Beyond FFT: Precision Vibration Tracking with FMCW Radar and Kalman Estimators

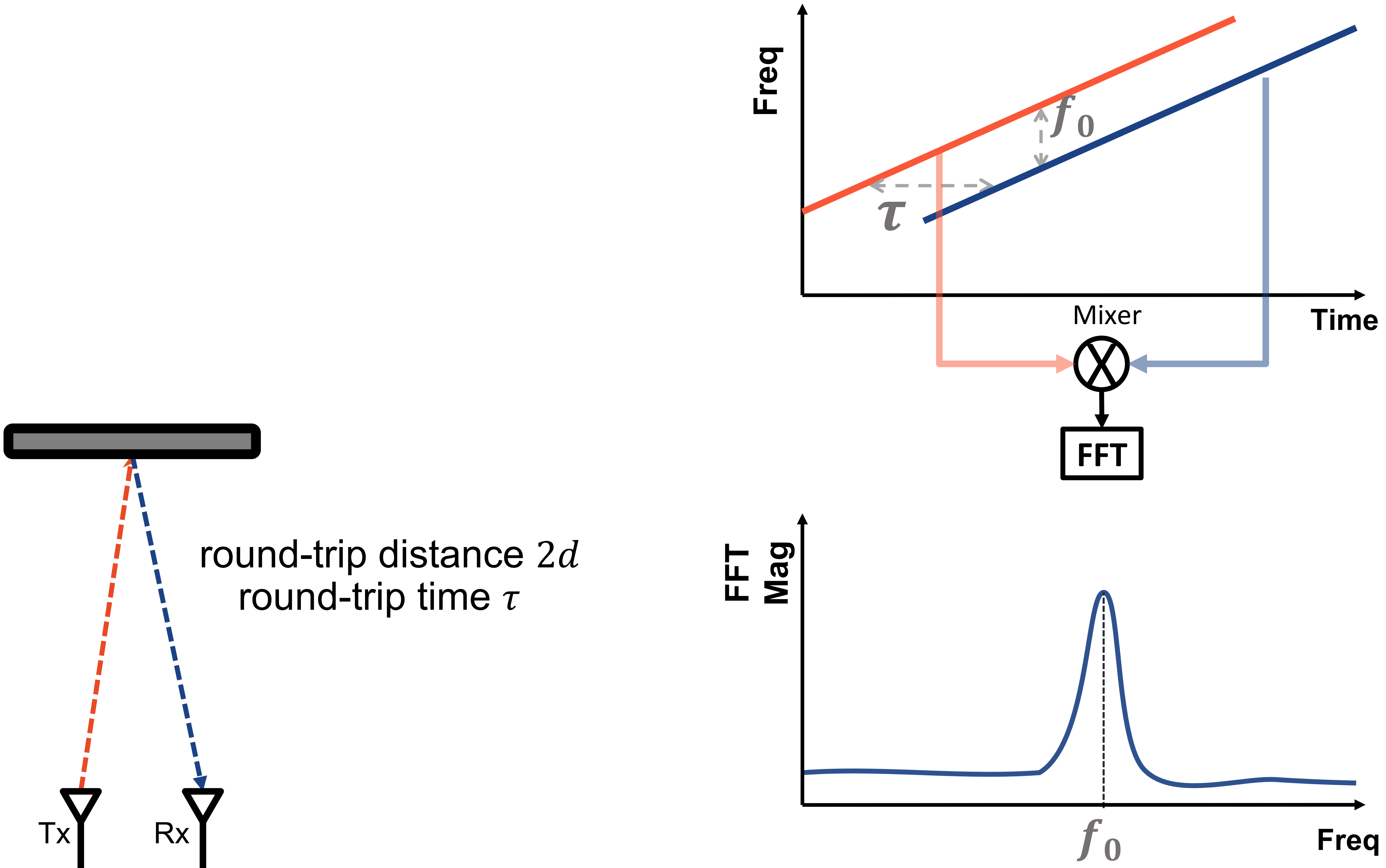
Thomas Moon

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

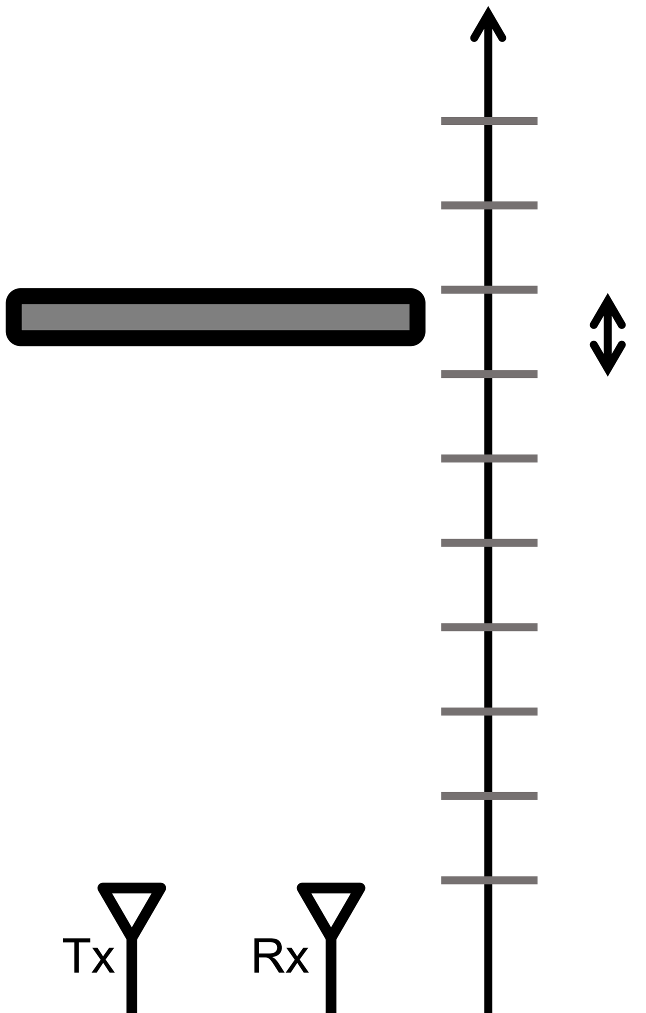
RadarConf'24
2024 IEEE RADAR CONFERENCE

Beyond FFT: Precision Vibration Tracking with FMCW Radar and Kalman Estimators

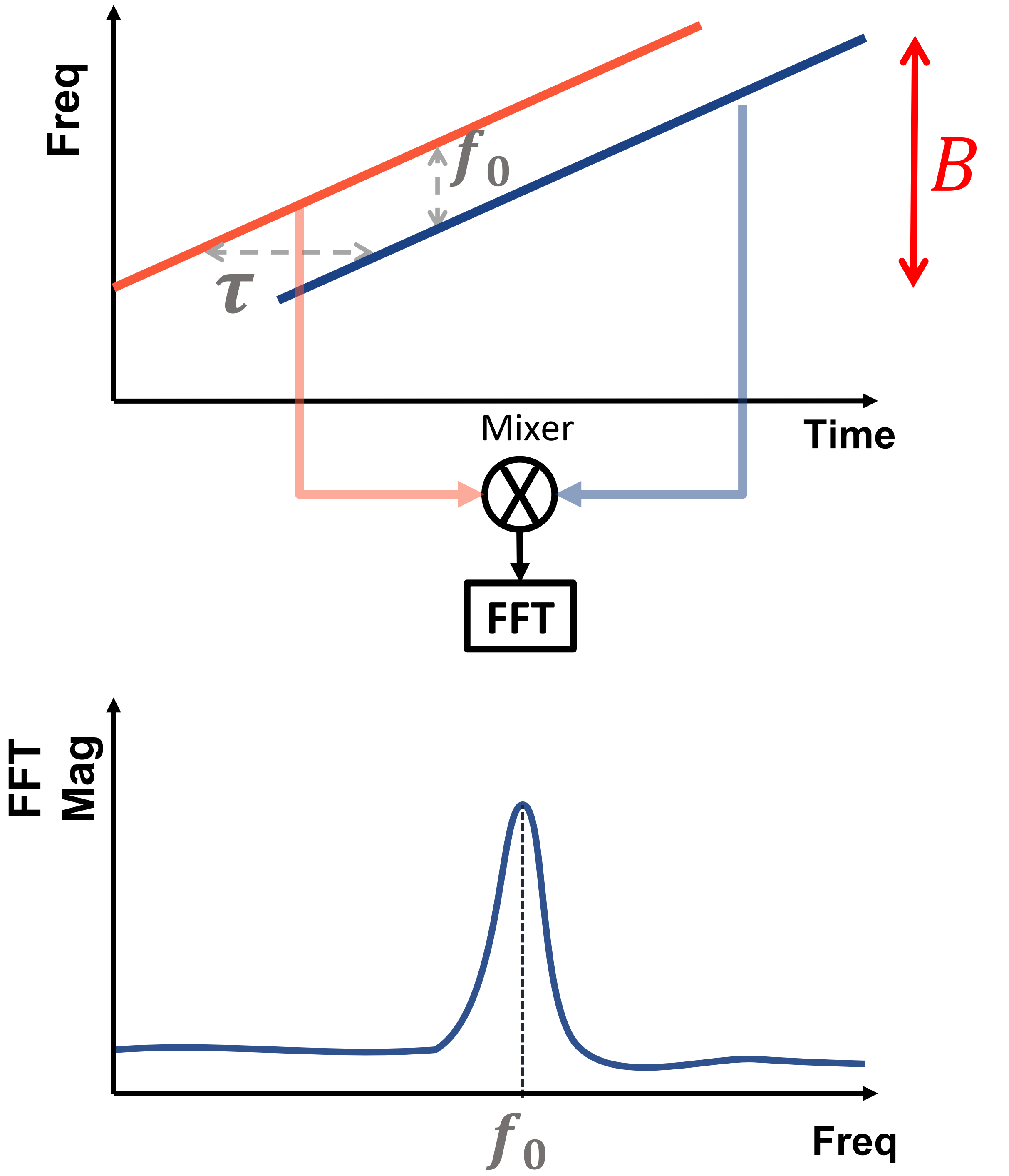
- What: This work is about Vibration Tracking using FMCW
- Why: Traditional FFT approach slows down the tracker
- How: Use Bayesian estimator (Kalman) tracking sample-by-sample

