Dual-Frequency Incoherent Subsampling Driven Test Response Acquisition of Spectrally Sparse Wideband Signals with Enhanced Time Resolution

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***Abstract*—In this paper, we propose a new test response acquisition technique for high-speed devices-based on dual-frequency incoherent sub-sampling and sparse signal reconstruction. The proposed technique enables reconstruction of spectrally sparse wideband signals such as multi-tone signals and short pseudo-random bit sequences (PRBS) with enhanced time/frequency resolution as opposed to current methods. The sampling hardware utilizes dual analog-to-digital converters (ADCs) and dedicated sampling frequency synthesizers with a common frequency reference. As compared to other compressive sampling architectures [1], the proposed hardware architecture is easy to implement at low cost since it does not require accurate sampling clock phase adjustment or random timing generation. For digital signal reconstruction, the proposed technique requires less number of waveform samples than conventional equivalent-time sampling techniques. In addition, the use of an resolution-enhanced discrete Fourier transform (DFT) frame and basis pursuit algorithms minimizes spectral leakage of incoherently sub-sampled signals. This co-design of sampling hardware and signal reconstruction algorithms enables testing of spectrally sparse wideband signals with enhanced time/frequency resolution.**

Keywords- Compressive Sampling, Undersampling, Incoherent Sampling, Signal Reconstruction, Multi-tone Testing, PRBS testing

I. Introduction

Periodic multi-tone signals and PRBSs are widely used as test signals for characterizing high-speed I/O and testing RF systems. [2][3][4] Capturing such high-speed multi-tone waveforms is getting more challenging, as the speed of signals becomes higher. In the domain of signal processing, there has been work on reconstruction of spectrally sparse signals using undersampling. [5][6][7] Recently, a signal reconstruction technique called compressive sampling (CS) [8], has been developed and is finding widespread use in many applications. The compressive sampling paradigm states that if a signal is sparse in a linear transform domain, it can be blindly reconstructed using a relatively small number of incoherent samples with very high probability. This has been applied to several fields to extract information from sparse representations of ill-posed problems [9][10][11]. In the testing field, our interest is in investigating how specific aspects of the compressive sampling paradigm can be used to capture high-speed, periodic and spectrally sparse wideband test response signals from high speed devices to reduce data acquisition and test capture instrumentation costs.

Most compressive sampling techniques involve the use of controlled randomness in clock edge timing to incoherently digitize the input signal. [1][12][13] In [14], random demodulation and random sampling are used as vehicles to implement the compressive sampling algorithm. In this context, generating a high-speed PRBS signal or a random sampling time-base “on-the-fly” is challenging and limits the speed of signal reconstruction. A large amount of storage and corresponding digital circuitry is needed for generating the required random timing signal and acquiring the test response data generated. In addition, multiple ADCs may be used to achieve a relatively high effective sampling rate, complicating the test procedure.

In this paper, a dual-frequency incoherent test response sampling scheme using two ADCs which does not require sampling phase alignment or random timing generation is proposed.. A key advantage of this architecture is that controlling the sampling time “on-the-fly” is not needed, thereby simplifying the test infrastructure considerably. Furthermore, a very high effective sampling rate can be obtained using the proposed test response data acquisition scheme. In addition to hardware design, the proposed scheme requires the use of associated back-end signal processing algorithms for reconstructing the acquired signal. We exploit *spectral sparsity* and *basis pursuit* to process the incoherent samples. One of the common limitations associated with incoherently sampling a signal in the frequency domain is spectral leakage. When performing incoherent sampling, if the spectral tones of the sampled signal do not correspond exactly to the DFT frequency bins, there is spectral leakage across the relevant frequency bins of the DFT. In the proposed high speed signal acquisition technique, an resolution-enhanced DFT frame, as described in [15], is used to increase the resolution and to reduce the mismatch between the DFT basis and the real tones. We also investigate the relation between incoherent sampling and the deterministic compressive sampling sensing matrices. [16]

