



# Advances and Challenges in Quantitative Verification and Synthesis for Cyber–Physical Systems

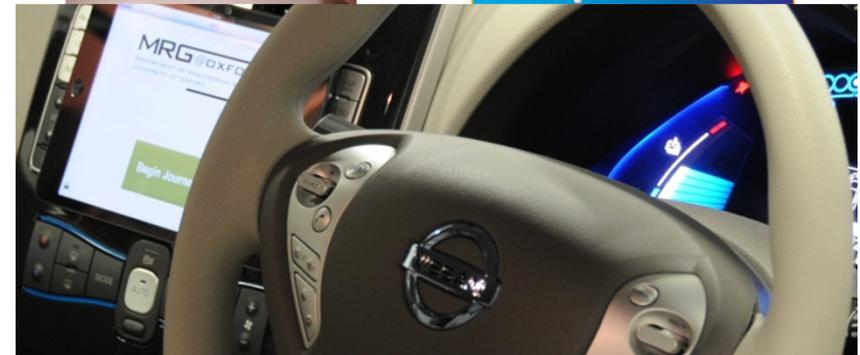
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SoSCYPS, Vienna, 11<sup>th</sup> Apr 2016

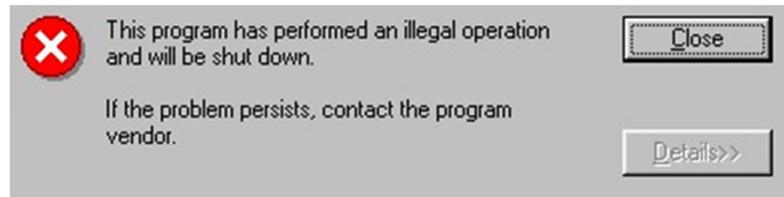
# Cyber-Physical Systems

- Autonomous vehicles
  - self-parking, self-driving cars...
- Patient monitoring
  - closed-loop infusion pumps...
- Robotic assistants
- Multitude of devices
  - networked
  - Internet-enabled
  - organised into communities
- Main characteristics
  - **sensors** and actuators are often integral
  - increasingly **autonomous** behaviour
  - **embedded** software controls **physical** processes
  - combining discrete, continuous **and** stochastic dynamics



# Are we safe?

- Embedded software at the heart of the device
- What if...
  - glucose monitor fails to trigger an alarm
  - infusion pump delivers the wrong dosage of drugs
  - self-parking car software crashes during the manouvre...