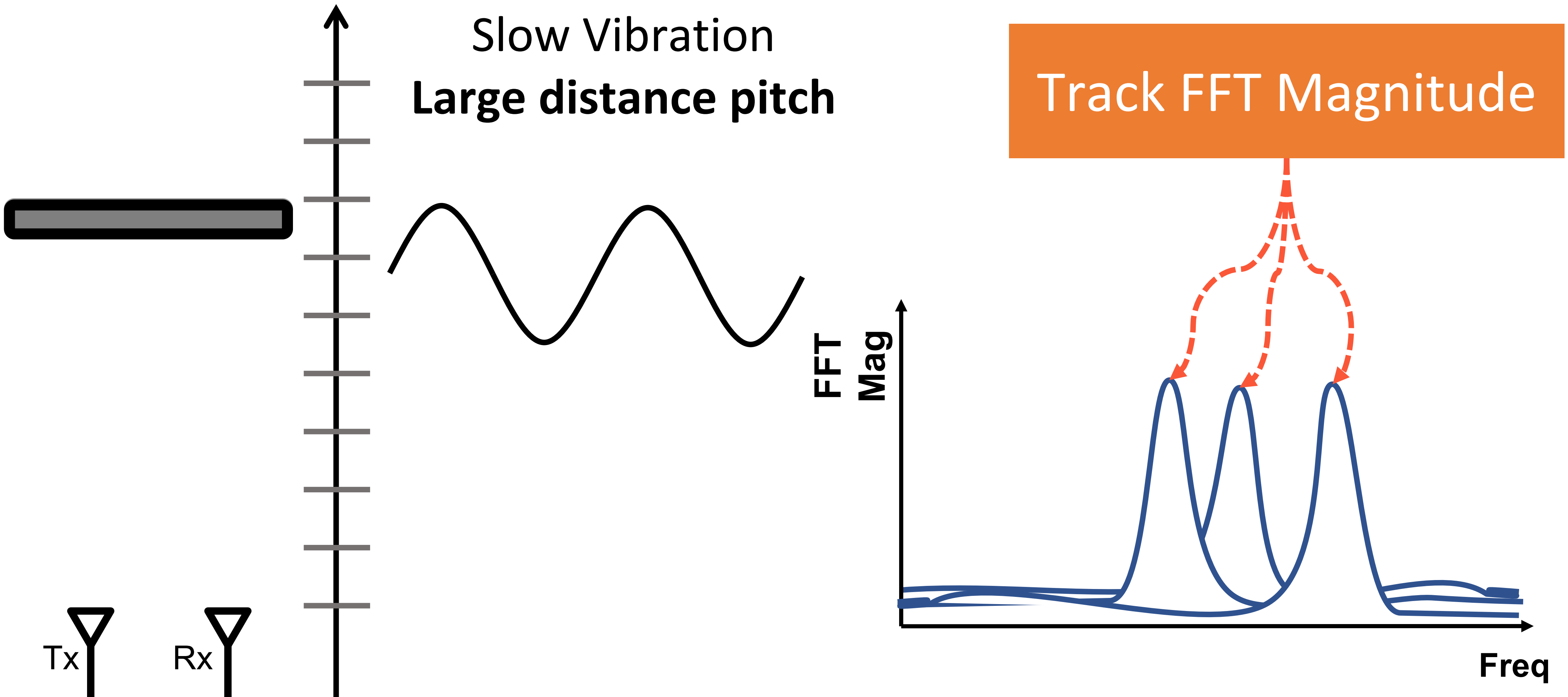


Freq
f_0
τ
Time
Mixer
FFT
FFT Mag
f_0
Freq
round-trip distance $2d$
round-trip time τ
Tx
Rx



