

Herding a Flock of Birds Approaching an Airport Using an Unmanned Aerial Vehicle

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Abstract

The problem of herding a flock of birds is posed in a graph theoretic framework. A novel algorithm, called the n-wavefront algorithm, is developed for enabling a single unmanned aerial vehicle to herd a flock of birds to a desired point. The technique is applied to the problem of diverting a flock of birds approaching an airport away from a protected zone around the airport. The n-wavefront algorithm is demonstrated in simulation and compared with existing strategies using graph-based metrics.

1 Introduction

The Federal Aviation Administration (FAA) documented 142,000 wild-life strikes at US airports between 1990 and 2013, including 11,000 in 2013 itself[1]. The FAA also notes that “*Ongoing research is being conducted to provide airport personnel with a set of passive and active methods to manage wildlife at airports by means of habitat modification, species deterrence and techniques for rapid dispersal of hazardous species when critical risks are encountered.*” [2]

A large number of passive and active methods are currently used for deterring birds from entering the airspace around airports, [3] which depend on the type of birds encountered at the airport. Passive techniques typically work by curtailing the food and water resources around the air field or laying bird-repellent grass swards. On the other hand, active techniques include the use of live ammunition and flares at airports. Usually, a combination of the aforementioned techniques is required for effective bird control, although they are not generally known to be very effective [4]. Using trained birds of prey like Peregrine falcons is the only technique proven to be effective in practice [5]. Robotic aircraft resembling birds of prey have been tested, but need a skilled human pilot (see Ref. [6]).

In this paper, we develop an active technique for bird control using an autonomous unmanned aircraft as a sentry. The technique is meant primarily for airports which witness organized bird activity in the form of flocks (e.g., migratory geese swarms are the primary cause of nuisance in the New York airports region, and led to the crash of US Airways 1549 in 2009).

A typical setting for the technique developed in the paper is shown in Figure 1, where a single sentry is used to divert oncoming flocks of birds (labeled as intruders) away from a no-fly zone around the airport. In a practical setting, the no-fly zone would be an area inside which the typical altitude of birds is similar to that of landing or climbing aircraft. At airports which consist of single or parallel runways, the no-fly zone could be reduced to a semi-cylindrical tube or a spherical cap around the runway.

1.1 Literature Review

Collective animal behaviors have long been a subject of interest to researchers from different fields including theoretical biology, ecology, sociology, and engineering. Flocking has been studied, broadly speaking, from three perspectives.

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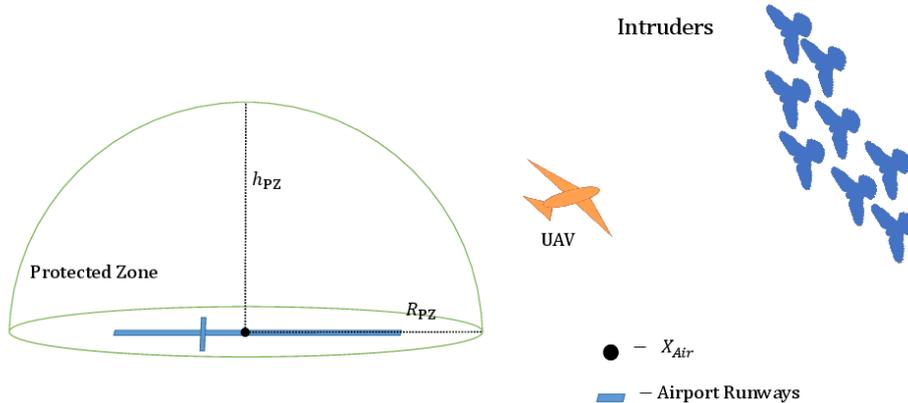


Figure 1: The setting for the problem addressed in this paper: establishing a no-fly zone around an airport using unmanned aerial vehicles as sentries against stray avian intruders.

1. Flock models: Reynolds [7] formulated simple motion behaviors for creating computer animations of bird flocks. These three simple behaviors or motion steers (viz. collision avoidance, velocity matching and flock centering) allow us to reproduce in simulation the stable collective motion seen in animal herds and flocks. Reynolds' model has been thoroughly studied and has formed the basis for other flocking models such as the self-propelled particle model [8, 9]. Hartman and Benes[10] added the concept of change of leadership to Reynolds' model which was essentially leader-free. Alongside theoretical modeling, attempts have been made to extract flocking rules, aggregate patterns and ordering from empirical studies on large flocks of birds and fish (e.g., the work by Ballerini and colleagues [11, 12] on flocks of starlings). A detailed review of collective animal behavior may be found in Sumpter [13].
2. Development of coordination algorithms: Motivated by the development of flocking algorithms, the robotics and controls community has developed several distributed, scalable coordination algorithms while trying to obtain mathematical guarantees on the stability of the formation.[14, 15, 16, 17, 18] The theoretical work on stability has been accompanied by parallel work on understanding the ordering and phase transitions in flocks [19] which has built upon analogies between flocks on the one hand, and fluid and magnetic systems on the other, which has led in turn to alternate, if somewhat mathematically complex, models for flocks.
3. Herding: Herding algorithms aim to establish external control on the flock behaviour. The most widely explored objective of herding is to move the flock in a certain prescribed manner, the so-called shepherding problem [20]. The literature on shepherding has generally tried to establish geometric principles for the shepherds steering the flock [21, 22, 23, 24, 25] and there have been some experimental demonstrations on ground-based robots [20, 22] as well. It is generally well-known that using a single herding agent (or sheep dog) can significantly reduce the chances of success [22]. Strombom *et al.* [26] recently identified "heuristics" in typical herding strategies with particular application to single herding agents. Conversely, Zheng[27] and Lee[28] have examined evasion strategies used by flocks against one or more predators.

1.2 Objectives and Contributions

The objective of this paper is to design a pursuit and herding strategy for diverting a flock of birds away from a specified area. This area is a spherical cap around an airport located on the ground and depicted in Figure 1. The problem addressed here is similar to that of containment of agents [29, 30], but the objective is more demanding than containment in that the herding algorithm achieves a desired motion of the center of mass of the flock while achieving guaranteed bounds on the size of the flock. Unlike the approach taken in the literature for similar problems, the problem of herding in this paper is posed as a problem of determining a specific subset of agents which need to be influenced, and the laws for influencing these agents. This formulation allows us to use graph theoretic tools such as vertex centrality for designing as well as analyzing herding algorithms.

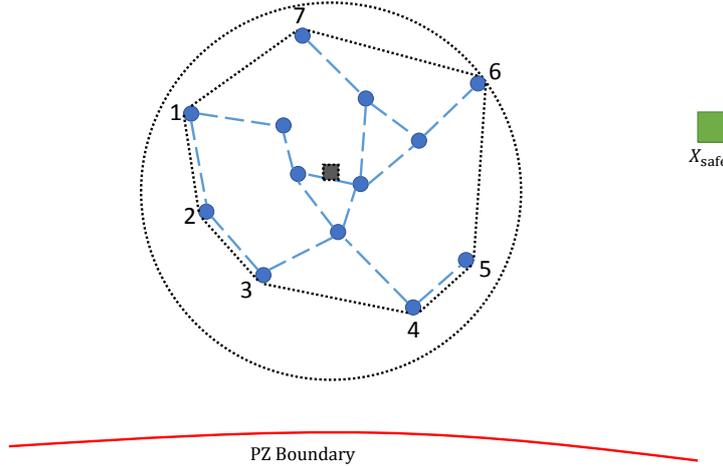


Figure 2: Neighborhood graph induced by the flock. The dotted circle is drawn with the flock centroid (grey filled square) as the center and flock diameter (D_{FD}) as the diameter. The blue dots represent birds and the dashed blue lines connecting them represent the edges of the neighborhood graph as seen in Eq. 2.

A standard flocking model similar to Reynolds [7] is used together with evasion laws similar to Zheng [27]. It is assumed that the sentries have complete information about the flock; this is a reasonable assumption given that UAV's can make use of avian radars which are being increasingly used at airports [31] to monitor avian intruders.

