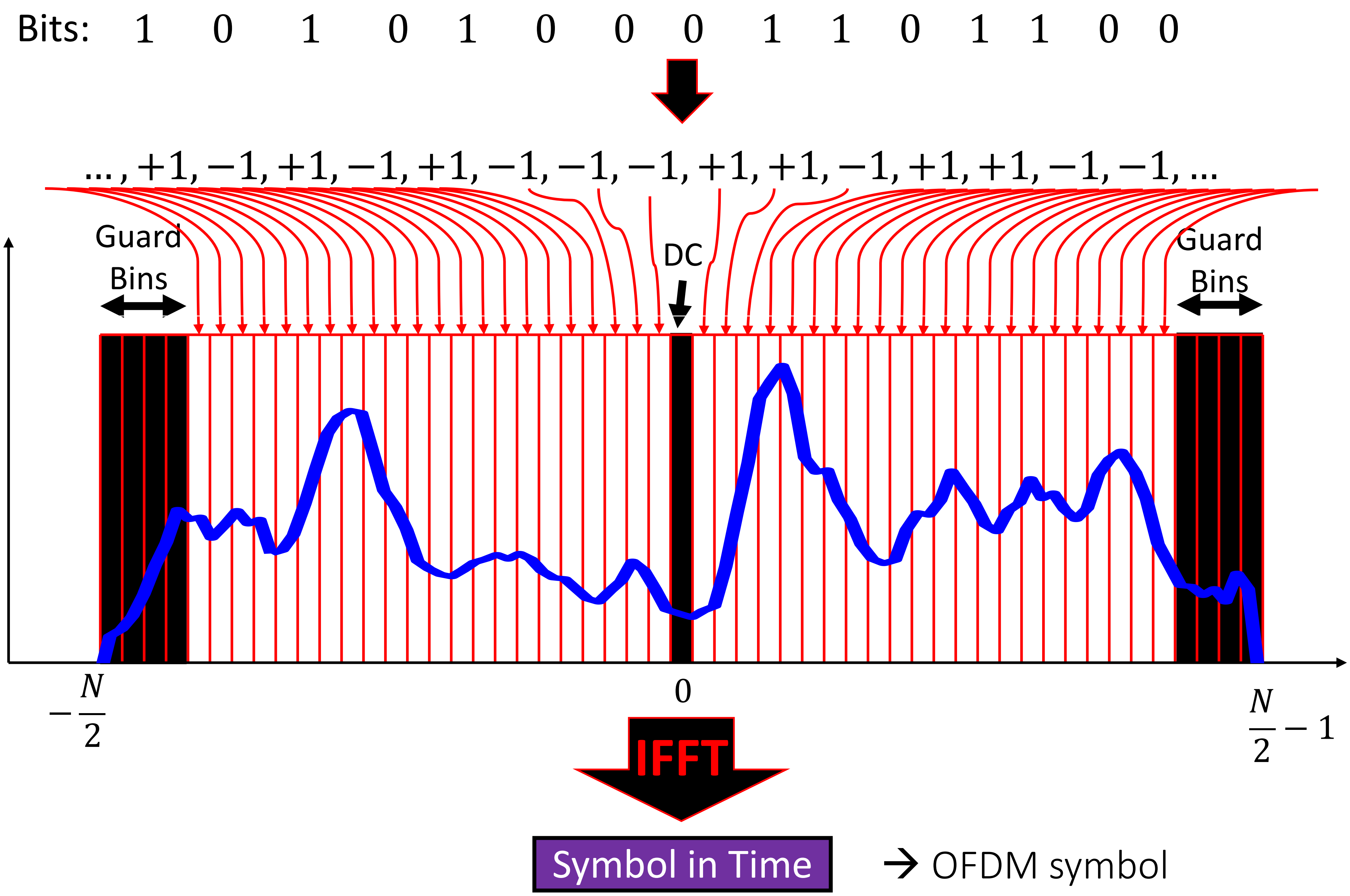
OFDM Symbol



→ OFDM symbol

Discrete Fourier Transform

N-Point DFT: $$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi kn}{N}}$$

N-Point IDFT: $$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi kn}{N}}$$

(Circular) Convolution property

$$y[n] = h[n] \otimes_N x[n] \quad \Leftrightarrow \quad Y[k] = H[k]\, X[k]$$

$$X[k] = \frac{Y[k]}{H[k]}$$

Channel effect can be neutralized
by a simple freqeucny domain equalizer

Discrete Fourier Transform

N-Point DFT: $X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi kn}{N}}$