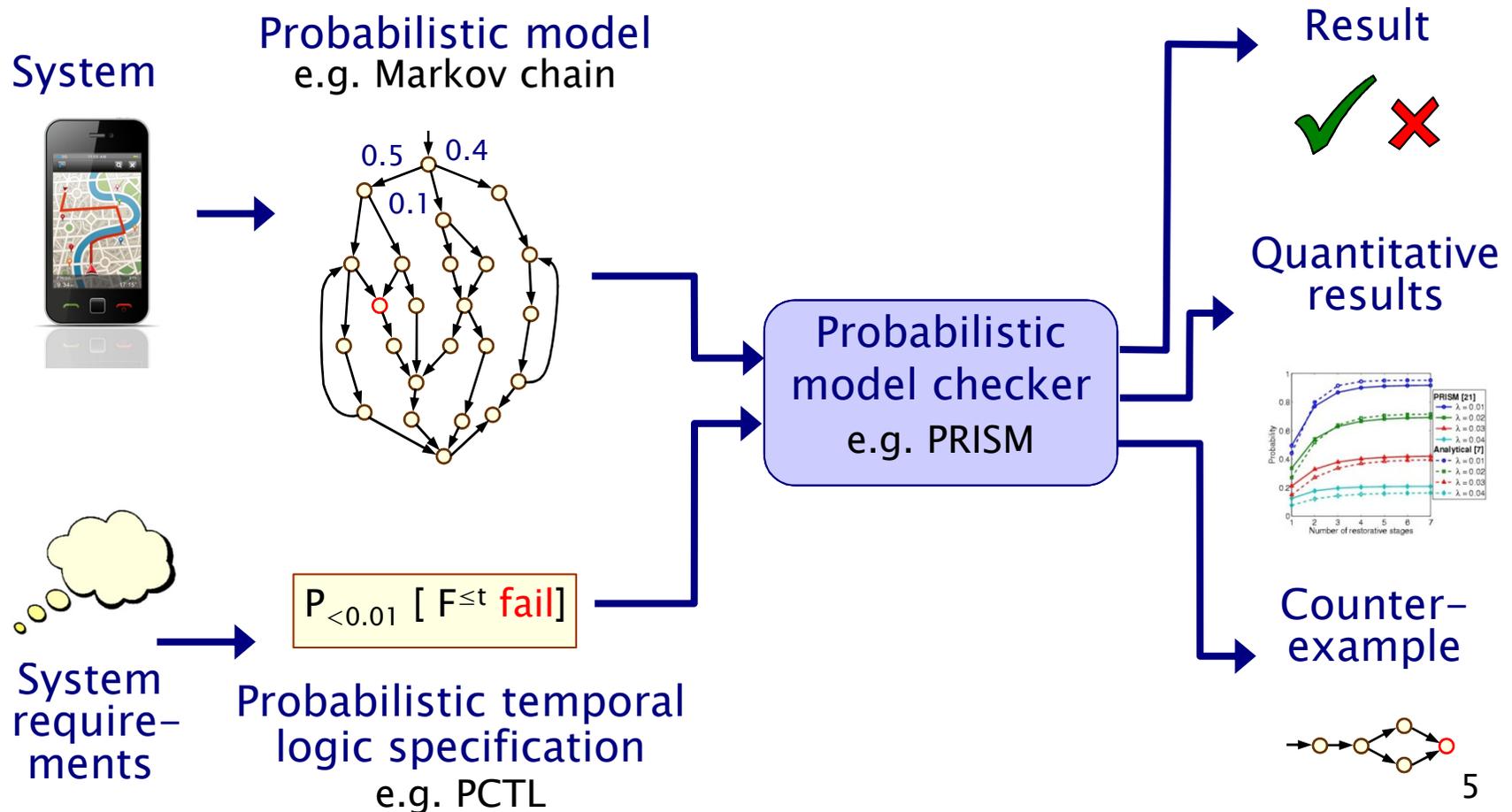
- Imagined or real?
  - February 2016: Nissan disables Leaf car app after discovering software could be **hacked**
  - May 2010 and each year since: FDA recalls programmable infusion pumps, over 710 patient deaths in five years, some because the **device's software malfunctioned**

# Software quality assurance

- Software is a **critical** component
  - embedded software failure costly and life endangering
- Need quality assurance methodologies
  - **model-based** development
  - **rigorous** software engineering
    - includes V&V
- Use formal techniques to produce guarantees for:
  - safety, reliability, performance, resource usage, trust, ...
  - (**safety**) “probability of failure to raise alarm is tolerably low”
  - (**reliability**) “the smartphone will never execute the financial transaction twice”
- Focus on automated, tool-supported methodologies
  - automated verification via **model checking**
  - **quantitative verification**

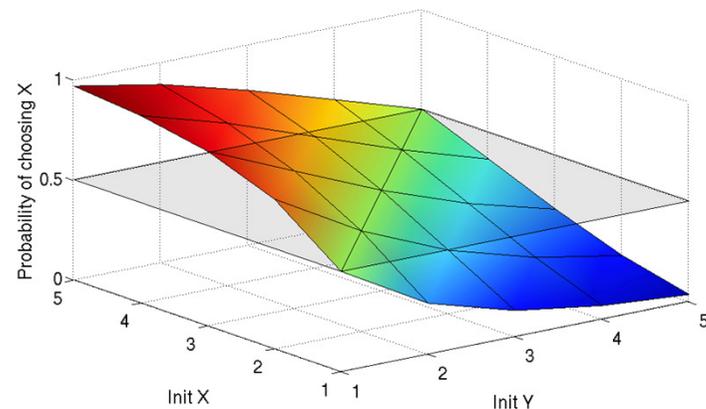
# Quantitative verification

Automatic verification (aka model checking) of **quantitative** properties of probabilistic system models



# PRISM – Property specification

- **Temporal logic**-based property specification language
  - temporal order of events, in conjunction with probability and cost/rewards
- Simple examples:
  - $P_{\leq 0.01} [ F \text{ “crash” } ]$  – “the probability of a crash is at most 0.01”
  - $S_{> 0.999} [ \text{“up”} ]$  – “long-run probability of availability is  $> 0.999$ ”
- Usually focus on **quantitative** (numerical) properties:
  - $R_{\{\text{“energy”}\}_{\max=?}} [ C^{\leq 7200} ]$  – “what is the worst-case expected energy consumption during the first 2 hours?”
  - then analyse trends in quantitative properties as system parameters vary