A novel herding technique, called the n-wavefront herding algorithm, is introduced in the paper. In particular, it is demonstrated that the n-wavefront algorithm not only ensures that the avian flocks do not intrude upon the airspace around the airport, but also improves the cohesiveness of the flock by simultaneously achieving both efficient herding and controlling the size of the flock. Being a boundary control type method, it ensures that the sentry maintains a safe distance from the flock, avoids flock fragmentation, and influences only the birds on the bounding convex hull, which makes it particularly useful in practical applications. The technique can be generalized to the case when multiple tasks are required to be performed on the boundary of the flock.

The paper is organized as follows. Preliminary concepts from graph theory are reviewed in Sec. 2. The problem formulation is presented in Sec. 3. The herding strategies, including the new n-wavefront algorithm, are described in Sec. 4. The herding strategies are illustrated using numerical simulations in Sec. 5.

2 Preliminaries

2.1 Notation for Flocking

Consider a set of N dynamic birds (also referred to as ‘birds’ following the relevant literature [32]) denoted by Γ . Let \mathbf{r}_{ij} denote the vector from the i^{th} bird to the j^{th} bird ($\mathbf{r}_{ij} = \mathbf{x}_j - \mathbf{x}_i$), such that $\mathbf{r}_{ij} = -\mathbf{r}_{ji}$. Given a vector \mathbf{r} , we let $\hat{\mathbf{r}}$ denote the unit vector along \mathbf{r} . We use the subscript ‘p’ to denote the pursuer, so that $\mathbf{x}_p^k \in \mathbb{R}^n$ denotes the position vector of the k^{th} pursuer, while \mathbf{r}_{ik}^p denote the vector from the i^{th} bird to the pursuer k .

Let R_0 denote the communication range or interaction distance between two birds. For a uniform circular disc sensor model, the neighborhood of the i^{th} bird is defined as,

$$\mathcal{N}_i = \{j \in N \mid \|\mathbf{x}_i - \mathbf{x}_j\| \leq R_0\}. \quad (1)$$

A limited field of view model can also be adopted, which restricts the sensing to a cone. We will use the uniform circular disc (in 2-D) and a uniform sphere (in 3D) for simplicity. The neighborhood sets together can be represented as a graph. In the case of uniform circular disc (or uniform sphere) sensor model this graph is an undirected graph, whereas using a limited field of view model creates a directed graph. We can define a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with the set of birds forming the vertex set \mathcal{V} , and pairs of communicating birds as the edge

set \mathcal{E} .

$$\begin{aligned} \mathcal{V} &\cong \{1, 2, \dots, i, \dots, N\} \\ \mathcal{E} &= \{(i, j) \in \mathcal{V} \times \mathcal{V} \mid j \in \mathcal{N}_i\}. \end{aligned} \quad (2)$$

2.2 Flock Metrics

We define flock metrics that enable us to characterize the overall swarm and measure performance of the herding methods. The metric include the flock centroid and flock diameter that describes the size of the flock. We borrow centralization from graph theory that allows us to bring out the internal dynamics of the swarm and distance to airport to look at the performance of herding.

The centroid of the flock is defined as the geometric centroid,

$$\mathbf{X}_C = \frac{1}{N} \sum_{i=1}^N \mathbf{x}_i. \quad (3)$$

The radius of the flock, R_{FD} , is the Euclidean radius of the smallest ball centered at \mathbf{X}_C such that $\|\mathbf{x}_i - \mathbf{X}_C\| \leq R_{FD}$ for all i . Alternatively one may visualize the radius R_{FD} as the maximum distance between any boid and the flock centroid. We define the flock diameter $D_{FD} = 2R_{FD}$ to represent the spread and distribution of boids in space. In this paper we show a strategy for controlling both the \mathbf{X}_C and R_{FD} of the flock.

In order to study the cohesion within the flock, we use the notion of degree centrality, which is the simplest centrality metric. Let $\deg(n_i)$ denote the degree of a vertex; i.e., the number of links incident upon the vertex. Since, our graph is based on neighborhood relationships, centrality tells us about the density or sparsity of the flock. It also allows us to find central boids in the flock. This information can be used to choose boids that should be influenced to affect the entire network. Then, the centrality of the complete graph (Θ_D) is given by Freeman's formula [33]:

$$\Theta_D(\mathcal{G}) = \frac{\sum_{n_i \in \mathcal{V}} [\deg(n^*) - \deg(n_i)]}{(N-1)(N-2)}, \quad (4)$$

where $\Delta(\mathcal{G}) = \max_{n_i \in \mathcal{V}} \{\deg(n_i)\}$ is the highest degree and $n^* = \{n_i : \deg(n_i) = \Delta(\mathcal{G})\}$ is the vertex with the highest degree.

The distance of the flock centroid from the airport ($d_{C,Air}$) is defined as a metric to study the proximity of the flock to the safe zone.

2.3 Consensus and Flocking

Let the N boids move in the standard n -dimensional Euclidean space with double integrator dynamics:

$$\dot{\mathbf{x}}_i = \mathbf{v}_i, \quad \dot{\mathbf{v}}_i = \mathbf{u}_i, \quad (5)$$

where $\mathbf{x}_i \in \mathbb{R}^n$ is the position of the i^{th} boid, $\mathbf{v}_i \in \mathbb{R}^n$ is its velocity vector, and $\mathbf{u}_i \in \mathbb{U} \subset \mathbb{R}^n$ is the control input. Bird flocking has been modeled like a consensus problem, where individual birds try to maintain a formation and move in harmony.

The benefit of using a simple consensus-like model with simple dynamics is that it makes the movement constraint free and it represents the best case dynamics or motion for the evaders (boids) while presenting the worst case scenario for the pursuers (robotic bird). It will ensure that the algorithms that we develop and test using this model are applicable and will perform as much if not better in experiments.

We use bird flock behaviors in order to simulate a flock of birds. Individual birds or boids as a part of a flock are guided by some 'steers' or forces. We use the guiding forces described in the widely used model by Reynolds [7] and augment it with predator-prey dynamics:

1. Separation Steer: The boids try to stay together while maintaining a safe threshold distance R_{safe} from each other. The separation control is given by,

$$\mathbf{u}_{i,\text{sep}} = \sum_{j \in \mathcal{N}_i} \left(1 - \frac{R_{\text{safe}}}{\|\mathbf{r}_{ij}\|}\right) \mathbf{r}_{ij} \quad (6)$$

2. Alignment Steer: Each boid tries to align its velocity with its neighbors. The alignment control can be described as,

$$\mathbf{u}_{i,\text{align}} = \begin{cases} \frac{\sum_{j \in \mathcal{N}_i} (\mathbf{v}_j - \mathbf{v}_i)}{\text{Card}(\mathcal{N}_i)}, & \text{Card}(\mathcal{N}_i) > 0 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

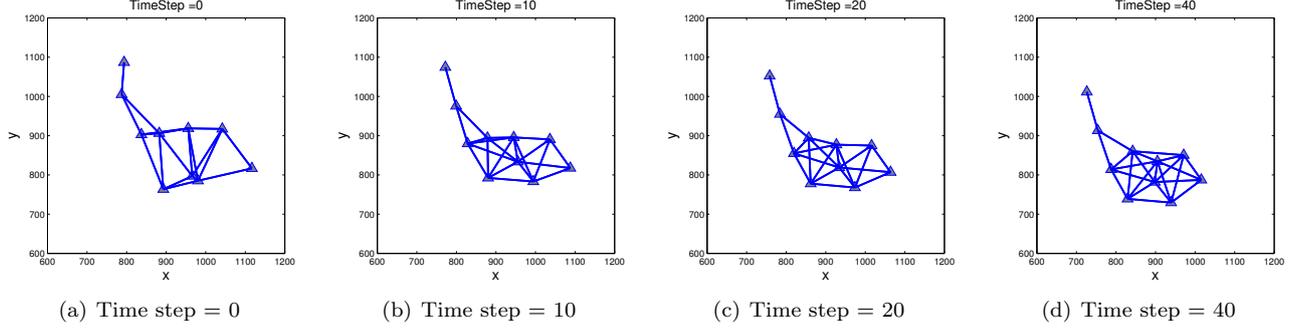


Figure 3: Flock formation in 2D based on Eq. (10) with 10 boids.

where $\text{Card}(\mathcal{N}_i)$ is the cardinality of the neighborhood set. Cardinality is the measure of number of elements in the set. In the process, each boid seeks to make its velocity equal to the average velocity of its neighbors.

