Probability with Engineering Applications
ECE 313 – Section C – Lecture 1

Lav R. Varshney

28 August 2017

AN INVESTIGATION

OF

THE LAWS OF THOUGHT,

ON WHICH ARE FOUNDED

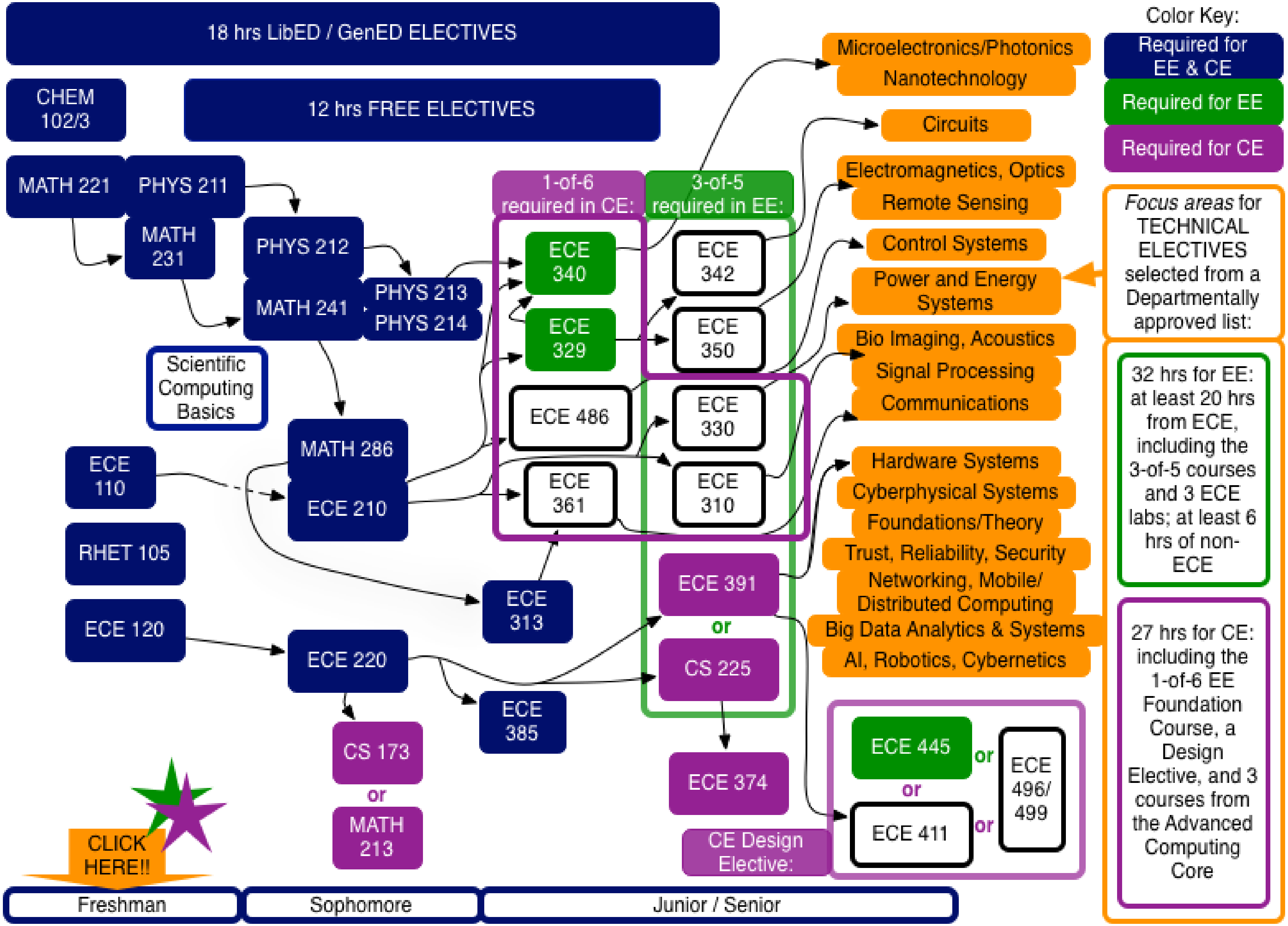
THE MATHEMATICAL THEORIES OF LOGIC
AND PROBABILITIES.

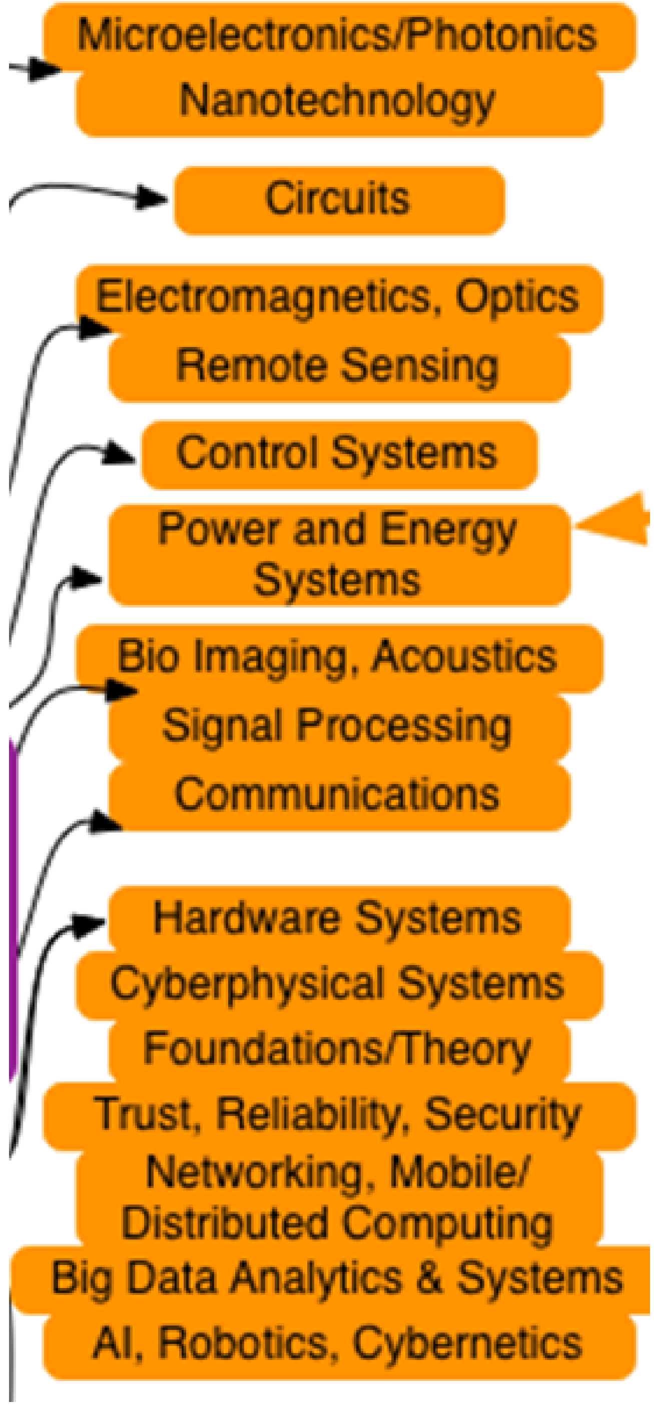
BY

GEORGE BOOLE, LL.D.
PROFESSOR OF MATHEMATICS IN QUEEN'S COLLEGE, CORK.

