Bonferroni's Inequality

(a) Prove that for any two events A and B, we have

$$P(AB) \geq P(A) + P(B) - 1$$

pf: we know from last time that $P(A \cup B) = P(A) + P(B) - P(AB)$
and that $P(A \cup B) \leq 1$, so the result follows directly by substitution

(b) Generalize to the case of n events: $A_1, A_2, \ldots, A_n$ by showing

$$P(A_1 A_2 A_3 \cdots A_n) \geq P(A_1) + P(A_2) + \cdots + P(A_n) - (n-1)$$

pf: use De Morgan's law to obtain:

$$\begin{aligned}
1 - P(A_1 A_2 \cdots A_n) &= P((A_1 A_2 \cdots A_n)^c) \\
&= P(A_1^c \cup \cdots \cup A_n^c) \\
&\leq P(A_1^c) + \cdots + P(A_n^c) \\
&= (1 - P(A_1)) + \cdots + (1 - P(A_n)) \\
&= n - P(A_1) - \cdots - P(A_n).
\end{aligned}$$