3. Goal Steer: Each boid tries to fly towards its goal (a fellow bird, food source or the direction of a pond etc.). The goal is usually defined for the flock as a whole. The steer due to this goal is given by:

$$\mathbf{u}_{i,\text{goal}} = c_1(\mathbf{x}^G - \mathbf{x}_i) + c_2(\mathbf{v}^G - \mathbf{v}_i) \quad (8)$$

Note that the goal can also be a leader of the flock, so that leader-following is included here.

4. Fear Steer: Each boid follows a two-prong strategy in response to predators (pursuers) within a fear radius R_{Fear} . This distance R_{Fear} is dependent on type of bird-predator combination and environmental conditions. It has to be estimated by observing flock-predator interactions or guessed. If a predator (or UAV/Robotic falcon) is beyond a critical distance $R_{\text{pred}} < R_{\text{Fear}}$, then the boid tries to accelerate radially away from the predator; otherwise, it moves tangentially in the hope of out-maneuvering the predator. The switching between these modes is smoothed using a hyperbolic tangent function:

$$s_i = \tanh \left(g_s \left(\frac{\|\mathbf{r}_{ik}^p\|}{R_{\text{pred}}} - 1 \right) \right)$$

where g_s is the gain which dictates the rapidness of the switch and \mathbf{r}_{ik}^p denotes the vector pointing from boid i to predator k . The steering control law is given by

$$\mathbf{u}_{i,\text{fear}} = \left(\frac{1 + s_i}{2} \right) \hat{\mathbf{r}}_{ik}^p + c_f \left(\frac{1 - s_i}{2} \right) \hat{\mathbf{r}}_{ik}^{p\perp} \quad (9)$$

where the gain c_f is a design parameter, $\mathbf{r}_{ik}^{p\perp} = \hat{\mathbf{k}} \times \mathbf{r}_{ik}^p$, and $\mathbf{k} = \mathbf{v}_p \times \mathbf{r}_{ik}^p$. Recall that $\hat{\cdot}$ denotes a unit vector along the given vector.

The overall guiding control can be given by a weighted sum of the above described behaviors:

$$\mathbf{u}_i = K_{\text{sep}} \mathbf{u}_{i,\text{sep}} + K_{\text{align}} \mathbf{u}_{i,\text{align}} + K_{\text{goal}} \mathbf{u}_{i,\text{goal}} + K_{\text{fear}} \mathbf{u}_{i,\text{fear}} + \mathbf{b}_i(t) \quad (10)$$

where $\mathbf{b}_i(t)$ is a disturbance term. The gains $\{K\}$ are chosen to ensure that the flock is stable. The flocking dynamics can be analyzed by considering the inter-boid separation $\mathbf{r}_{ij} = \mathbf{x}_j - \mathbf{x}_i$. The dynamics of the inter-boid separation are given by

$$\begin{aligned} \ddot{\mathbf{r}}_{ij} = & -K_{\text{sep}} \left(\sum_{k \in \mathcal{N}_j} \left(1 - \frac{R_{\text{safe}}}{\|\mathbf{r}_{kj}\|} \right) \mathbf{r}_{kj} - \sum_{k \in \mathcal{N}_i} \left(1 - \frac{R_{\text{safe}}}{\|\mathbf{r}_{ki}\|} \right) \mathbf{r}_{ki} \right) + K_{\text{align}} \left(\frac{\sum_{k \in \mathcal{N}_j} \mathbf{v}_{kj}}{\text{Card}(\mathcal{N}_j)} - \frac{\sum_{k \in \mathcal{N}_i} \mathbf{v}_{ki}}{\text{Card}(\mathcal{N}_i)} \right) \\ & - K_{\text{goal}} (c_1 \mathbf{r}_{ij} + c_2 \dot{\mathbf{r}}_{ij}) + K_{\text{fear}} (\mathbf{u}_{j,\text{fear}} - \mathbf{u}_{i,\text{fear}}) + \mathbf{b}_j(t) - \mathbf{b}_i(t) \end{aligned} \quad (11)$$

where $\mathbf{v}_{ij} = \mathbf{v}_j - \mathbf{v}_i$. The stability of the flock without the goal seeking steer and predator prey dynamics has been proven by Tanner [16] paraphrased in Theorem 1.

Theorem 1. [16] Consider a system of N mobile agents with dynamics Eq. (13), each steered by control law Eq. (14). Let both the position and velocity graphs be time-varying, but always connected. Then all pairwise velocity differences converge asymptotically to zero, collisions between the agents are avoided, and the system approaches a local extremum of agent potentials.

$$\dot{\mathbf{r}} = (B_{\mathcal{K}_N} \otimes I_2)v \quad (12)$$

$$\dot{v} \in^{a.e.} K[u] \quad (13)$$

$$u_i = - \sum_{j \in (N)_c(i)} (v_i - v_j) - \sum_i^N \nabla_{r_i} V_i \quad (14)$$

When the predator p is sufficiently far from the flock so that for every pair of neighbors i, j , the distance $r_{pi} \approx r_{pj} = r_{p,ij}$, it can be shown that

$$\mathbf{u}_{j,\text{fear}} - \mathbf{u}_{i,\text{fear}} = \left(\frac{1}{r_{p,ij}} \right) \mathbf{r}_{ij}$$

The fear term thus tends to destabilize the flock by driving the agents away from each other. The flock can be stable as long as K_{sep} and K_{goal} are large enough.

Consider the case where $c_1 = 0$ in the goal-seeking behavior. This corresponds to the case where the goal is located sufficiently far off so that the boids try to maintain a specific heading at all times. If the two agents i and j are neighbors, then the first term on the right hand side of Eq. (11) can be written as

$$-K_{\text{sep}} \left(2 \left(1 - \frac{R_{\text{safe}}}{\|\mathbf{r}_{ij}\|} \right) \mathbf{r}_{ij} + \sum_{k \in \mathcal{N}_j \setminus i} \left(1 - \frac{R_{\text{safe}}}{\|\mathbf{r}_{kj}\|} \right) \mathbf{r}_{kj} - \sum_{k \in \mathcal{N}_i \setminus j} \left(1 - \frac{R_{\text{safe}}}{\|\mathbf{r}_{ki}\|} \right) \mathbf{r}_{ki} \right)$$

By adding the fear term to the first term in the above expression, it is evident that the fear term tends to increase the inter-boid distance in the flock by detracting from the constant term (i.e., 2). The inter-agent separation remains bounded as long as $K_{\text{sep}} > K_{\text{fear}}/2r_{p,ij}$, which imposes a bound on the choice of K_{sep} and K_{fear} as a function of R_{pred} defined above: $K_{\text{fear}} < 2K_{\text{sep}}r_{p,ij}$. Figures 3 and 4 show flocking in 2-D and 3-D space, respectively, under Eq. (10). The stability of the flock can be compromised when the falcon approaches the flock to within R_{pred} of one or more boids; the stability analysis of the resulting dynamics is not addressed in this paper. The proof of flocking without predator-prey dynamics is presented in Theorem 2 presented from Ref. [15]. It is shown that flocking based on separation, alignment and velocity matching steers results in connected structures known as α -lattices and quasi α -lattices.