$$\Delta d = \frac{c}{2B}$$

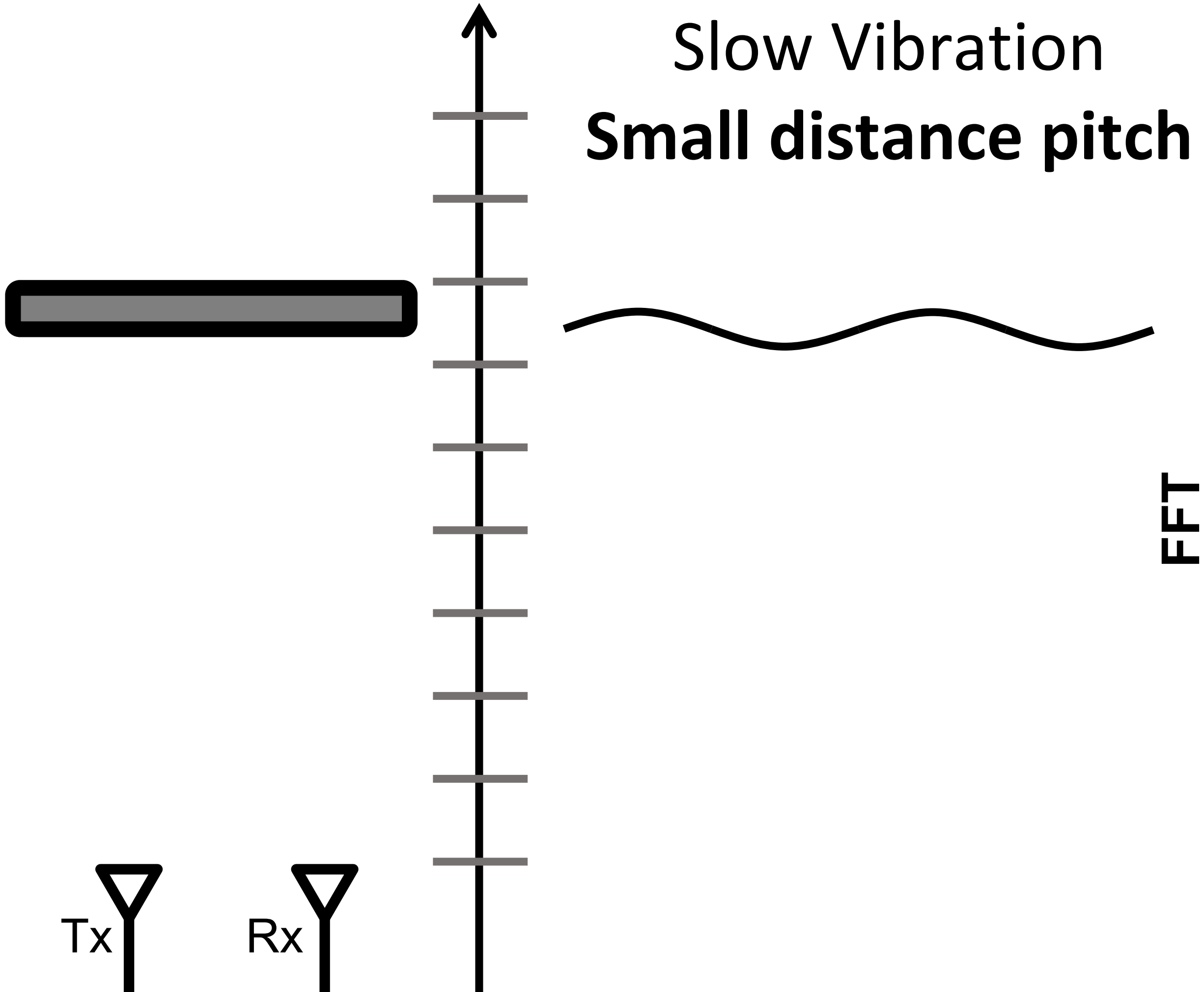


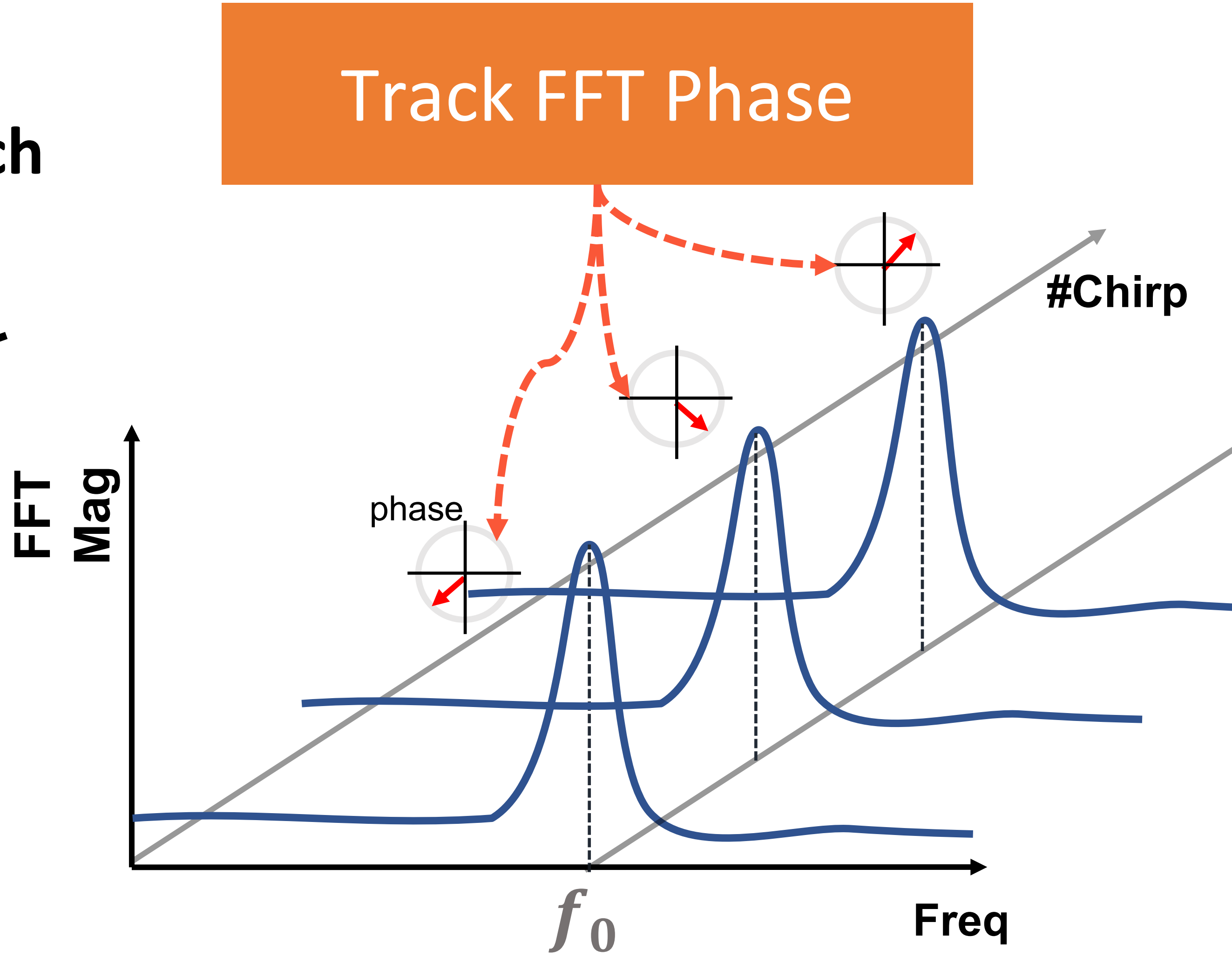

Slow Vibration
Large distance pitch
Track FFT Magnitude
FFT Mag
Freq
Tx
Rx

No Change in FFT Magnitude 😟

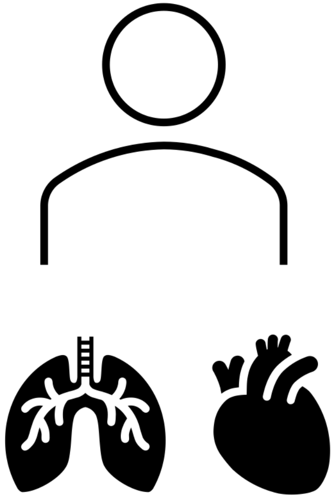
However,

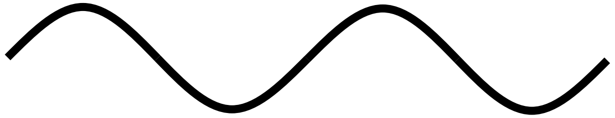
FFT Phase changes!





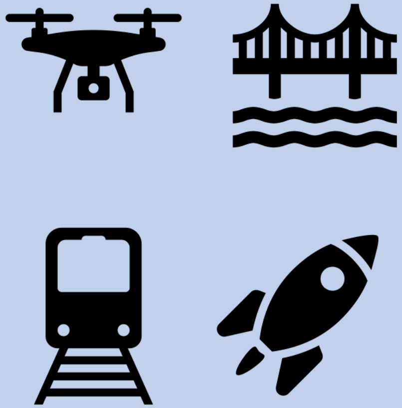
FMCW radar can also measure Vibration, but…

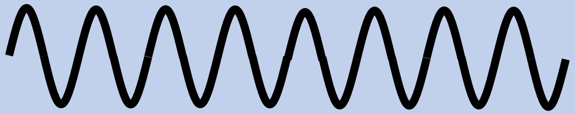




Slow frequency

Fourier Transfrom
Mag/Phase

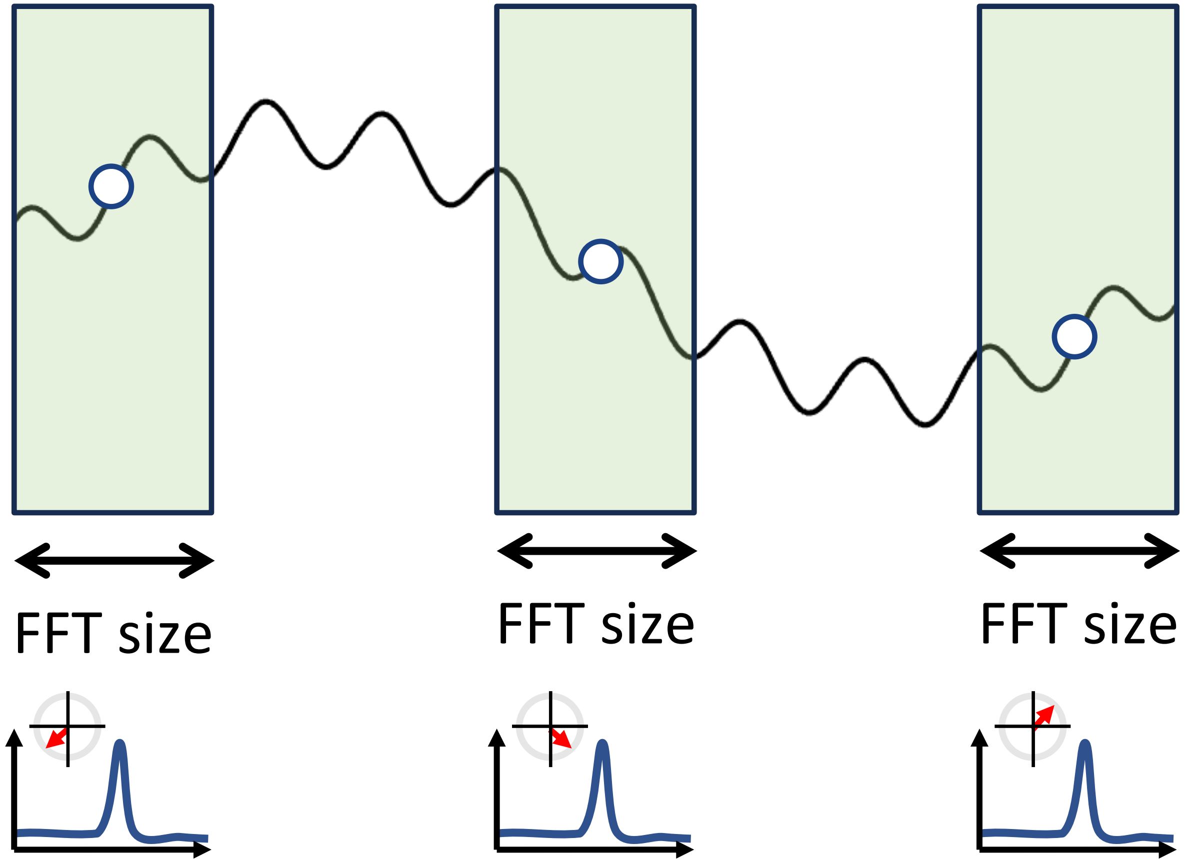




Fast frequency

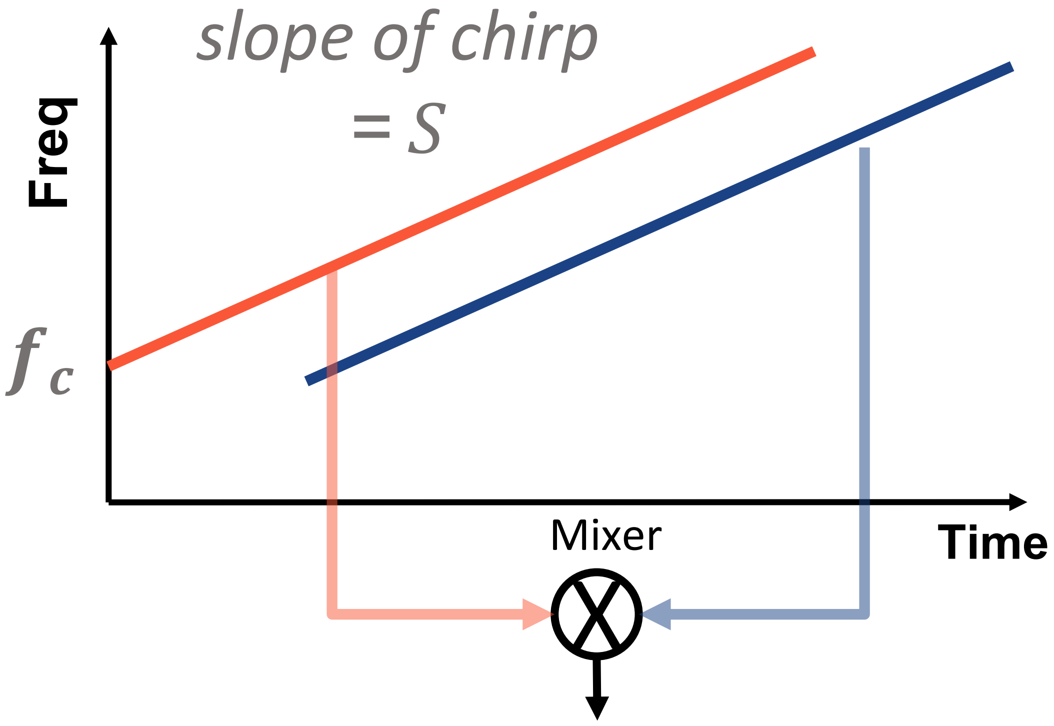
???