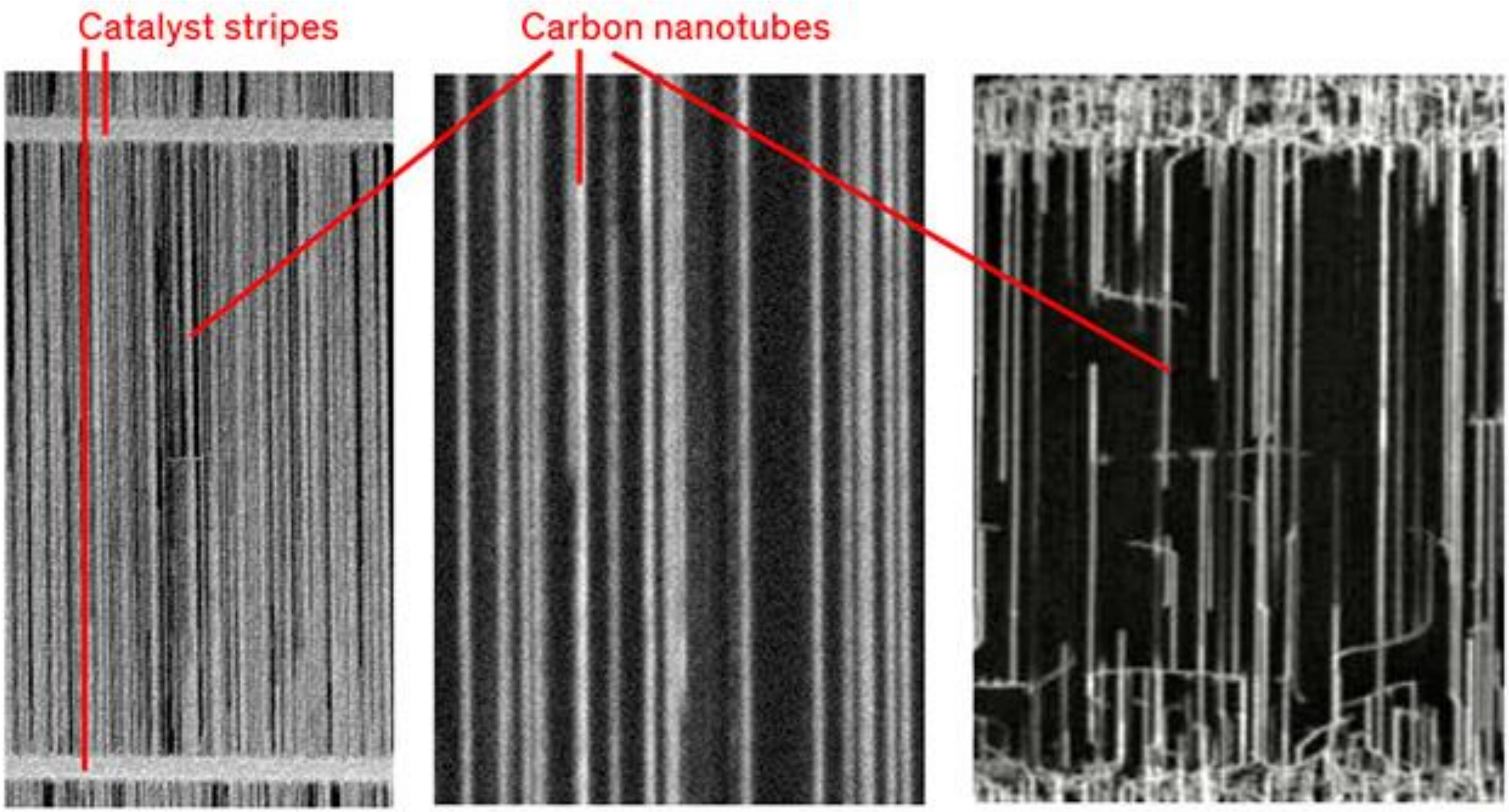
LONDON:
WALTON AND MABERLY,
UPPER GOWER-STREET, AND IVY-LANE, PATERNOSTER-ROW.
CAMBRIDGE: MACMILLAN AND CO.
1854.

THE MATHEMATICAL THEORIES OF LOGIC
AND PROBABILITIES.


18 hrs LibED / GenED ELECTIVES
CHEM 102/3
12 hrs FREE ELECTIVES
MATH 221
PHYS 211
MATH 231
PHYS 212
MATH 241
PHYS 213
PHYS 214
Scientific Computing Basics
MATH 286
ECE 110
ECE 210
RHET 105
ECE 120
ECE 220
CS 173
or
MATH 213
ECE 313
ECE 385
1-of-6 required in CE:
3-of-5 required in EE:
ECE 340
ECE 329
ECE 486
ECE 361
ECE 342
ECE 350
ECE 330
ECE 310
ECE 391
or
CS 225
ECE 374
CE Design Elective:
ECE 445
or
ECE 411
or
or
ECE 496/ 499
Microelectronics/Photonics
Nanotechnology
Circuits
Electromagnetics, Optics
Remote Sensing
Control Systems
Power and Energy Systems
Bio Imaging, Acoustics
Signal Processing
Communications
Hardware Systems
Cyberphysical Systems
Foundations/Theory
Trust, Reliability, Security
Networking, Mobile/ Distributed Computing
Big Data Analytics & Systems
AI, Robotics, Cybernetics
Color Key:
Required for EE & CE
Required for EE
Required for CE
Focus areas for TECHNICAL ELECTIVES selected from a Departmentally approved list:
32 hrs for EE: at least 20 hrs from ECE, including the 3-of-5 courses and 3 ECE labs; at least 6 hrs of non-ECE
27 hrs for CE: including the 1-of-6 EE Foundation Course, a Design Elective, and 3 courses from the Advanced Computing Core
CLICK HERE!!
Freshman
Sophomore
Junior / Senior


Microelectronics/Photonics
Nanotechnology
Circuits
Electromagnetics, Optics
Remote Sensing
Control Systems
Power and Energy Systems
Bio Imaging, Acoustics
Signal Processing
Communications
Hardware Systems
Cyberphysical Systems
Foundations/Theory
Trust, Reliability, Security
Networking, Mobile/ Distributed Computing
Big Data Analytics & Systems
AI, Robotics, Cybernetics