Theorem 2. [15] Consider a group of α agents using the flocking protocol with $c_1, c_2 > 0$ (from the goal steer term) and structural dynamics \sum . Assuming that the initial velocity mismatch and inertia are finite. Then, the following statements hold.

$$\sum = \begin{cases} \dot{x} = v \\ \dot{v} = -\nabla U_\lambda(x) - D(x)v \end{cases} \quad (15)$$

1. The group remains cohesive for all $t \geq 0$
2. Almost every solution of \sum asymptotically converges to an equilibrium point $(x_\lambda^*, 0)$ where x_λ^* is the local minimum of $U_\lambda(x)$

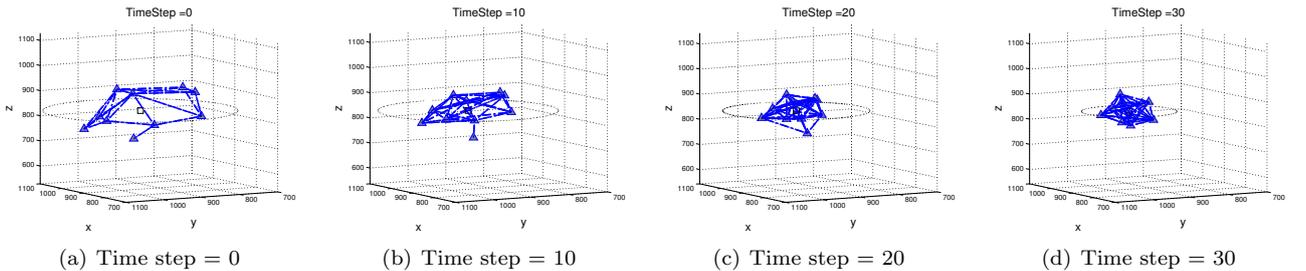


Figure 4: Flock formation in 3D based on Eq. (10) with 10 boids.

3. All agents asymptotically move with same velocity
4. Assume the initial structural energy of the particle system is less than $(k+1)c^*$ with $c^* = \psi_\alpha(0)$ (ψ_α represents the pairwise attractive/repulsive smooth potential) and $k \in \mathbb{Z}_+$. Then, at most k distinct pairs of α agents could possibly collide

2.4 Equations of Motion for Robotic Falcon

The equations of motion of the pursuing UAV sentry are given by [34][35]:

$$\begin{aligned}
\dot{x}_p &= V \cos \gamma \cos \chi, & \dot{y}_p &= V \cos \gamma \sin \chi, & \dot{h}_p &= V \sin \gamma, \\
\dot{V} &= T \cos \alpha - \eta V^2 C_D(\alpha) - g \sin \gamma, \\
\dot{\gamma} &= \left(\eta V C_L(\alpha) + \frac{T \sin \alpha}{V} \right) \cos \mu - \frac{g \cos \gamma}{V}, \\
\dot{\chi} &= \left(\eta V C_L(\alpha) + \frac{T \sin \alpha}{V} \right) \frac{\sin \mu}{\cos \gamma}.
\end{aligned} \tag{16}$$

where V is the pursuer's flight speed, γ is the flight path angle, χ is the global heading angle, and h denotes the altitude. The thrust T , angle of attack α and bank angle η are the control inputs for the pursuer. The term $\eta = \rho g / (2\bar{W})$, where ρ is the density of air, and \bar{W} is the wing-loading of the sentry.

The maximum achievable level speed of the pursuer is of special interest. It occurs typically at low α so that $T \cos \alpha \approx T$ and $T \sin \alpha \ll kV^2 C_L$. Moreover, at low α , C_L and C_D can be written as,

$$C_L = C_{L_0} + C_{L\alpha}\alpha, \quad C_D = C_{D_0} + kC_L^2$$

If T_{\max} is the maximum attainable thrust, then the maximum speed V_{\max} is found by solving the equation

$$T_{\max} = \eta V_{\max}^2 \left(C_{D_0} + \frac{k g^2}{\eta^2 V_{\max}^4} \right) \tag{17}$$

3 Problem Formulation

We consider the case where a single pursuer UAV has to divert and herd a single flock of birds (safely) while controlling their size. The pursuer UAV has to avoid collision with birds to ensure their safety and maintain a minimum "Safe Distance". The flock diameter has to be controlled and kept below a threshold maximum ($D_{\text{FD,Max}}$). The flock is assumed to obey Eq. (10) with suitably chosen gains.

Let us consider an airport whose center is located at $\mathbf{X}_{\text{air}} = (X, Y, Z)^T \in \mathbb{R}^3$ in a three dimensional Euclidean space. The area around the airport to be protected will be referred to as "Protected Zone (PZ)". The PZ can be defined as a spherical cap centered at \mathbf{X}_{air} with a radius R_{PZ} and an altitude of h_{PZ} . The values of R_{PZ} and h_{PZ} are such that PZ covers the entire airport with required safety zones in all directions. The $x-y$ projection of PZ will be a circle with a radius R_{PZ} (see Figure 1). We assume that birds are not allowed to fly inside the PZ. We also assume that the birds inherently meant to fly to some point inside the PZ.

The safe point \mathbf{X}_{safe} is essentially a herding goal selected and provided to the UAV (robotic falcon). The UAV aims to herd the flock to this herding goal, \mathbf{X}_{safe} . We define a safe point as the point which satisfies the following criteria:

1. Starting from the safe point, the flock, obeying Eq. (10), will never enter PZ; i.e., $\mathbf{x}_i(t) \cap \text{PZ} = \emptyset \forall t > T$ if $\frac{\sum_{\mathcal{V}} \mathbf{x}_i(T)}{N} \approx \mathbf{X}_{\text{safe}}$ for some T large enough.
2. The distance between circumferential points of PZ and the safe point satisfies $\|\mathbf{X}_{\text{safe}} - R_{\text{PZ}}\| > \delta$ for some $\delta > 0$. The distance δ has to be chosen to be at least equal to the expected radius of the flock.

The simplest way to choose \mathbf{X}_{safe} is as follows: first, draw tangent lines from the global goal point to the $x-y$ projection of PZ. Next, identify the two points located radially away at a distance δ from the intersection of the tangent with PZ. These are two candidates for \mathbf{X}_{safe} ; the one closest to the flock at the start of the pursuit may be chosen as the safe point.

The problem formulation is to design herding laws for the sentry which allow it to:

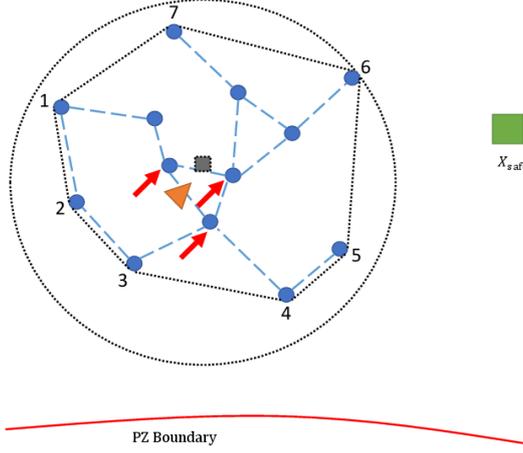


Figure 5: Robotic falcon (orange triangle) tracks centroid of the flock and pushes the boids closest to it towards goal and away from the boundary. The red arrows show the nodes that are influenced.

