Probability with Engineering Applications
ECE 313 – Section C – Lecture 13

Lav R. Varshney

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Example 3

- Suppose we have a weighted coin, that shows heads with some unknown probability p each time it is flipped
- We flip it some large and known number n times and heads shows on k of the flips
- Find the ML estimate of p

Reliability of parameter estimate?

- How reliable an estimator of probability of heads can we form from relative frequency of occurrence in first 1000 tosses of a coin that is actually fair?
- Estimate the probability that the relative frequency will differ from ½ by at least 0.05.

Polling

- https://www.youtube.com/watch?v=suBMSdJklrc

Markov inequality

- If Y is a nonnegative random variable, then for $c > 0$,

$$Pr\{Y \geq c\} \leq \frac{E[Y]}{c}$$

Proof

Spelling of Name [edit]

Just for fun I entered 36 different spellings of his name into Google and tabulated the number of hits. Three forms of the first syllable - Ch, Tch, and Tsch, and twelve forms of the last syllable - shev, sheff, shov, shoff, chev, cheff, chov, choff, schev, scheff, schov and schoff. The middle syllable was eby. Results are in the table below. Then I discovered that the first syllable could also be sh, tsh or kh, the middle syllable could be eby, ebi, ebie, ebe, ebye or eba. Also the last syllable could be shef, shof chef, chof, schef or schof. This gives a total of 648 spellings, and no, I didn't. PAR 03:32, 13 August 2006 (UTC)

Chebyshev	1 350 000	Chebychev	126 000	Tchebycheff	45 300
Tchebychev	19 700	Tschebyscheff	13 400	Tchebyshev	12 900
Chebyschev	11 700	Chebycheff	776	Tchebysheff	653
Tchebyscheff	641	Tschebychev	454	Tschebycheff	360
Chebysheff	348	Tschebyshev	228	Chebyshov	158
Tschebysheff	148	Tchebyschev	143	Chebyscheff	104
Tschebyschev	83	Tchebyshov	18	Chebyschoff	9
Tschebyschoff	9	Tchebychoff	8	Chebyshoff	7
Tschebyschov	6	Tchebychov	5	Tchebyschoff	2
Chebychov	1	Chebychoff	1	Tschebyshov	1
Tschebyshoff	1	Tchebyshoff	1	Chebyschov	0
Tschebychov	0	Tschebychoff	0	Tchebyschov	0

Chebychev inequality

- If X is random variable with finite mean μ and variance σ^2, then for any $d > 0$,

$$Pr\{|X - \mu| \geq d\} \leq \frac{\sigma^2}{d^2}$$

- Alternatively, letting $d = a\sigma$,

$$Pr\{|X - \mu| \geq a\sigma\} \leq \frac{1}{a^2}$$

Proof

Confidence Intervals

- Use Chebyshev for binomial random variable with parameters n and p, which has mean np and std $\sqrt{np(1-p)}$:

$$Pr\{|X - np| \geq a\sigma\} \leq \frac{1}{a^2}$$

$$Pr\left\{\left|\frac{X}{n} - p\right| < \frac{a\sigma}{n}\right\} \geq 1 - \frac{1}{a^2}$$

$$Pr\left\{p \in \left(\hat{p} \pm a\sqrt{\frac{p(1-p)}{n}}\right)\right\} \geq 1 - \frac{1}{a^2}$$