N-Point IDFT: $x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi kn}{N}}$

(Circular) Convolution property

$$y[n] = h[n] \otimes_N x[n] \quad \Leftrightarrow \quad Y[k] = H[k]\,X[k]$$

In reality, $h[n]$ is a LTI discrete-time system

(Linear) Convolution property

$$y[n] = h[n] * x[n]$$

OFDM Cyclic Prefix

Append last *n* symbols to beginning

Cyclic prefix will trick the channel to perform circular convolution

(Circular) Convolution property

$$y[n] = h[n] \otimes_N x[n] \quad \Leftrightarrow \quad Y[k] = H[k]\,X[k]$$

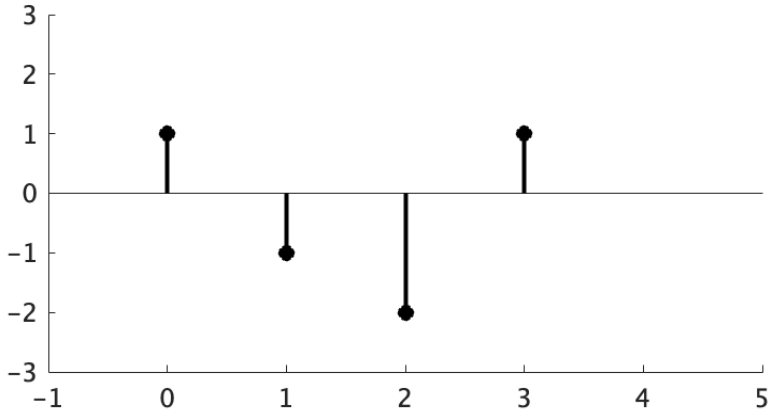
In reality, $h[n]$ is a LTI discrete-time system

