Probability with Engineering Applications ECE 313 – Section C – Lecture 2

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Logistics

- Text: ECE 313 course notes
  - Hard copy sold through ECE Store
  - Soft copy available on the course website
- Other reference books on reserve in Grainger, see also books by T. L. Fine (2006) and by D. P. Bertsekas and J. N. Tsitsiklis (2002)

- Class website: https://courses.engr.illinois.edu/ece313

Grading

- 10% Homework. MasterProbo checkpoints
- 50% Midterm exams (2 @ 25% each)
- 40% Final Examination

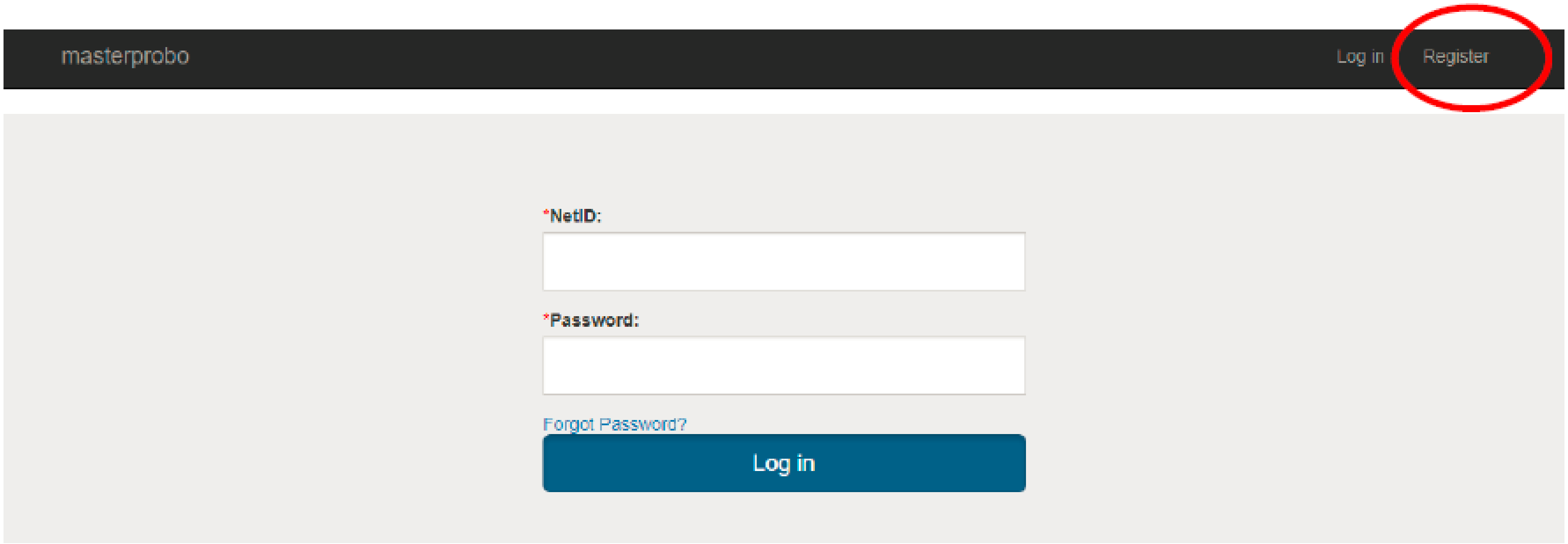
Exams

- Midterm I: October 11, 8:45pm–10pm
  – 1 cheat sheet
- Midterm II: November 15, 8:45pm–10pm
  – 1 cheat sheet
- Final: December 18, 7pm–10pm
  – 2 cheat sheets

- Further infomation on course website

MasterProbo: Online homework

- Pass checkpoint by Tuesday 11:59pm (First checkpoint: 9/12)
- https://courses.engr.illinois.edu/ece313/Homework.html



Missing homework

- What happens if don't pass checkpoint by deadline?

