2012 IEEE 30th VLSI Test Symposium (VTS)

Low-Cost High-Speed Pseudo-Random Bit Sequence Characterization Using Nonuniform Periodic Sampling in the Presence of Noise

Thomas Moon, Nicholas Tzou, Xian Wang, Hyun Choi*, Abhijit Chatterjee
School of ECE, Georgia Tech, USA
* Samsung Electronics

***Abstract*— In this paper, we propose a wideband signal reconstruction scheme for testing high-speed pseudo random bit sequences (PRBSs) in the presence of jitter noise using incoherent sampling. The proposed approach exploits synchronous multirate sampling (SMRS) hardware and multicoset back-end signal processing algorithms. The SMRS hardware consists of multiple analog-to-digital converters (ADCs) whose sampling frequencies are synchronized with a common frequency reference and can be individually configured. The optimal sampling frequency of each ADC is chosen based on the input signal information and sampling hardware specifications. As compared to other sampling hardware used for multicoset signal reconstruction, the proposed approach uses less number of ADCs and does not require accurate sampling clock phase adjustment. In the digital signal reconstruction, the input waveform is reconstructed by the multicoset signal processing algorithms and the phase noise of each tone of the PRBS test signal is measured.**

***Index Terms*— Nonuniform periodic sampling, signal representation, Analog-to-digital converters, jitter noise, phase noise, PRBS**

I. INTRODUCTION

HIGH-SPEED signal acquisition and jitter characterization involves large testing cost. As new multi-GHz digital circuits are designed and fabricated, novel high-speed signal acquisition techniques have been introduced to determine the fidelity of high-speed signals and underlying circuitry. By the well-known Whittaker, Kotelnikov and Shannon (WKS) theorem, a signal band-limited to H Hertz can be recovered with 2H samples per second [1]. In [2], recent advances in the design of ADCs running at 3.6Gsps with 12-bit resolution are described. There are, however, many design/manufacturing issues involved in the ability to deliver higher speed ADCs beyond 3.6Gsps. To resolve this problem, the use of multiple interleaved ADCs to achieve an effective Nyquist sampling rate beyond that possible via direct sampling has been investigated in the past [3], [4]. Furthermore, if the spectrum within the bandwidth of the input signal is not fully occupied, periodic nonuniform sampling (or multicoset sampling) has been shown to be an efficient method for reconstructing the signal because the sampling rate can be lower than the Nyquist rate [5], [6]. In recent research [7], [8], a modulated wideband converter uses an analog mixing circuit to recover wideband sparse signals at sub-Nyquist rates. Even though the objective of this work is targeted towards blind multiband signal reconstruction, the corresponding hardware design is challenging due to the use of multiple high quality front-end RF mixers. Moreover, the hardware cost of such a data acquisition can be high due to the number of RF channels involved.

In [9], a high-speed periodic signal acquisition technique using incoherent sub-sampling is presented. The analog frequency of the input signal can be estimated by switching the sampling rate of an ADC. The basic idea that exploits different sampling rates is developed in this paper. In contrast to [9], this paper focuses on signal reconstruction in the presence of jitter noise assuming the frequency of the input signal is already estimated by using the techniques in [9]. The algorithm in this paper assumes the following conditions:1) The PRBS signal is band-limited and 2) The bit period and the bit rate of the PRBS are roughly known a priori before sampling and signal reconstruction.

The main contribution of this paper is in proposing a low-cost hardware scheme to reconstruct a high-speed pseudo-random bit sequence (PRBS) signal in the presence of jitter noise using intelligent undersampling and signal reconstruction algorithms. Signal reconstruction in the presence of jitter noise in a non-ideal PRBS signal using a sub-Nyquist sampling rate is not trivial because the spectrum of a PRBS signal is not sparse. However, the proposed signal reconstruction algorithm based on non-uniform periodic sampling, also known as *multicoset sampling*, is an efficient way to capture the information contained in the multiband spectrum of the signal. Furthermore, the signal reconstruction algorithm is mapped to a low cost synchronous multirate sampling (SMRS) for the hardware implementation, as compared to conventional hardware design for multicoset sampling. In this paper, we define “dual-sampling” as a special case of SMRS with two channels. The dual-sampling system exploits two different sampling frequencies across two different ADCs. By adjusting the sampling frequencies, various combinations of sampling patterns can be achieved. Further,

978-1-4673-1074-1/12/$31.00 ©2012 IEEE

based on the active frequency bands of the input signal, the most optimal sampling frequencies for efficient signal reconstruction can be determined. To summarize, the followings are key benefits of this research:

- It is possible, by our methodology, to reconstruct PRBS signals with significant amount of jitter noise accurately using sub-Nyquist sampling rate.
- As opposed to current multicoset sampling techniques, only two ADCs (channels) are used in our approach and each of these ADCs may be run at lower than the Nyquist rate.