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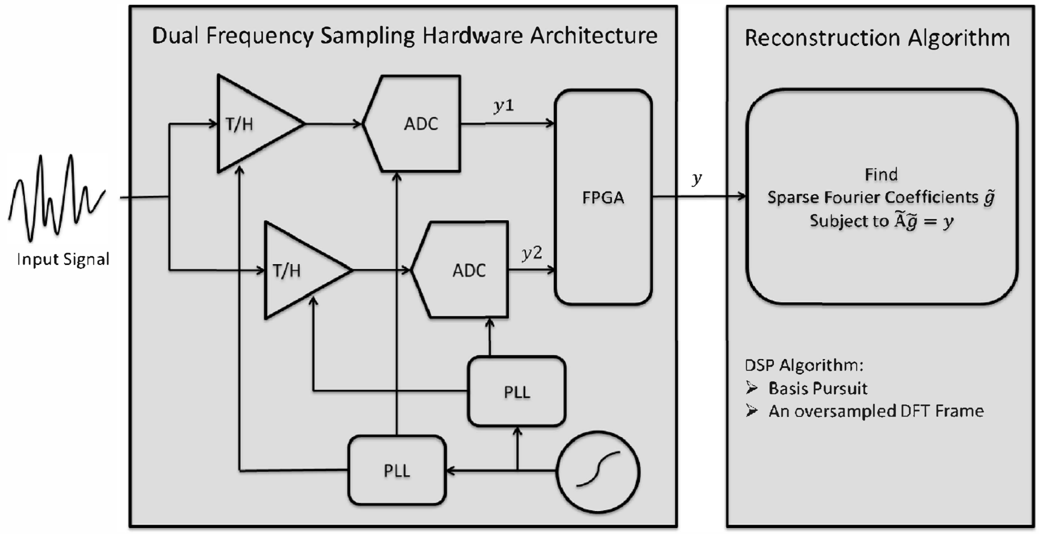

Fig. 1 Block Diagram of the Proposed System

There has been research in the past on the subject of undersampling and reconstruction of high speed signals using two different frequencies. [17] The proposed method is different from prior work in several ways. First, we reconstruct the actual signal during the signal capture time interval. Therefore, estimating the fundamental frequency needed to reconstruct the signals not necessary. In addition, the number of samples used is significantly less than equivalent time sampling and the method proposed in [17]. In Section 3, we also show that the eye diagram of a noisy PRBS signal can be obtained using the proposed reconstruction algorithm.

In summary, we propose a dual-frequency hardware architecture and associated signal reconstruction algorithm. By co-designing the test hardware and software, we can reconstruct wideband spectrally sparse signal with enhanced spectral resolution using a small number of incoherent samples. The rest of the paper is arranged as follows. In Section 2, we discuss the proposed dual-frequency sampling scheme and compare the proposed scheme with random sampling. We also explore the relationship between basis pursuit and coherent sampling. In Section 3, we show the applications of the proposed scheme to reconstruction of test PRBS and multi-tone signals. Conclusions are discussed in Section 4.

II. METHODOLOGY

There are two parts in the proposed sampling schemes – hardware sampling architecture and DSP reconstruction algorithm.

A. Dual-Rate Direct Sampling

A incoherent dual-frequency direct sampling system is used to obtain irregularly spaced samples of input test signals as shown in Fig. 1. The test signal is divided into two channels and individually fed to the wideband track-and-hold (T/H) amplifiers and digitized by the two identical analog-to-digital converters (ADCs). The proposed system uses a dual-frequency synthesizer that contains two independent phase lock loops (PLLs) and the common reference time-base. The dual-frequency synthesizer generates two independent sampling time-bases and feeds them into the ADCs as sampling clocks. The digitized test signals are then captured by a field programmable gate array (FPGA) and digitally interleaved and processed using the proposed DSP algorithm, which is discussed in the next section. The block diagram of the test architecture is shown as Fig. 1.