# Quantitative probabilistic verification

- **What's involved**
  - specifying, extracting and building of quantitative models
  - graph-based analysis: reachability + qualitative verification
  - numerical solution, e.g. linear equations/linear programming
  - typically computationally more **expensive** than the non-quantitative case
- **The state of the art**
  - fast/efficient techniques for a range of probabilistic models
  - feasible for models of up to  $10^7$  states ( $10^{10}$  with symbolic)
  - extension to probabilistic real-time systems
  - abstraction refinement (CEGAR) methods
  - compositional verification
  - statistical model checking
  - **tool support** exists and is widely used, e.g. **PRISM**, **UPPAAL**

# PRISM

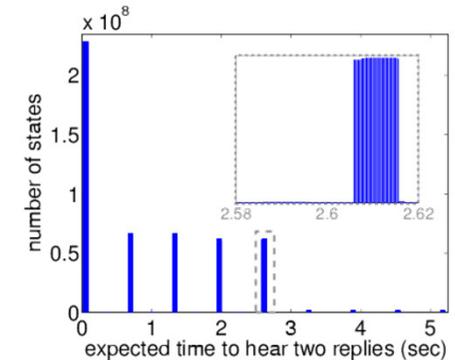
- **PRISM: Probabilistic symbolic model checker**
  - developed at Birmingham/Oxford University, since 1999
  - free, open source software (GPL), runs on all major O/s
  - simple but flexible high-level modelling language
- **Various efficient model checking engines and techniques**
  - symbolic methods (binary decision diagrams and extensions)
  - explicit-state methods (sparse matrices, etc.)
  - statistical model checking (simulation-based approximations)
- **Graphical user interface**
  - editors, simulator, experiments, graph plotting
- **See: <http://www.prismmodelchecker.org/>**
  - downloads, tutorials, case studies, papers,



# Quantitative verification in action

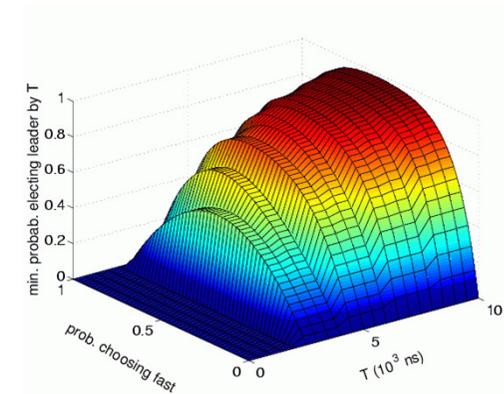
- Bluetooth device discovery protocol

- frequency hopping, randomised delays
- low-level model in PRISM, based on detailed Bluetooth reference documentation
- numerical solution of 32 Markov chains, each approximately 3 billion states
- identified **worst-case** time to hear one message, 2.5 seconds



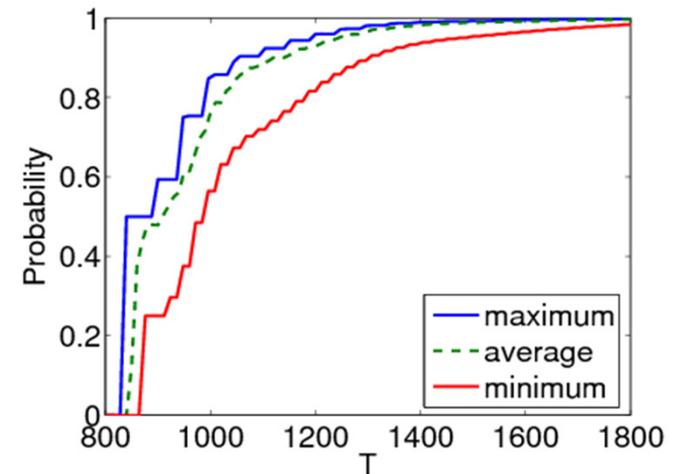
- FireWire root contention

- wired protocol, uses randomisation
- model checking using PRISM
- optimum probability of leader election by time  $T$  for various coin biases
- demonstrated that a **biased coin** can improve performance