In Section II, we describe our proposed approach to reconstruct a PRBS signal in the presence of jitter noise. In Section III, we explain background knowledge pertaining to PRBS signals and multicoset sampling. The proposed unique hardware architecture with its dual-sampling system and PRBS jitter noise reconstruction methodology using multicoset sampling are described in Section IV. Numerical experiments are summarized in section V. Finally conclusions are presented.

II. Approach

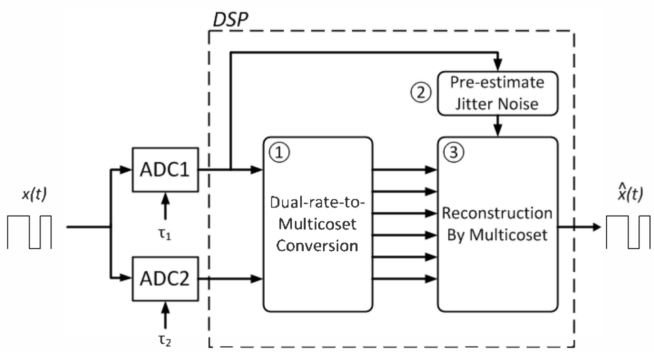

Fig. 1. Block diagram of multicoset dual-sampling system.

Fig. 1 shows the proposed dual-frequency wideband sampling system for multicoset-based signal reconstruction. The two ADCs shown in Fig. 1 are run in parallel to each other and simultaneously capture the input signal with their programmable sampling frequencies. The sampling frequencies are independently programmed but synchronized with a common frequency reference. The discrete waveforms from the outputs of the ADCs are acquired by the digital signal processor (DSP) that performs multirate-to-multicoset conversion and multicoset-based signal reconstruction. In the multirate-to-multicoset conversion (block①), the sample sets obtained from the two sampling channels are merged into one set whose samples are irregularly spaced. The irregular samples are classified into cosets and sent to the reconstruction block with p channels (data sets). Coarse jitter noise estimation (block②) is performed by analyzing the samples from a single ADC before detailed multicoset based signal/jitter reconstruction is performed. The jitter noise information helps in the reconstruction of the input signal. The reconstruction block (block③) recovers the input signal by solving linear equations for the p channels (data sets).

The detailed process of merging the samples from the two channels is presented in Fig. 2, which shows the case for the normalized sampling periods of 3 and 4 for the first ADC (ADC1) and the second ADC (ADC2), respectively. Once the two different clocks are synchronized with a common frequency reference (t=0), the sampling pattern repeats by a fixed period (L=12). The sampling pattern in a period (L=12) is translated to the cosets in multicoset sampling. In this example, the cosets are $\{0,3,4,6,8,9\}$ and the total number of cosets is 6.

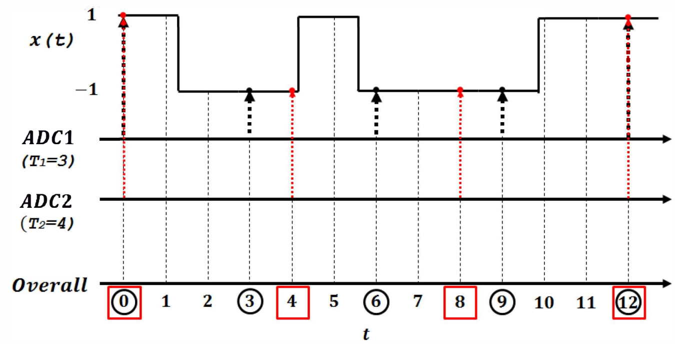

Fig. 2. Samples from ADC1(T_1=3) and ADC2(T_2=4) are merged into uniform grid with period L=12. The cosets are $\{0,3,4,6,8,9\}$.

III. Preliminaries

In this section, we provide background on PRBS test signals [10] in Section II-A and multicoset sampling [11], [12] in Section II-B.

A. PRBS

A PRBS signal consists of a pseudo-random bit sequence that repeats every N clock cycles and is denoted as *PRBS-N*. A PRBS is usually generated by linear feedback shift register (LFSR) with m flip-flops. Since the period of a PRBS generated by an LFSR with m flip-flops is limited to $2^m - 1$, a PRBS-N is called a maximal-length-sequence when $N = 2^m - 1$.