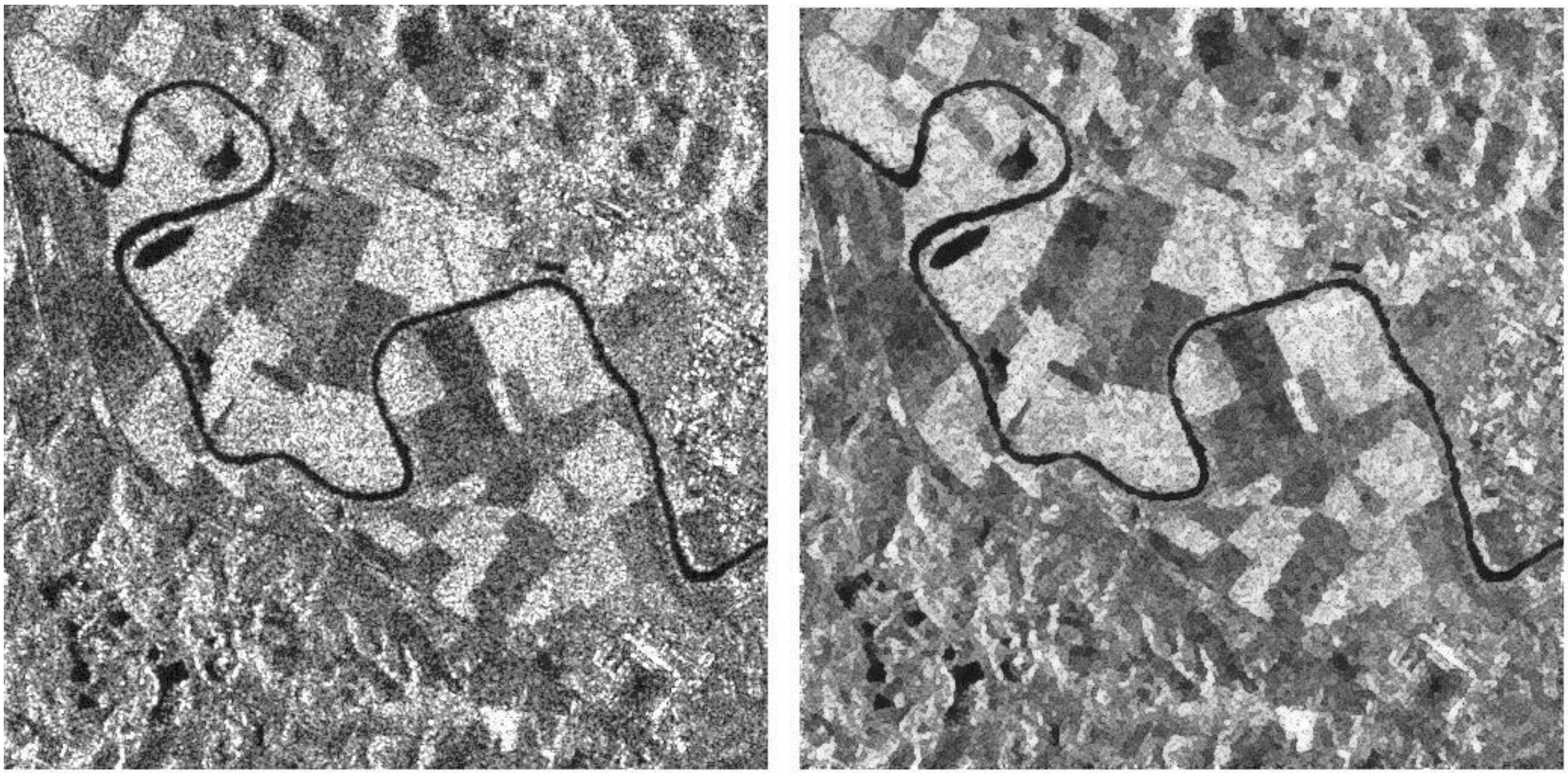
Carbon Nanotube Computers



Carbon nanotubes can be grown in parallel lines, but imperfections do occur.

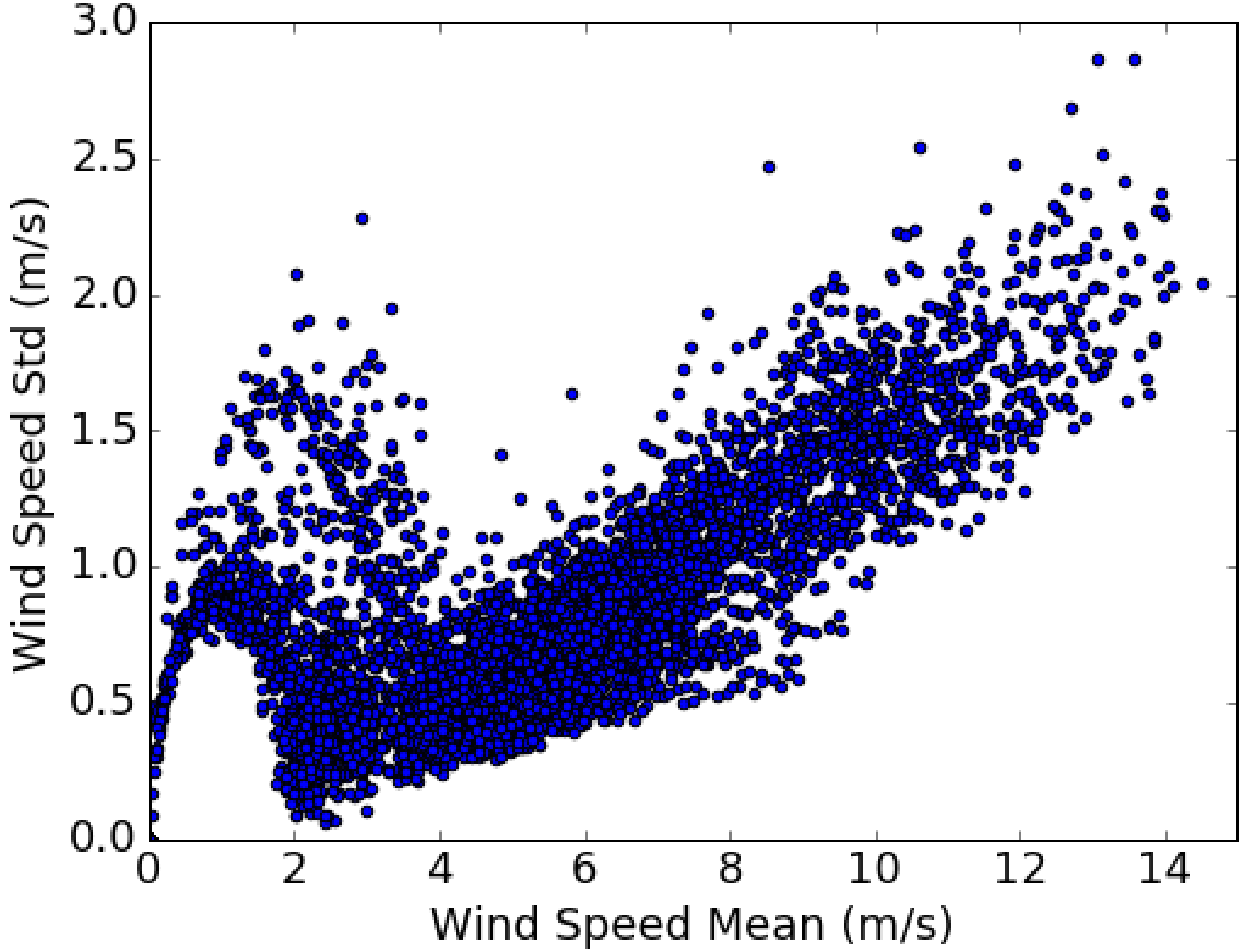
[M. Shulaker, H.-S. P. Wong, and S. Mitra, "How We'll Put a Carbon Nanotube Computer in Your Hand," *IEEE Spectrum*, Jun. 2016.]