# Quantitative verification – Status

- Tools/techniques widely applicable, since **real** software/systems **are** quantitative
  - extensions/adaptations of model-based frameworks
  - new application domains
- Analysis “quantitative” & “exhaustive”
  - strength of mathematical proof
  - best/worst-case scenarios, **not** possible with simulation
  - identifying trends and anomalies
- **But**
  - the modelling phase time-consuming and error prone
  - often used as a debugging tool
  - scalability continues to be hard to overcome

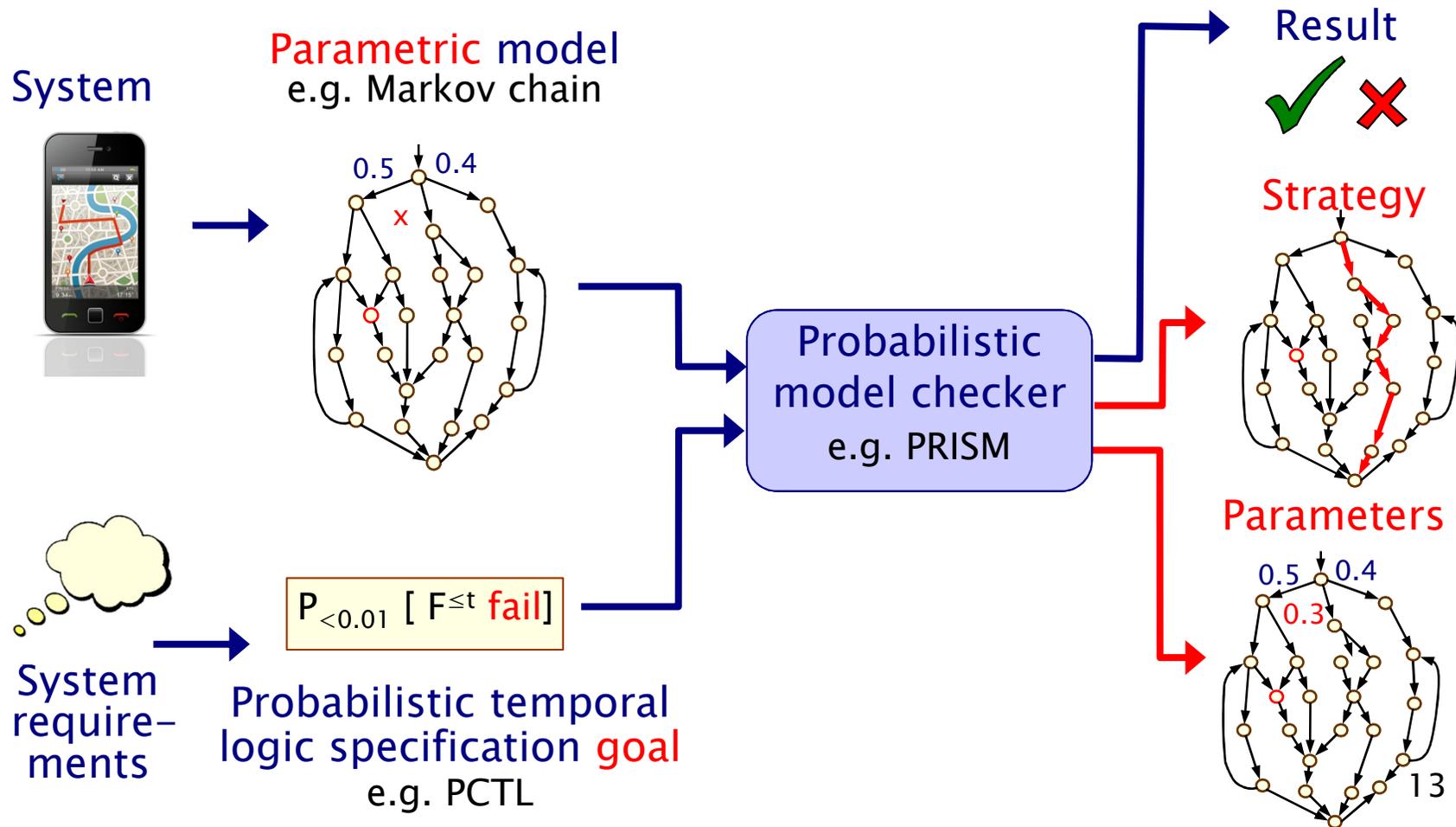


# From verification to synthesis

- Move towards quantitative **model synthesis**
  - i.e. “can we **construct** a model to guarantee that a given quantitative property is satisfied?”
  - instead of “does the model satisfy a given quantitative property?”
  - advantage: **correct-by-construction**
- We initially focus on simpler problems
  - strategy (controller) synthesis
  - parameter synthesis
  - etc
- Many application domains
  - robotics (controller synthesis from LTL/PCTL)
  - security (generating attacks or counter-strategies)
  - smart grid (optimal policy synthesis, coordination)