A: Pass – 10 points

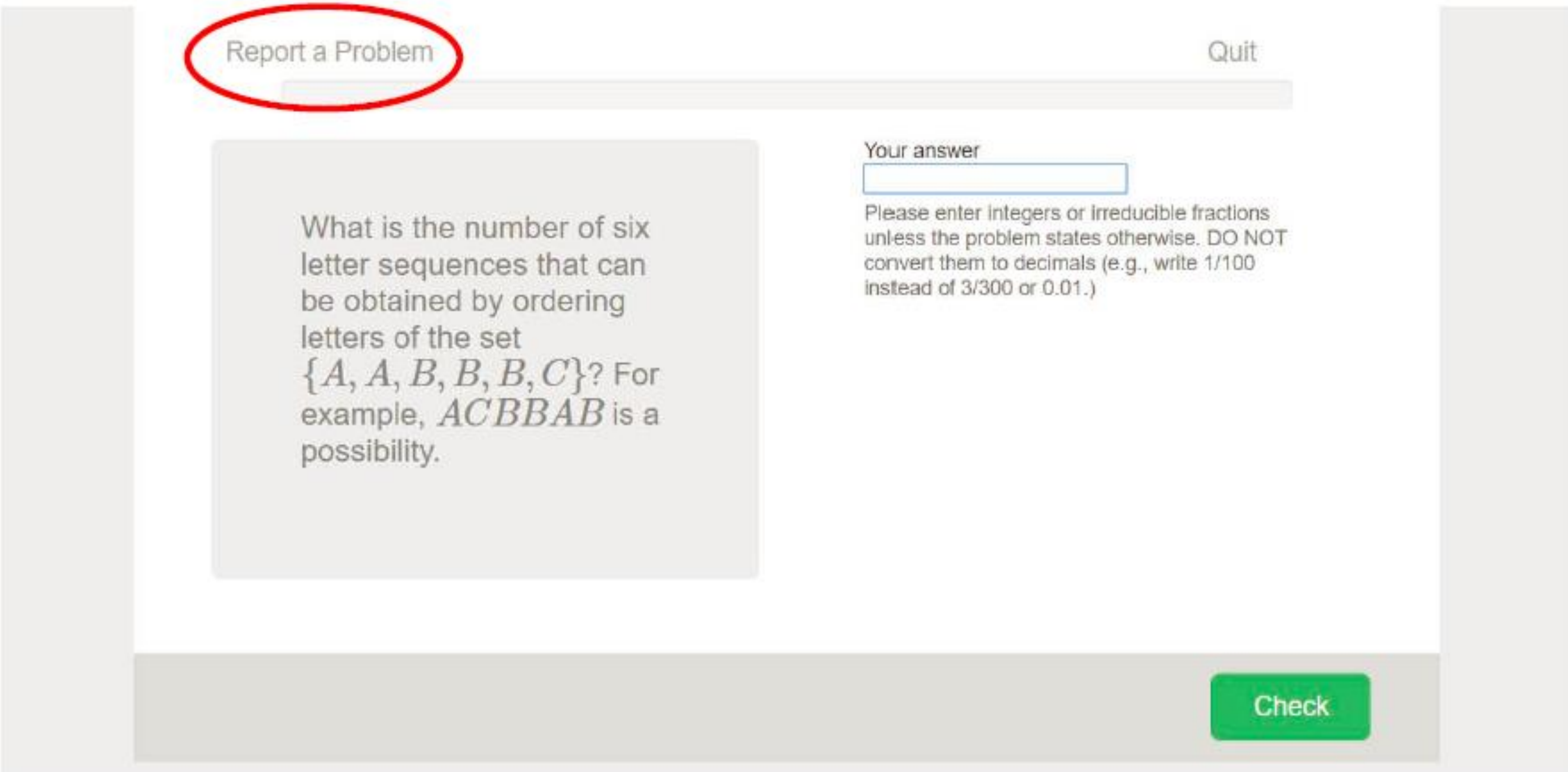
otherwise: $\frac{\#\, probos\ earned}{total\ \#\, probos\ for\ this\ checkpoint}$ x 10

Only top 9 out of 12 scores will be used to compute grade

No extension of deadlines

Tech support

Best way to report a problem



Tech support

Other issues:
Such as “can’t register. Says I’m not enrolled”

E-mail: masterprobos@gmail.com

With proper headings to expedite response
[enrollment]
[technical]

Do NOT e-mail individual professor or TA

MasterProbo

- Your *personal coach* for probability
- Focus you on problem-solving
- Detect your weakness and practice adaptively

Safety

- Run > Hide > Fight

- https://www.youtube.com/watch?v=8j0_8PCWASE

Kolmogorov’s axiomatic approach

[http://gozips.uakron.edu/~decamer/math_history_pages/ANKolmogorov/ANK1.jpg]

- outcomes
- events
- probabilities

Kolmogorov's axiomatic approach

Let Ω denote the *sample space*, the set of possible outcomes.