- (a) Move the flock to \mathbf{X}_{safe}

$$\|(\mathbf{X}_C(t) - \mathbf{X}_{\text{safe}})\| \leq \epsilon \quad \forall t \geq T, \text{ for a sufficiently large } T \quad (18)$$

- (b) Avoid fragmentation of the flock

$$D_{\text{FD}}(t) < D_{\text{FD,Max}} \quad \forall t \geq 0 \quad (19)$$

- (c) Avoid collision with the flock

$$\|(\mathbf{x}_p(t) - \mathbf{x}_i(t))\| > \text{Safe Distance} \quad \forall i \in \Gamma, \forall t \geq 0 \quad (20)$$

- (d) Keep the flock outside PZ at all times in the process of herding to \mathbf{X}_{safe}

$$\mathbf{x}_i(t) \cap \text{PZ} = \emptyset \quad \forall t \geq 0 \quad (21)$$

4 Herding Strategies

Herding is a type of shepherding behavior where the objective is to steer a group of entities from one location to another. Other types of shepherding behaviors include patrolling, collecting and coverage [21]. In this section, we summarize the herding strategies used in this paper in conjunction with the problem statement defined in Sec. 3. The herding strategy is designed for a double integrator model of the pursuer, with the idea that the resulting trajectory can act as a virtual leader for the pursuer. The control inputs in Eq. (16) can be found using approximate dynamic inversion [36]. The herding strategies are all of the following form:

$$\mathbf{u}_p = \begin{cases} V_{\text{max}} \hat{\mathbf{r}}_{p,\text{target}}, & \text{if } \|\mathbf{r}_{p,\text{target}}\| > \text{threshold} \\ k_{p,1} \left(1 - \frac{R_{\text{Fear}}}{\|\mathbf{r}_{p,\text{target}}\|}\right) \hat{\mathbf{r}}_{p,\text{target}} + k_{p,2}(\mathbf{X}_{\text{safe}} - \mathbf{x}_p) & \text{otherwise} \end{cases} \quad (22)$$

where ‘target’ denotes the target, $\mathbf{r}_{p,\text{target}} = \mathbf{x}_{\text{target}} - \mathbf{x}_p$, R_{Fear} denotes the fear radius defined in Fear Steer point in Sec. 2.3, \mathbf{X}_{safe} is as defined in Sec. 3 and V_{max} is the maximum permissible speed of the pursuer (which would be slightly smaller than Eq. (17) to leave some margin for error.) The target could be an actual boid (such as when the target is the central boid in the flock) or a point in space (such as when the target is the centre of mass of the flock). The threshold distance is a design parameter, but is chosen to equal the collision-avoidance threshold, R_{Fear} , of the boids. The gains $k_{p,1}$ and $k_{p,2}$ are currently prescribed on a case-by-case basis. It is worth noting that $k_{p,2}$ is usually much smaller than the gain $k_{p,1}$, since keeping distance from the flock is essential. Severe breaches of this distance criteria may result into flock fragmentation and result into unacceptable values of R_{FD} (Eq. 19).

The herding strategies presented here cover different ways of choosing the target location at any given point in time.

4.1 Centroid Push

Centroid push strategy involves tracking the flock centroid and pushing the boids that are closest from the centroid towards the herding direction and away from the PZ. The centroid push has been illustrated in Figure 5 and the described in Algorithm 1. In this strategy, note that the pursuer tracks the centroid which may not be an actual boid. The availability of radar data is especially critical for centroid push.

The control on pursuer due to centroid push strategy is given by.

$$\mathbf{u}_p = (\mathbf{X}_C - \mathbf{x}_p) + l_1(\mathbf{X}_{\text{safe}} - \mathbf{x}_p), \quad (23)$$

where the first component tries to track the flock centroid and the second component pushes towards the safe point.

4.2 n-Wavefront Herding Algorithm

The n-wavefront algorithm selects n boids on the boundary that need to be influenced (pushed or controlled) by the UAV (robotic falcon) using Eq. (22) to achieve the desired flock centroid trajectory and the flocking diameter D_{FD} . These boids are selected from the convex hull (or “the boundary”, shown in black dotted lines in Figure 2) of the flock, thereby ensuring that the flock does not cause panic and fragmentation among boids. It is called the n-Wavefront algorithm because it involves selection of n boids that are on the boundary or the wavefront of the flock. Let the set of boundary boids that are to be influenced by the UAV be denoted by ACTN (Actively Controlled Nodes). Possible ACTN entities are shown with numerical labels in Figure 2. The n-Wavefront herding algorithm is written in Algorithm 3

The selection of the n boundary boids is based on user defined priorities and choices. The following policies can be used for selecting influencing boids. The algorithm for selecting influencing boids is given in Algorithm 2.

1. Boundary Keeping: Influencing the boids that are closest to the Protection Zone (PZ).
2. Herding: Influencing the boids that are farthest away from the desired herded location.
3. Hybrid: Influencing a few boids that are selected to prioritize boundary keeping and the rest that are selected to herd the flock to a specific location. This case also includes scenarios with more than one boundary to keep.

Let us define a parameter λ that represents the ratio of importance or weights given to boundary keeping and herding towards goal (\mathbf{X}_{safe}). Note that λ is the ratio of number of influencing nodes selected for boundary keeping to the total n boundary nodes selected. Several metrics other than the distance can also be used to select influencing nodes. Centrality is one such interesting metric that can be used in selecting nodes. Such a metric provides us most connected or central nodes on the convex hull that are potentially more effective than others.

$$\lambda = \frac{n_{\text{BoundaryKeeping}}}{(n_{\text{BoundaryKeeping}} + n_{\text{HerdingGoal}})} \quad (24)$$

```

1: Input:  $\mathbf{x}_{p,t=k}, \mathbf{X}_C, \mathbf{X}_{\text{safe}}$ 
2: Output:  $\mathbf{x}_p$ 
3: ctr = 1
4: while  $\|\mathbf{X}_C - \mathbf{X}_{\text{safe}}\| \geq \text{eps}$  do
5:   if  $\|\mathbf{x}_p - \mathbf{x}_C\| \leq \text{eps}$  then
6:      $l_1 = 1$ 
7:   else
8:      $l_1 = 0$ 
9:   end if
10:  Compute  $\mathbf{u}_p$  using Eq. 23
11:   $\mathbf{v}_p = \mathbf{v}_p + \mathbf{u}_p$ 
12:   $\mathbf{x}_p = \mathbf{x}_p + \mathbf{v}_p$ 
13: end while

```

Algorithm 1: Herding using Centroid Push Algorithm

```

1: Input: Convex Hull,  $\mathbf{X}_{\text{safe}}$ , PZ,  $n$ ,  $\eta$ 
2: Output: Active Nodes (ACTN)
3: ACTN = {}, ctr = 1
4: while ctr < n do
5:   if ctr  $\leq \lfloor \eta n \rfloor$  then
6:     ACTN =  $\{j \in \text{Convex Hull} : \min_j \{d(\mathbf{x}_j, \text{PZ})\}\}$    $\dots$   $\{d(a,b)$  is the distance between points a and b $\}$ 
7:     ctr = ctr + 1
8:     Convex Hull = Convex Hull  $\setminus \{j\}$ 
9:   else
10:    ACTN =  $\{j \in \text{Convex Hull} : \min_j \{d(\mathbf{x}_j, \mathbf{X}_{\text{safe}})\}\}$ 
11:    ctr = ctr + 1
12:    Convex Hull = Convex Hull  $\setminus \{j\}$ 
13:   end if
14: end while

```

Algorithm 2: n-Wavefront Selection Algorithm.

where $(n_{\text{BoundaryKeeping}} + n_{\text{HerdingGoal}}) = n$, i.e., the the total boundary nodes selected at any point of time.

This algorithm is novel in the sense that it does not uses a containment like strategy discussed in Refs. [37], [25]. Boids are not herded by enclosing them with a bounding formation and then controlling the trajectory of that formation. The containment strategies developed before demand more than one controlling robot. We overcome that deficiency here. Herding can be achieved even by a single controlling robot.

The pursuer uses a strategy to maintain distance from the flock and influence boids in ACTN gently. The control input based on the n-Wavefront herding algorithm is given by,

$$\mathbf{u}_p = k_0 \left(1 - \frac{(R_{\text{Fear}} + R_{\text{FD}})}{\|(\mathbf{X}_C - \mathbf{x}_p)\|}\right) (\mathbf{X}_C - \mathbf{x}_p) + k_1 (\mathbf{x}_{\text{ACTN}} - \mathbf{x}_p) + k_2 (\mathbf{X}_{\text{safe}} - \mathbf{x}_p), \quad (25)$$

where R_{Fear} is the threshold beyond which boids do not fear the predator, R_{FD} is the flock diameter, \mathbf{X}_C is the flock centroid, \mathbf{x}_p is the position of the pursuer, \mathbf{X}_{safe} is the safe point defined in Sec. 3 and \mathbf{x}_{ACTN} is the location of the boid selected in ACTN. The first term of Eq. (25) represents the pursuer maintaining a distance (attraction when pursuer is far away and repulsion if pursuer is close to the flock) equal to sum of the fear threshold of flock R_{Fear} and the flock radius R_{FD} from the flock centroid. The second term is attractive (consensus like) to the boundary boids selected by n-Wavefront based selection. The third term is attraction towards the safe point, \mathbf{X}_{safe} , when the pursuer is close to the influencing node. One should note that here the distance maintaining control is much higher than the other two components, i.e., k_0 is atleast a couple of order of magnitude higher than k_1 and k_2 .