Speckle in SAR Imagery



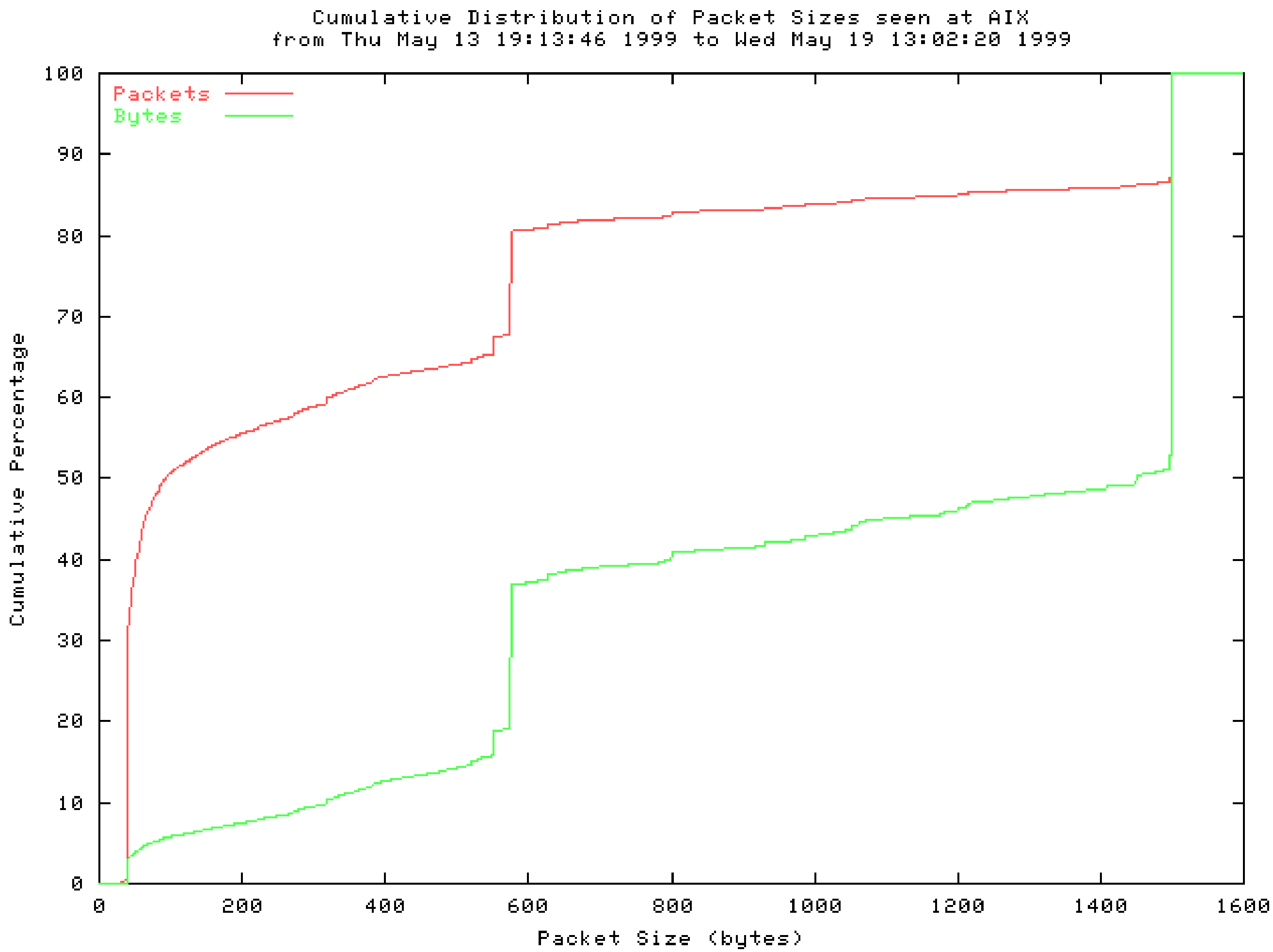
[https://opticks.org/display/opticksExt/SAR+Processing+Plug-in]

