Probability with Engineering Applications ECE 313 – Section C – Lecture 10

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Bernoulli random variable

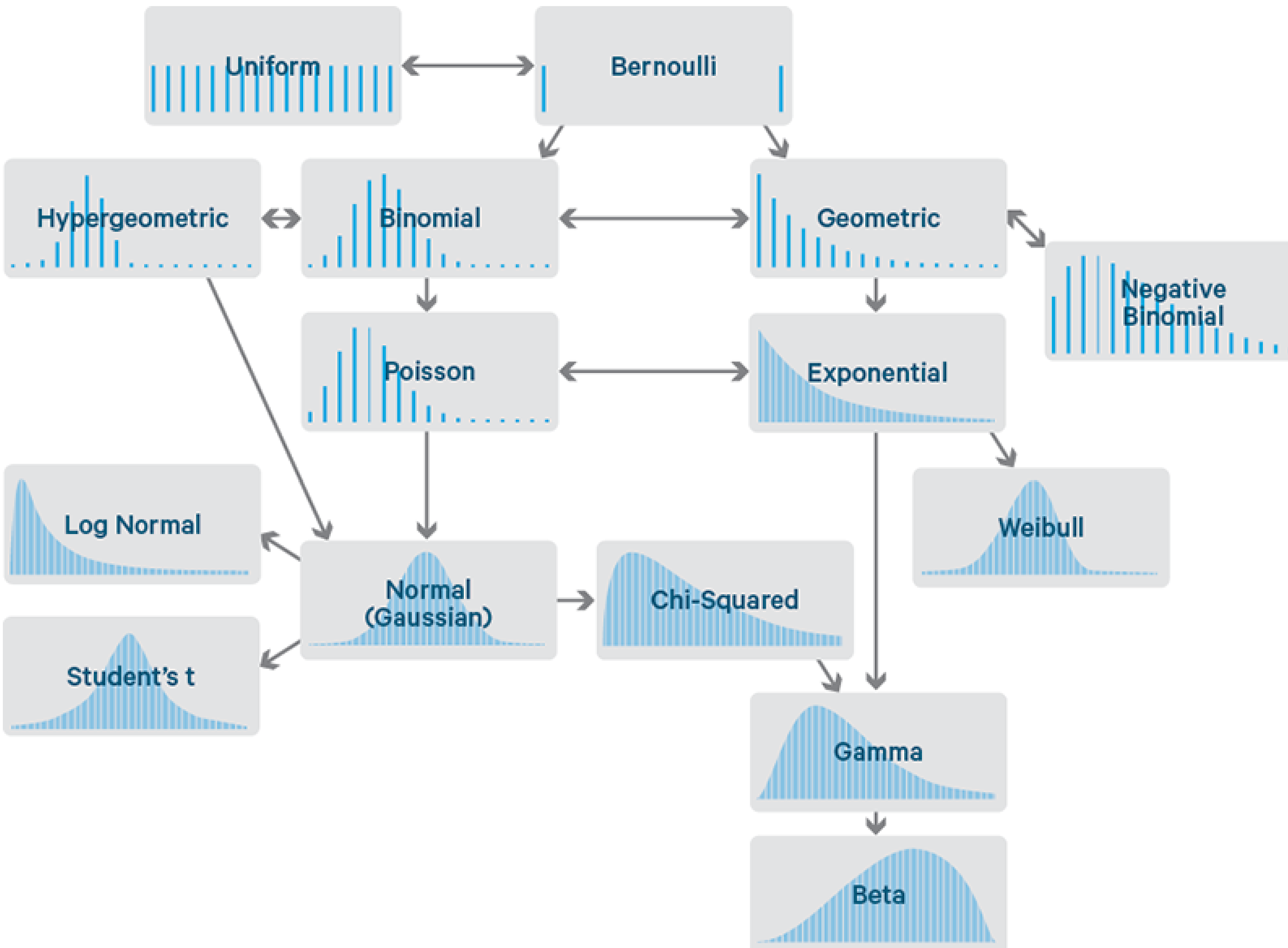
- Consider tossing a coin that comes up heads with probability p and tails with probability $1-p$.
- The Bernoulli random variable maps these two outcomes to the numbers 0 and 1:

$$X = \begin{cases} 1, & \text{heads} \\ 0, & \text{tails} \end{cases}$$

- Corresponding pmf is therefore:

