STAT 400 : DISCUSSION SECTION BD3 AND BD4 HANDOUT

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1. Theory and Discussions

1.1. Last Quiz problem (Solution will be uploaded).

1.2. PMF, Expectation, its physical significance.

Theorem 1. *Linearity property of Expectation*

Proof. $E[\alpha g(x) + \beta h(x)] = \sum_{x=1}^{\infty}(\alpha g(x) + \beta h(x))f(x) = \alpha\sum_{x=1}^{\infty}g(x)f(x) + \beta\sum_{x=1}^{\infty}h(x)f(x) = \alpha E[g(x)] + \beta E[h(x)]$ □

Problem 2. Mean and Variance under scale and origin transformation. Let X be a random variable with mean $\mu$and variance σ^2. And let Y=$X - \mu$. W=$\frac{X}{c}$, Z=$\frac{X-\mu}{\sigma^2}$. What are E(Y) and E(Z)

- E(Y)=E(X-μ)=E(X)-μ=0
- E(W)=E($\frac{X}{c}$)=$\frac{E(x)}{c}$
- E(Z)=E($\frac{X-\mu}{\sigma^2}$)=$\frac{E(X-\mu)}{\sigma^2}$.

1.3. Homework problems Hints and Discussions.

Problems to be discussed (solution will NOT be provided)- Three prisoner problem, same as the Monty hall game we played in class. 1-6-10, same as the false positive rate medical test problems we did in class. 1-6-8. Think in line of the building collapse example I gave in class. 1-5-16. To be explained in class.

1.4. Bernoulli and Binomial pmf.

Bernoulli experiement- A random experiment with binary outcomes (Recall from first class: A random experiment has fixed outcomes, but we don't know which one will happen in one particular run of the experiment. Bernoulli trials are a sequence of repeated bernoulli experiments. A coin toss once is a bernoulli experiment and repeated coin tosses are bernoulli trials. (Think of one more example of Bernoulli experiment).

If X follows Bernoulli with probability of success p. PMF is given by $P(X = x) = p^x(1-p)^{(1-x)}$, $x = 0, 1$. E(X)=p. Var(x)=pq . (All derivations in class).

Now let X is total number of successes in n successive Bernoulli trails. X follows Binomial (n,p) distribution. PMF is $P(X = x) = (nCx)p^x(1-p)^{(n-x)}$. E(X)=$np$, Var (X)=npq.

2. Problems

Problem 3. A warranty is written on a product worth $10,000 so that the buyer is given $8000 if it fails in the first year, $6000 if it fails in the second, $4000 if it fails in the third, $2000 if it fails in the fourth, and . zero after that. The probability of the product's failing in a year is 0.1; failures are independent of those of other years. What is the expected value of the warranty?

Sol. Simple application of definition of expectation. Calculate the probabilities for each year and multiply by the value the warranty generates for that year. So E(warranty)= $8000*0.1+6000*0.1*0.9+4000*0.1*0.9^2+2000*0.1*0.9^3 = 800+540+324+145.8 = 1809.8$

Problem 4. Let the random variable X be the number of days that a certain patient needs to be in the hospital. Suppose X has the p.m.f. .

$$f(x) = \begin{cases} \frac{(5-x)}{c} & x = 1,2,3,4 \\ 0 & otherwise \end{cases}$$

If the patient is to receive \$200 from an insurance company for each of the first two days in the hospital and \$100 for each day after the first two days, what is the expected payment for the hospitalization?

Sol. $\sum_{x=1}^{4} f(x) = 1$ Implies, $c = 10$. Let Y be a random variable which denotes the payment received., so

$$Y = \begin{cases} 200X & x = 1,2 \\ 400 + 100(X-2) & x = 3,4 \end{cases}$$

The probability mass function remain same for both the variables. So we calculate expectation as

$$\sum Y.f(x) = 200f(1) + 400f(2) + 500f(3) + 600f(4) = 80 + 120 + 100 + 60 = 360$$

Problem 5. In a lab experiment involving inorganic syntheses of molecular precursors to organometallic ceramics, the final step of a five-step reaction involves the formation of a metal-metal bond. The probability of such a bond forming is p = 0.20. Let X equal the number of successful reactions out of n = 10 such experiments. (a) Find the probability that X is at most 4. (b) Find the probability that X is at least 5. (c) Find the probability that X is equal to 6. (d) Give the mean, variance, and standard deviation

Sol. From the mess of words, what we can make out is there is a certain event (Bond formation implied successful reaction. So probabilistically they are same event) that occurs with probability of 0.20. So, we have a RV, X which is number of successes out of n=10 experiments where success probability is p=0.20. We are to find the probability distribution of X. Clearly X follows Binomial(10,0.20) distribution.

What is the PMF of Binomial? Write it down. The PMF gives you P(X=x) where x takes values 0 to 10. Now

(1) P(X at most 4)=$P(X \leq 4)$ =P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)=0.9672.
(2) $P(X \geq 5)$=1-$P(X < 5)$=1- answer in part (a)=0.03279.
(3) P(X=6)=0.0055
(4) Use formula. Mean $np = 10 * 0.2 = 2$, and variance is $npq = 10 * 0.2 * 0.8 = 1.6$. Sd is $\sqrt{1.6} = 1.26$