Wind Speed and Turbulence



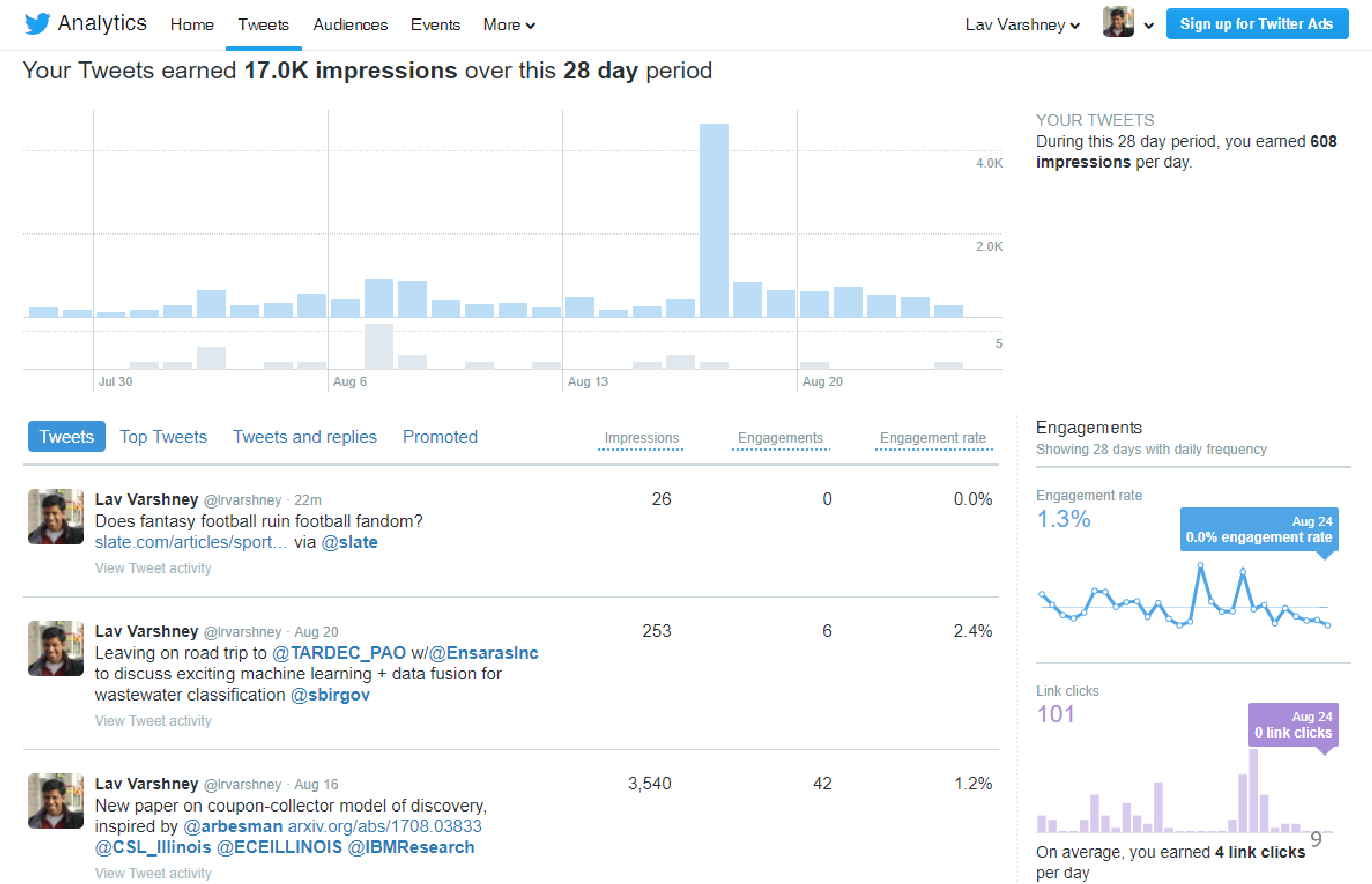
[V. B. Krishna, University of Illinois at Urbana-Champaign]

IP Packet Sizes (NASA Ames)

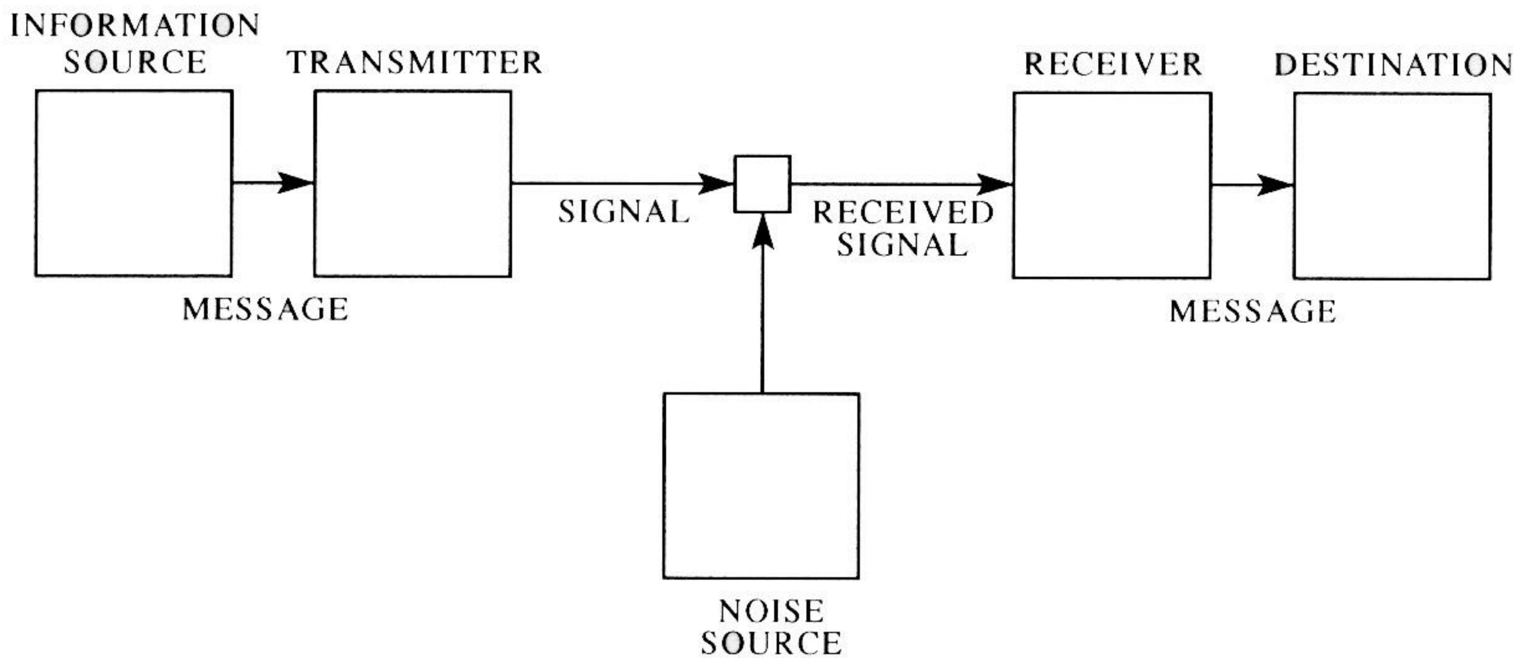


[http://www.caida.org/research/traffic-analysis/AIX/plen_hist/]

