

Properties of binomial random variable.

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(normalization, mean, variance).

One approach is to use the binomial theorem:

$$(x+y)^n = \sum_{r=0}^n \binom{n}{r} x^r y^{n-r}$$

$$\text{and so } \sum_{r=0}^n \binom{n}{r} = 2^n$$

$$\text{for our setting: } \sum_{k=0}^n \binom{n}{k} p^k (1-p)^{n-k} = (p+1-p)^n = 1.$$

One can also check this using the Taylor series approach in the text.

$$\begin{aligned} E[X] &= \sum_{k=0}^n k \binom{n}{k} p^k (1-p)^{n-k} \\ &= \sum_{k=1}^n k \binom{n}{k} p^k (1-p)^{n-k} \\ &= np \sum_{k=1}^n \frac{(n-1)!}{(n-k)!(k-1)!} p^{k-1} (1-p)^{n-k} \\ &= np \sum_{\ell=0}^{n-1} \binom{n-1}{\ell} p^{\ell} (1-p)^{n-1-\ell} \quad \text{where } \ell = k-1 \\ &= np \end{aligned}$$

An alternate approach is based on the fact $X = B_1 + B_2 + \dots + B_n$ where B_i are Bernoulli with mean p . Thus by linearity, $E[X] = np$.

The Bernoulli variables have variance $p(1-p)$, so $\text{var}[X] = np(1-p)$.

Use the binomial pmf to evaluate the probability that in a message of length 5, there will be less than 2 errors when probability of error on single symbol is 0.1.

For each symbol there is or is not an error, and we care about either 0 or 1 error.

$$P_X(0) = \binom{5}{0} p^0 (1-p)^5 = 1 \cdot 1 \cdot (0.9)^5 = 0.59049$$

$$P_X(1) = \binom{5}{1} p^1 (1-p)^4 = 5 \cdot (0.1) \cdot (0.9)^4 = 0.32805$$

so total is 0.91854.

In a certain bit transmission system sending $\{0,1\}$ observed that probability of making 2 errors in 10 bits is .0045. What is probability of making error on first bit?

Consider binomial distribution with parameter p and $n=10$.
evaluate $P_X(2)$

$$P_X(2) = \binom{10}{2} p^2 (1-p)^8 = .0045$$

want to solve for p

$$p^2 (1-p)^8 = .0045 \cdot \frac{2! 8!}{10!} = .0045 \cdot \frac{2}{90} = 0.0001$$

numerically solving for p yields $p = 0.0104$.

The following table shows the results of the experiment. The first column is the number of trials, the second column is the number of successes, and the third column is the probability of success.

$$\begin{aligned}
 P(X=0) &= \binom{10}{0} (0.1)^0 (0.9)^{10} = 0.9^{10} = 0.3487 \\
 P(X=1) &= \binom{10}{1} (0.1)^1 (0.9)^9 = 10 \cdot 0.1 \cdot 0.9^9 = 0.3771 \\
 P(X=2) &= \binom{10}{2} (0.1)^2 (0.9)^8 = 45 \cdot 0.01 \cdot 0.9^8 = 0.2818 \\
 P(X=3) &= \binom{10}{3} (0.1)^3 (0.9)^7 = 120 \cdot 0.001 \cdot 0.9^7 = 0.1969 \\
 P(X=4) &= \binom{10}{4} (0.1)^4 (0.9)^6 = 210 \cdot 0.0001 \cdot 0.9^6 = 0.1172 \\
 P(X=5) &= \binom{10}{5} (0.1)^5 (0.9)^5 = 252 \cdot 0.00001 \cdot 0.9^5 = 0.0547 \\
 P(X=6) &= \binom{10}{6} (0.1)^6 (0.9)^4 = 210 \cdot 0.000001 \cdot 0.9^4 = 0.0238 \\
 P(X=7) &= \binom{10}{7} (0.1)^7 (0.9)^3 = 120 \cdot 0.0000001 \cdot 0.9^3 = 0.0080 \\
 P(X=8) &= \binom{10}{8} (0.1)^8 (0.9)^2 = 45 \cdot 0.00000001 \cdot 0.9^2 = 0.0027 \\
 P(X=9) &= \binom{10}{9} (0.1)^9 (0.9)^1 = 10 \cdot 0.000000001 \cdot 0.9 = 0.0009 \\
 P(X=10) &= \binom{10}{10} (0.1)^{10} (0.9)^0 = 1 \cdot 0.0000000001 \cdot 1 = 0.0000
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$$P(X=0) = \frac{1}{10} \cdot 0.9^{10} = \frac{1}{10} \cdot 0.3487 = 0.03487$$

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