Fast Vibration: Challenge 1



FFT Needs a batch of samples
→ Fail to capture the rapid vibration

Fast Vibration: Challenge 2

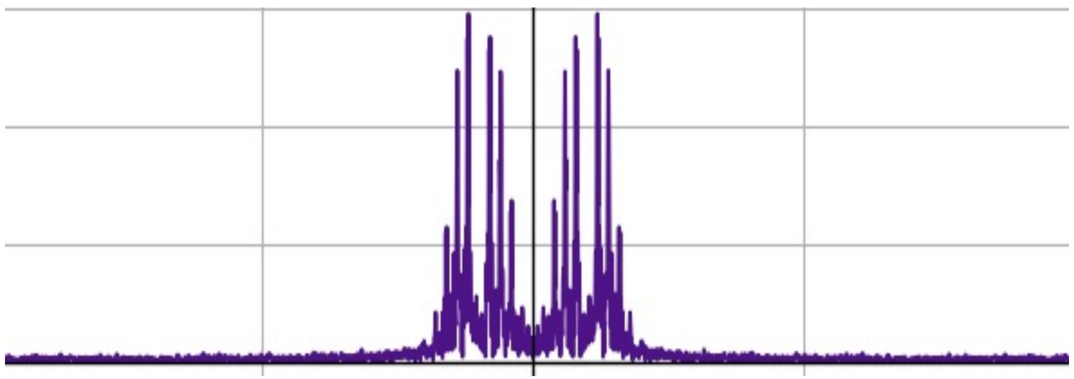


$$x(t) = e^{j2\pi(S\tau(t)t + f_c\tau(t))}$$

$$= e^{j(2\pi f(t)t + \phi(t))}$$

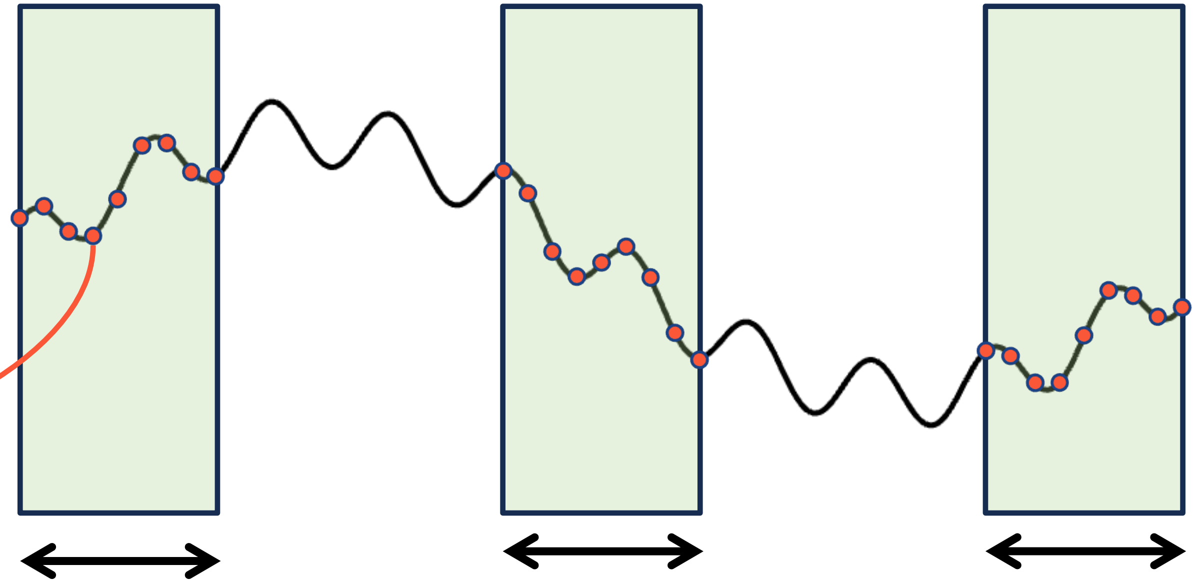
Frequency Modulation (FM) + Phase Modulation (PM)

Wideband FM/PM Spectrum



Spectrum is too complicated

Proposed: State Equations for FMCW



$$x(t) = e^{j2\pi(S\tau(t)t + f_c\tau(t))}$$

phase of a sample (Not phase of N-FFT)

$$\theta(t) = 2\pi S\tau(t)t + 2\pi f_c\tau(t)$$

phase of a sample (Not phase of N-FFT)

$$\theta(t) = 2\pi S\tau(t)t + 2\pi f_c\tau(t)$$

observation

unknown vibration

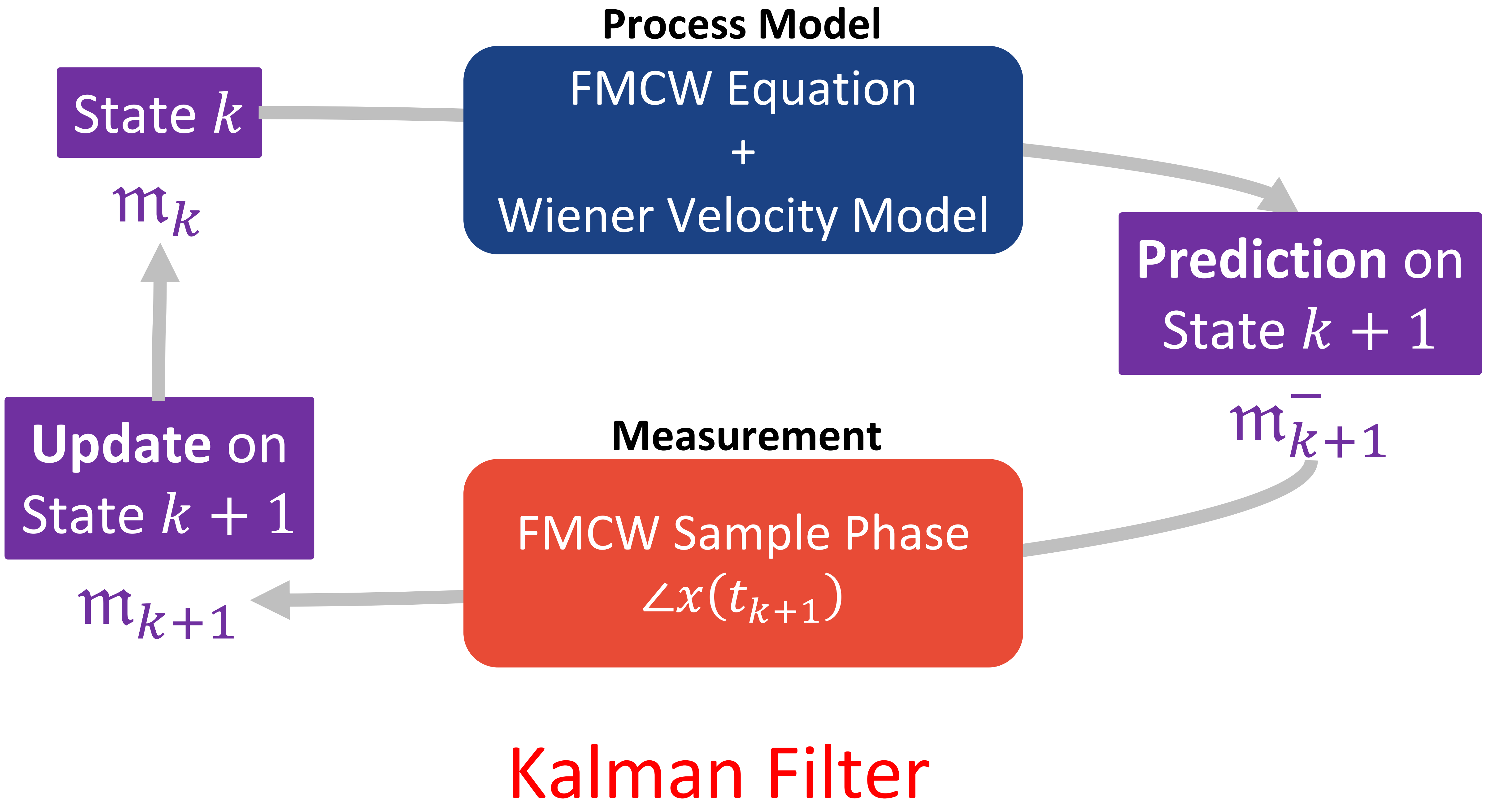
State

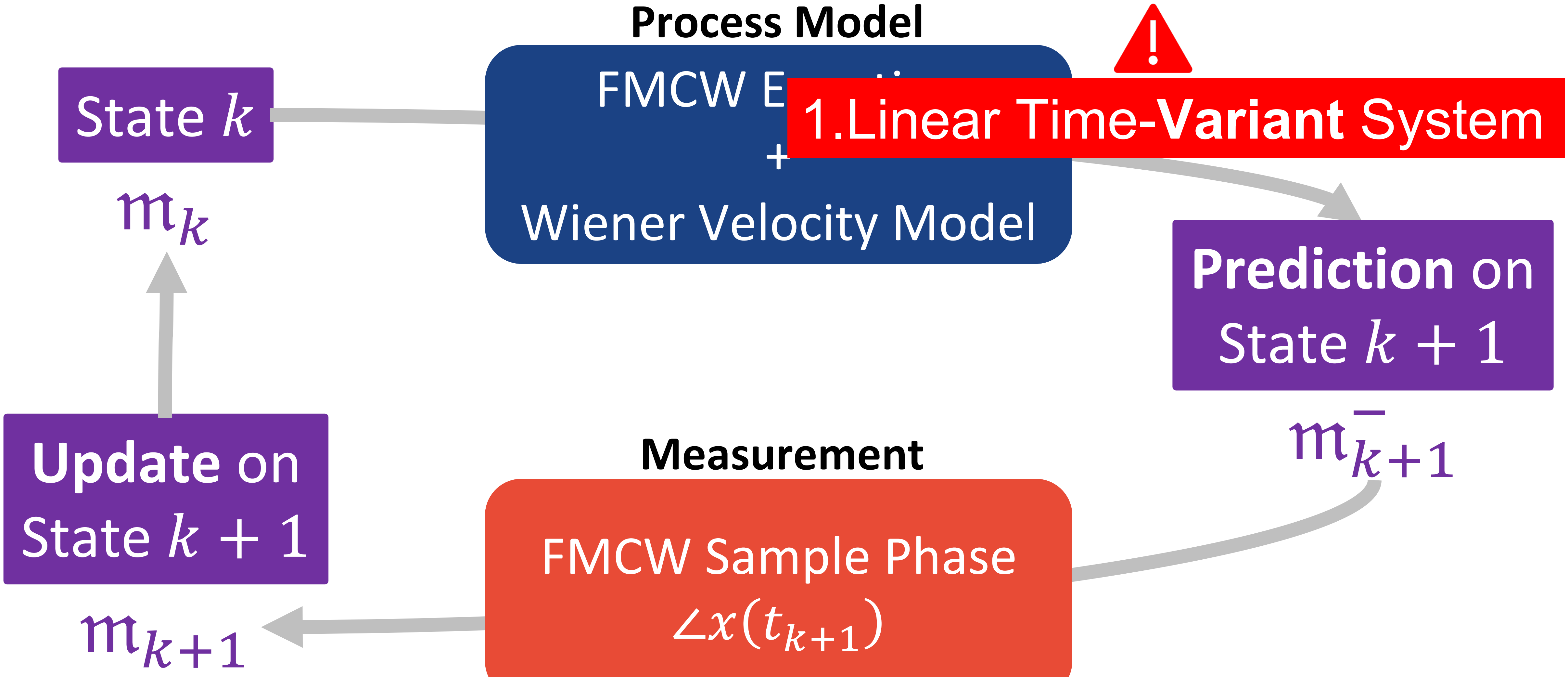
$$\mathfrak{m}_k = \begin{bmatrix} \theta(t_k) \\ \tau(t_k) \\ \tau'(t_k) \end{bmatrix}$$