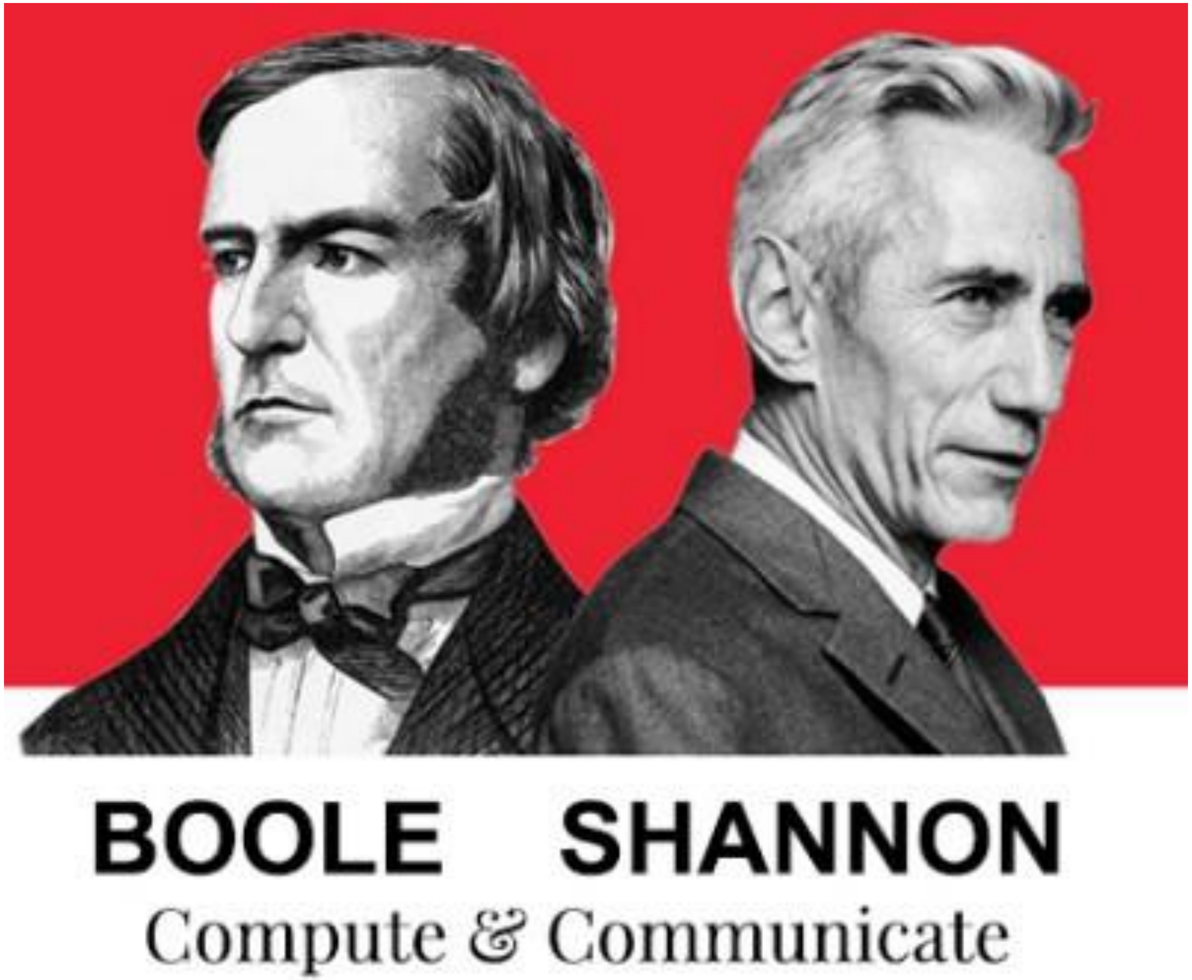
Social Media Popularity



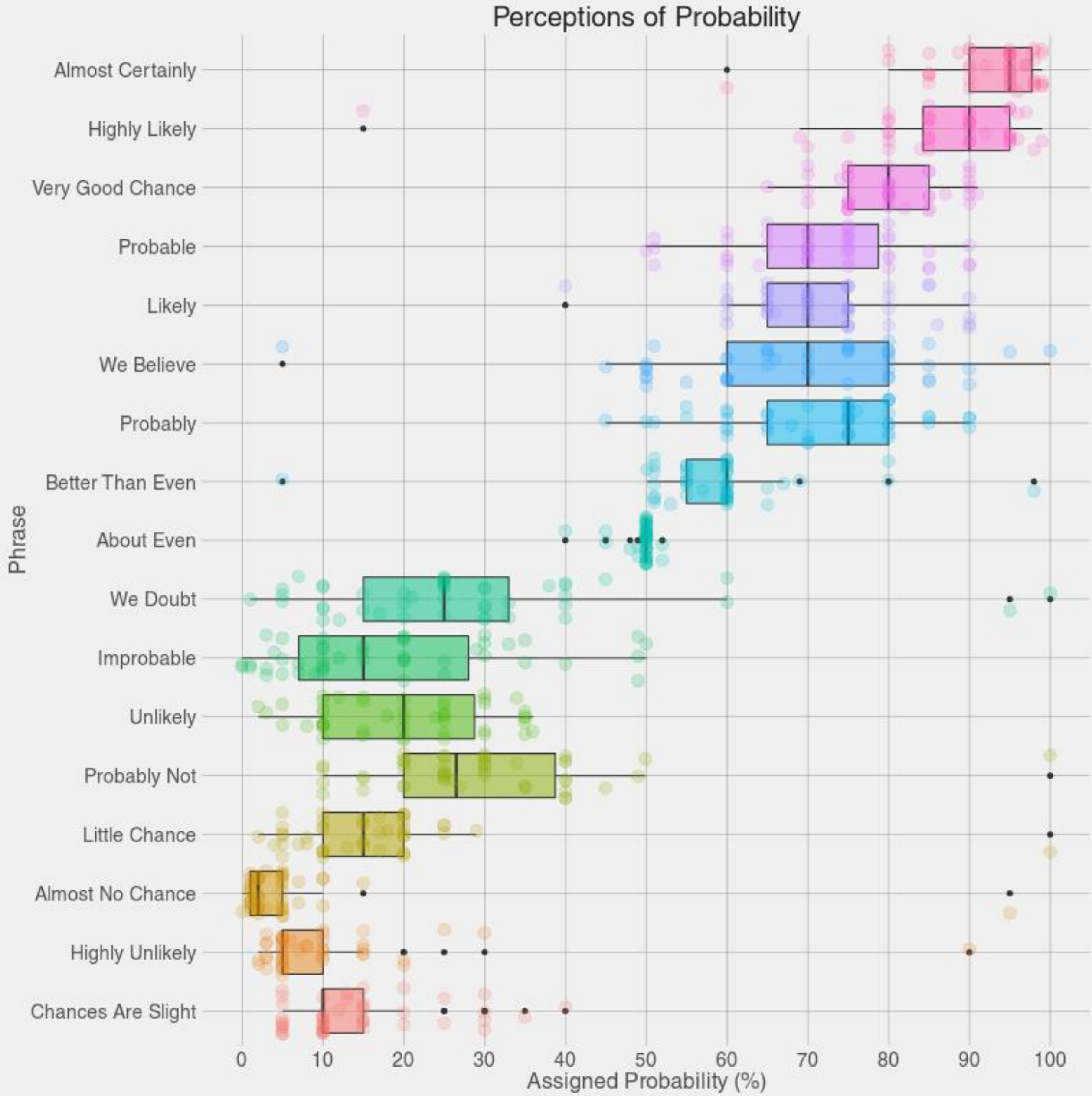
The Problem of Communication



[C. E. Shannon, "A Mathematical Theory of Communication," *Bell Syst. Tech. J*., Jul. 1948.]


BOOLE
SHANNON
Compute & Communicate


GREAT MINDS
DON'T THINK ALIKE



Sherman Kent, a director of the CIA's Office of National Estimates conducted an experiment with 23 NATO military officers accustomed to reading intelligence reports. The goal was to understand how to mathematize probabilistic language.

[Donald P. Steury, et al., "Probability," in *Sherman Kent and the Board of National Estimates: Collected Essays*, Washington, DC: Center for Study of Intelligence, CIA, 1994. Replotted at https://github.com/zonination/perceptions.]

