AN INVESTIGATION
OF
THE LAWS OF THOUGHT,
ON WHICH ARE FOUNDED
THE MATHEMATICAL THEORIES OF LOGIC
AND PROBABILITIES.

BY
GEORGE BOOLE, LL.D.
PROFESSOR OF MATHEMATICS IN QUEEN'S COLLEGE, CORK.

LONDON:
WALTON AND MABERLY,
UPPER GOWER STREET, AND IVY-LANE, PATERNOSTER-ROW.
CAMBRIDGE: MACMILLAN AND CO.
1854.
Carbon nanotubes can be grown in parallel lines, but imperfections do occur.

Speckle in SAR Imagery

[https://opticks.org/display/opticksExt/SAR+Processing+Plug-in]
Wind Speed and Turbulence

[V. B. Krishna, University of Illinois at Urbana-Champaign]
IP Packet Sizes (NASA Ames)


[http://www.caida.org/research/traffic-analysis/AIX/plen_hist/]
Social Media Popularity

Your Tweets earned 17.0K impressions over this 28 day period.

During this 28 day period, you earned 608 impressions per day.

<table>
<thead>
<tr>
<th>Date</th>
<th>Impressions</th>
<th>Engagements</th>
<th>Engagement rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aug 13</td>
<td>253</td>
<td>6</td>
<td>2.4%</td>
</tr>
<tr>
<td>Aug 20</td>
<td>3,540</td>
<td>42</td>
<td>1.2%</td>
</tr>
<tr>
<td>Aug 24</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Engagements
Showing 28 days with daily frequency

Engagement rate
1.3%
0.0% engagement rate

Link clicks
101
0 link clicks

On average, you earned 4 link clicks per day.
The Problem of Communication

[INFOGRAPHIC]

BOOLE  SHANNON
Compute & Communicate
Sherman Kent, a director of the CIA’s Office of National Estimates conducted an experiment with 23 NATO military officers accustomed to reading intelligence reports. The goal was to understand how to mathematize probabilistic language.

Chevalier de Méré

The French gambler Chevalier de Méré suspected that (1) was higher than (2), but his mathematical skills were insufficient to show why. He posed the question to Pascal.

(1) The probability of getting at least one “6” in four rolls of a single 6-sided die.

(2) The probability of at least one double-six in 24 throws of two dice.
Chevalier de Méré

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(1) The probability of getting at least one “6” in four rolls of a single 6-sided die.

\[ 1 - \left( \frac{5}{6} \right)^4 \approx 0.5177 \]

(2) The probability of at least one double-six in 24 throws of two dice.

\[ 1 - \left( \frac{35}{36} \right)^{24} \approx 0.4914 \]
Powerball (23 August 2017)

$758.7 million jackpot

Five white balls are drawn without replacement from a drum that holds 69 balls, each bearing a number between 1 and 69, where order does not matter. Then, a red Powerball is drawn from a drum holding 26 balls, each bearing a number between 1 and 26.
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Powerball (23 August 2017)

What is the probability of winning the jackpot?

Number of possible outcomes is:

$$\binom{69}{5} \binom{26}{1} = \frac{69 \times 68 \times 67 \times 66 \times 65}{5 \times 4 \times 3 \times 2 \times 1} \times \frac{26}{1} = 292201338$$

So odds of winning is:

$$\frac{1}{292201338}$$

Five white balls are drawn without replacement from a drum that holds 69 balls, each bearing a number between 1 and 69, where order does not matter. Then, a red Powerball is drawn from a drum holding 26 balls, each bearing a number between 1 and 26.
### Powerball (23 August 2017)

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<th>MATCHING COMBINATION</th>
<th>PRIZES</th>
<th>CURRENT ODDS (1 IN ...)</th>
<th>PREVIOUS ODDS (1 IN ...)</th>
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<tbody>
<tr>
<td>5 white balls and the Powerball <em>The grand prize</em></td>
<td>$292,201,338</td>
<td>175,223,510</td>
<td></td>
</tr>
<tr>
<td>5 white balls</td>
<td>$1,000,000</td>
<td>11,688,054</td>
<td>5,153,633</td>
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<tr>
<td>4 white balls and the Powerball</td>
<td>$50,000 (formerly $10,000)</td>
<td>913,129</td>
<td>648,976</td>
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<tr>
<td>4 white balls</td>
<td>$100</td>
<td>36,525</td>
<td>19,088</td>
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<tr>
<td>3 white balls and the Powerball</td>
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$758.7 million jackpot