# Quantitative verification and synthesis

Automatic **synthesis** of correct-by-construction strategies and models from **quantitative** properties/goals



# The challenges of CPS

- **Autonomous behaviour:** electronic agents make decisions and act independently of humans, e.g. search and rescue
- **Constrained resources:** low power, processor speed and memory capacity, intermittent connectivity
- **Adaptiveness:** systems have to adapt to changing requirements in predictable fashion
- **Monitoring and control of physical processes:** needed in autonomous transport, robotic planning, implantable medical devices such as pacemakers, etc
- **Communities of agents:** need to model cooperation, competition and resource sharing, necessitating game-theoretic approaches
- **Interfacing with the natural world:** biosensing and DNA/molecular computation have important applications in disease detection and drug delivery

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# This lecture...

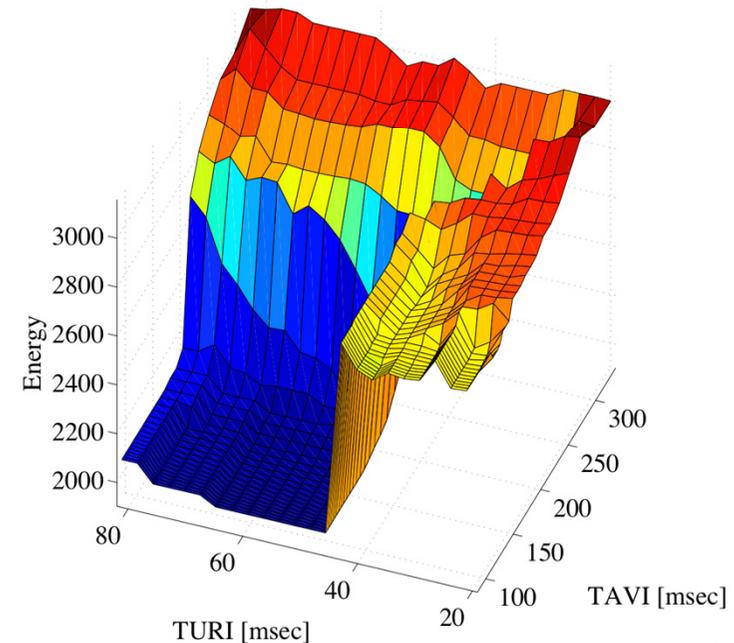
- Selected recent advances in quantitative verification
- **Pacemaker case study:** monitoring and control of physical processes
  - non-linear hybrid dynamics, stochasticity
  - (privacy and security essential)
  - ECG used in biometrics
  - optimal parameter synthesis
  - introducing HeartVerify
- **Stochastic games** to model cooperation, competition and resource sharing
  - strategy synthesis in adversarial situations
  - multi-objective specifications
  - introducing PRISM-games
- Beyond PRISM...

# Case study: Cardiac pacemaker

- Develop model-based framework
  - **timed automata** model for pacemaker software [Jiang et al]
  - hybrid heart models in **Simulink**, adopt synthetic ECG model (non-linear ODE) [Clifford et al]
- Properties
  - (basic safety) maintain 60–100 beats per minute
  - (advanced) detailed analysis **energy usage**, plotted against timing parameters of the pacemaker
  - **parameter synthesis**: find values for timing delays that optimise energy usage



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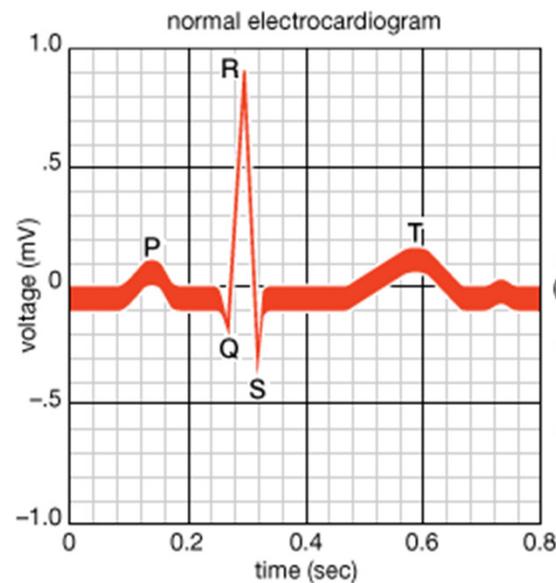


