1. An LTI system is described by the difference equation

\[ y[n] = x[n] + x[n - 6] \]

(a) Compute and sketch its magnitude and phase response
(b) Determine its output to inputs
i. \( x[n] = \cos \frac{\pi}{5} n + 3 \sin \left( \frac{\pi}{3} n + \frac{\pi}{10} \right) \)
ii. \( x[n] = 1 + 5 \cos \left( \frac{2\pi}{5} n + \frac{\pi}{4} \right) \)

2. The response of a real LTI system for input

\[ x[n] = 3 + \cos \left( \frac{\pi}{4} n + 10^\circ \right) + \sin \left( \frac{\pi}{3} n + 25^\circ \right) \]

is

\[ y[n] = 12 + 2 \cos \left( \frac{\pi}{4} n + 55^\circ \right) . \]

Determine the system response \( \tilde{y}[n] \) for input

\[ \tilde{x}[n] = 3 + 2 \sin \left( \frac{\pi}{4} n + 30^\circ \right) + 10 \cos \left( -\frac{\pi}{3} n + 45^\circ \right) . \]

3. A continuous-time signal \( x_c(t) = \cos(\Omega_0 t) \) was sampled at a rate of 100 samples/sec (or 100 Hz) to produce \( x[n] = \cos(2\pi n/3) \). Find the three lowest possible different values of \( |\Omega_0| \) that could produce the sequence \( x[n] \). Are there more possible values?

4. The continuous-time signal \( x_c(t) \) has the Fourier transform shown below. The signal \( x_c(t) \) is sampled with a sampling period of \( T \) to produce the discrete-time signal \( x[n] = x_c(nT) \).

(a) Sketch the DTFT \( X_d(\omega) \) of \( x[n] \) for \( -2\pi < \omega < 2\pi \) for the sampling frequencies \( F_s \) of (i) 2 kHz and (ii) 3 kHz and (iii) 1 kHz.
(b) What is the Nyquist rate (minimum sampling rate needed to avoid aliasing) for the signal \( x_c(t) \)?
(c) Is the signal \( x[n] \) real-valued? Why or why not?