Let $x(t)$ be a maximal length PRBS-N with its binary symbols 0 and 1 mapped to the levels -1 and +1. The period of $x(t)$ is $T_b = NT_c$, where T_c is the duration of a single bit. The autocorrelation of $x(t)$ is obtained by $R_x(\tau) = \frac{1}{T}\int_{-T_b/2}^{T_b/2} x(t)\,x(t-\tau)\,dt$. Applying the balance property which states that the number of 1 symbols is always one more than the number of 0 symbols in a maximal-length-sequence,

$$R_x(\tau) = \begin{cases} 1 - \frac{N+1}{NT_c}|\tau|, & |\tau| \le T_c \\ -\frac{1}{N}, & for\ the\ remainder\ of\ the\ period \end{cases} . \quad (1)$$

By taking the Fourier transform of $R_x(\tau)$ in (1), the spectrum of $x(t)$ is

$$P_x(f) = \frac{1}{N^2}\delta(f) + \frac{1+N}{N^2}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} sinc^2\left(\frac{n}{N}\right)\delta\left(f - \frac{n}{NT_c}\right) . \quad (2)$$

Thus, the spectrum of $x(t)$ consists of an infinite number of tones with an envelope given by the square of the sinc function. In each lobe with a frequency range between m/T_c and

$(m+1)/T_c$ for integer m, the number of tones corresponding to the lobe is always *N-1* excluding the dc component.

B. Multicoset Sampling

Let $x(t)$ be a continuous real-valued function of *t*. The Fourier transform of $x(t)$ is

$$X(f) = \int_{-\infty}^{\infty} x(t) \exp(-j2\pi ft)\, dt.$$

It is assumed that $x(t)$ is a band-limited signal, $X(f) = 0, f \notin F$. The spectral support F of $x(t)$ indicates the region in frequency domain that contains the energy of the signal $x(t)$. It is defined as

$$F = \pm \bigcup_{i=1}^{n} [a_i \cap b_i]$$

where $0 \leq a_1 < b_1 < \cdots < a_n < b_n \leq \frac{1}{T}$. For example, if a multiband signal $x(t)$ has its energy in band 100Hz~200Hz and 500Hz~550Hz, it is clear that $a_1 = 100, b_1 = 200, a_2 = 500, b_2 = 550$. Uniform sampling with a frequency of *1/T*, called the Nyquist rate, which guarantees no aliasing and results in a perfect reconstruction of the signal $x(t)$.

In multicoset sampling, some of the samples are chosen from the Nyquist rate grid. The chosen samples are defined by the pattern from a set $C = \{c_1, \ldots c_p\}$ and *L*,

$$x_{c_i}(n) = \begin{cases} x(nT), & n = kL + c_i, k \in Z \\ 0, & otherwise \end{cases}$$

where $0 \leq c_1 < c_2 < \ldots < c_p \leq L-1$ and $0 < p \leq L$. The $x_{ci}(n)$ are sampled from the uniform sampling sequence spacing by *LT*. The *i*-th entry of the set *C*, c_i, is called the *i*-th active coset. The integer *L* is the period of the pattern and the integer *p* is the number of cosets in the set *C*.

The discrete-time Fourier transform of the $x_{ci}(n)$ is

$$X_{c_i}(e^{-j2\pi fT}) = \frac{1}{LT} \sum_{r=0}^{L-1} X(f + \frac{r}{LT}) \exp\left(\frac{j2\pi rc_i}{L}\right), f \in F_0$$

where $F_0 = [0, \frac{1}{LT})$.

Let the signal $x(t)$ be composed of n bounded bands such that

$$0 = a_1 < b_1 < a_2 < \cdots < a_n < b_n = \frac{1}{T}$$

which are called the active bands of $x(t)$. We define the set Γ as:

$$\Gamma = \left\{a_i - \frac{LTa_i}{LT} : 1 \leq i \leq n\right\} \cup \left\{b_i - \frac{LTb_i}{LT} : 1 \leq i \leq n\right\}.$$

The increasing-order elements of set Γ are $\{\lambda_1, \lambda_2, \ldots, \lambda_M\}$ where $M \leq 2n$. We add an element $\lambda_{M+1} = 1/LT$ to define the intervals G_m whose borders are λ_M and λ_{M+1}.

$$G_m = [\lambda_m, \lambda_{m+1}), 1 \leq m \leq M.$$

The active bands of $x(t)$ are a combination of G_m shifted by *r/LT* with some integer *r*.

The spectral index set k_m is defined as

$$k_m = \{r | X(f + \frac{r}{LT}) \subset F, f \in G_m\},$$

and its *l*-th element is denoted by $k_m(l)$. We define the number of elements in each K_m as q_m which indicates the number of overlaps of the active bands.

