The Inverse-Deformation Approach to Fracture

Timothy J Healey

Department of Mathematics, Cornell University

Abstract:

We propose a one-dimensional, nonconvex elastic constitutive model with higher gradients that can predict spontaneous fracture at a critical load via a bifurcation analysis. It overcomes the problem of discontinuous deformations without additional fields, such as damage or phase-field variables, and without pre-existing cracks. A central idea is the employment of the inverse deformation, which is a piecewise smooth mapping even when the original deformation has discontinuities describing crack openings; the infinite strain associated with fracture corresponds to zero inverse strain. We exploit this via the inverse-deformation formulation of finite elasticity due to Shield and Carlson, which includes the strain gradient in the energy. The latter endows spontaneously cracked surfaces with thin concentrated surface energy. Unlike the standard Lagrangian description, the inverse formulation lends itself well to methods of nonlinear analysis. We illustrate with two problems under hard tensile loading: (1) a single brittle bar; (2) a composite bar comprising a brittle material coupled with an elastic core. In both cases, we show that spontaneous fracture occurs along a global bifurcating solution branch at a critical value of the macroscopic stretch. This leads to a sudden drop to zero stress in problem (1). In problem (2), the brittle material fractures in a periodic pattern of multiple cracks – in consonance with recent experiments on composite filaments. Problem (2) is more difficult; the periodically broken material continues to interact with the elastic core as the cracks open.

Bio:

Tim Healey holds degrees in Civil Engineering from the University of Missouri, Columbia (BS 1976) and the University of Illinois, Urbana-Champaign (MS 1978, PhD. 1985). He worked for a structural engineering firm in Los Angeles 1978-80. He did post-doctoral work in Math at U. Maryland (1985-86) before joining the faculty at Cornell University shortly thereafter. His research is at the crossroads of nonlinear elasticity and mathematical analysis of PDE/calculus of variations.

He held positions in the Department of Theoretical & Applied Mechanics (1985-2008), including serving as Chair that department (2000-2008), and in the Departments of Mathematics and Mechanical & Aerospace Engineering (2009-2014). He currently holds a full-time appointment in the Department of Mathematics. A total of 19 students have obtained their PhD degrees under his mentorship. He has given numerous invited presentations and keynote addresses throughout his career and has enjoyed nearly continuous support from the National Science Foundation (DMS) for his research. He is the recipient of four teaching prizes at Cornell. He serves on several editorial boards and has served in various leadership roles on committees and organizations representing the science of mechanics. He has held many visiting positions, including 1-year distinguished visiting professorships at Universidad de Los Andes (Bogotá, Colombia) and Ecole Polytechnique (Palaiseau, France).

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1310 Yeh Student Center