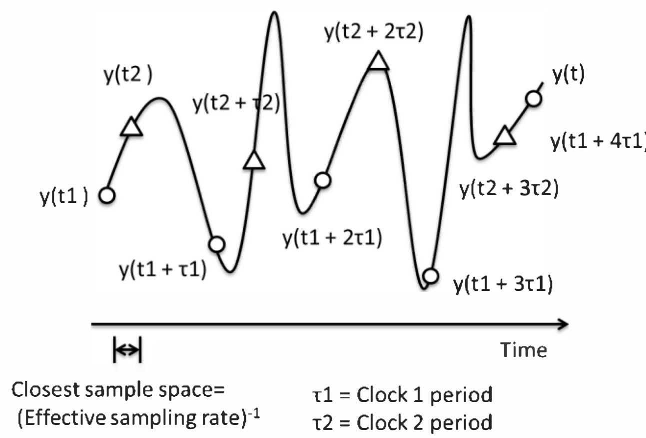

Fig. 2 Obtaining Superset y by sequencing two sets of samples

The two clocks from the two PLLs have two different sampling periods, $\tau1$ and $\tau2$. By properly synchronizing the PLLs and aligning the two sets of samples, $y1$ and $y2$, from the two ADCs, we obtain an irregularly spaced superset y as shown in Fig. 2. The sample time difference between two adjacent samples varies periodically, and the reciprocal of the smallest difference is the effective sampling rate. As one can see from Fig. 2, using two clocks with a small frequency offset, we can achieve a very high effective sampling rate. Since the proposed hardware is designed to under-sample wideband signals, the T/Hs before the two ADCs must above appropriate input bandwidth. The sampling time of ADCs should be adjusted such that they acquire data when the T/Hs are holding the signal. Using the proposed hardware architecture, we can under-sample wideband signal with a high effective sampling rate and obtain irregularly spaced samples without generating random timing.

B. Basis Pursuit and Resolution-Enhanced DFT Frame

In order to reconstruct the input test signal from the digitized samples acquired using the incoherent dual-frequency sampling hardware, we exploit the spectrum-sparse nature of the signal and model the problem as linear equations. Let

$$Ax = y \quad (1)$$

where y is the $m \times 1$ irregularly spaced sample superset (e.g. y is the vector consisting of the sequence of data points of Fig. 2 in increasing order of time), x is a $n \times 1$ vector of sparse Fourier coefficients, and A is the $n \times n$ associate Fourier basis sampling at corresponding sampling time of the superset y. $m < n$ in this undersampling scheme. Each element of the sensing matrix A, $a_{p,q}$, is

$$a_{p,q} = e^{j(q-1)\omega t_p}, where\ 1 \leq p \leq m, 1 \leq q \leq n \quad (2)$$

The system can thus be written as

$$\begin{bmatrix} 1 & e^{j\omega t_1} & e^{j2\omega t_1} & \cdots & e^{j(n-1)\omega t_1} \\ 1 & e^{j\omega t_2} & e^{j2\omega t_2} & & e^{j(n-1)\omega t_2} \\ & \vdots & & \ddots & \vdots \\ 1 & e^{j\omega t_m} & e^{j2\omega t_m} & \cdots & e^{j(n-1)\omega t_m} \end{bmatrix} \begin{bmatrix} \\ x \\ \\ \end{bmatrix} = \begin{bmatrix} y[1] \\ y[2] \\ \vdots \\ y[m] \end{bmatrix} \quad (3)$$

Where $y = [y[1], y[2], \ldots, y[m]]^T$

Since we are targeting reconstruction of multi-tone wideband signals, the unknown vector, x, is sparse. Thus, we can solve this ill-posed problem using basis pursuit. That is

$$\min_{\tilde{x} \in \mathcal{R}^n} \|\tilde{x}\|_{l1}\ subject\ to\ A\tilde{x} = y \quad (4)$$

We can obtain $\tilde{x}$ by solving this ill-posed equation, and the time-domain waveform is inverse Fourier Transform of $\tilde{x}$.

In incoherent sampling, spectral leakage in the discrete frequency domain is a problem. When the measured tones are not placed at the center of the DFT bins, spectral leakage degrades measurement performance since basis pursuit exploit sparsity in the frequency domain. To solve this problem, we enhance the frequency domain resolution of the system of equations by replacing the DFT basis with an k-times resolution-enhanced DFT frame and rewrite the sensing matrix as $\tilde{A}$, where each element of $\tilde{A}$, $\widetilde{a_{p,q}}$ is

$$\widetilde{a_{p,q}} = e^{j(q-1)\omega t_p/k}, where\ 1 \leq p \leq m, 1 \leq q \leq kn \quad (5)$$