# Modelling of the heart

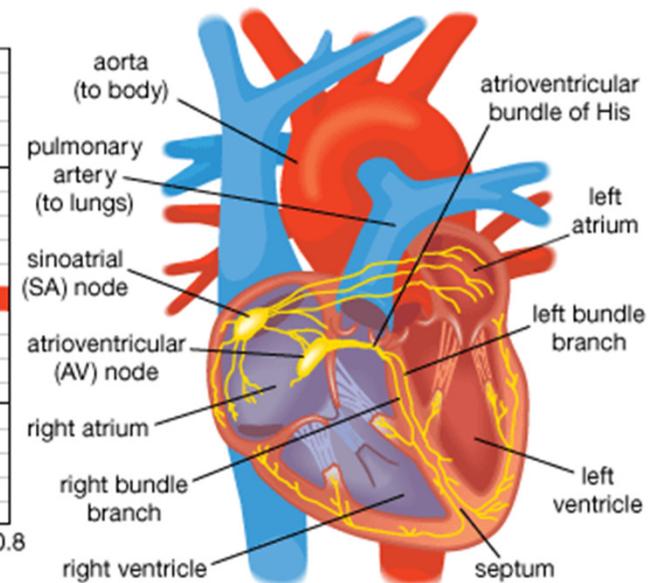
- The heart maintains blood circulation by contracting the atria and ventricles
  - spontaneously generates electrical signal (action potential)
  - conducted through cellular pathways into atrium, causing contraction of atria then ventricles
  - repeats, maintaining 60–100 beats per minute