Replace with a random process

→ Wiener velocity model

$$\frac{d\tau'(t)}{dt} = w(t)$$


Process Model
FMCW Equation
+
Wiener Velocity Model
State k
$\mathfrak{m}_k$
Prediction on State $k+1$
$\mathfrak{m}_{k+1}^{-}$
Measurement
FMCW Sample Phase $\angle x(t_{k+1})$
Update on State $k+1$
$\mathfrak{m}_{k+1}$
Kalman Filter



Kalman Filter

Solution 1: Find Transition Matrix for **LTV** system

$$x(t) = e^{j2\pi(S\tau(t)t + f_c\tau(t))} \quad + \quad \frac{d\tau'(t)}{dt} = w(t)$$

→ Stochastic Differential Equation (SDE)

$$\frac{d}{dt}\begin{bmatrix}\theta(t)\\ \tau(t)\\ \tau'(t)\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 2\pi S & 2\pi(f_c + St)\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}}_{F(t)}\begin{bmatrix}\theta(t)\\ \tau(t)\\ \tau'(t)\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix} w(t)$$

State k → A_k (Transition Matrix) → **Prediction on State $k+1$**

$\mathfrak{m}_k$ → $\mathfrak{m}^-_{k+1}$

$$\neq \exp(F\Delta t) = I + F\Delta t + \frac{1}{2}(F\Delta t)^2 + \cdots$$

Only hold for **Time-Invariant**

Solution 1: Find Transition Matrix for **LTV** system

Using a fundamental matrix, we can find A_k for **LTV** system.

A fundamental matrix arranges n-solutions of SDE, given n independent initial states.

3 indep init state

$$\begin{bmatrix}1\\0\\0\end{bmatrix} \quad \begin{bmatrix}0\\1\\0\end{bmatrix} \quad \begin{bmatrix}0\\1\\1\end{bmatrix}$$

$$A_k = \Phi(t_k)\Phi^{-1}(0)$$

Solutions of SDE

$$\left[\begin{bmatrix}1\\0\\0\end{bmatrix} \begin{bmatrix}2\pi St\\1\\0\end{bmatrix} \begin{bmatrix}2\pi St^2 + 2\pi(S+f_c)t\\t+1\\1\end{bmatrix}\right] = \Phi(t)$$

fundamental matrix

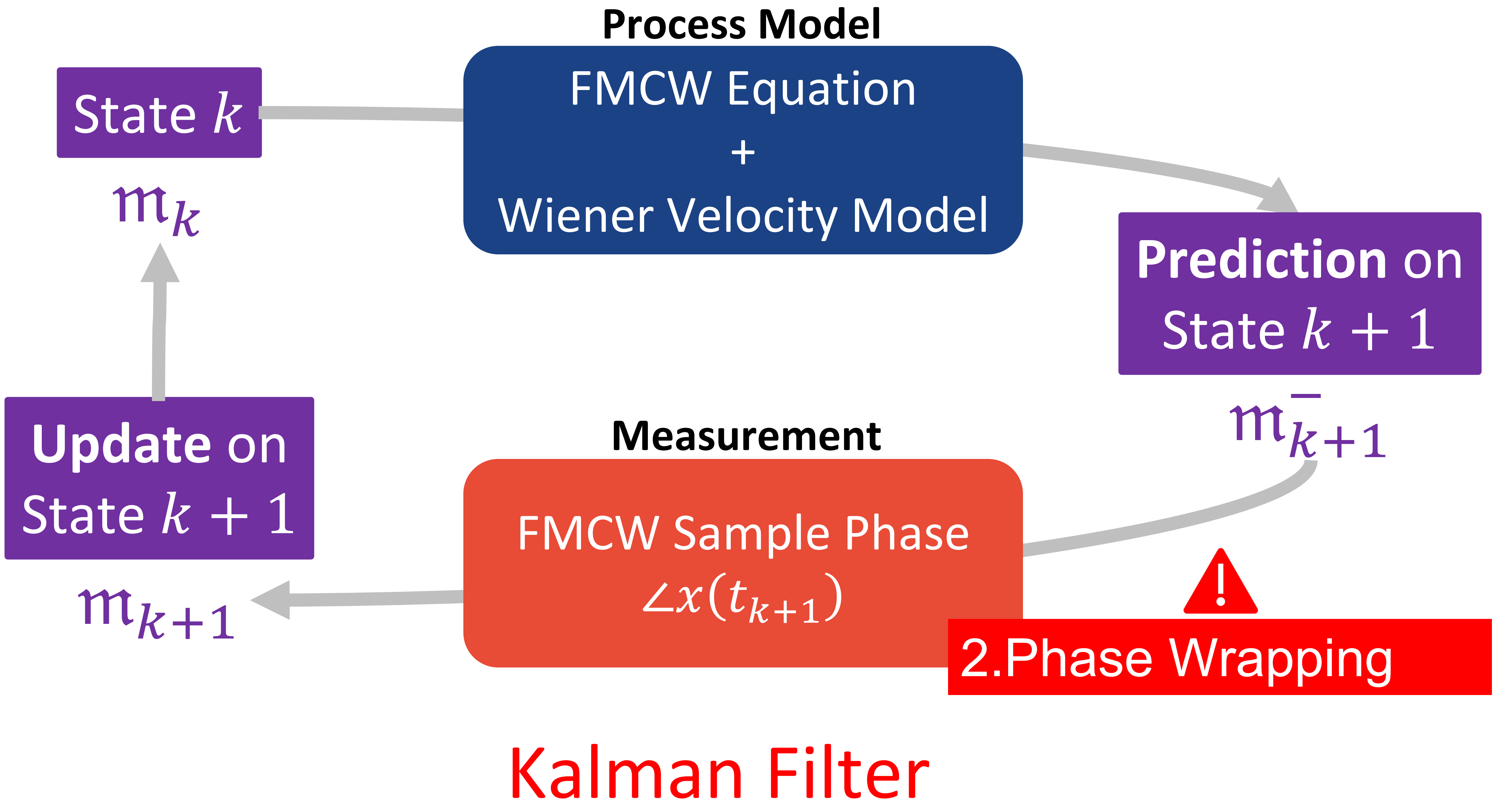
Comparison transition matrices

(correct) Using the fundamental matrix,

$$A_k = \begin{bmatrix} 1 & 2\pi S & 2\pi f_c \Delta t + 2\pi S \Delta t\ t_k \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}$$

(incorrect) Using $\exp(F\Delta t)$,

$$A_k = \begin{bmatrix} 1 & 2\pi S & 2\pi f_c \Delta t + 2\pi S \Delta t\ t_k + 2\pi S (\Delta t)^2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}$$

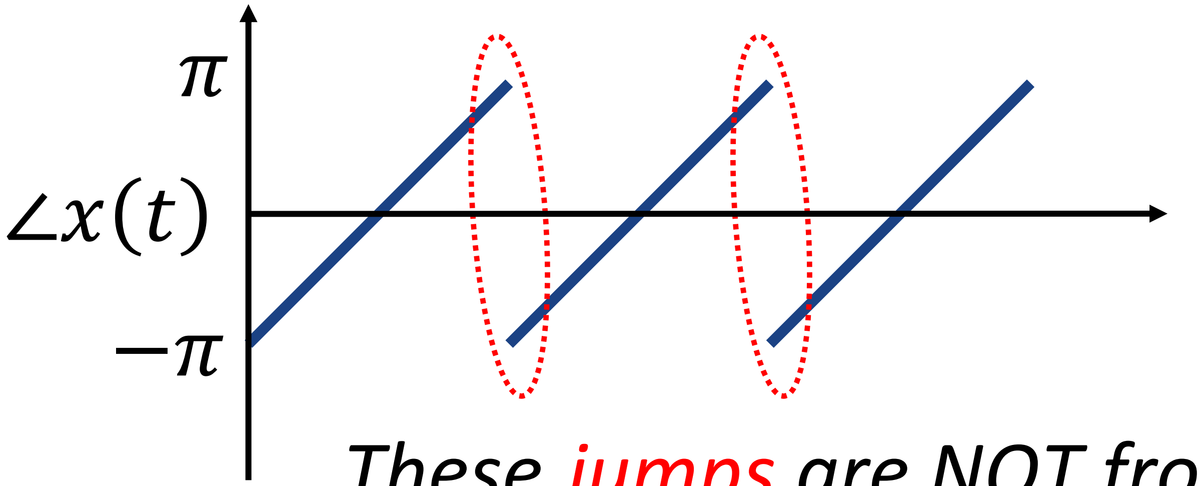

Process Model
State k
FMCW Equation
+
Wiener Velocity Model
$\mathfrak{m}_k$
Prediction on State $k+1$
$\mathfrak{m}_{k+1}^{-}$
Update on State $k+1$
Measurement
FMCW Sample Phase
$\angle x(t_{k+1})$
$\mathfrak{m}_{k+1}$
2.Phase Wrapping
Kalman Filter