The system equation then becomes

$$\begin{bmatrix} 1 & e^{j\omega t_1/k} & e^{j2\omega t_1/k} & \cdots & e^{j(nk-1)\omega t_1/k} \\ 1 & e^{j\omega t_2/k} & e^{j2\omega t_2/k} & & e^{j(nk-1)\omega t_2/k} \\ & \vdots & & \ddots & \vdots \\ 1 & e^{j\omega t_m/k} & e^{j2\omega t_m/k} & \cdots & e^{j(nk-1)\omega t_m/k} \end{bmatrix} \begin{bmatrix} \\ g \\ \\ \end{bmatrix} = \begin{bmatrix} y[1] \\ y[2] \\ \vdots \\ y[m] \end{bmatrix} (6)$$

where g is the vector of coefficients associated with the frame. Applying basis pursuit to solve the equations,

$$\min_{\tilde{x} \in \mathcal{R}^n} \|\tilde{g}\|_{l1}\ subject\ to\ \tilde{A}\tilde{g} = y \quad (7)$$

The solution $\tilde{g}$ has k-times higher spectrum resolution than $\tilde{f}$, and the time domain waveform is the inverse Fourier Transform of $\tilde{g}$. The spectral leakage problem is thus reduced since the difference between the center frequency of the measured tones and the nearest DFT bins is reduced. Reducing spectral leakage can also increase the sparsity of the measured signal. This leads to better signal reconstruction because basis pursuit exploits signal sparsity to find the solution to the ill-posed problem. The result is shown as Fig.3. We can see from the figure that the reconstruction mean square error (MSE) is reduced as the resolution-enhancing factor k increases, so does the computation for the basis pursuit. Using the proposed reconstruction algorithm, one can improve the reconstruction performance of basis pursuit and reduce the spectral leakage by increasing the spectrum resolution using an resolution-enhanced DFT frame.

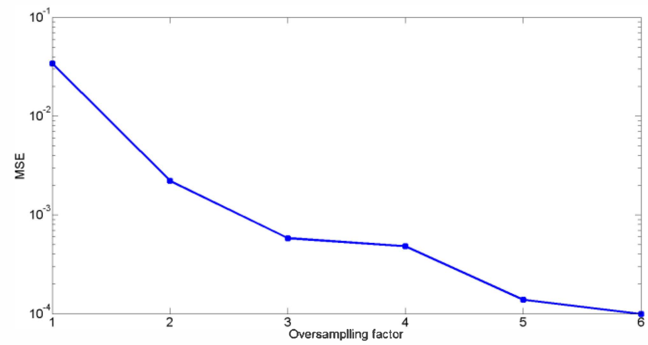

Fig. 1 Reconstruction MSE vs Resolution-Enhancing Factor k

C. Sampling Time-base Optimization

The sensing matrix $\tilde{A}$ obtained from dual-rate sampling is deterministic. In comparison, the optimal random sampling samples the signal at arbitrary random timing intervals. Researchers have attempted to facilitate compressive sampling based reconstruction using deterministic sampling methods while minimizing the complexity of signal reconstruction. [18] An analysis of the use of a deterministic sensing matrix for compressive sampling is presented in [16]. In our methodology, the sensing matrix corresponds to an irregular sampling of the Fourier frame. Based on [16], for sensing matrices that have the statistically restricted isometry property, three conditions have to be satisfied. For a Fourier basis, it is easy to verify the first two conditions. To verify the third condition, we calculate the square of the sum of each column of the sensing matrix $\tilde{A}$ and plot the result as shown in Fig. 4.

We can see from the figure that peaks occur at multiples of the two sampling frequencies. This means that when the input signals contain frequencies which are multiples of either of the sampling frequencies, , the sampling is coherent, and there is no way to distinguish the DC component from these frequencies. Therefore, to have successful reconstruction, the sampling frequencies should be incoherent with the input signal. In other words, the input signal should not contain frequencies in restricted bands, which are multiples of the ADC sampling frequencies as shown in Fig. 5.