- Abnormalities in electrical conduction

- missed/slow heart beat (Bradycardia)
- treatable with pacemakers

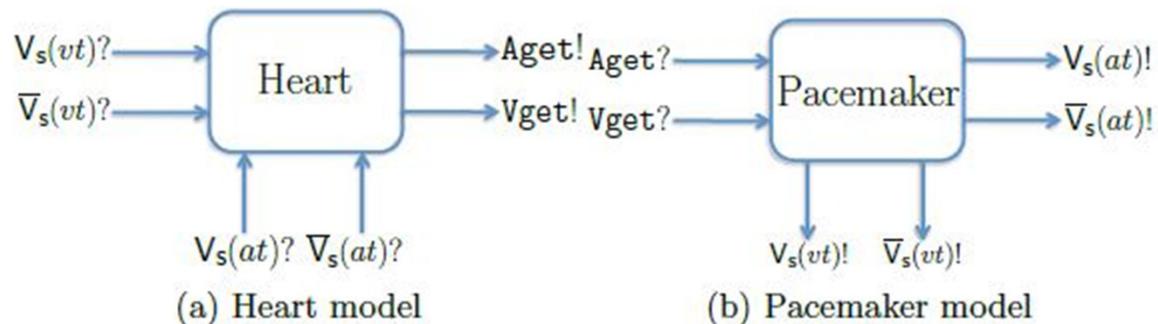


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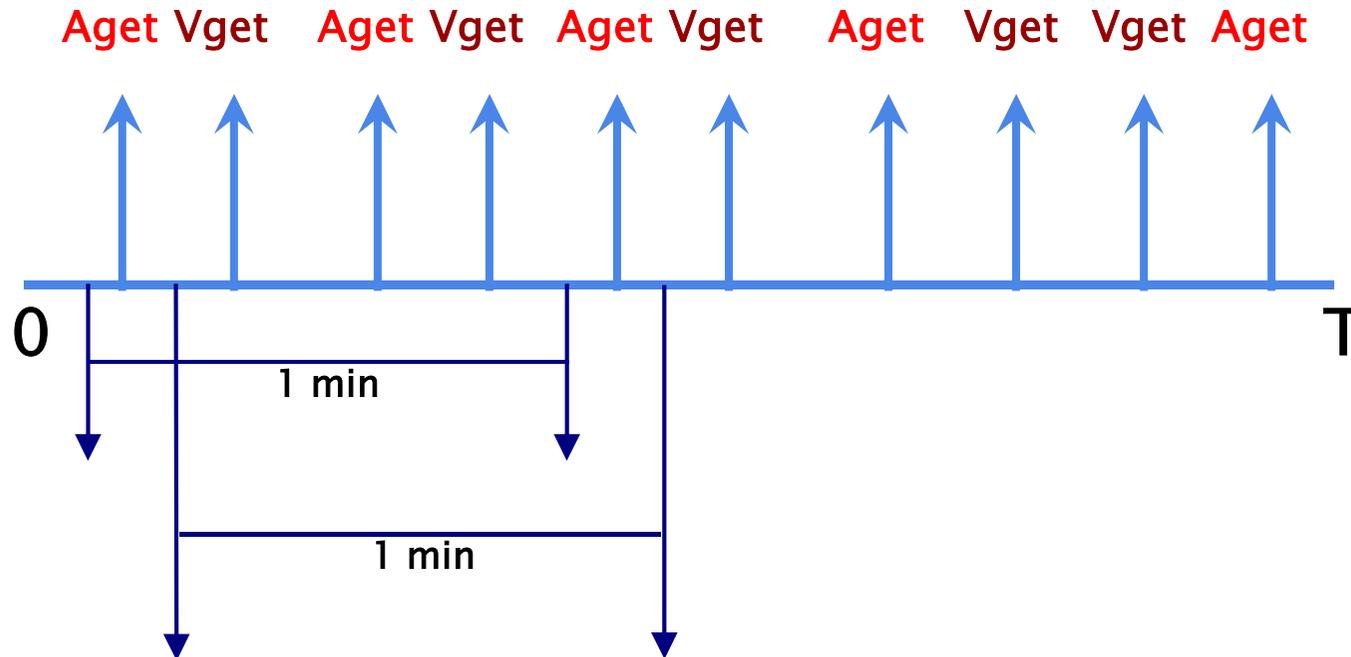
# Model-based framework

- We advocate a **model-based framework**
  - **models** are networks of communicating **hybrid** I/O automata, realised in Matlab Simulink
    - discrete mode switching and continuous flows: electrical conduction system
    - **quantitative**: energy usage and battery models
    - **patient-specific** parameterisation
  - framework supports **plug-and-play composition** of
    - heart models (timed/hybrid automata, some stochasticity)
    - pacemaker models (timed automata)



Quantitative Verification of Implantable Cardiac Pacemakers over Hybrid Heart Models.  
Chen *et al*, *Information and Computation*, 2014

# Property specification: Counting MTL



$$\square^{[0, \tau]} (\#_0^\tau Vget \geq B_1 \wedge \#_0^\tau Vget \leq B_2)$$

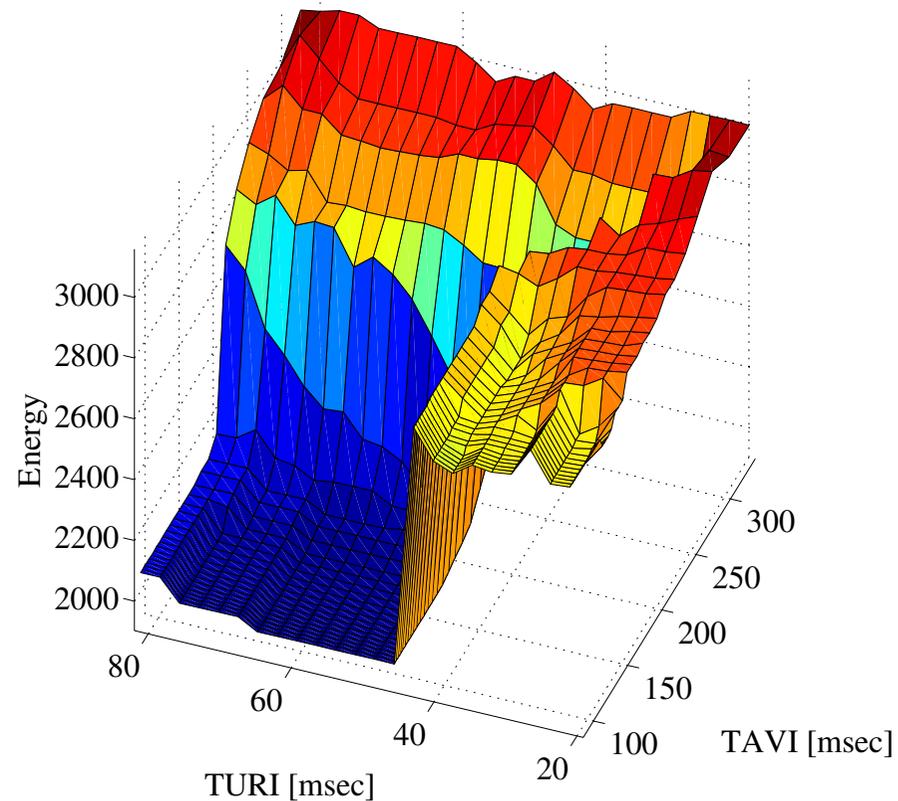
**Safety** ‘for any 1 minute window, heart rate is in the interval [60,100]’

