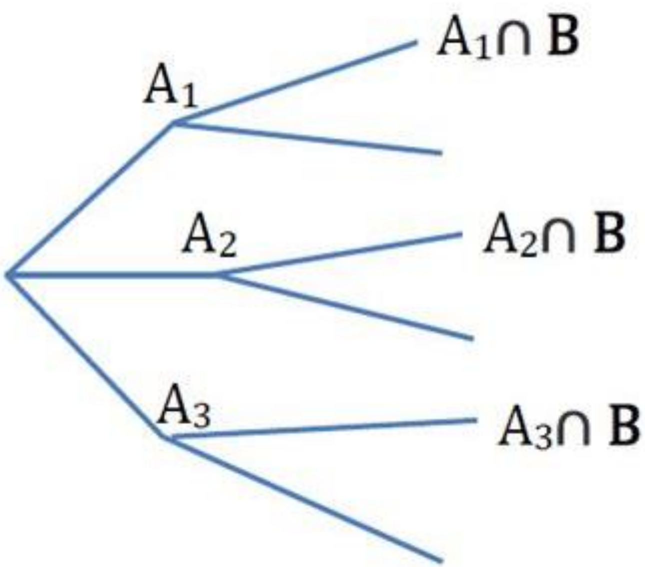
$$E[X|A] = \sum_i u_i P(X = u_i|A)$$

Also conditional version of LOTUS:

$$E[g(X)|A] = \sum_i g(u_i) P(X = u_i|A)$$

Total Probability Theorem





[http://slideplayer.com/slide/6247946/]

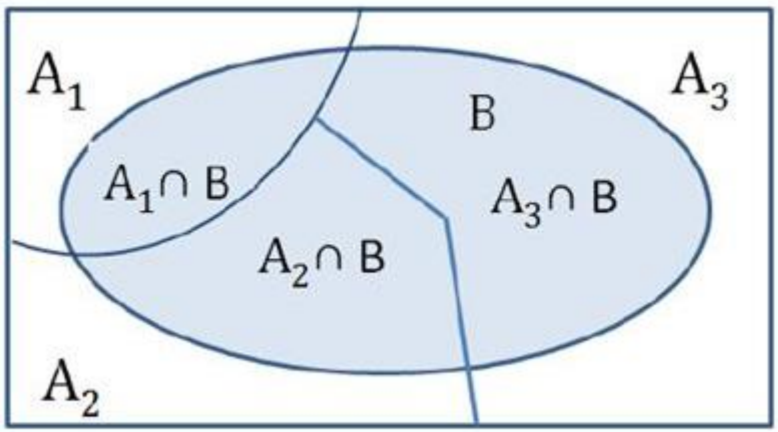
Let $A_1, A_2, \dots, A_n$ be disjoint events that form a partition of the sample space and assume $P(A_i) > 0$ for all i.

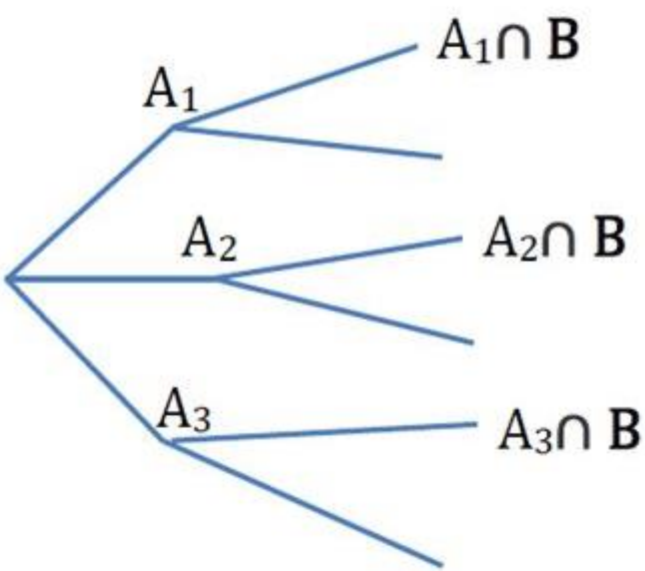
Then, for any event B,

$$P(B) = \sum_i P(A_i B)$$

$$P(B) = \sum_i P(B|A_i)P(A_i)$$

Total Probability Theorem





Intuitively, we are partitioning the sample space into a number of scenarios. Then the probability that B occurs is a weighted average of its conditional probability under each scenario, where each scenario is weighted by its (unconditional) probability

[http://slideplayer.com/slide/6247946/]

Response of Stochastic System

- Consider a stochastic system that takes input event A and transforms it into output event B
  - Described by specifying $P(B|A)$ for all pairs of events in the algebras
- Inputs are described by $P(A)$
- Want to specify the system response, $P(B)$

Response of Stochastic System

- Want to assess the probability of an output, response, or "effect" B in terms of the probabilities of possible inputs, excitations, or "causes" $\{A_i\}$ and knowledge of the probabilistic cause-effect mechanism
- B could be the event of a set of possible outputs of a stochastic system (e.g. symptoms of a disease) and $\{A_i\}$ a list of possible inputs (i.e. diseases) to the system
- The list of causes $\{A_i\}$ is assumed complete and without duplication, so forms a partition of Ω

Total Probability Theorem

Let $A_1, A_2, \ldots, A_n$ be disjoint events that form a partition of the sample space and assume $P(A_i) > 0$ for all i. Then, for any event B,

$$P(B) = \sum_i P(B|A_i)P(A_i)$$

Also extends to expectation, for a random variable X:

$$E[X] = \sum_i E[X|A_i]P(A_i)$$

Bayes Rule for Inference

- There are a number of "causes" that may result in a certain "effect"; we observe the effect and want to infer the cause
- Reversing the order of conditioning from a description of cause → effect, to one for inference from effect → cause

Bayes Rule for Inference

Theorem Let $A_1, A_2, \dots, A_n$ be disjoint events that form a partition of the sample space, and assume $P(A_i) > 0$ for all i. Then for any event B such that $P(B) > 0$:

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(B)} = \frac{P(A_i)P(B|A_i)}{\sum_i P(B|A_i)P(A_i)}$$

Proof Use definition of conditional probability $P(AB) = P(A|B)P(B) = P(B|A)P(A)$ and total probability.