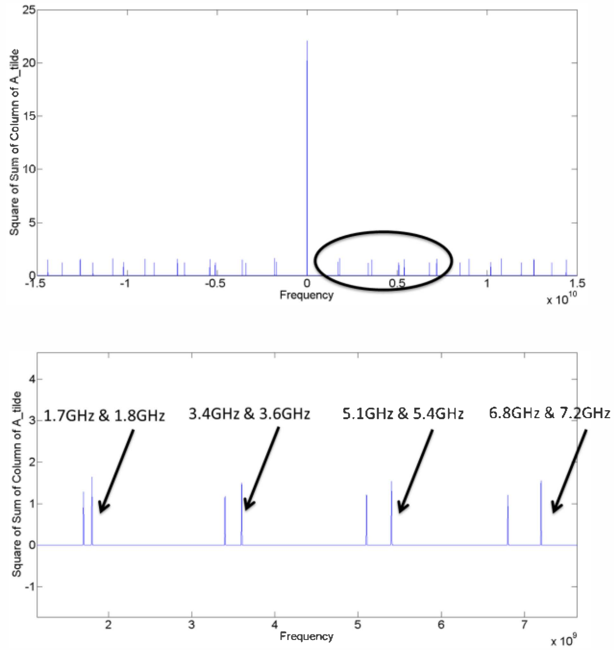

Fig. 4 Square of sum of each column of sensing matrix

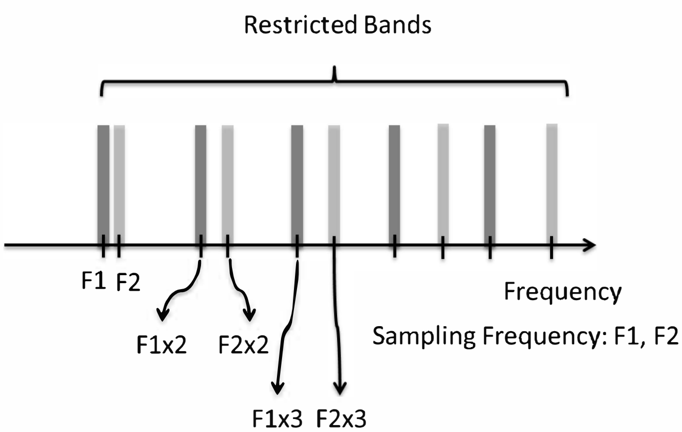

Fig.5 Square of sum of each column of sensing matrix

We compare the reconstruction mean square error per sample of the proposed scheme with the optimal random sampling scheme using the same number of samples. The result is shown as Fig. 6. In this simulation, the two sampling clocks have frequencies of 1.7 GHz and 1.8 GHz, and 400 incoherent samples are used to blindly reconstruct a 30 GHz wideband multi-tone signal. As we can see from the plot, the reconstruction error difference of the proposed sampling scheme and the optimal random sampling scheme is less than 0.1%.

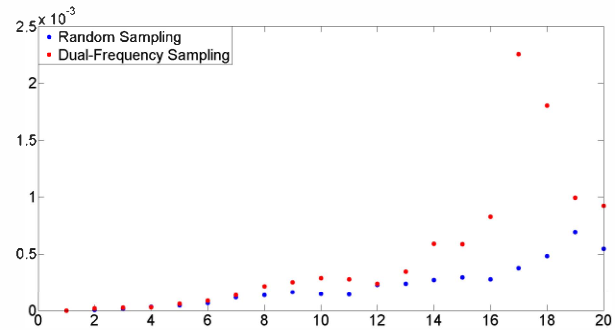

Fig. 6 Dual-Frequency Sampling vs Optimal Random Sampling

III. Simulation Result

We demonstrate our method in simulation by reconstructing an 8-tone signal and a PRBS-15 signal. In the simulation setup, the dual frequency we use is 1.7 GHz and 1.8GHz and use 400 incoherent samples. The effective sampling rate is therefore $\left(\frac{1}{1.7\ \mathrm{GHz}}-\frac{1}{1.8\ \mathrm{GHz}}\right)^{-1} = 30.6\ \mathrm{GHz}$. Basis pursuit and a 5-time resolution-enhanced DFT frame are used in reconstruction.