The reconstruction equation can be expressed in matrix form as below:

$$y(f) = A_m x_m(f), \quad f \in G_m, \quad 1 \leq m \leq M \tag{3}$$

where $y(f)$ is a *p*-length vector with the *i*th element of $X_{c_i}(e^{-j2\pi fT})$, and $x_m(f)$ is a q_m-length vector with the *k*th element given by:

$$[x_m(f)]_k = X\left(f + \frac{k_m(k)}{LT}\right). f \in G_m$$

The matrix A_m is *p*-by-q_m matrix with *ik*th element defined by

$$[A_m]_{ik} = \frac{1}{LT} \exp\left(\frac{j2\pi c_i k_m(k)}{L}\right). \tag{4}$$

The reconstruction of the original signal is achieved by solving $x_m(f)$ in the linear system for all m. Given $y(f)$ which is the vector of spectra obtained by the active cosets, the uniqueness of the solution depends on the rank of the matrix A_m. If the matrix has full-column rank, the solution is unique, if it exists. Since the matrix A_m is a submatrix of the *L*-by-*L* DFT matrix by (4), the matrix A_m has full-column rank if $p \geq q_m$. Thus, the necessary condition for a unique solution for $x(f)$ is that *p* be greater than or equal to q_m for all m,

$$p \geq q_m, \quad 1 \leq m \leq M$$

IV. DUAL-SAMPLING FOR MULTICOSET

In this section, we describe our proposed approach and algorithm in detail.

A. Dual-rate-to-Multicoset Conversion

Let τ_1 and τ_2 be the two different sampling periods for the ADCs and T_1 and T_2 be the two normalized different sampling periods in integer.

$$T_1 = \tau_1 R$$
$$T_2 = \tau_2 R$$

where *R* is a real number that makes T_1 and T_2 coprime.

In dual-sampling, the least common multiple (LCM) of T_1 and T_2 becomes *L*. The C_1 and C_2 are the set of samples from each sampling period T_1 and T_2, respectively.

$$C_1 = \{c_i | c_i = T_1 i : 0 < c_i \leq L, i \in \mathbb{Z}^+\}$$
$$C_2 = \{c_i | c_i = T_2 i : 0 < c_i \leq L, i \in \mathbb{Z}^+\}$$

The space between the elements, or cosets, of each set C_1 and C_2 is regular. The total sampling pattern set, C_{total}, is obtained by taking the union of the set C_1 and C_2.

$$C_{total} = C_1 \cup C_2$$

The Nyquist frequency can be defined by the grid space of the samples in C_{total},

$$f_N = \frac{1}{T} = \frac{1}{1/R} = R$$

To formulate the number of elements in C_{total}, denoted by p_{total}, we first define the positive integer K_1 and K_2 as

$$T_1 = K_1 G$$
$$T_2 = K_2 G$$

where *G* is the greatest common divisor of T_1 and T_2. Since *L* is the LCM of T_1 and T_2, it reduces to

$$L = K_1 K_2 G = \frac{T_1 T_2}{G}$$

Then, we can express the number of cosets in C_1, denoted by p_1,

$$p_1 = \frac{L}{T_1} = K_2$$

Similarly, p_2 is

$$p_2 = \frac{L}{T_2} = K_1$$

By the definition of C_1 and C_2, they share only one coset, c_{p1}=c_{p2}. Thus, p_{total} is

$$p_{total} = K_1 + K_2 - 1$$

As the pattern of cosets is fixed in dual sampling system, it is not possible to optimize the cosets by directly adjusting the location of them. The rigidity of coset pattern is a disadvantage of dual sampling system. The condition number of the matrix A_m in (3) is one of indicators for the quality of the reconstructed signal [14]. As the condition number grows, the reconstructed signal quality is degraded due to the error amplification. Thus, the location of cosets and the spectral index set k_m affects the quality of the reconstructed signal because the matrix A_m is dependent on them.

B. *Pre-estimation of jitter noise in PRBS*

The quality of PRBS signal can be degraded by its jitter noise. As we express the spectrum of PRBS in (2), it is composed of infinite number of tones at the fundamental frequency and its harmonics. When a tone is degraded by jitter noise, the spectrum of the tone is surrounded by its noise skirt. The noise skirt is symmetrical across the center frequency of the tone [13]. We simulate jitter noise as follows. Assume that the samples from an ideal PRBS signal, $x[n]$, is corrupted by the jitter noise, $\phi[n]$. Then, the jitter-noised PRBS, $x[n]$, is

$$x\ n\ = x\ n + \phi[n]$$

The jitter noise $\phi[n]$ can be described as a random walk process.

$$\phi\ n\ = \phi\ n - 1\ + \sigma Y$$

where σ is a constant and Y is a random variable defined by

$$P\ Y = 1\ = 0.5\ and\ P\ Y = -1\ = 0.5$$

The constant σ can be interpreted as the amount of jitter noise. If σ is a large number, the sample points will be far away from the ideal points due to its jitter noise.