$$p_X(x) = \begin{cases} p, & x = 1 \\ 1-p, & x = 0 \end{cases}$$

Common probability distributions

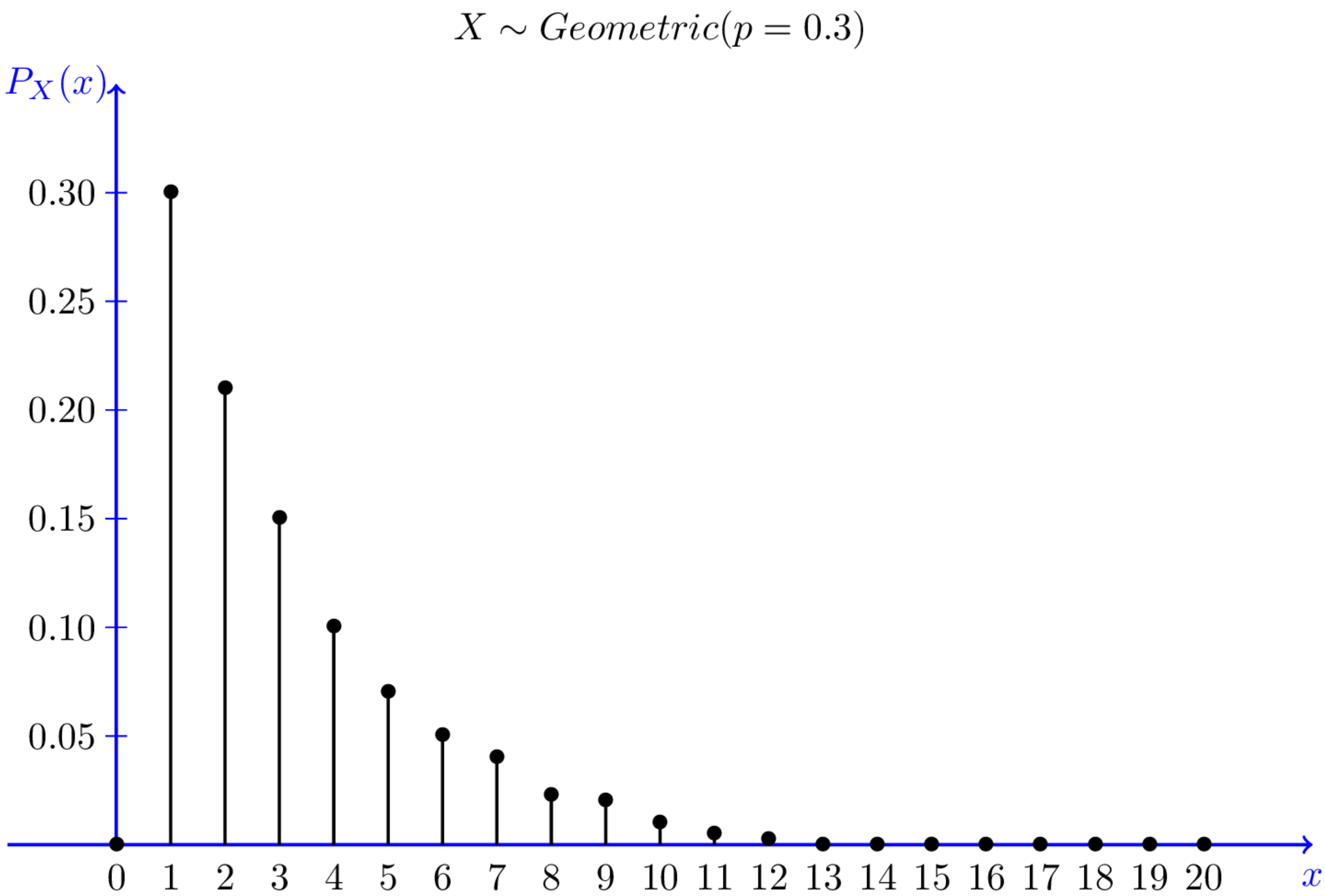


[https://blog.cloudera.com/blog/2015/12/common-probability-distributions-the-data-scientists-crib-sheet/]

Geometric random variable

- Suppose we repeatedly and independently toss a weighted coin (from last lecture) until a head comes up for the first time. The geometric random variable X is the number of tosses needed for a head to come up for the first time, with pmf:

$$p_X(k) = (1-p)^{k-1}p, \qquad k = 1, 2, \ldots$$



[https://www.probabilitycourse.com/chapter3/3_1_5_special_discrete_distr.php]

Geometric random variable

- Check normalization (using geometric series sum, recall you've seen this in calculus)

$$\sum_{k=1}^{\infty} p_X(k) = \sum_{k=1}^{\infty} (1-p)^{k-1} p = p \sum_{k=0}^{\infty} (1-p)^k$$

$$= p \frac{1}{1-(1-p)} = 1$$

Key problem

- You just rented a large house and the owner gave you five keys, one for each of the five doors of the house. Unfortunately all keys look alike, so to open the front door, you try them at random.
- Find the pmf of the number of trials you need to open the door, under the following alternative assumptions:
  1. After an unsuccessful trial, you mark the corresponding key, so you never try it again
  2. At each trial you are equally likely to choose any key
- Repeat problem when the owner gives you an extra duplicate key for each of the five doors

Banach's smoking problem

- A smoker mathematician carries one matchbox in his right pocket and one in his left pocket. Each time he wants to light a cigarette, he selects a matchbox from either pocket with probability ½, independently of earlier selections. The two matchboxes have initially n matches each. What is the pmf of the number of remaining matches at the moment the mathematician reaches for a match and discovers the corresponding matchbox is empty?