(Linear) Convolution property

$$y[n] = h[n] * x[n]$$

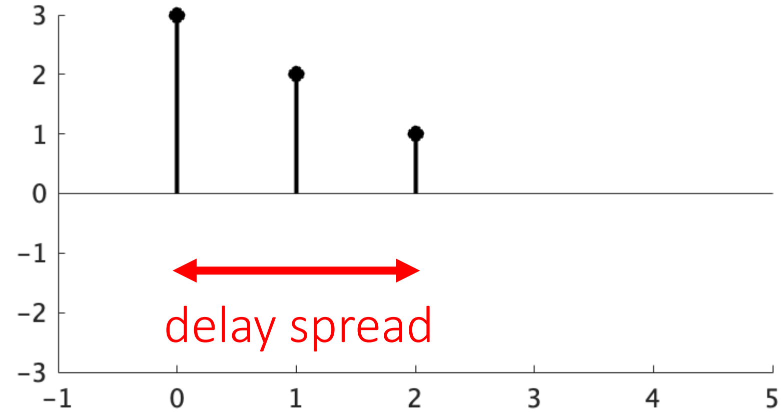
OFDM symbol without CP

$x[n]$



Multi-tap channel

$h[n]$

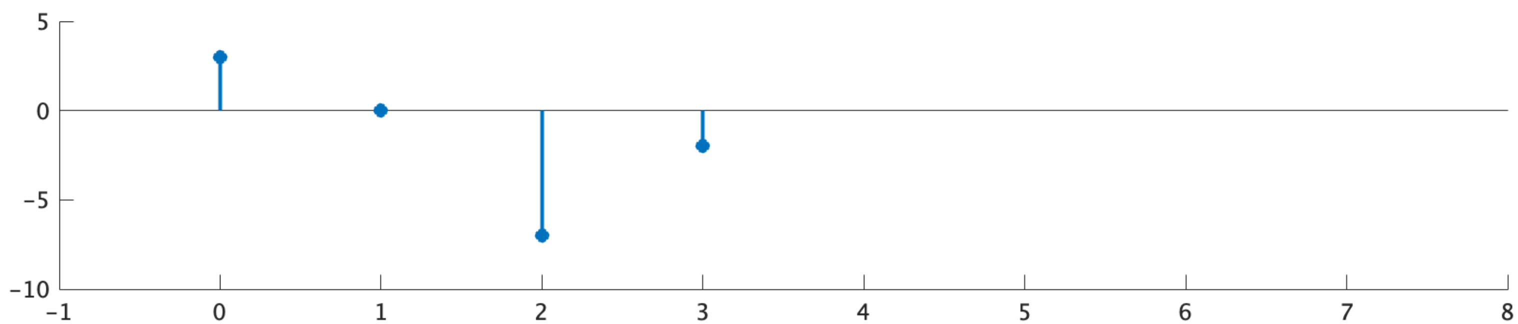


Linear convolution and Circular convolution does not yield same results

→ Cannot use Frequency EQ

