Properties of binomial random variable.

(normalization, mean, variance).

One approach is to use the binomial theorem:

$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r}$$

and so $\sum_{r=0}^{n} \binom{n}{r} = 2^n$

for our setting: $\sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} = (p+1-p)^n = 1.$

One can also check this using the Taylor series approach in the text.

$$E[X] = \sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k}$$

$$= \sum_{k=1}^{n} k \binom{n}{k} p^k (1-p)^{n-k}$$

$$= np \sum_{k=1}^{n} \frac{(n-1)!}{(n-k)!\,(k-1)!} p^{k-1} (1-p)^{n-k}$$

$$= np \sum_{\ell=0}^{n-1} \binom{n-1}{\ell} p^\ell (1-p)^{n-1-\ell} \quad \text{where } \ell = k-1$$

$$= np$$

An alternate approach is based on the fact $X = B_1 + B_2 + \cdots + B_n$ where B_i are Bernoulli with mean p. Thus by linearity, $E[X] = np$.

The Bernoulli variables have variance $p(1-p)$, so $\mathrm{var}[X] = np(1-p)$.

Use the binomial pmf to evaluate the probability that in a message of length 5, there will be less than 2 errors when probability of error on single symbol is 0.1.

for each symbol there is or is not an error, and we care about either 0 or 1 error.

$$P_X(0) = \binom{5}{0} p^0 (1-p)^n = 1 \cdot 1 \cdot (0.9)^5 = 0.59049$$

$$P_X(1) = \binom{5}{1} p^1 (1-p)^4 = 5 \cdot (0.1) \cdot (0.9)^4 = 0.32805$$

so total is 0.91854.

In a certain bit transmission system sending $\{0,1\}$ observed that probability of making 2 errors in 10 bits is .0045. What is probability of making error on first bit?

consider binomial distribution with parameter p and $n = 10$.

evaluate $P_X(2)$

$$P_X(2) = \binom{10}{2} p^2 (1-p)^8 = .0045$$

want to solve for p

$$p^2 (1-p)^8 = .0045 \cdot \frac{2!\,8!}{10!} = .0045 \cdot \frac{2}{90} = 0.0001$$

numerically solving for p yields $p = 0.0104$.