Assuming the amount of jitter noise in the input PRBS is unknown, the estimation of jitter noise helps to construct active bands in multicoset sampling. In the presence of the jitter noise, the spectrum of the center frequency of each tone is spread into its skirt. To reconstruct the jitter noise information, the active bands are estimated to contain the support of the skirt. If the estimated active bands do not cover the actual active bands of the signal, the reconstructed signal is in error.

As the jitter noise increases, less energy is located on the tones. Fig. 3 shows two aliased spectrums of PRBS-7 with 4GHz-bitrate signal sampled at 1.8GHz. In case of the ideal PRBS, most of its energy is located at the aliased tones. The energy of the PRBS with jitter noise, however, spreads out near the tones.

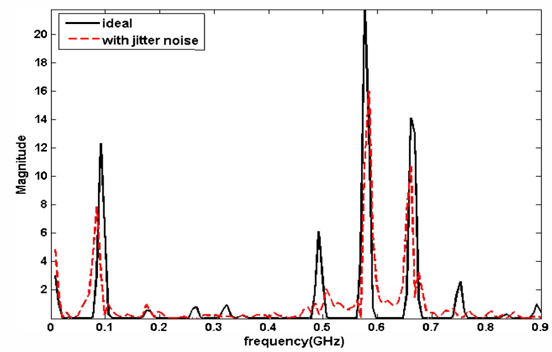

Fig. 3. Aliased spectrum of PRBS-7 with 4GHz-bitrate sampled at 1.8GHz. Ideal signal and jitter-noised signal are plotted.

The jitter noise can be approximately quantified by observing the ratio between the norms of the spectrum near the tones and that of the total spectrum. Let $X_a(f)$ be an aliased spectrum in the frequency range $F_t = [0, f_u/2)$, where f_u is the sampling rate under the Nyquist rate. As we know the fundamental frequency of PRBS and the sampling frequency, the center frequencies of the aliased tones can be computed as follows.

Let f_k be the center frequency of the aliased tone from the k-th tone of a PRBS. The k-th tone of the PRBS is originally located at frequency kf_0, where f_0 is the fundamental frequency of the PRBS. The center frequency, f_k, can be expressed as

$$f_k = mod\ m, 2\ \cdot \frac{f_u}{2} + \ -1\ ^{mod\ m,2}\ \cdot mod(kf_0, \frac{f_u}{2})$$

where $m =\ 2kf_0/f_u\ $. The union of the intervals for the aliased tone is

$$F_a = \bigcup_{k=1}^{\infty}\ f_k - \epsilon, f_k + \epsilon$$

where ϵ is the margin to cover the tone. The ratio, β_p, is

$$\beta_p = \frac{X_a\ f_a\ \ _p}{X_a\ f_t\ \ _p}, \qquad f_a \in F_a\ and\ f_t \in F_t$$

which stands for the ratio between the L_p norms of the spectrum near the aliased tones and that of the overall in sampling frequency domain. As the amount of jitter noise increases, the ratio β will decrease.

C. *Optimal dual-sampling frequencies selection*

Dual-sampling system can instantiate many different multicoset designs by changing the relationship between its two different sampling frequencies. Though the coset pattern is fixed, the spectral index k_m is changeable because the spectral index depends on L and T. Thus, q_m is also flexible with the choice of the two sampling frequencies. Our objective is to achieve maximum p and minimum q_m, which will lead more robust solution in (3). The total number of cosets, p, increases as the sampling frequencies are close. However, the uniform sampling space LT also increases with high possibility of aliasing. In other words, q_m will grow creating many columns in the matrix A_m. Thus, we define the ratio, γ,

$$\gamma = \frac{\max\ q_m}{p_{total}}$$

and the optimized sampling frequencies will generate the smallest γ.

To simplify the problem, we fix one of the sampling frequencies to have the maximum frequency that the ADC can accept. By sweeping the second sampling frequency, γ for each sampling frequency can be computed. The optimal sampling frequency for the second ADC is obtained by choosing the sampling frequency corresponding the minimum γ.