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

An *event A* is a subset of Ω, a member of the power set 2^{Ω}.

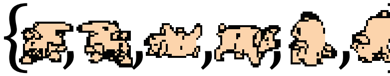
A = rolled an even number

Each event A has an associated probability, $P(A)$

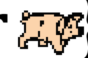
$P(A) = 1/2$

Kolmogorov's axiomatic approach

Let Ω denote the *sample space*, the set of possible outcomes.

$\Omega = \{$  $\}$

An *event A* is a subset of Ω, a member of the power set 2^{Ω}.

$A = \{$  or  $\}$

Each event *A* has an associated probability, *P*(*A*)

$P(A) = 0.224 + 0.088 = 0.312$

Experiment

- A random experiment $\mathcal{E}$ is characterized by three components:
  - Possible outcomes (sample space), Ω
  - Collection of sets of outcomes of interest, $\mathcal{F}$
  - Numerical assessment of likelihood of occurrence of each outcome of interest, P

Desiderata for sample space Ω

- Von Mises (1919):
  - List the possible outcomes of the experiment;
  - Do so without duplication;
  - At a level sufficient for our interests; and
  - List is complete in a practical sense, though usually not complete regarding all logically or physically possible outcomes

Example of sample space

- Specify an appropriate sample space for the following random experiments

Instantaneous voltage at 120VAC electric outlet measured at a randomly chosen time

The number of photons emitted by an LED operating at 1 mW in 1 second

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Instantaneous voltage at 120VAC electric outlet measured at a randomly chosen time

$$\left[-120\sqrt{2} \approx -170, 170 \approx 120\sqrt{2}\right] \subset \mathbb{R}$$