Solution 2: Phase Unwrapping

Measured Sample: $x(t) = e^{j2\pi(S\tau(t)t + f_c\tau(t))}$

$-\pi \leq \angle x(t) \leq \pi$

Unwrapping $\angle x(t)$



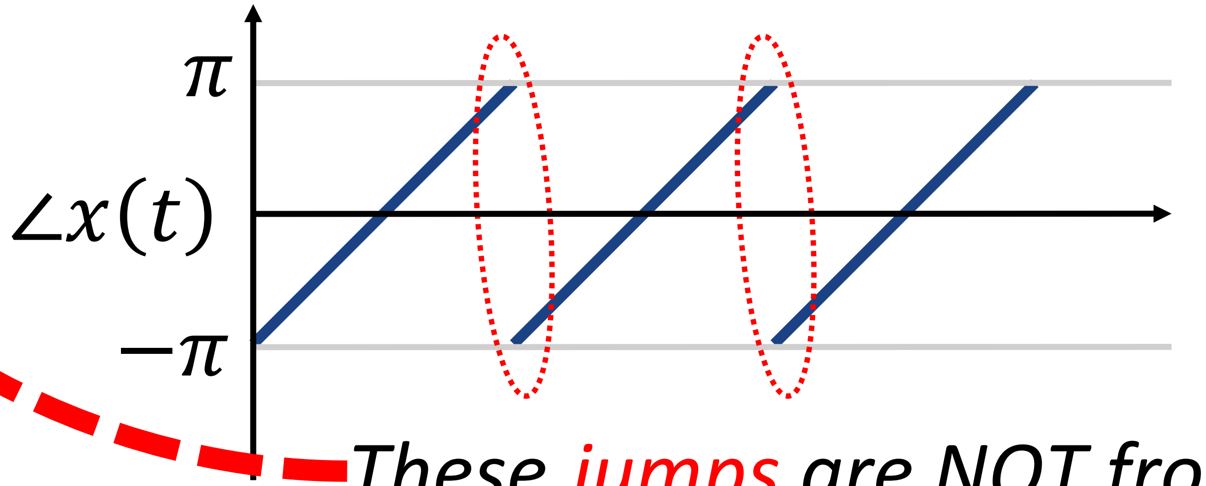
These jumps are NOT from $\tau(t)$

Tracking Limit

Measured Sample: $x(t) = e^{j2\pi(S\tau(t)t + f_c\tau(t))}$

What if it's not true?