The active band-locations can be estimated with the above assumptions. In addition, the coarse information about jitter noise acquired in Section III-B will also be applied to construct the active band. We denote Δ_m as the side bandwidth of the active band for the m-th tone. Suppose the main-lobe and the first side-lobe of PRBS is the target reconstruction region in frequency domain, then the active band-locations can be expressed as follows.

$$\mathop{}_{m=1}^{2N-1} [\ mf_0 - \Delta_m\ ,\ mf_0 + \Delta_m \quad , \qquad m \neq N$$

where f_0 is the fundamental frequency of the PRBS and N is the period of the PRBS.

V. Numerical Simulations

We first start by demonstrating pre-estimation of jitter noise of a PRBS. In computer simulation, a 4-Gbps PRBS-7 was digitized at a sampling rate of 1.8-Gsps. In Fig. 4, where L_1 norm is chosen for β, we can see β_1 decreases as σ grows. As σ represents the amount of jitter noise, the coarse estimation of jitter noise is accomplished by calculating β.

In Fig. 5, we fix the first sampling frequency of 1.8GHz and sweep the second sampling frequency to find the minimum γ. We drop some points when L is too large that the computation for maximum q_m takes enormous time. We can see many small γs are clustered when the second sampling frequency is close to the first one. The optimal sampling frequency 1.755GHz is chosen with the minimum $\gamma = 0.3333$.

We examine a jitter-noised PRBS-15 and a jitter-free PRBS-15. Both signals have 4GHz bitrate and are dual-sampled with optimal frequencies 1.8GHz and 1.755GHz. Our target is to reconstruct the signals with the main lobe (0~4GHz) and the first side lobe (4GHZ~8GHz) in frequency domain. The active bands are constructed to obtain 13MHz sideband for each tone, which covers enough amounts of skirts due to the jitter noise. The jitter-noise is assumed to be $\sigma = 0.001$. Fig. 6 shows the spectrums of the original signal without jitter noise and reconstructed signal between the fundamental and the second tone. Fig. 7 zooms in the two tones. The tones are not ideal peaks because of incoherent sampling. Fig. 8 compares the two signals in time domain. It is clear that the reconstructed signal closely matches the original signal. In Fig. 9, the jitter-noised signal is tested. The spectrums of the original jitter-noised signal and the reconstructed signal are shown. Fig. 10 shows the spectrum of the fundamental tone and that of the second tone, respectively. Fig. 11 compares the jitter-noised original and reconstructed signals in time domain. Though the reconstruction result of jitter-noised case is not as good as the result of jitter-free case, the rising and falling edges of the reconstructed signal correspond to the original signal. The simulation results implies that the tones of PRBS-15 signal band-limited to 8GHz including the main-lobe and the first side-lobe can be reconstructed with the jitter noise information. The reconstruction is achieved by dual-sampling using only 1.8GHz and 1.755GHz sampling frequencies, while the Nyquist rate is 16GHz.

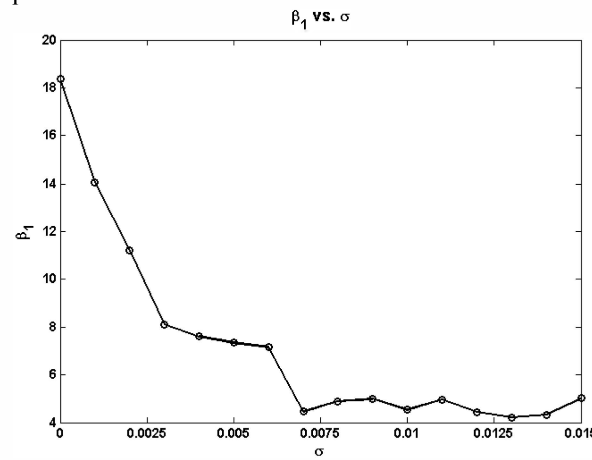

Fig. 4. Trend of $\boldsymbol{\beta_1}$ with $\boldsymbol{\sigma}$. 4GHz-bitrate PRBS-7 sampled at 1.8GHz.