where $\hat{p} = X/n$

Confidence Intervals

- Note that $\hat{p}$ is random and so the interval

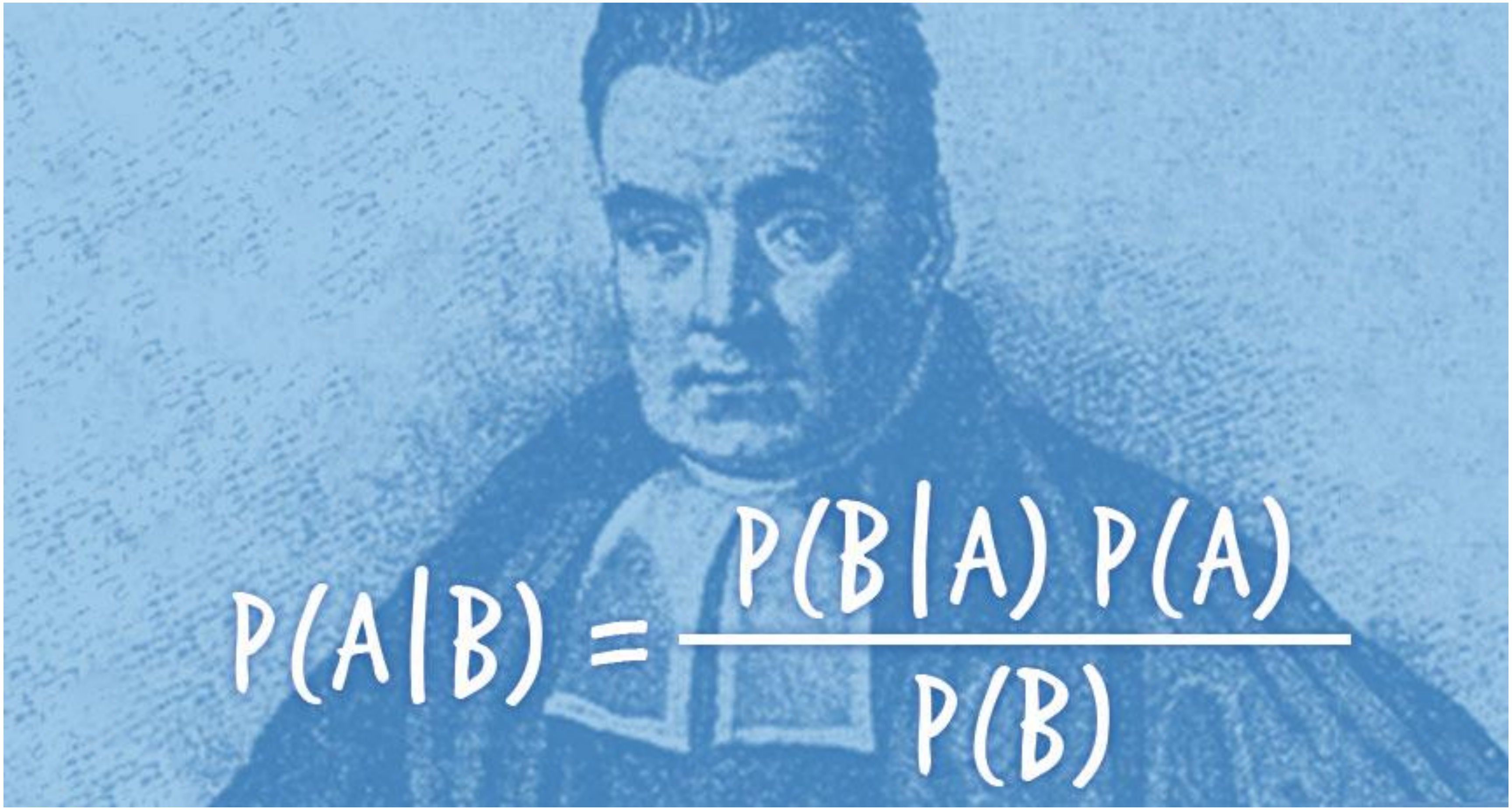
$$\left(\hat{p} \pm a\sqrt{\frac{p(1-p)}{n}}\right)$$

is also random. Since the interval depends on the unknown parameter p it is not quite appropriate. Since $p(1-p) \leq \frac{1}{4}$ for any p, we can use that to get:

$$Pr\left\{p \in \left(\hat{p} \pm \frac{a}{2\sqrt{n}}\right)\right\} \geq 1 - \frac{1}{a^2}$$

The power of *induction*, my dear Watson

Next time



[https://www.sciencenews.org/article/bayesian-reasoning-implicated-some-mental-disorders]