If $\tau(t)$ causes more than $\pm\pi$ change, the state estimation fails…



These jumps are NOT from $\tau(t)$

Tracking Limit

$$\frac{-1-M}{L} < velocity < \frac{1-M}{L}$$

$$L = \frac{4f_0\Delta t}{c} \qquad M = \frac{4S\Delta t d_0}{c}$$

For TI mmWave setup,

$$f_0 = 77GHz$$

$$S = 30\text{MHz}/\mu s$$

$$\frac{1}{\Delta t} = 3MHz$$

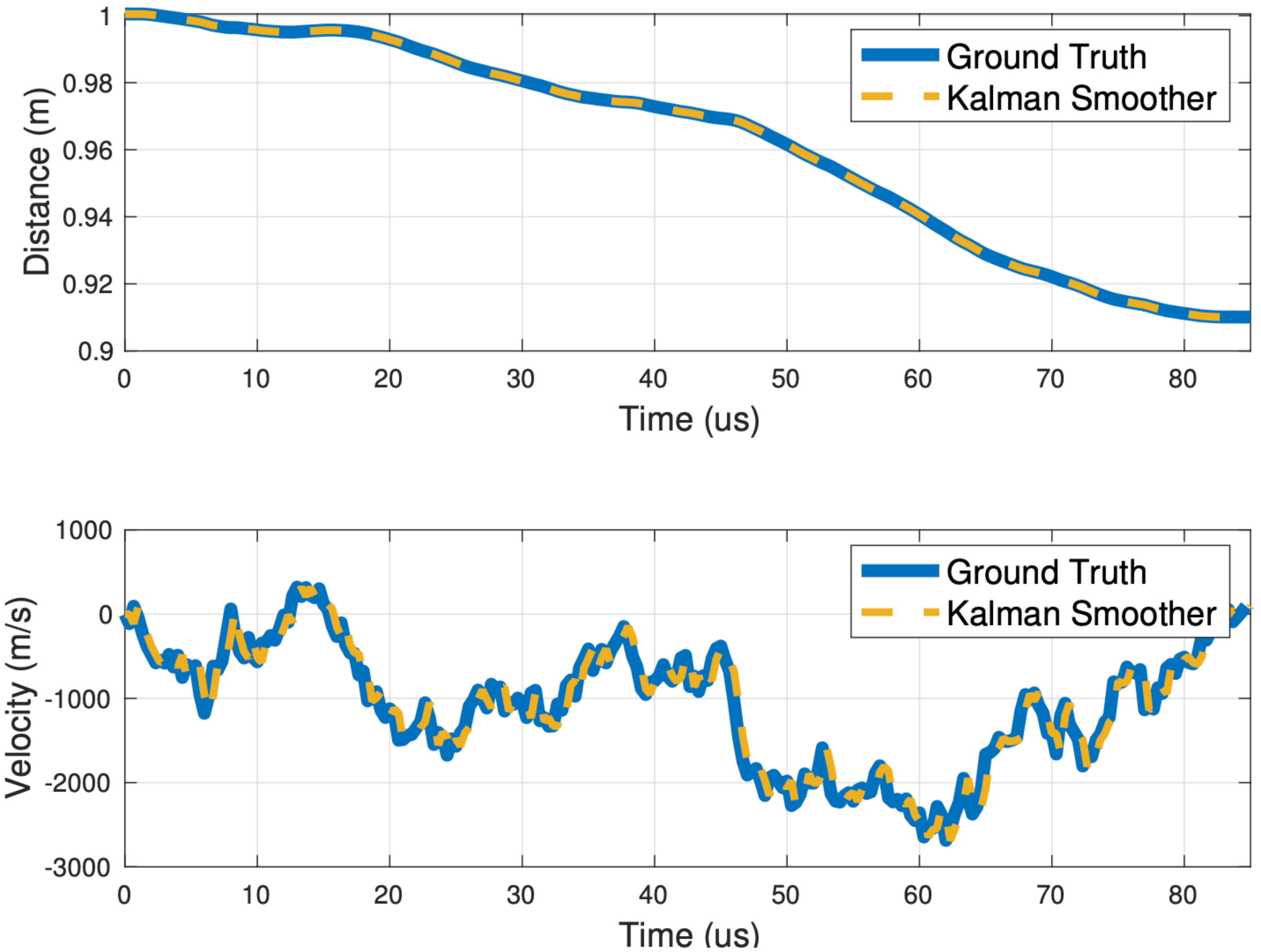
$$-3{,}312m/s < velocity < 2{,}533m/s$$

To orbit the Earth, escape velocity = 11,000m/s

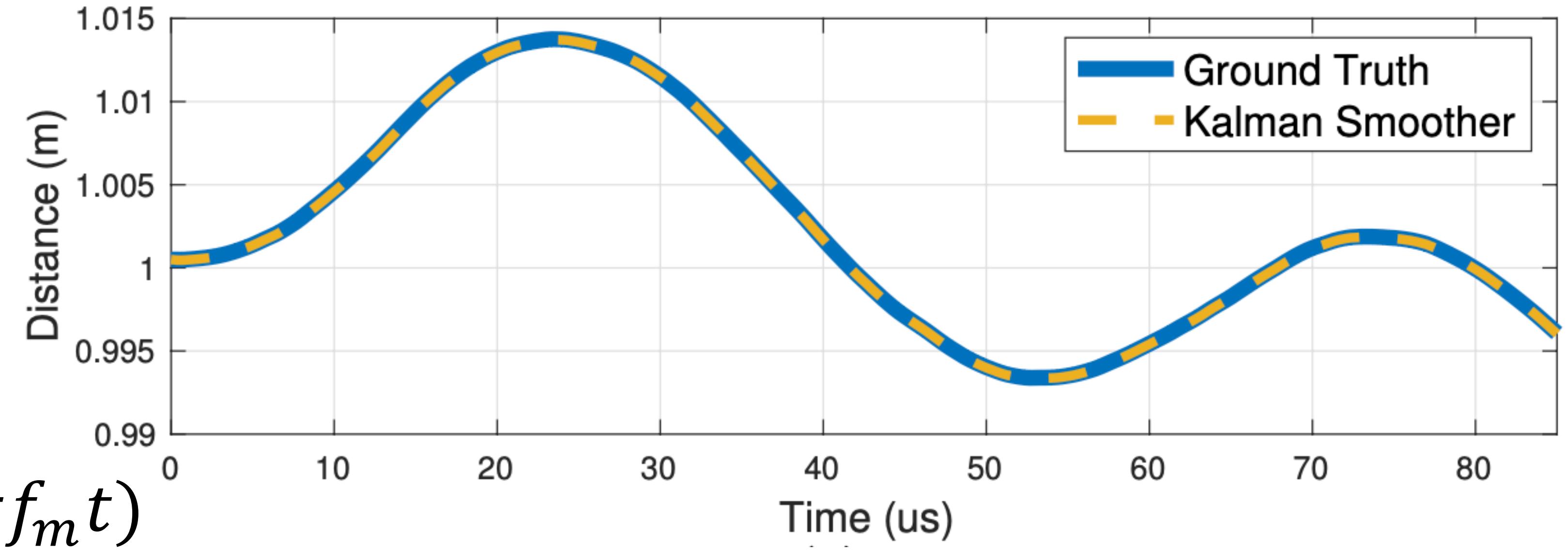
Simulation Results

Scenario 1

- $\frac{d\tau'(t)}{dt} = w(t) \sim N(0, \sigma^2)$
- Vib in tracking limit

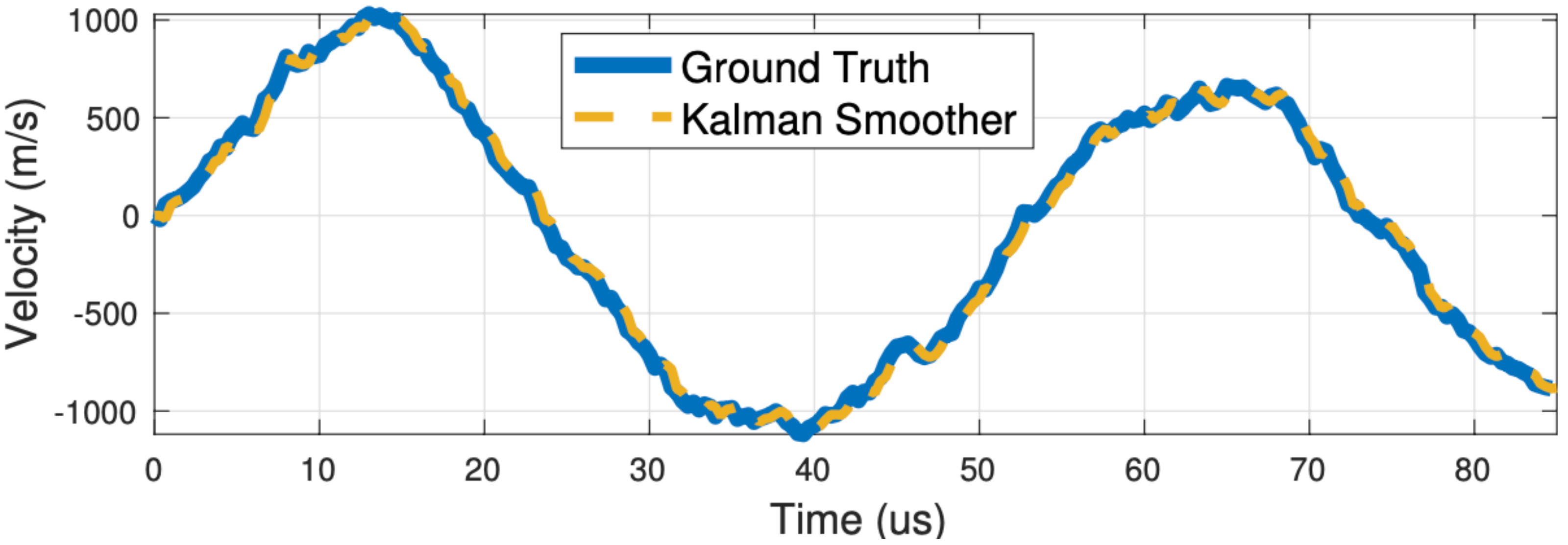


Simulation Results



Scenario 2

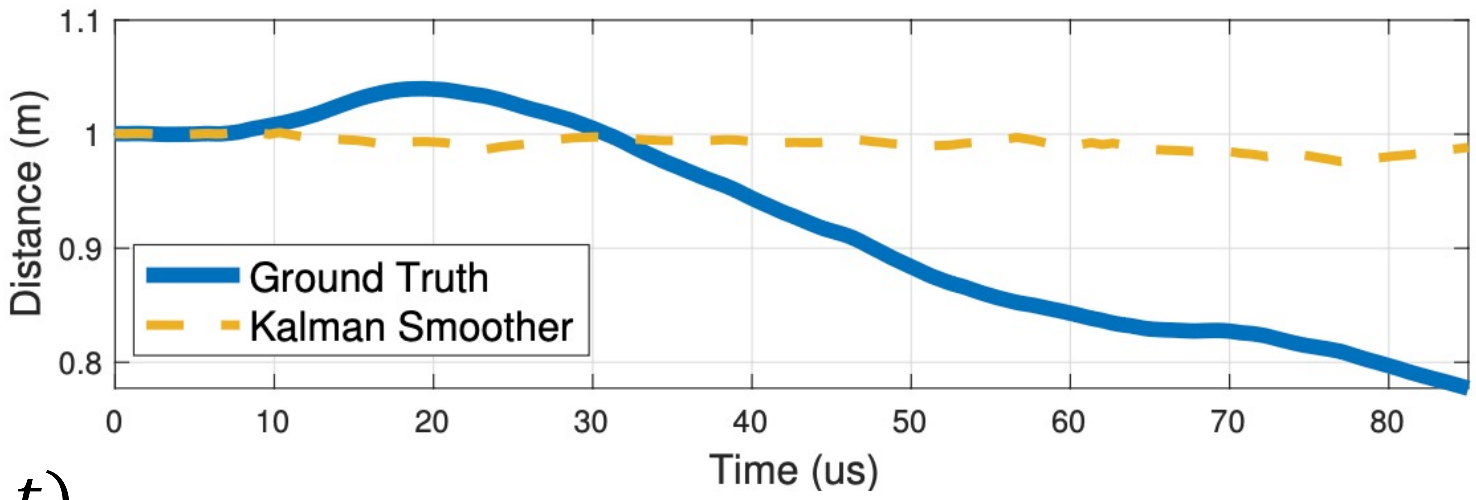
- $\frac{d\tau'(t)}{dt} = w(t) + Asin(2\pi f_m t)$
- Vib in tracking limit

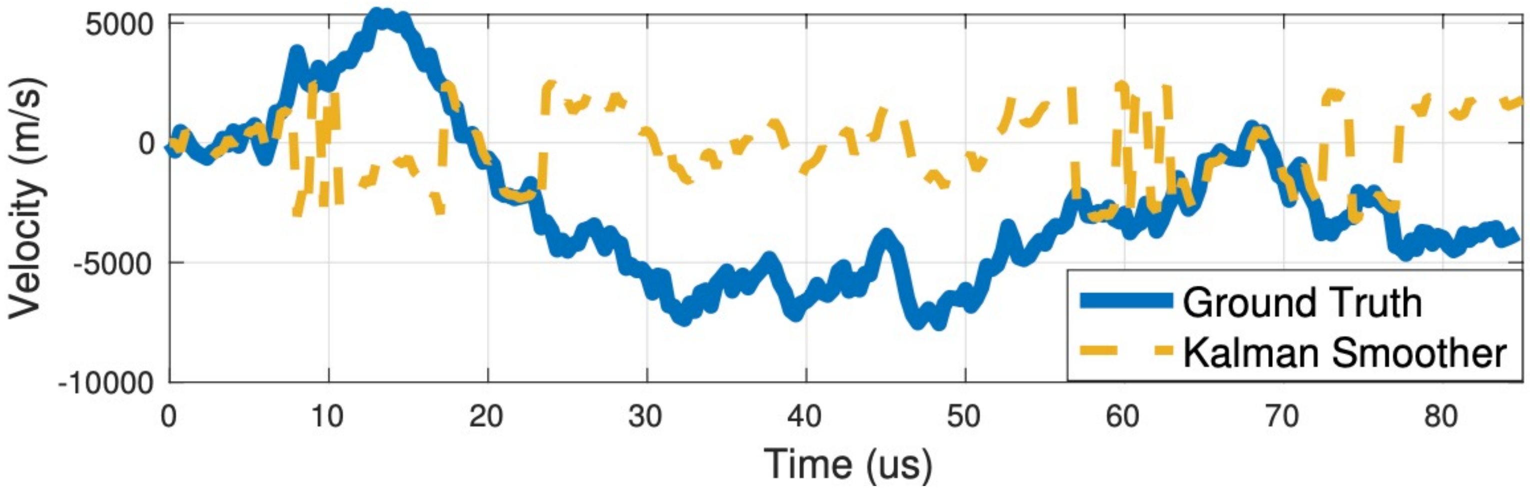


Simulation Results

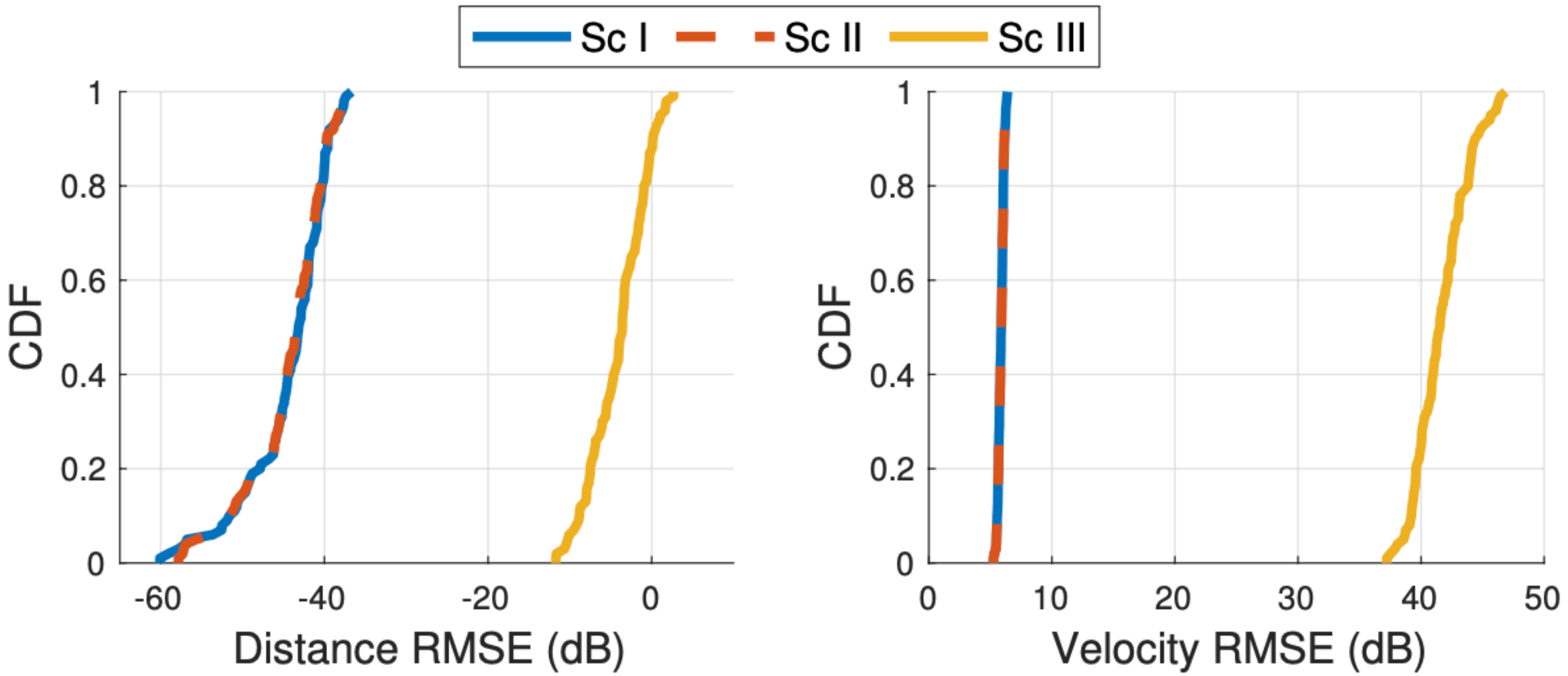
Scenario 3

- $$\frac{d\tau'(t)}{dt} = w(t) + A\sin(2\pi f_m t)$$
- Vib out of tracking limit





