MINICONFERENCE ON ANALYSIS AND PROBABILITY IN SUB-RIEMANNIAN SPACES

This one-day online miniconference on analysis and probability in sub-Riemannian spaces is being held to replace a Special Session on this topic scheduled for the AMS Spring Central Sectional Meeting at Purdue University. The in-person Special Session was canceled due to the COVID-19 pandemic.

The organizers are **Jeremy Tyson** (University of Illinois at Urbana-Champaign) and **Jing Wang** (Purdue University). The miniconference will be held on **Saturday, April 4, 2020** on Zoom.

There will be a Zoom informational/help session on **Friday, April 3, 2020** from 4:00–5:00 pm, Eastern time. Zoom links for both the help session and the conference will be sent to the participants.

All times listed in the following table are Eastern Daylight Time.

Time	Speaker
9:00-9:15	gathering
9:15-9:20	welcome
9:20-9:35	Fabrice Baudoin (Univ. of Connecticut)
9:45-10:00	Li Chen (Univ. of Connecticut)
10:10-10:25	Phanuel Mariano (Univ. of New Haven)
10:25-10:45	"coffee break"
10:45-11:00	Valentino Magnani (Univ. of Pisa)
11:10-11:25	Rob Neel (Lehigh Univ.)
11:35-11:50	Nathan Fisher (Tufts Univ.)
12:00-12:15	Nate Eldredge (Univ. of North Colorado)
12:15-1:45	lunch/dinner break
1:45-2:00	Fernando Róman García (UIUC)
2:15-2:30	Maria Gordina (Univ. of Connecticut)
2:30-3:00	conclusion and social time

Titles and Abstracts

- Fabrice Baudoin (Univ. of Connecticut), *Sub-Laplacian comparison theorems in H-type sub-Riemannian manifolds*

  Abstract. On H-type sub-Riemannian manifolds we establish sub-Hessian and sub-Laplacian comparison theorems which are uniform for a family of approximating Riemannian metrics converging to the sub-Riemannian one. We also prove a sharp sub-Riemannian Bonnet–Myers theorem that extends to this general setting results previously proved on contact and quaternionic contact manifolds.

- Li Chen (Univ. of Connecticut), *Second Riesz transforms on some Lie groups*

  Abstract. In this talk, we study explicit L^p bounds for second order Riesz transforms on Heisenberg groups $\mathbb{H}^{2n+1}$ and $\mathbb{SU}(2)$ via martingale transform techniques. In particular, Beurling–Ahlfors operators on $\mathbb{H}^3$ and $\mathbb{SU}(2)$ are discussed. This is joint work with Fabrice Baudoin.

- Nate Eldredge (Univ. of North Colorado), *Transportation cost and functional inequalities for sub-Riemannian heat semigroups*

  Abstract. Regularizing properties of a heat semigroup P_t can be studied in terms of optimal transport: for instance, under appropriate conditions, the action of P_t on probability measures may be a contraction with respect to the Wasserstein distance. Recent work of Luise and Savaré obtained an interesting contraction result relating Wasserstein-type distances to the Hellinger distance, which is equivalent to total variation. However, their results are in the setting of $\mathrm{RCD}(K,\infty)$ spaces, which involve curvature constraints that rule out sub-Riemannian manifolds. We discuss how these results can be generalized to include many sub-Riemannian examples, as well as an adaptation of their techniques that gives information about entropic distances. The key ingredients turn out to be reverse Poincaré and reverse logarithmic Sobolev inequalities.

- Nathan Fisher (Tufts Univ.), *Sub-Finsler metrics and the Heisenberg group at infinity*

  Abstract. In this talk, we will discuss a class of sub-Finsler metrics on the real Heisenberg group $H(\mathbb{R})$, arising as asymptotic cones of word metrics on the integer Heisenberg group $H(\mathbb{Z})$ for various generating sets. We will describe new results on boundaries for these Carnot-Carathéodory metrics, with applications to the study of random walks on $H(\mathbb{Z})$. This is joint work with Sebastiano Nicolussi Golo.

- Maria Gordina (Univ. of Connecticut), *Hypoelliptic diffusions in infinite dimensions*

  Abstract. We will review a number of recent developments concerning what can be considered as infinite-dimensional hypoelliptic diffusions. One of the settings is Wiener or heat kernel measures on infinite-dimensional sub-Riemannian manifolds such as the Heisenberg group. Another is path spaces over sub-Riemannian manifolds. Smoothness of measures (understood as quasi-invariance) will be discussed.

- Valentino Magnani (Univ. of Pisa), *Area formulas for intrinsic regular submanifolds in the Heisenberg group*

  Abstract. We present an explicit integral formula for the spherical measure of intrinsic regular submanifolds in any Heisenberg group. Since these submanifolds are not smooth in the classical sense, our approach is based on intrinsic differentiability of the parametrizing mapping and a chain rule for intrinsic differentiable mappings. A delicate lemma to compute the Jacobian of projections among vertical subgroups also plays an important role. These results have been obtained in collaboration with Francesca Corni.

- Phanuel Mariano (Univ. of New Haven), *The Cheng–Yau gradient estimate for Carnot groups and sub-Riemannian manifolds*

  Abstract. We recall the Cheng–Yau gradient inequality and its implications in the Riemannian setting. We discuss how the Cheng–Yau inequality can be proven on two classes of sub-Riemannian manifolds: Carnot groups and sub-Riemannian manifolds satisfying a generalized curvature-dimension inequality. Two approaches will be presented: a coupling approach and an analytic approach using a curvature-dimension condition.

- Rob Neel (Lehigh Univ.), *Extending Brownian motion to a family of Grushin-type singularities*

  Abstract. We consider a one-parameter family of Grushin-type singularities on surfaces, and discuss the possible diffusions that extend Brownian motion to the singularity. Stochastic analysis gives a quick proof and clear intuition for the fact that a diffusion extending Brownian motion can only cross the singularity for an intermediate range of the parameter. When crossing is possible, one must specify the behavior at the singularity. In the case when the singularity consists of one point, we give a complete description of these diffusions, and we describe a best extension, which respects the isometry group of the surface and also realizes the unique symmetric one-point extension of the Brownian motion, in the sense of Chen–Fukushima. This extension, however, does not correspond to the bridging extension, which was introduced by Boscain–Prandi, when they previously considered self-adjoint extensions of the Laplace-Beltrami operator on the Riemannian part for these surfaces. We clarify that several of the extensions they considered induce diffusions that are carried by the Martin compactification at the singularity, which is much larger than the (one-point) metric completion. In the case when the singularity is more than one-point, a complete classification of diffusions extending Brownian motion would be unwieldy. Nonetheless, we again describe a best extension which respects the isometry group, and in this case, the diffusion corresponds to the bridging extension.

- Fernando Román García (UIUC), *A Fourier coefficient approach to Hausdorff dimension in the Heisenberg group*

  Abstract. In this talk, I discuss a recently developed alternative formulation of the Fourier transform on the Heisenberg group, and use it to show that energies of measures can be computed via integrals on an appropriate frequency space. This in turn opens the possibility of using Fourier methods in the computation of Hausdorff dimensions of sets.