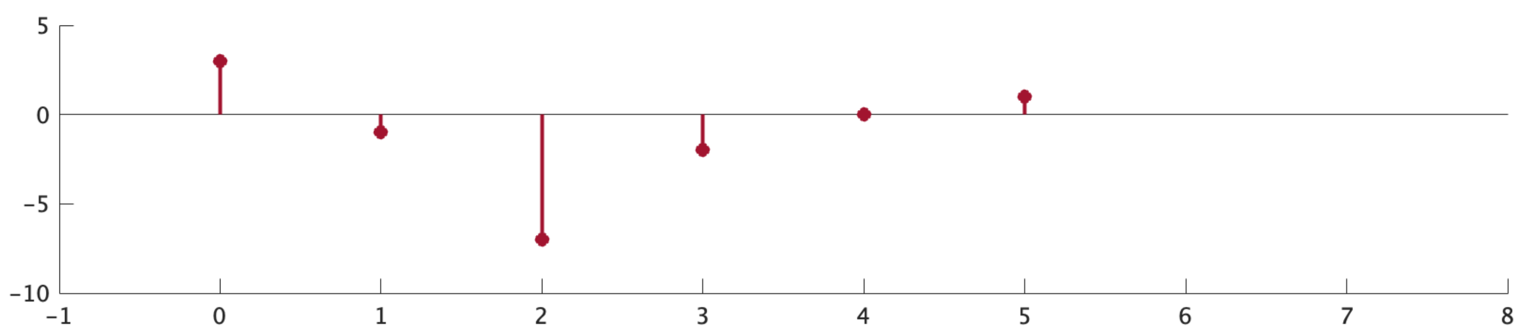
circular conv

$h[n] \otimes_N x[n]$



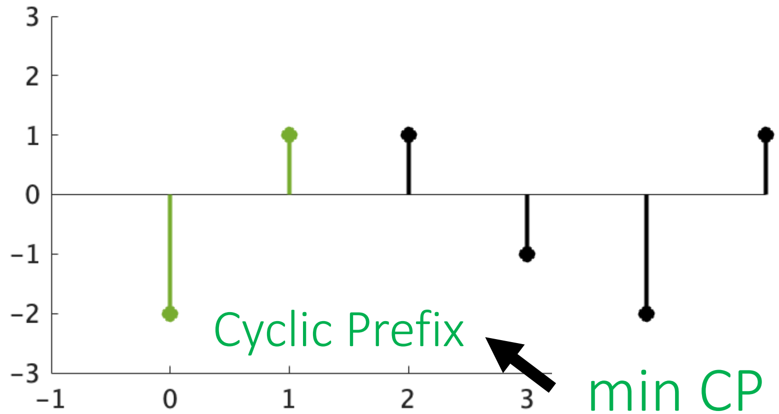
linear conv

$h[n] * x[n]$



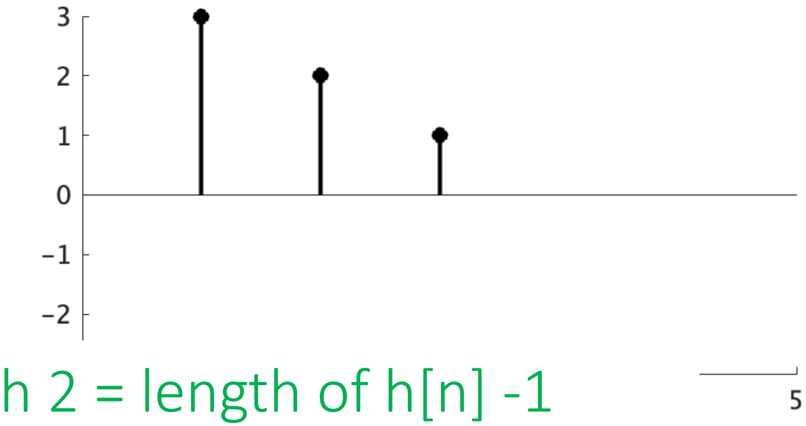
OFDM symbol with CP

$x_{cp}[n]$



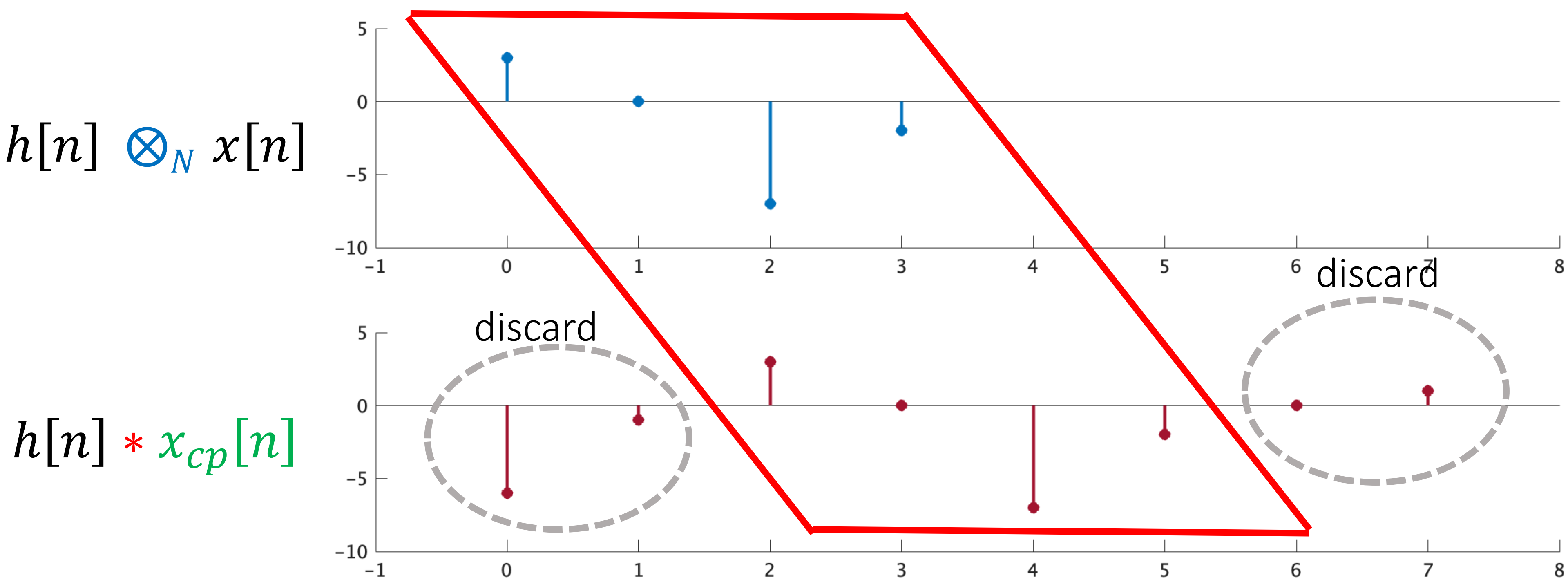
Multi-tap channel

$h[n]$