In Figure 2, for vanilla boundary keeping, an implementation of 2-wavefront algorithm will select node 4 and node 3 for influencing since they are closest to the boundary. Alternatively, for achieving the objective of herding the bird flock towards the herding goal, an implementation of 2-wavefront algorithm will select node 1 and node 2 as they are farthest from the goal and a ‘push’ to them will result in the flock moving towards

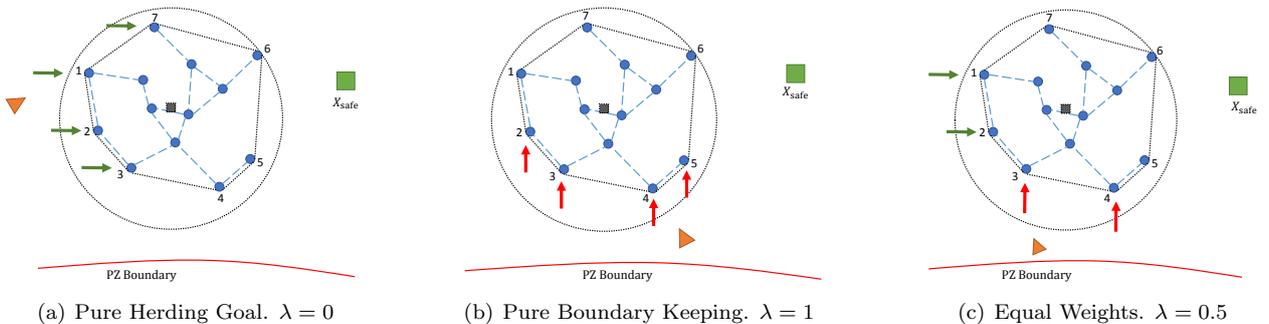


Figure 6: Selection of influencing boundary node based on n-wavefront algorithm. Nodes selected with boundary keeping policy are shown with red arrows, while nodes selected with herding goal policy are shown in green.

```

1: Input:  $\mathbf{x}_{p,t=k}, \mathbf{X}_C, \mathbf{X}_{\text{safe}}, \text{ACTN}, R_{\text{Fear}}, R_{\text{FD}}, \text{eps}$ 
2: Output:  $\mathbf{x}_p$ 
3:  $\text{ctr} = 1$ 
4: while  $\|\mathbf{X}_C - \mathbf{X}_{\text{safe}}\| \geq \text{eps}$  do
5:   if  $\|\mathbf{x}_p - \mathbf{x}_{\text{ACTN}(\text{ctr})}\| \geq R_{\text{Fear}}$  then
6:      $k_1 = 1; k_2 = 0$ 
7:   else
8:      $k_1 = 1; k_2 = 1$ 
9:   end if
10:  Compute  $\mathbf{u}_p$  using Eq. 25
11:   $\mathbf{v}_p = \mathbf{v}_p + \mathbf{u}_p$ 
12:   $\mathbf{x}_p = \mathbf{x}_p + \mathbf{v}_p$ 
13:  if  $\|\mathbf{x}_p - \mathbf{x}_{\text{ACTN}(\text{ctr})}\| \leq R_{\text{Fear}} - \text{eps}$  then
14:     $\text{ctr} = \text{ctr} + 1$ 
15:  end if
16: end while

```

Algorithm 3: Herding using n-Wavefront Algorithm

goal. In a separate scenario that gives importance to both boundary keeping and forced herding, a 2-wavefront algorithm will select a pair out of $\{(1,4),(1,3),(2,4),(2,3)\}$ depending on the weights: (1,4) and (2,3) will be selected if both have equal weights (selection among the both will depend upon the distance the robotic bird has to travel as it influences nodes sequentially); (2,4) if boundary keeping is more important, and (1,3) if herding is more important.

5 Simulation Results and Discussion

Numerical simulations of the herding algorithms are performed in MATLAB. A flock of birds ($N = 10, 20$ and 30) are trying to enter the PZ around an airport. We first show the ability of the robotic falcon to herd the flock of birds away from the airport and towards a predefined safe point, using the n-Wavefront herding algorithm (using Algorithms 2, 3) with 4 influencing nodes and $\lambda = 0, 0.5$ and 1 . Thereafter, we compare the new algorithm with a centroid push.

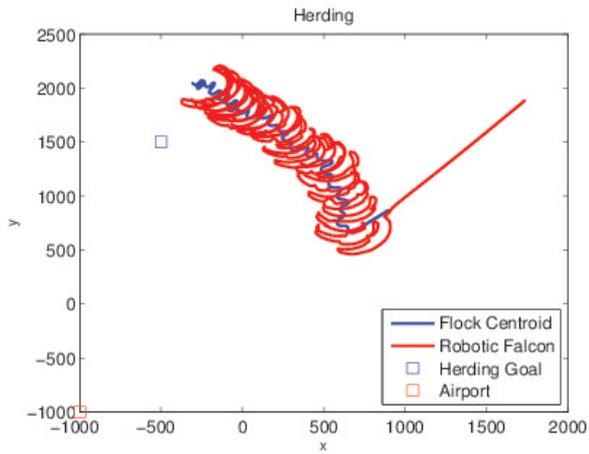
We show simulations for both 2D and 3D flock. Figures 7, 8, and 9 show control over the flock centroid (\mathbf{X}_C) in 2D, the flock diameter (D_{FD}) and graph centralization ($\Theta(\mathcal{G})$, see Eq. (4)) measures as functions of time. Figure 10 shows similar results for 3D. Interestingly, the graph centralization ($\Theta(\mathcal{G})$, see Eq. (4)) in case of 3D goes to zero often, implying that for large amounts of time the degree of boids equals max degree. This implies that the robotic bird in 3D scenario can “round-up” or collect boids much more effectively. No collisions were observed between the boids and the pursuing UAV.

Figure 11 shows the results for a simulation of the centroid-push algorithm. The robotic bird perfectly tracks the center of mass of the flock. It however fails to take the flock to the desired herding location. The centroid push strategy does not allow us to keep the flock cohesive. The boids maintain a distance almost equal to R_{Fear} from the pursuer. This also results in much higher flock diameter as compared to the n-Wavefront herding algorithm. It exceeds the D_{FD} desired value.

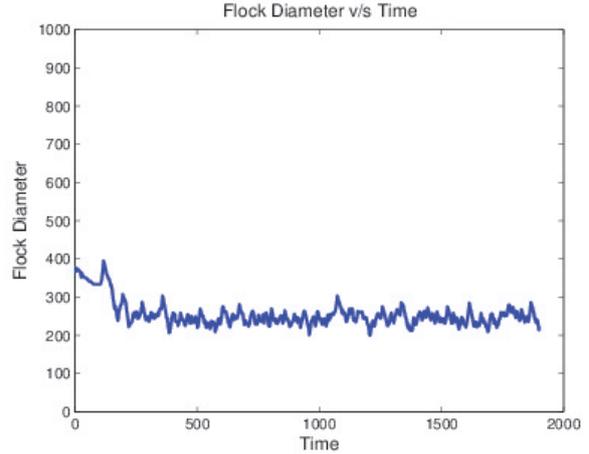
Interestingly enough, the trajectory of the sentry under the n-wavefront algorithm has some parallels with the heuristics proposed by Strobom [26]. The sentry’s motion can be broken into two segments: rounding and driving; the former is used for keeping the herd cohesive, and the latter to drive it toward the desired location. It remains an open problem to check whether the n-wavefront algorithm *always* gives rise to such a composite motion, and whether the availability of multiple pursuers modifies or eliminates these heuristics.