A. Application in Reconstructing multi-tone signals

We first demonstrate our method by reconstructing multi-tone signals. 8 tones, as shown in Fig. 10, are randomly selected from DC to 14 GHz, which is about half of the effective sampling rate and combined to be the input signal. It is then fed into the dual-frequency sampling system. Using 400 samples, we can reconstruct the signal with negligible error as shown in Fig. 7 and Fig. 8. We can see from the Fig. 10 that the tones spread 14GHz wide, and the time domain waveform is perfectly reconstructed with sampling rate 3.5Gsps (1.7Gsps + 1.8Gsps)

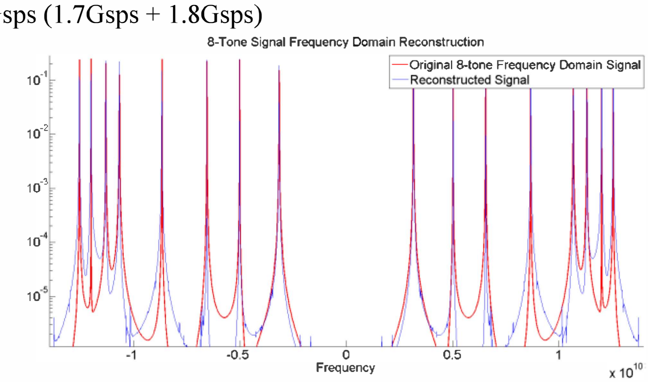

Fig. 7 8-Tone Signal Frequency Domain Reconstruction

B. Application in Reconstructing PRBS

We now apply the proposed method to 4GSps, 15-bit PRBS reconstruction. The phase-noise injected PRBS signal is first filtered by an 8GHz low-pass filter and then sampled by the dual-frequency sampling system. As can be seen the time domain reconstruction Fig. 9, 400 incoherent samples can be used to reconstruct the PRBS with negligible error. The phase noise of the noisy PRBS can be captured as shown in the frequency domain reconstruction Fig. 10.