min CP length 2 = length of h[n] -1

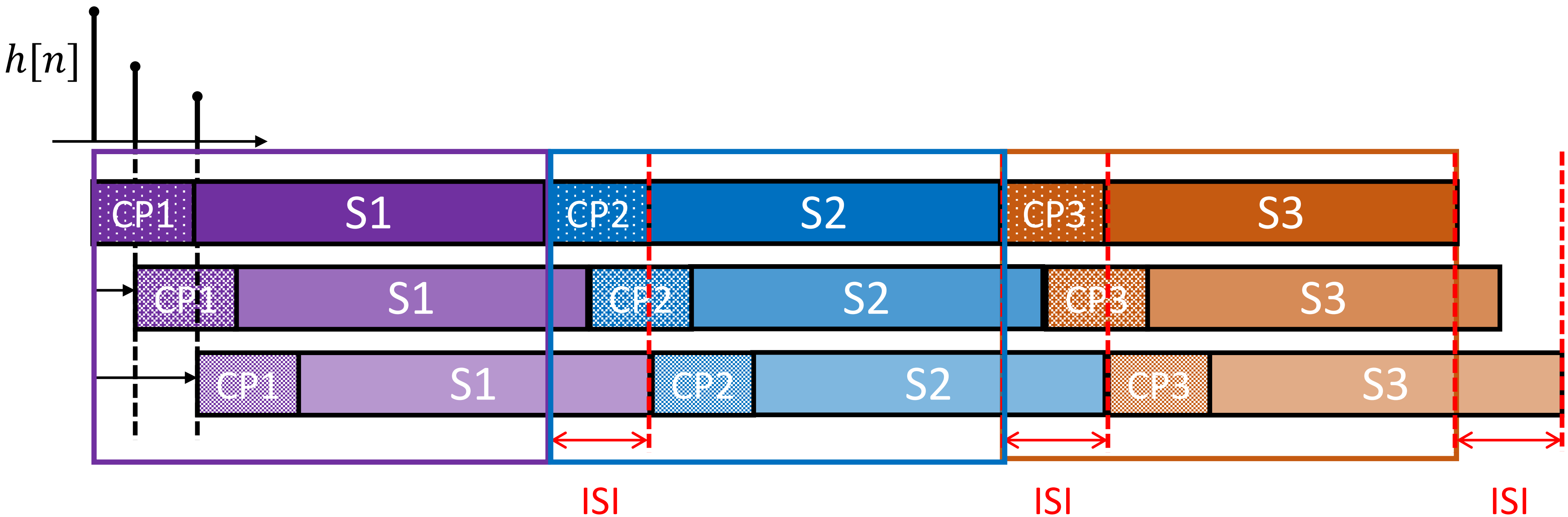
Trick the channel to perform circular convolution
by adding Cyclic Prefix



OFDM Cyclic Prefix

Cyclic Prefix:

- Preserves Circular Convolution property, $Y[k] = H[k]\,X[k]$
- Deals with Inter-Symbol-Interference

