Properties of Variance

$$\begin{aligned}
Var(X+b) &= E[(X+b-E[X+b])^2] \\
&= E[(X+b-E[X]-b)^2] \\
&= E[(X-E[X])^2] \\
&= Var(X)
\end{aligned}$$

so shifting around doesn't change spread.

$$\begin{aligned}
Var(aX) &= E[(aX-E[aX])^2] \\
&= E[(aX-aE[X])^2] \\
&= a^2 E[(X-E[X])^2] \\
&= a^2 Var(X),
\end{aligned}$$

so multiplying r.v. by a multiplies variance by a^2.

The random variable $\frac{X-\mu_X}{\sigma_X}$ is called the standardized version of X, with mean 0 and variance 1.

$$E\left[\frac{X-\mu_X}{\sigma_X}\right] = \frac{1}{\sigma_X}\left(E[X]-\mu_X\right) = 0$$

$$Var\left[\frac{X-\mu_X}{\sigma_X}\right] = \frac{1}{\sigma_X^2} E[(X-\mu_X)^2] = \frac{\sigma_X^2}{\sigma_X^2} = 1.$$

Exercise 1

$$P(A\mid B) = \frac{P(A\cap B)}{P(B)} = \frac{p(5)}{p(2)+p(3)+p(5)} = \frac{.05}{.05+.05+.05} = \frac{1}{3}.$$

Exercise 2

The sample space is

$$\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

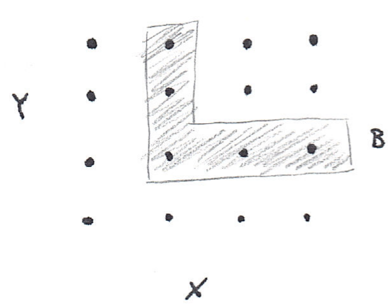
Event B consists of four of these: $B = \{HHH, HHT, HTH, HTT\}$ so $P(B) = \frac{4}{8} = \frac{1}{2}$

Event $A\cap B$ consists of three elements: $\{HHH, HHT, HTH\}$ so $P(AB) = \frac{3}{8}$

Then $P(A\mid B) = \dfrac{P(AB)}{P(B)} = \dfrac{3/8}{4/8} = \dfrac{3}{4}$.

Exercise 3

We can determine $P(AB)$ and $P(B)$ by counting as before, or alternatively directly count size of sets AB and B and divide.



event includes the 5 shaded ~~events~~ samples

The set $A = \{\max(X,Y) = m\}$ shares two elements with B if $m = 3, 4$, shares one element if $m=2$ and none if $m=1$, so

$$P(A\mid B) = \begin{cases} 2/5 & \text{for } m = 3 \text{ or } 4 \\ 1/5 & \text{for } m = 2 \\ 0 & \text{for } m = 1 \end{cases}.$$