Stat 400 : Discussion section BD3 and BD4 Handout 11

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1 Must haves in your formula sheet

- ***All formula for CI, testing,sample size "Hypothesis Testing & Confidence Interval Visual Guide" or Tables from my handouts***
- ***Briefly write down the process of MoM and MLE estimation. Formula for Bias and variance***
- ***Some integration/series sum formula if you need to.***
- ***Joint, marginal, conditional distribution formula for discrete and continuous cases***
- ***Interpretation of CI, p value, independence, correlation,iid***
- ***Formula for Expectation, variance, MGF, CLT***
- ***Rules to recognize discrete distribution from word problems-from previous handouts***

2 Selected problems from last final

- An engineering development laboratory conducted an experiment to investigate the life characteristics of a new solar heating panel, designed to have a useful life of at least 5 years with probability p=0.95. A random sample of 21 such solar panels was selected and the useful life of each was recorded. What is the probability that exactly 10 will have a useful life of at least 5 years?
- A manufacturer uses electrical fuses in an electronic system. The fuses are purchased in large lots and tested sequentially until the first defective fuse is observed. Assume that the proportion of defective fuses in the lot is 0.09. What is the probability that the first defective fuse will be one of the first 30 fuses tested?
- Suppose that the number of children per family X follows a logarithmic distribution with $0 < \theta < 1$.

$$f(X = x) = -\frac{\theta^x}{xln(1-\theta)}, x = 1, 2, 3...$$

(a) Consider the number of children in two families, X_1 and X_2 with $\theta_1 = 0.5$ and $\theta_2 = 0.5$. If the number of children in each family is independent, what is the probability that both families have more than 1 child? (b). What is the probability that at least 1 of the two families has more than 1 child? (c). The moment generating function for the logarithmic distribution is:

$$M_t = \frac{ln(1-\theta exp(t))}{ln(1-\theta)}$$

For the same two families mentioned above, what is the expected total number of children in both families?

- Let $x_1$be a sample of size 1 from a continuous uniform distribution over $(1, \theta)$, such that $1 \leq x \leq \theta$. Show that (a) x_1 is a biased estimator of θ and compute the bias. (b) $2(x_1 - 0.5)$ is an unbiased estimator of θ. (c) Find the variance of $2(x_1 - 0.5)$.

- Let $X_1, X_2, ..., X_n$ be a random sample of size n from distributions with the given probability density function. $x > 0$, $\theta > 0$. Find maximum likelihood estimator

$$f(x, \theta) = \frac{\theta^4}{12} x exp(-\theta\sqrt{x})$$

- Suppose we roll 11 fair 6-sided dice where each side has an equal probability of occurring. What is the probability that there are exactly 3 2's showing and exactly 3 3's showing?

- In the past, the mean monthly long-distance telephone bill in a certain area was \$17.85. After an advertising campaign, a random sample of 20 household bills was taken, the sample mean was \$19.35 and sample standard deviation of \$3.88. Assume that the phone bills are approximately normally distributed. (a) Is there enough evidence that the mean monthly long-distance telephone bill has changed. Perform the appropriate test at a 1% significance level. Report p value. (b) Construct a 95% confidence interval for the overall mean monthly long-distance telephone bill. (c) Construct a 95% confidence interval for the overall standard deviation of monthly long-distance telephone bills.

- (borrowed from practice problems by Alex) Suppose joint pdf of X and Y is

$$f(x, y) = \begin{cases} Cxy^2 & 0 < y < x < 1 \\ 0 & o.w \end{cases}$$

(a) Find the value of C that would make it a valid probability model? (b) Find $P(X > 2Y)$ (c) Find P$(X + Y < 1)$ (d) Find marginal densities of X and Y. Are X and Y independent?