Chevalier de Méré

The French gambler Chevalier de Méré suspected that (1) was higher than (2), but his mathematical skills were insufficient to show why. He posed the question to Pascal.

(1) The probability of getting at least one “6” in four rolls of a single 6-sided die.

(2) The probability of at least one double-six in 24 throws of two dice.

Chevalier de Méré

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(1) The probability of getting at least one "6" in four rolls of a single 6-sided die.

$$1 - \left(\frac{5}{6}\right)^4 \approx 0.5177$$

(2) The probability of at least one double-six in 24 throws of two dice.

$$1 - \left(\frac{35}{36}\right)^{24} \approx 0.4914$$

Powerball (23 August 2017)

$758.7 million jackpot

Five white balls are drawn without replacement from a drum that holds 69 balls, each bearing a number between 1 and 69, where order does not matter. Then, a red Powerball is drawn from a drum holding 26 balls, each bearing a number between 1 and 26.

Powerball (23 August 2017)

What is the probability of winning the jackpot?

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Powerball (23 August 2017)

What is the probability of winning the jackpot?

Number of possible outcomes is:

$$\binom{69}{5}\binom{26}{1} = \frac{69 \times 68 \times 67 \times 66 \times 65}{5 \times 4 \times 3 \times 2 \times 1} \times \frac{26}{1} = 292201338$$

So odds of winning is:

$$\frac{1}{292201338}$$

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Powerball (23 August 2017)

MATCHING COMBINATION	PRIZES	CURRENT ODDS (1 IN ...)	PREVIOUS ODDS (1 IN ...)
5 white balls and the Powerball	The grand prize	292,201,338	175,223,510
5 white balls	$1,000,000	11,688,054	5,153,633
4 white balls and the Powerball	$50,000 (formerly $10,000)	913,129	648,976
4 white balls	$100	36,525	19,088
3 white balls and the Powerball	$100	14,494	12,245
3 white balls	$7	580	360
2 white balls and the Powerball	$7	701	706
1 white balls and the Powerball	$4	92	111
The Powerball	$4	38	55

$758.7 million jackpot

[A. Horton, "How Powerball manipulated the odds to make another massive jackpot," *Washington Post*, 22 Aug. 2017. https://www.washingtonpost.com/news/wonk/wp/2017/08/22/how-powerball-manipulated-the-odds-to-make-another-massive-jackpot]

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What is the expected payout for buying a $2 ticket, with a $758.7 million jackpot?

$$\frac{\$758.7M}{292201338} + \frac{\$1M}{11688054} + \frac{\$50000}{913129} + \frac{\$100}{36525} + \frac{\$100}{14494} + \frac{\$7}{580} + \frac{\$7}{701} + \frac{\$4}{92} + \frac{\$4}{38}$$

$$\$2.597 + \$0.086 + \$0.055 + \$0.003 + \$0.007 + \$0.012 + \$0.010 + \$0.044 + \$0.105$$

$2.92

Powerball (23 August 2017)

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(At $500 million jackpot, expected payout is $2.03)

How would things look under the old rules?

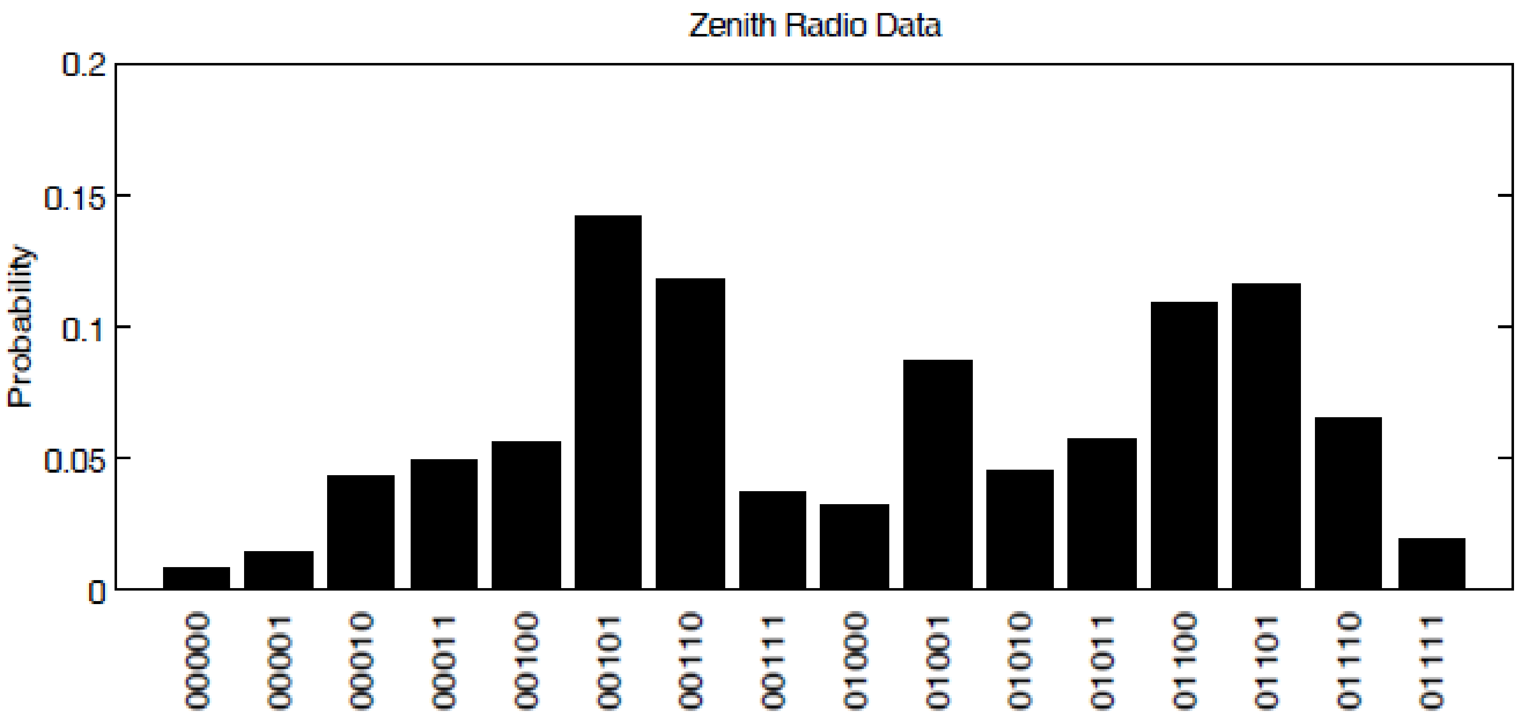
Preventing Ties

- Choice of numbers does not affect odds of winning, but it does affect odds of having to share prize, if people are manually choosing numbers
- People do not choose possible numbers with equal probability
  - Zenith radio telepathy experiment

[https://www.sciencefriday.com/articles/you-dont-need-esp-to-predict-behavior/]

