Proof of Markov Ineq for discrete r.v.:

Consider Y with support $u_1, u_2, \ldots$.

$$E[Y] = \sum_i u_i \, p_Y(u_i)$$ split the u_i into those less than c, and $\geq c$.

$$\geq \sum_{i: u_i < c} 0 \cdot p_Y(u_i) + \sum_{i: u_i \geq c} c \, p_Y(u_i)$$

lower bounded by 0 — lower bounded by c.

$$= c \sum_{i: u_i \geq c} p_Y(u_i) = c \, P\{Y \geq c\}$$

Equality iff $p_Y(0) + p_Y(c) = 1$, so generally quite loose

Proof of Chebyshev

Apply Markov with $Y = |X - \mu|^2$ and $c = d^2$.