6 Conclusion

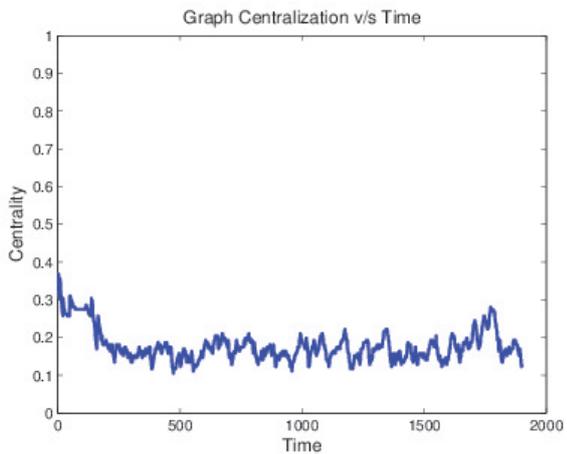
This paper addressed the problem of diverting a flock of birds away from a prescribed area such as an airport. The flock was modelled using a combination of existing flocking protocols coupled with predator-prey dynamics in birds. Herding performance metrics were defined. Flock centroid, flock diameter, flock (neighborhood graph) centralization and distance to airport were used as metrics for measuring performance of herding algorithms.



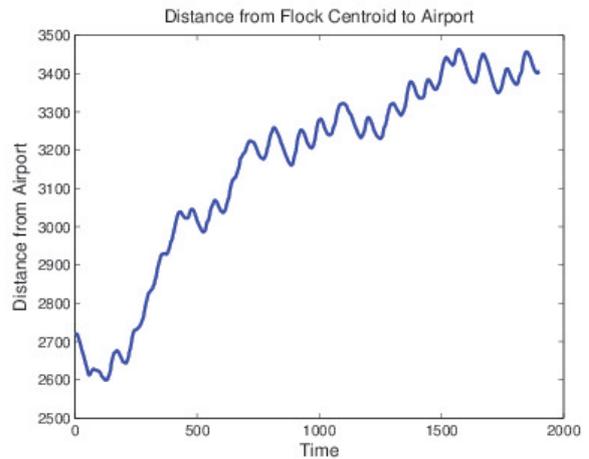
(a) Herding



(b) Flock Diameter



(c) Graph Centralization



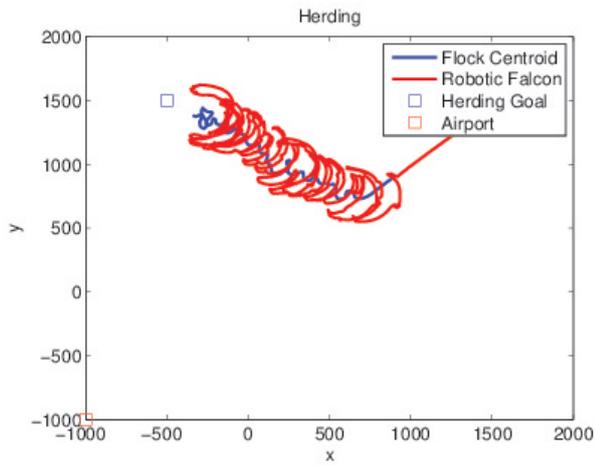
(d) Distance to airport

Figure 7: Herding with a n-Wavefront algorithm. $N = 20$ birds, $n = 4$, $\lambda = 0.5$.

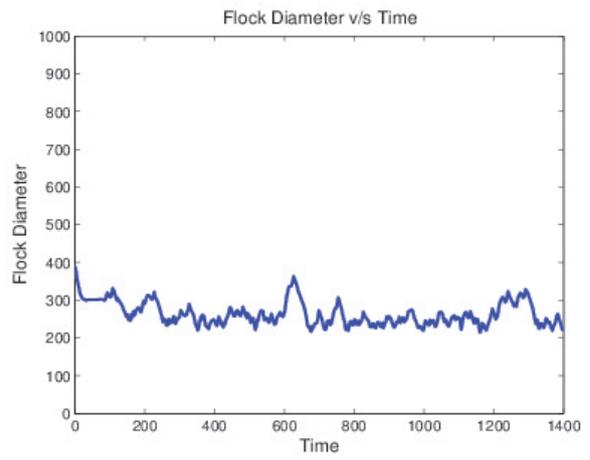
The herding problem was posed in graph theoretic framework using herding performance metrics. A novel boundary control strategy, called the n-wavefront algorithm, was introduced in the paper for enabling a single pursuer UAV to safely herd the flock without fragmenting it and while interacting only with agents on the boundary of the flock. The algorithm was successfully demonstrated in numerical simulations.

Acknowledgement

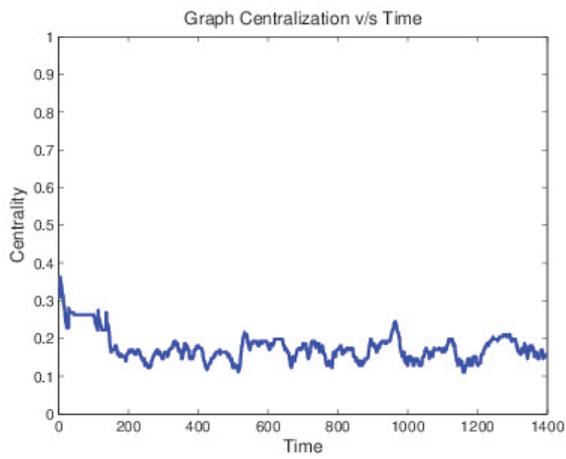
This work was supported by the National Science Foundation Career Award NSF IIS-1253758.