Telepathy Experiment

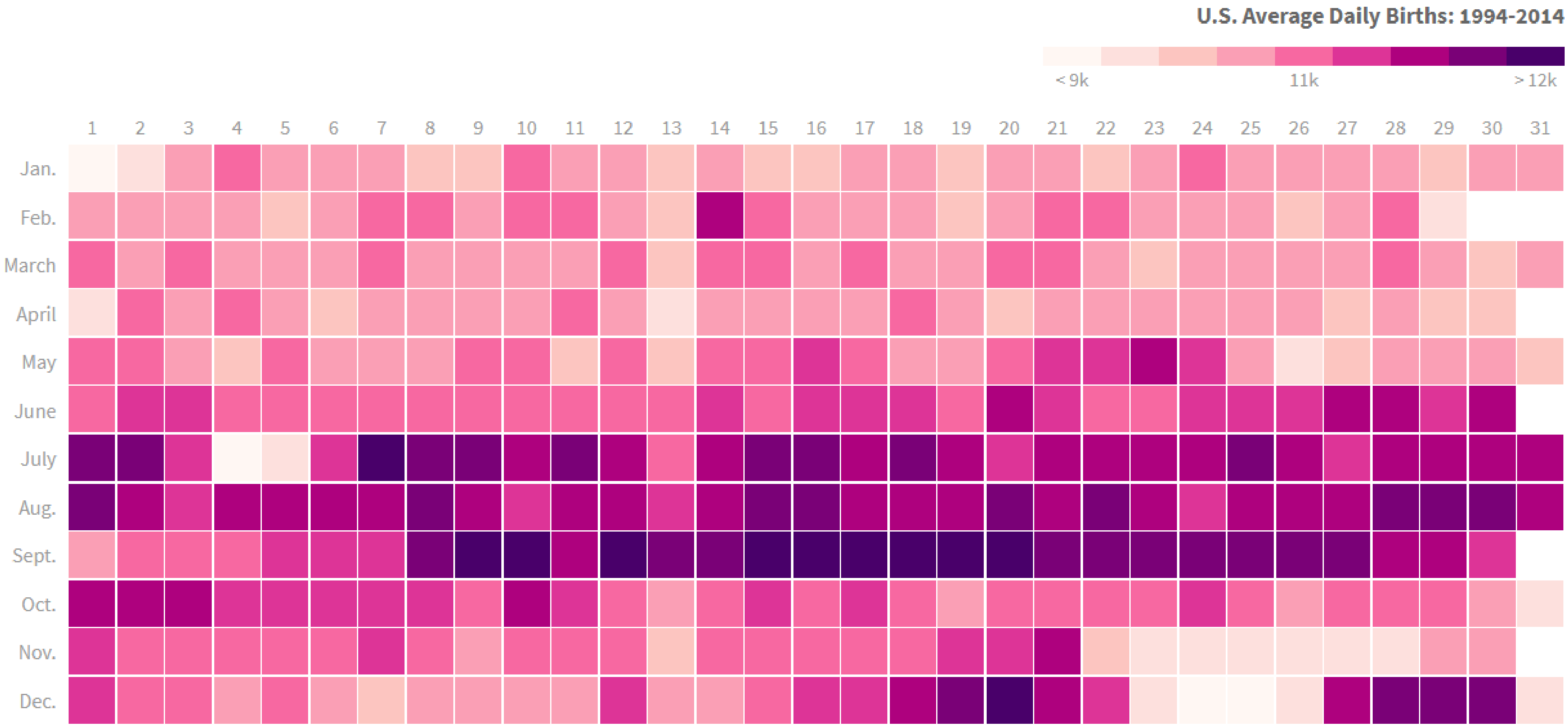
Original Zenith radio data, representing responses of 20,099 participants; sequences are collapsed over the initial choice, represented by 0.



[L. D. Goodfellow, "A psychological interpretation of the results of the Zenith radio experiments in telepathy," *Journal of Experimental Psychology*, vol. 23, pp. 601–632, 1938. As plotted by T. L. Griffiths and J. B. Tenenbaum, "Randomness and coincidences: Reconciling intuition and probability theory," in *Proceedings of the 23rd Annual Conference of the Cognitive Science Society*, Edinburgh, Aug. 2001.]

Birthdays

Treat phenomena as probabilistic at the population level, even if underlying phenomenon is not



[U.S. National Center for Health Statistics (1994-2003); U.S. Social Security Administration (2004-2014) — via FiveThirtyEight
Credit: Matt Stiles/The Daily Viz (http://thedailyviz.com/2016/09/17/how-common-is-your-birthday-dailyviz)]

Kolmogorov’s Axiomatic Approach

[http://gozips.uakron.edu/~decamer/math_history_pages/ANKolmogorov/ANK1.jpg]

- outcomes
- events
- probabilities

Kolmogorov's Axiomatic Approach

Let Ω denote the *sample space*, the set of possible outcomes.

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

An *event* A is a subset of Ω, a member of the power set 2^{Ω}.

A = rolled an even number

Each event A has an associated probability, $P(A)$

$P(A) = 1/2$

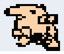
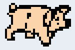
Astragali and Pass the Pigs

[http://www.pitt.edu/~pittcntr/Being_here/last_donut/donut_2014-15/02-27-15_dice.html]

[https://en.wikipedia.org/wiki/Pass_the_Pigs]

Pass the Pigs

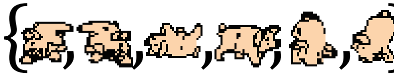
The approximate relative frequencies of the various positions for a single pig, using a standardized surface, a trap-door rolling device, and a sample size of 11,954, are:

Position		Percentage
Side (no dot)		34.9%
Side (dot)		30.2%
Razorback		22.4%
Trotter		8.8%
Snouter		3.0%
Leaning Jowler		0.61%

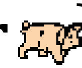
[J. C. Kern, "Pig Data and Bayesian Inference on Multinomial Probabilities," *Journal of Statistics Education*, vol. 14, no. 3, 2006.

Kolmogorov’s Axiomatic Approach

Let Ω denote the *sample space*, the set of possible outcomes.

$\Omega = \{$  $\}$

An *event A* is a subset of Ω, a member of the power set 2^{Ω}.

$A = \{$  or  $\}$

Each event *A* has an associated probability, *P*(*A*)

$P(A) = 0.224 + 0.088 = 0.312$

Problem to Consider

- If Alice tosses a coin until she sees a head followed by a tail, and Bob tosses a coin until he sees two heads in a row, then on average, Alice will require four tosses while Bob will require six tosses (try this at home!), even though head-tail and head-head have an equal chance of appearing after two coin tosses.

[E. Klarreich, “Mathematicians Discover Prime Conspiracy,” *Quanta Magazine*, 13 March 2016.]

- Class website: https://courses.engr.illinois.edu/ece313
- You cannot log into masterprobo with your U of I password; you need to “register” first
- Read the “Homework” page carefully