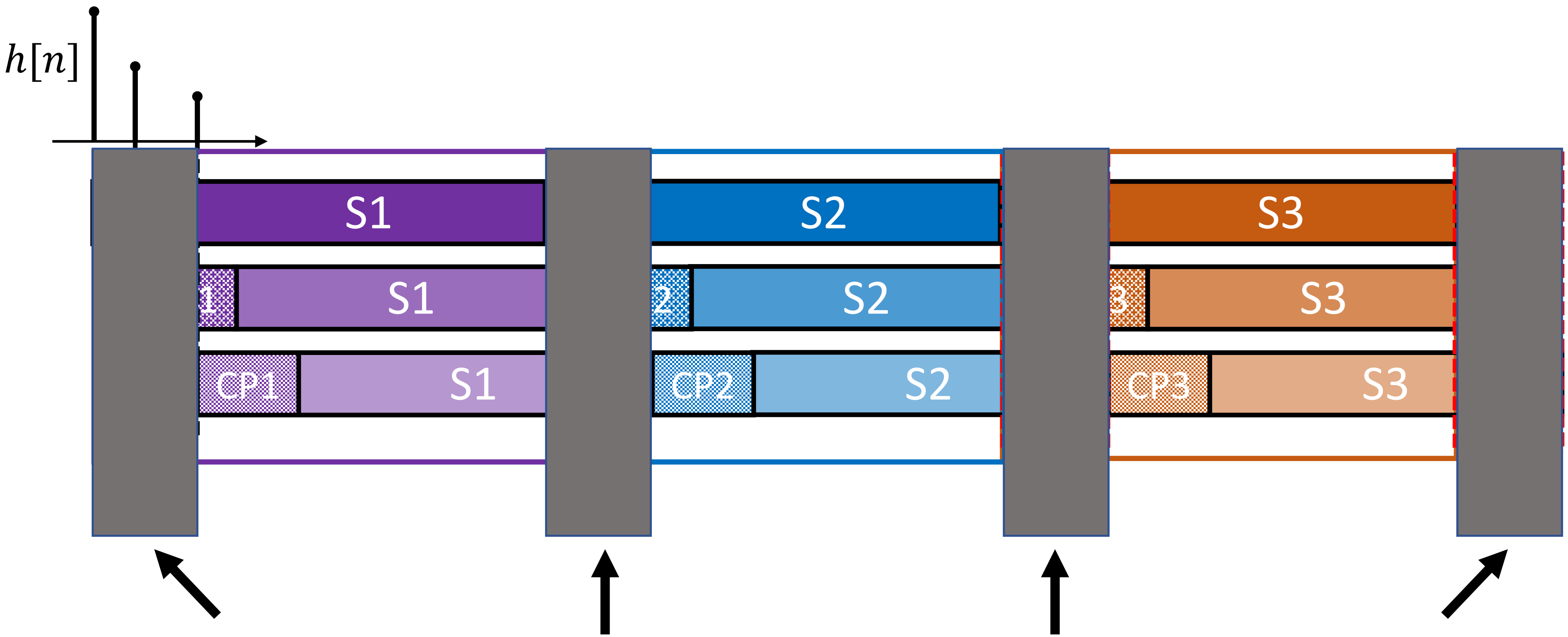


OFDM Cyclic Prefix

Cyclic Prefix:

- Preserves Circular Convolution property, $Y[k] = H[k]\,X[k]$

- Deals with Inter-Symbol-Interference



Discarding Cyclic Prefix will remove ISI

OFDM Coarse CFO Estimation & Correction

- Use Preamble to estimate CFO

$$y_1[n] = x[n]e^{-j2\pi\Delta f_c nT_s}$$

$$y_2[n] = x[n]e^{-j2\pi\Delta f_c(nT_s+NT_s)}$$

- Compute: $A = \sum_{t=1}^{N} y_1^*[n]y_2[n] = \sum_{t=1}^{N} x[n]^* x\,[n]e^{-j2\pi\Delta f_c NT_s}$

$$= e^{-j2\pi\Delta f_c NT_s}\sum_{t=1}^{N}|x[n]|^2 \Rightarrow \Delta f_c = -\frac{\angle A}{2\pi NT_s}$$

OFDM Coarse CFO Estimation & Correction

- Use Preamble to estimate CFO

$$y_1[n] = x[n]e^{-j2\pi\Delta f_c n T_s}$$

$$y_2[n] = x[n]e^{-j2\pi\Delta f_c (nT_s + NT_s)}$$

- Compute: $A = \sum_{t=1}^{N} y_1^*[n] y_2[n]$ ➡ $\Delta f_c = -\frac{\angle A}{2\pi N T_s}$

- Correct CFO: $y[n] \times e^{j2\pi\Delta f_c n T_s}$

OFDM Channel Estimation

- Use Preamble to estimate the channel

$$y[n] = h[n] \otimes_N x[n] \quad \Longleftrightarrow \quad Y[k] = H[k]\, X[k]$$