The number of photons emitted by an LED operating at 1 mW in 1 second

$$\mathbb{Z}_+$$

Fitch’s five card trick

Mathellaneous

Norman Do

A Mathemagical Card Trick

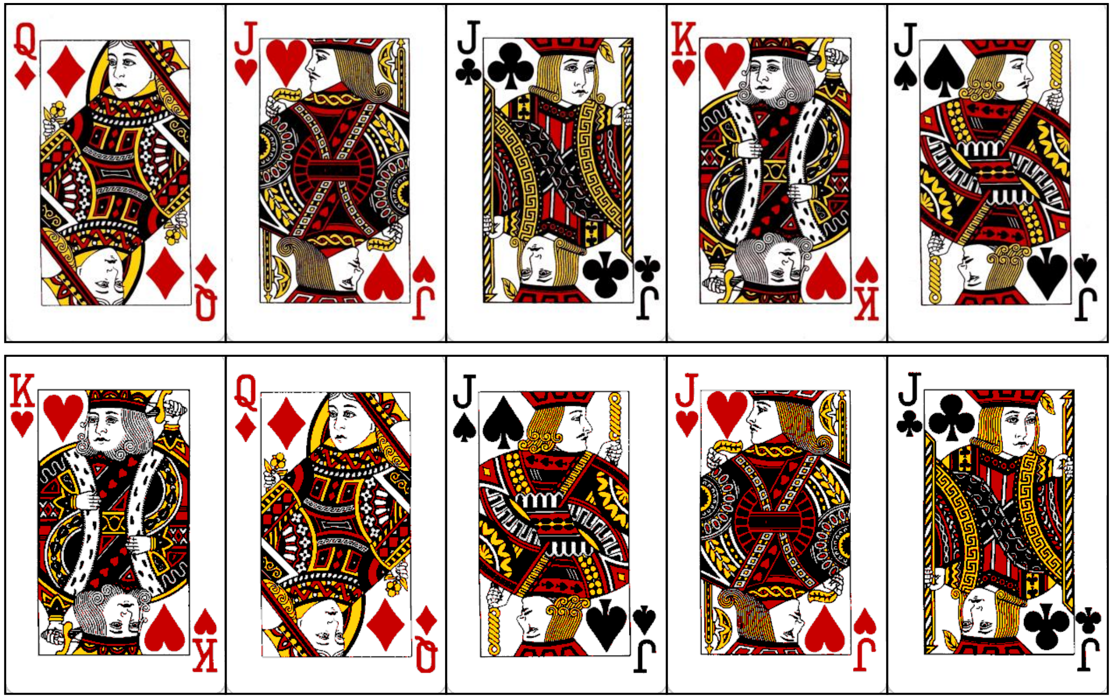
Fitch’s five card trick

1 Card Trick

Here I have a normal deck of 52 playing cards... take them and have a look for yourself. I would like you to choose five, any five, of your favourite cards and remove them from the deck. Now, being careful not to show me, pass those five cards to my lovely assistant. She will reveal four of them by placing them face up on the table — the $4\spadesuit$, the $Q\clubsuit$, the $5\heartsuit$ and then the $10\diamondsuit$ — but leave the identity of the remaining card known only to you and herself. And the identity of that card is none other than the $9\spadesuit$!

Few people would fail to be amazed by this card trick, especially when performed live.

The same or not the same?



Poker — Same

Magic — Not the Same

Events and combinations of events

- Use an event language where statements are represented by sets, constructed using Boolean set operations (complementation, union, and intersection)

$$
\begin{aligned}
A^c &= \text{complement of } A \\
AB &= A \cap B \\
A \subset B &\leftrightarrow \text{any element of } A \text{ is also an element of } B \\
A - B &= AB^c \\
I_A(x) &= \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{else} \end{cases}
\end{aligned}
$$

K-map

	B^c	B
A^c	A^cB^c	A^cB
A	AB^c	AB

Event algebra $\mathcal{F}$

- Collection of subsets of Ω such that the event axioms are satisfied:
  1. Nonempty (equivalent to $\Omega \in \mathcal{F}$)
  2. Closed under complementation, so if A is an event then A^c is an event (if $A \in \mathcal{F}$ then $A^c \in \mathcal{F}$)
  3. Closed under finite union, so if A and B are both events, then $A \cup B$ is an event (if $A, B \in \mathcal{F}$ then $A \cup B \in \mathcal{F}$)

Examples of event algebras

- Suppose $\Omega = \{1, 2, 3\}$. List three valid event algebras that correspond

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1. The smallest $\mathcal{F} = \{\emptyset, \Omega\}$, where $\emptyset$ is the empty set
2. The largest $\mathcal{F}$ is the set of all subsets of Ω known as the power set and denoted by $2^{\Omega} = \big\{\emptyset, \Omega, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}\big\}$
3. An intermediate example is $\mathcal{F} = \big\{\emptyset, \Omega, \{1\}, \{2, 3\}\big\}$

Other properties of event algebras

4. The empty set $\emptyset$ must be an event, since $\emptyset = \Omega^c$
5. If A and B are events, then AB is also an event, due to De Morgan's law: $AB = (A^c \cup B^c)^c$

Probability measure P

- The probability measure is a mapping $\mathcal{F} \to [0, 1]$ that satisfies the following axioms:

1. For any event A, $P(A) \geq 0$
2. If $A, B \in \mathcal{F}$ and if A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$
3. The probability of the sample space satisfies $P(\Omega) = 1$

Properties of probability measures

4. $P(A^c) = 1 - P(A)$
5. For any event A, $P(A) \leq 1$
6. $P(\emptyset) = 0$
7. If $A \subset B$ then $P(A) \leq P(B)$
8. $P(A \cup B) = P(A) + P(B) - P(AB)$

Where do probability measures come from?

- Frequentist
- Beliefs
- Generative models, e.g. symmetry
- (Central limit and related theorems)

Problem to Consider

- If Alice tosses a coin until she sees a head followed by a tail, and Bob tosses a coin until he sees two heads in a row, then on average, Alice will require four tosses while Bob will require six tosses (try this at home!), even though head-tail and head-head have an equal chance of appearing after two coin tosses.

[E. Klarreich, “Mathematicians Discover Prime Conspiracy,” *Quanta Magazine*, 13 March 2016.]