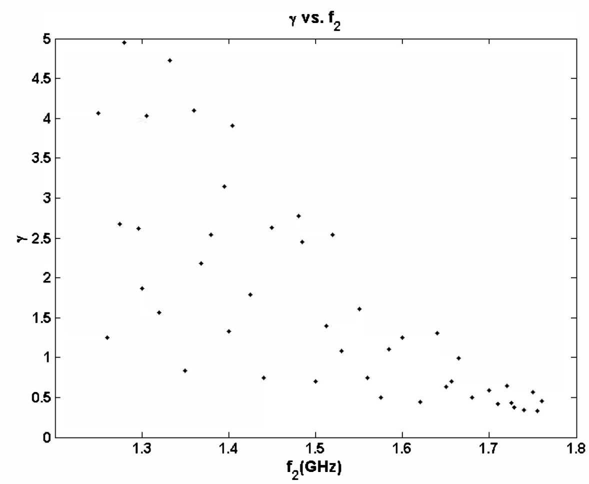

Fig. 5. $\boldsymbol{\gamma}$ converges as the second sampling frequency(f_2) approaches to the first sampling frequency(f_1=1.8GHz)

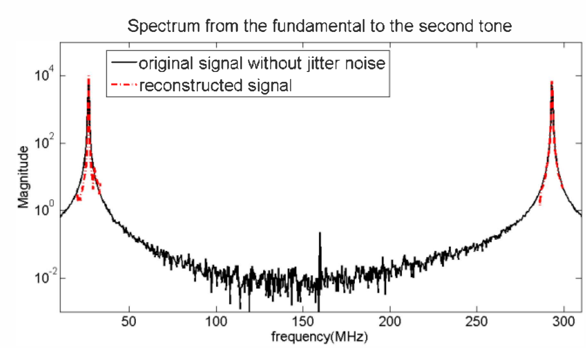

Fig. 6. Comparison of the spectrum between the jitter-free original signal and reconstructed signal.

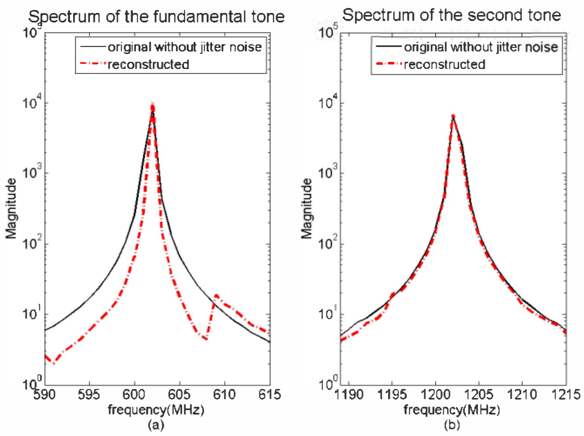

Fig. 7. The fundamental tone (a) and the second tone (b) are zoomed in Fig. 6.

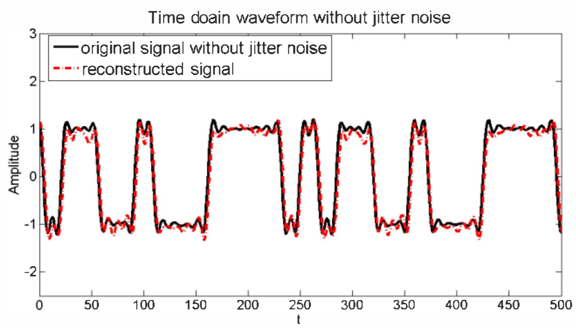

Fig. 8. The jitter-free original signal and reconstructed signal are compared in time-domain.

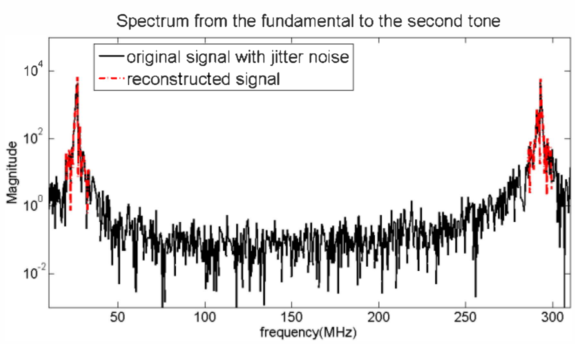

Fig. 9. Comparison of the spectrum between the jitter-noised original signal and reconstructed signal.

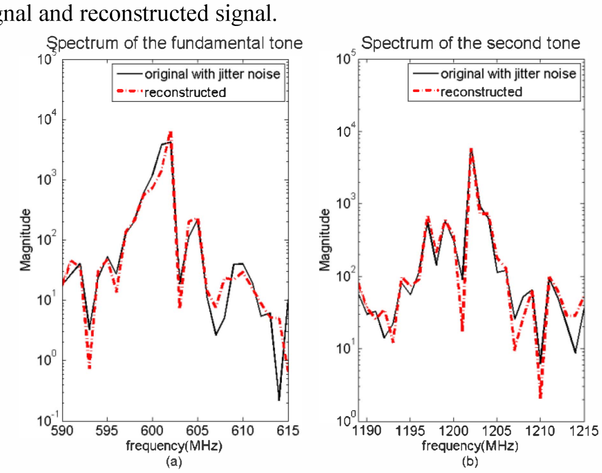

Fig. 10. The fundamental tone (a) and the second tone (b) are zoomed in Fig. 9.