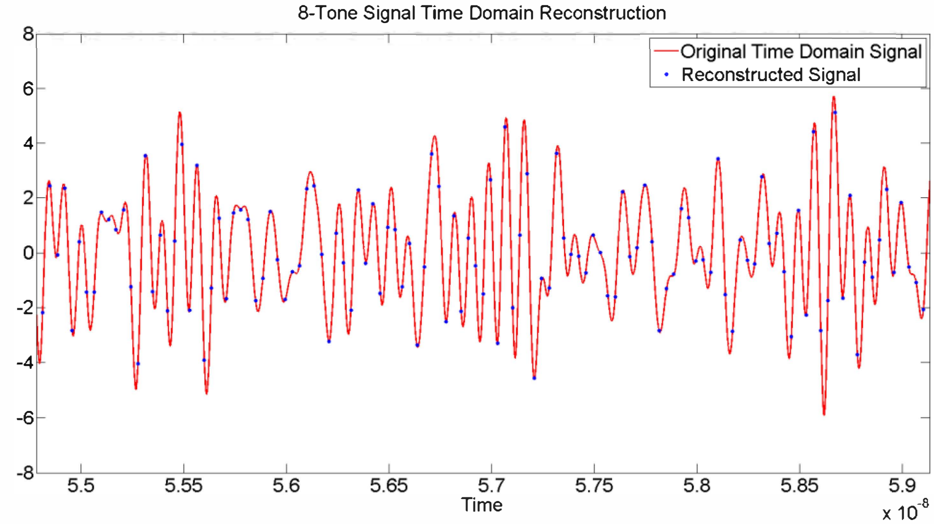

Fig. 8 8-Tone Signal Time Domain Signal Reconstruction

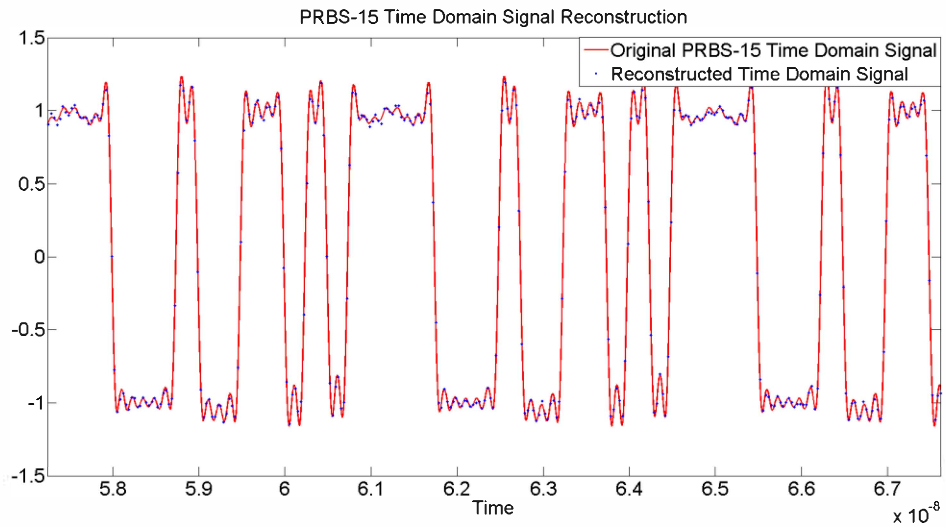

Fig. 9 PRBS-15 Time Domain Signal Reconstruction

The eye diagram is then reconstructed easily since we already reconstruct the whole time domain signal as shown in Fig. 11. By using the proposed hardware-software co-design method, the number of samples needed to reconstruct the PRBS signal is significantly less than traditional equivalent time sampling technique.

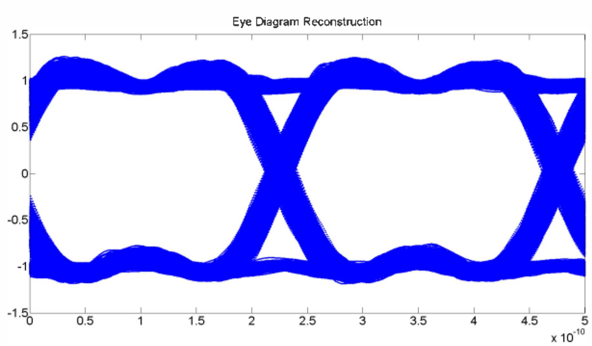

Fig. 11 Eye Diagram

IV. Conclusion

We have proposed an under-sampling scheme to reconstruct wideband multi-tone or PRBS signals using a small number of incoherent samples. We also explore the relation between basis pursuit and incoherent sampling. The hardware is currently under verification and the performance of the proposed hardware and software design will be released soon.

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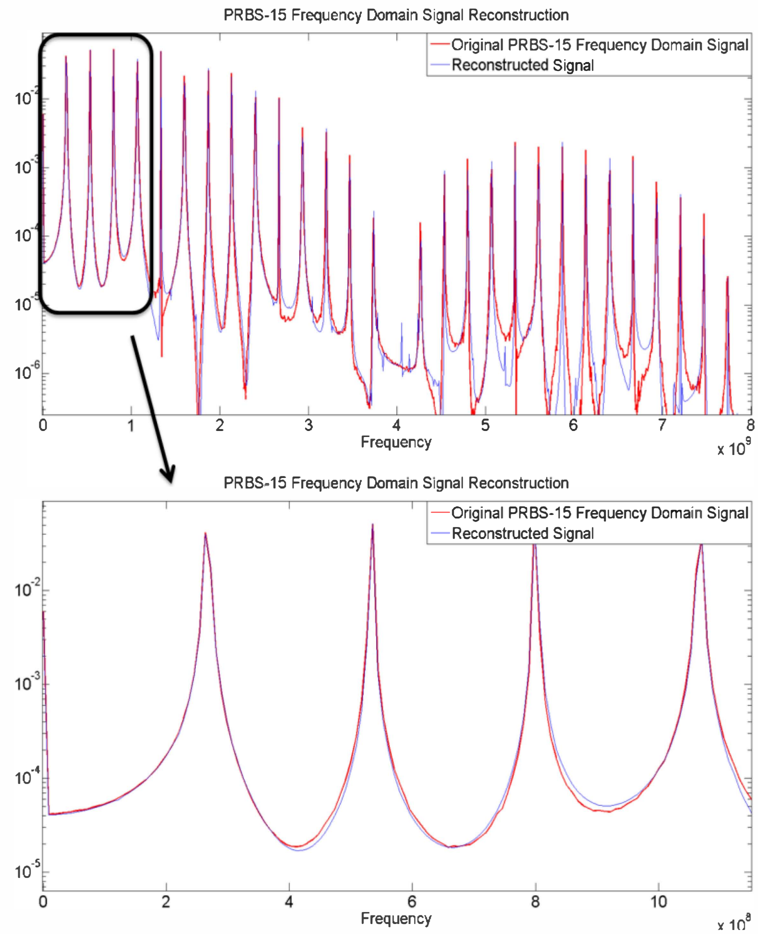

Fig. 10 PRBS-15 Frequency Domain Reconstruction

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