k	0	1	2	…	N-1
$X[k]$	+1	-1	-1	…	+1
$Y[k]$	$H[0]$	$-H[1]$	$-H[2]$	…	$H[N-1]$

- Estimate: $\widetilde{H}[k] = \dfrac{Y[k]}{X[k]}, k = 0,1,\dots, N-1$

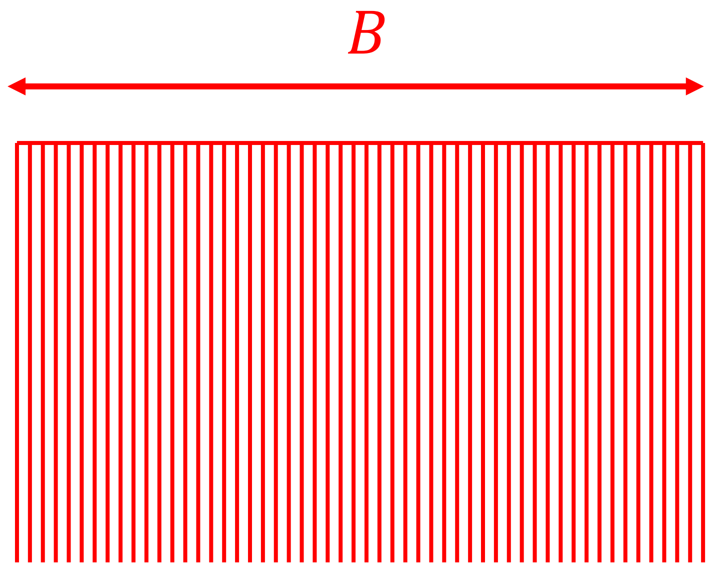
- Use two preambles to average noise: $\widetilde{H}[k] = \dfrac{Y_1[k] + Y_2[k]}{2\, X[k]}$

Case study: 802.11a WiFi

- Carrier frequency = 5GHz
- Channel bandwidth B (1/symbol rate)= 20MHz
- # subcarriers N = 64
- # null tones = 16
- Length of CP = 16 symbols

What is subcarrier bandwidth B_N?

$$B_N = \frac{20MHz}{64} = 312.5kHz$$



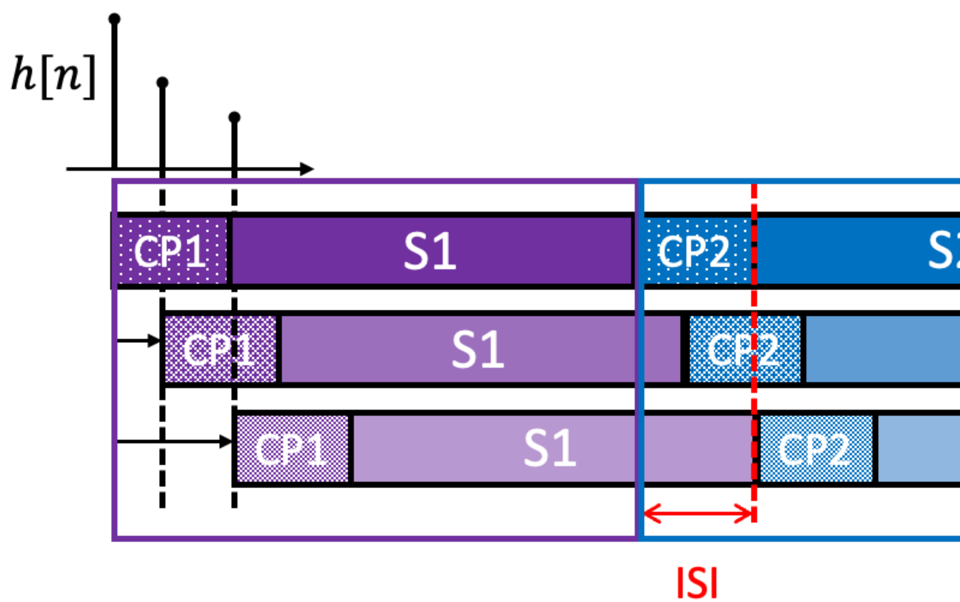
Case study: 802.11a WiFi

- Carrier frequency = 5GHz
- Channel bandwidth B (1/symbol rate)= 20MHz
- # subcarriers N = 64
- # null tones = 16
- Length of CP = 16 symbols

What is the maximum delay spread for which ISI is removed?

→ time duration of a single CP

$$T_{spread} < 16 \frac{1}{20MHz} = 800ns$$



Max delay in a typical large building ≈ 300ns

Case study: 802.11a WiFi

- Carrier frequency = 5GHz
- Channel bandwidth B (1/symbol rate)= 20MHz
- # subcarriers N = 64
- # null tones = 16
- Length of CP = 16 symbols

What is the data rate if 4-QAM is used?

$$\frac{\frac{2\ bits}{1\ symbol}(64-16)\ data\ symbols}{(16+64)\frac{1}{20MHz}}$$

$$= 24Mbps$$