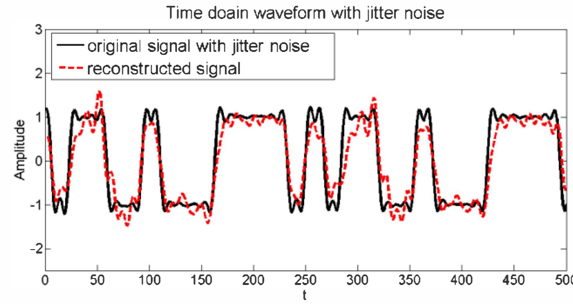

Fig. 11. The jitter-noised original signal and reconstructed signal are compared in time-domain.

VI. Conclusion

In this paper, we propose a new hardware architecture and reconstruction algorithm to recover the jitter noise information of PRBSs. While the conventional multicoset sampling requires large numbers of ADCs and accurate phase adjustment of sampling clocks, our proposed system needs much less number of ADCs without any phase adjustment. The simulation results show the jitter-noised high-speed PRBSs can be reconstructed with much lower sampling speeds than the Nyquist rate. In future work, we will present more robust methodology to connect the pre- estimation of jitter noise to our current algorithm. Furthermore, the quality of reconstruction will be evaluated in concrete way and compared with other optimal multicoset patterns. Finally, we will show the measurement result with our hardware.

Acknowledgement

The authors would like to thank Samsung Electronics for their insights and support.

References

[1] C.E.Shannon, “Communication in the presence of noise.” Proc, IRE, vol. 37, pp.10-21, Jan. 1949.

[2] Scott Kulchycki, “Software Defined Radio: Don’t Talk to Me about ENOBs”

[3] J. J. Brown, "Multi-channel sampling of low-pass signals," IEEE Trans on Circuits and Systems, vol. 28, no. 2, pp. 101-106, February 1981.

[4] M. Kozak and I. Kale, Oversampled Delta-Sigma Modulators: Analysis, Applications and Novel Topologies. Kluwer Academic Publishers, 2003.

[5] P. Feng and Y. Bresler, “Spectrum-blind minimum-rate sampling and reconstruction of multiband signals,” in Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing. Atlanta, GA, May 1996.

[6] B. Foster and C. Herley, “Exact reconstruction from periodic nonuniform sampling of signals with arbitrary frequency support,” in Proc.IEEE Int. Conf. Acoustics, Speech, and Signal Processing. Detroit, MI, May 1998.

[7] M. Mishali, Y. C. Eldar, O. Dounaevsky and E. Shoshan, "Xampling: Analog to Digital at Sub-Nyquist Rates", IET Circuits, Devices & Systems, vol. 5, no. 1, pp. 8-20, Jan. 2011.

[8] M. Mishali and Y. C. Eldar, "Blind Multi-Band Signal Reconstruction: Compressed Sensing for Analog Signals", IEEE Trans. on Signal Processing, vol. 57, no. 3, pp. 993-1009, March 2009.

[9] Hyun Choi, Gomes, A.V. and Chatterjee, A., “Signal Acquisition of High-Speed Periodic Signals Using Incoherent Sub-Sampling and Back-End Signal Reconstruction Algorithms”, IEEE Trans. on VLSI Systems, vol. 19, no. 7, July 2011.

[10] Simon Haykin, Communication Systems, John Wiley & Sons.

[11] Y.-C. Jeng, “Digital spectra of nonuniformly sampled signals: Fundamentals and high-speed waveform digitizers,” IEEE Trans. Instrum. Meas.,vol. 37, no. 2, pp. 245–251, Jun. 1988.

[12] R. Venkataramani and Y. Bresler, ‘‘Perfect reconstruction formulas and bounds on aliasing error in sub-Nyquist nonuniform sampling of multiband signals,’’ IEEE Trans. Inf. Theory, vol. 46, no. 6, pp. 2173- 2183, Sep. 2000.

[13] Q. Huang, “Phase noise to carrier ratio in LC oscillators,” IEEE Trans. Circuits Syst. I, Fundam. Theory Appl., vol. 47, no. 7, pp. 965–980, Jul.2000.

[14] L. Berman and A. Feuer, “Robust patterns in recurrent sampling of multiband signals,” IEEE Trans. Signal Process., vol. 56, no. 6, pp.2326–2333, Jun. 2008.