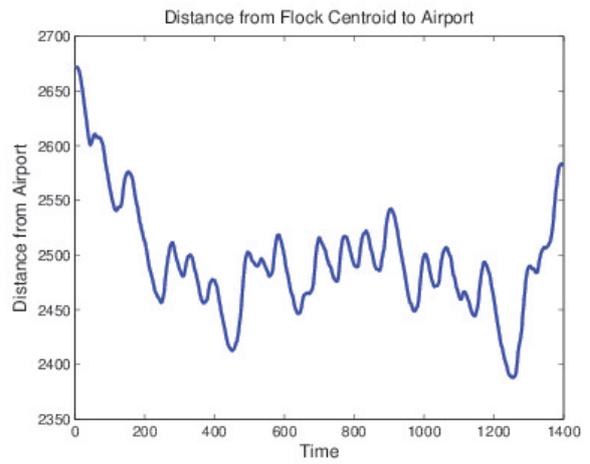
(a) Herding



(b) Flock Diameter

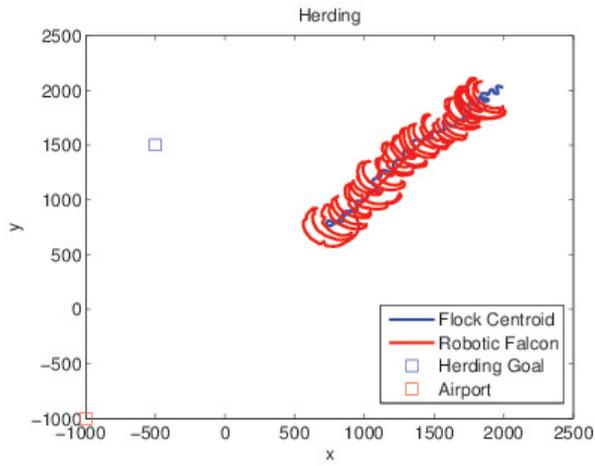


(c) Graph Centralization

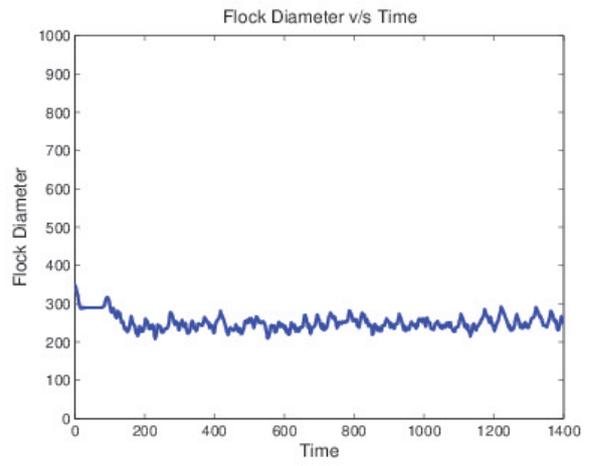


(d) Distance to airport

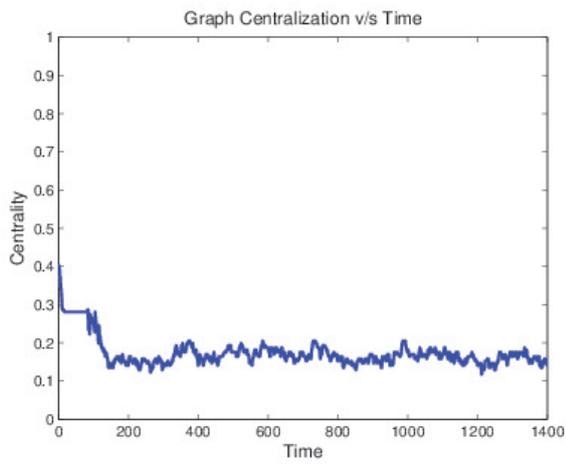
Figure 8: Herding with a n-Wavefront algorithm. $N = 20$ birds, $n = 4$, $\lambda = 0$.



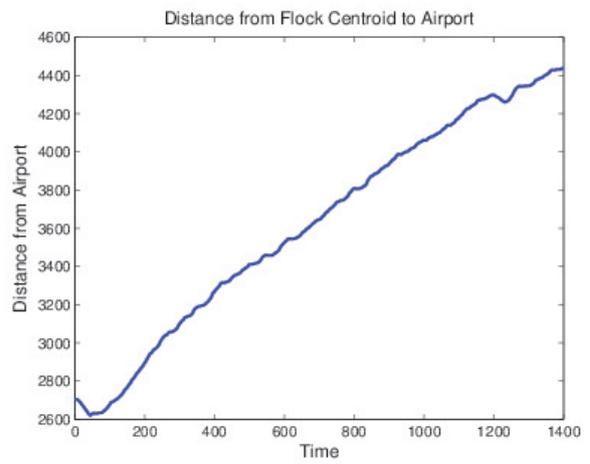
(a) Herding



(b) Flock Diameter

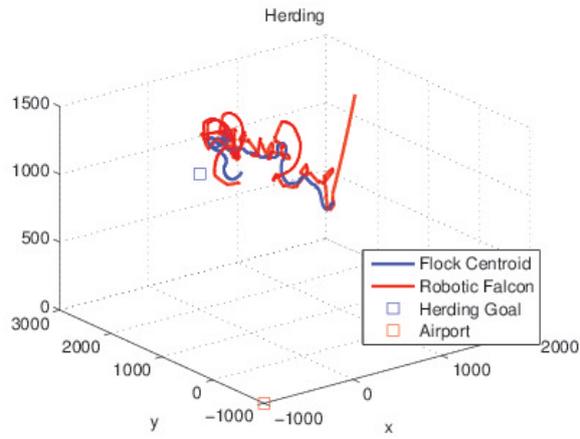


(c) Graph Centralization

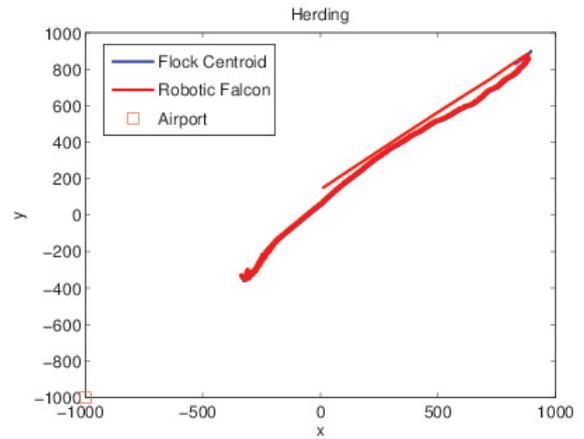


(d) Distance to airport

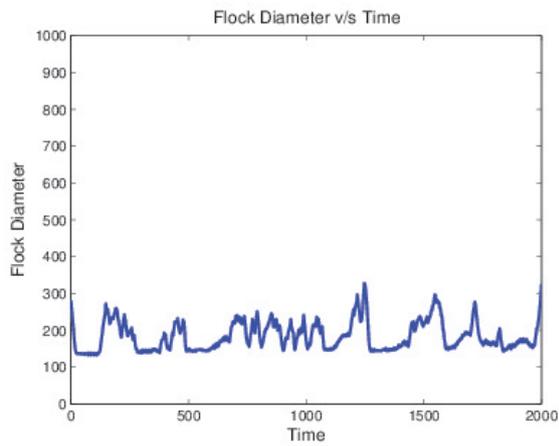
Figure 9: Herding with a n-Wavefront algorithm. $N = 20$ birds, $n = 4$, $\lambda = 1$.



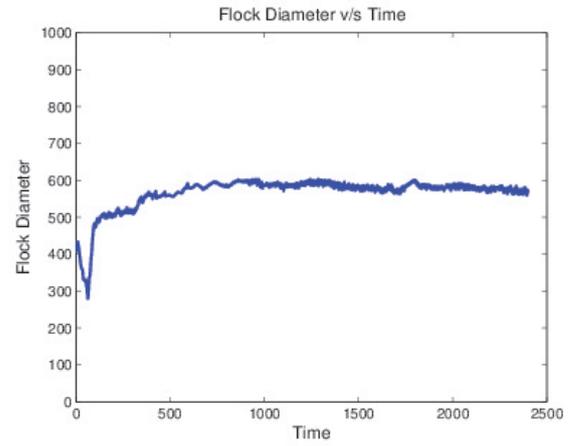
a) Herding



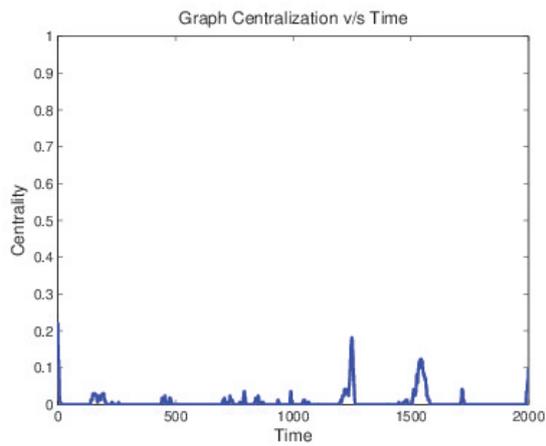
a) Herding



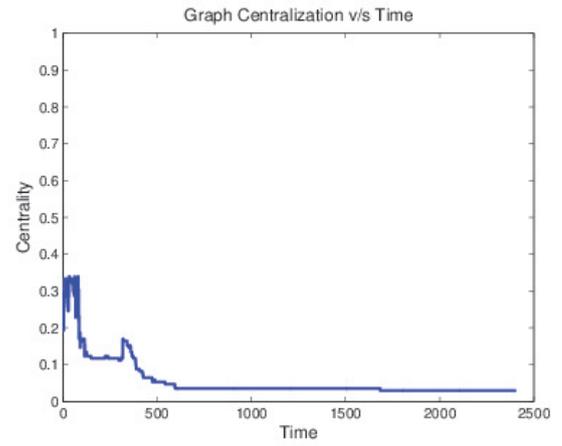
b) Flock Diameter



b) Flock Diameter



c) Graph Centralization



c) Graph Centralization

Figure 10. Herding with a n-Wavefront algorithm. $N = 20$ birds, $n = 4$, $\lambda = 0$.

Figure 11. Boundary keeping using Centroid Push strategy. $N = 20$ birds

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