Simulation Results



Monte Carlo Runs

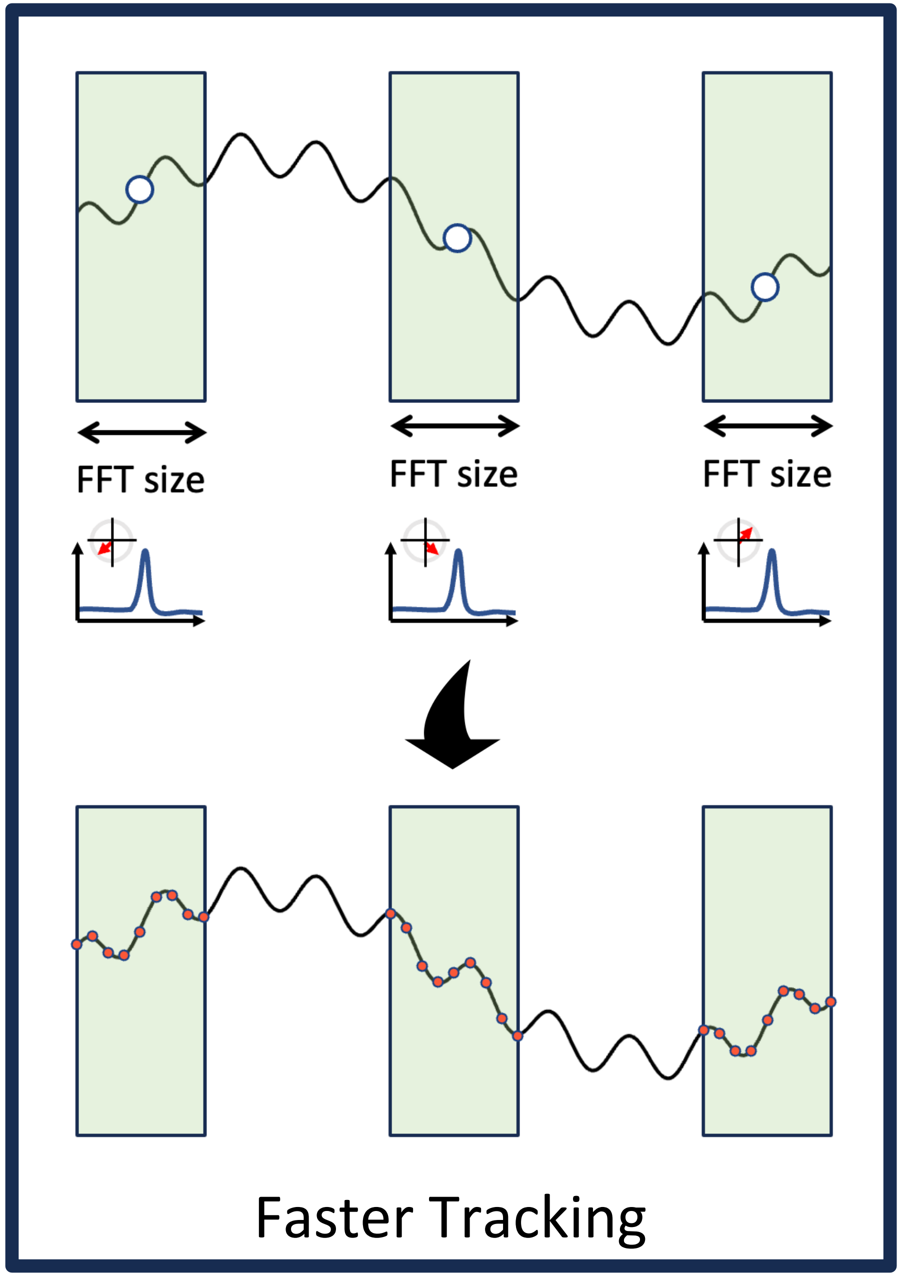
- 100 runs for each scenario.
- Empirical CDF of RMSE for distance and velocity of the target

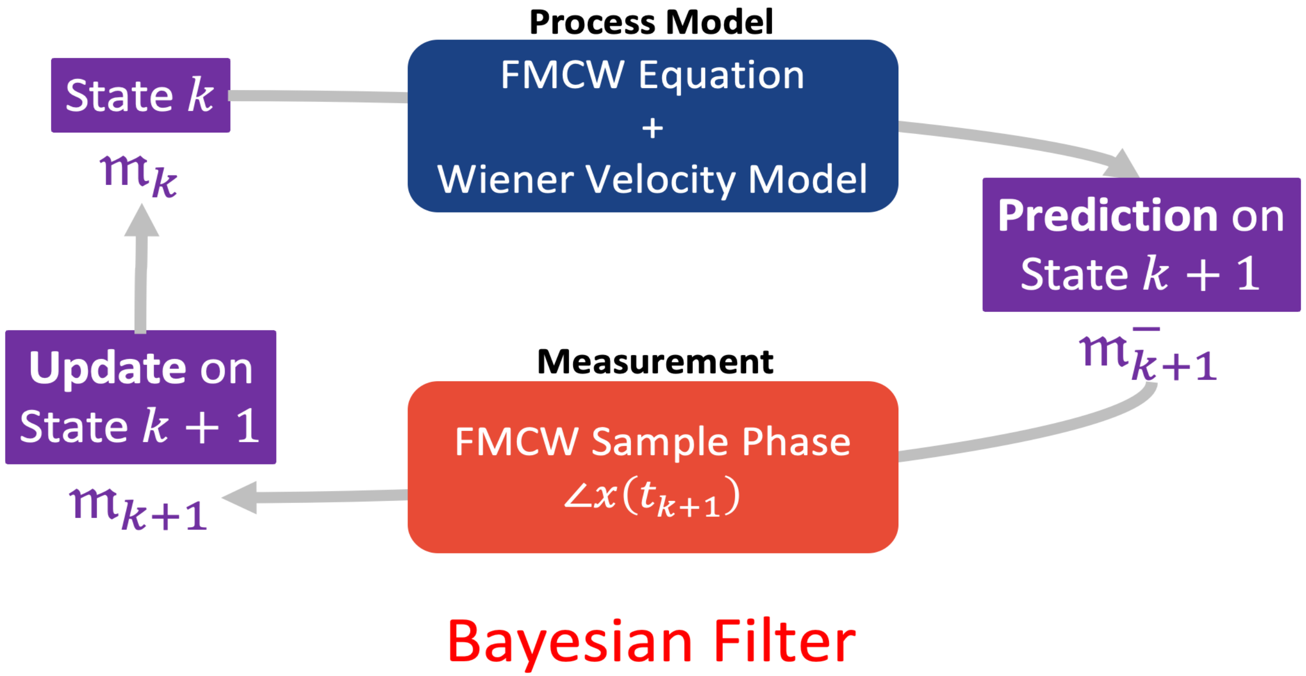
Future Works

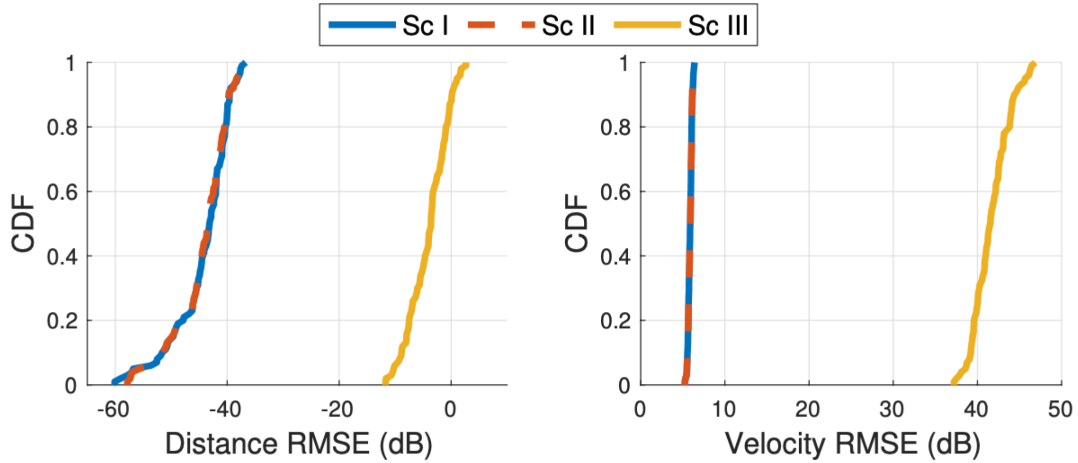
- Estimation of Kalman filter parameters
- Non-Gaussian noises
- Hardware measurements

  (specularity, clutters)

Conclusion


FFT size
FFT size
FFT size
Faster Tracking


Process Model
FMCW Equation
+
Wiener Velocity Model
State k
$\mathfrak{m}_k$
Prediction on State $k+1$
$\mathfrak{m}^-_{k+1}$
Measurement
FMCW Sample Phase
$\angle x(t_{k+1})$
Update on State $k+1$
$\mathfrak{m}_{k+1}$
Bayesian Filter


Sc I
Sc II
Sc III
CDF
Distance RMSE (dB)
CDF
Velocity RMSE (dB)

Beyond FFT: Precision Vibration Tracking with FMCW Radar and Kalman Estimators

Thank You
Questions?

Thomas Moon

tmoon@illinois.edu

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN