UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering
ECE 310 Digital Signal Processing – Summer 2022

Homework 5

Prof. Moon **Due: 10 pm, July 19, 2022**

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1. An LTI system is described by the difference equation

$$y[n] = x[n] + x[n-6]$$

   (a) Compute and sketch its magnitude and phase response
   (b) Determine its output to inputs
      i. $x[n] = \cos\frac{\pi}{5}n + 3\sin\left(\frac{\pi}{3}n + \frac{\pi}{10}\right)$
      ii. $x[n] = 1 + 5\cos\left(\frac{2\pi}{5}n + \frac{\pi}{4}\right)$

2. The response of a real LTI system for input

$$x[n] = 3 + \cos\left(\frac{\pi}{4}n + 10^\circ\right) + \sin\left(\frac{\pi}{3}n + 25^\circ\right)$$

   is

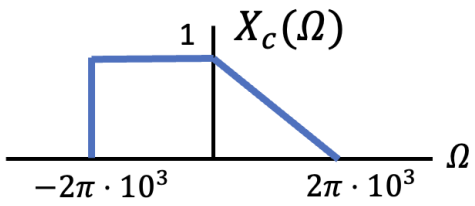
$$y[n] = 12 + 2\cos\left(\frac{\pi}{4}n + 55^\circ\right) .$$

   Determine the system response $\tilde{y}[n]$ for input

$$\tilde{x}[n] = 3 + 2\sin\left(\frac{\pi}{4}n + 30^\circ\right) + 10\cos\left(-\frac{\pi}{3}n + 45^\circ\right) .$$

3. A continuous-time signal $x_c(t) = \cos(\Omega_0 t)$ was sampled at a rate of 100 samples/sec (or 100 Hz) to produce $x[n] = \cos(2\pi n/3)$. Find the three lowest possible different values of $|\Omega_0|$ that could produce the sequence $x[n]$. Are there more possible values?

4. The continuous-time signal $x_c(t)$ has the Fourier transform shown below. The signal $x_c(t)$ is sampled with a sampling period of T to produce the discrete-time signal $x[n] = x_c(nT)$.



   (a) Sketch the DTFT $X_d(\omega)$ of $x[n]$ for $-2\pi < \omega < 2\pi$ for the sampling frequencies F_s of (i) 2 kHz and (ii) 3 kHz and (iii) 1 kHz.
   (b) What is the Nyquist rate (minimum sampling rate needed to avoid aliasing) for the signal $x_c(t)$?
   (c) Is the signal $x[n]$ real-valued? Why or why not?