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What is the expected payout for buying a $2 ticket, with a $758.7 million jackpot?
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\[
\frac{\$758.7M}{292201338} + \frac{\$1M}{11688054} + \frac{\$50000}{913129} + \frac{\$100}{36525} + \frac{\$100}{14494} + \frac{\$7}{580} + \frac{\$7}{701} + \frac{\$4}{92} + \frac{\$4}{38}
\]

\[
\$2.597 + \$0.086 + \$0.055 + \$0.003 + \$0.007 + \$0.012 + \$0.010 + \$0.044 + \$0.105
\]

\[
\$2.92
\]
Powerball (23 August 2017)

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(At $500 million jackpot, expected payout is $2.03)

How would things look under the old rules?
Preventing Ties

• Choice of numbers does not affect odds of winning, but it does affect odds of having to share prize, if people are manually choosing numbers

• People do not choose possible numbers with equal probability
  – Zenith radio telepathy experiment

[https://www.sciencefriday.com/articles/you-dont-need-esp-to-predict-behavior/]
Telepathy Experiment

Original Zenith radio data, representing responses of 20,099 participants; sequences are collapsed over the initial choice, represented by 0.

Treat phenomena as probabilistic at the population level, even if underlying phenomenon is not
Kolmogorov’s Axiomatic Approach

• outcomes
• events
• probabilities

[http://gozips.uakron.edu/~decamer/math_history_pages/ANKolmogorov/ANK1.jpg]
Kolmogorov’s Axiomatic Approach

Let \( \Omega \) denote the sample space, the set of possible outcomes.
\[
\Omega = \{1, 2, 3, 4, 5, 6\}
\]

An event \( A \) is a subset of \( \Omega \), a member of the power set \( 2^\Omega \).
\[
A = \text{rolled an even number}
\]

Each event \( A \) has an associated probability, \( P(A) \)
\[
P(A) = 1/2
\]
Astragali and Pass the Pigs


[https://en.wikipedia.org/wiki/Pass_the_Pigs]
Pass the Pigs

The approximate relative frequencies of the various positions for a single pig, using a standardized surface, a trap-door rolling device, and a sample size of 11,954, are:

<table>
<thead>
<tr>
<th>Position</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side (no dot)</td>
<td>34.9%</td>
</tr>
<tr>
<td>Side (dot)</td>
<td>30.2%</td>
</tr>
<tr>
<td>Razorback</td>
<td>22.4%</td>
</tr>
<tr>
<td>Trotter</td>
<td>8.8%</td>
</tr>
<tr>
<td>Snouter</td>
<td>3.0%</td>
</tr>
<tr>
<td>Leaning Jowler</td>
<td>0.61%</td>
</tr>
</tbody>
</table>

Kolmogorov’s Axiomatic Approach

Let $\Omega$ denote the *sample space*, the set of possible outcomes.

$$\Omega = \{\text{outcome}_1, \text{outcome}_2, \text{outcome}_3, \text{outcome}_4\}$$

An *event* $A$ is a subset of $\Omega$, a member of the power set $2^\Omega$.

$$A = \{\text{outcome}_1 \text{ or } \text{outcome}_2\}$$

Each event $A$ has an associated probability, $P(A)$.

$$P(A) = 0.224 + 0.088 = 0.312$$
Problem to Consider

• If Alice tosses a coin until she sees a head followed by a tail, and Bob tosses a coin until he sees two heads in a row, then on average, Alice will require four tosses while Bob will require six tosses (try this at home!), even though head-tail and head-head have an equal chance of appearing after two coin tosses.

• Class website:  
https://courses.engr.illinois.edu/ece313

• You cannot log into masterprobo with your U of I password; you need to “register” first

• Read the “Homework” page carefully