Event counting not expressible in MTL ( Metric Temporal Logic)

# Framework functionality

- Broad range of techniques
  - Monte-Carlo **simulation** of composed models
    - with (confidence level) guarantees for non-linear flows
  - (approximate) **quantitative verification** against variants of MTL
    - to ensure property is satisfied
  - **parametric** analysis
    - for **in silico** evaluation, to reduce need for testing on patients
  - automated **synthesis** of optimal timing parameters
    - to determine delays between paces so that energy usage is optimised for a given patient
  - **patient-specific** parameterisation
  - **hardware-in-the-loop** simulation
    - parameter optimisation with respect to **real** energy measurements
- See <http://www.veriware.org/pacemaker.php>

# Energy consumption



**Efficiency** “energy consumed must be below some fixed level”

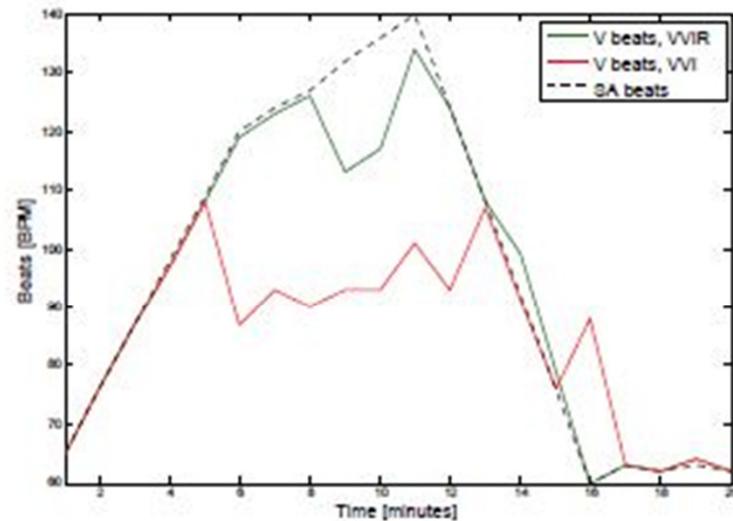
Battery charge in 1 min under Bradycardia, varying timing parameters

Based on **real** power measurements

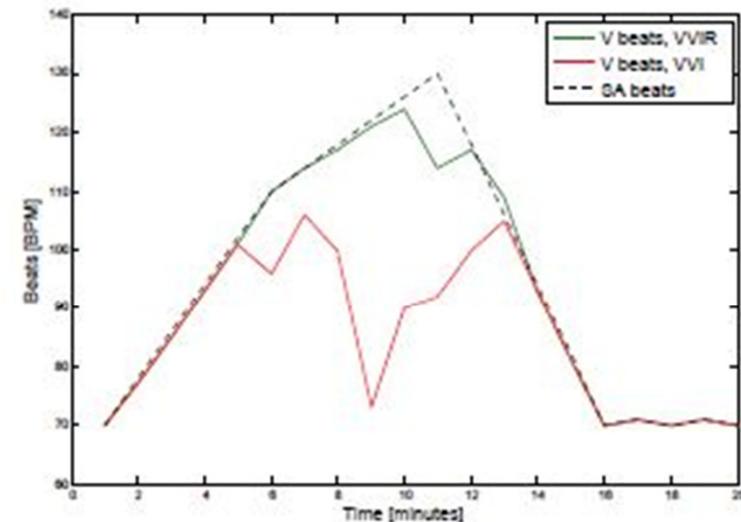
[Hardware-in-the-loop simulation and energy optimization of cardiac pacemakers.](#)

Barker *et al*, In *Proc EMBC*, 2015

# Modulation during physical activity



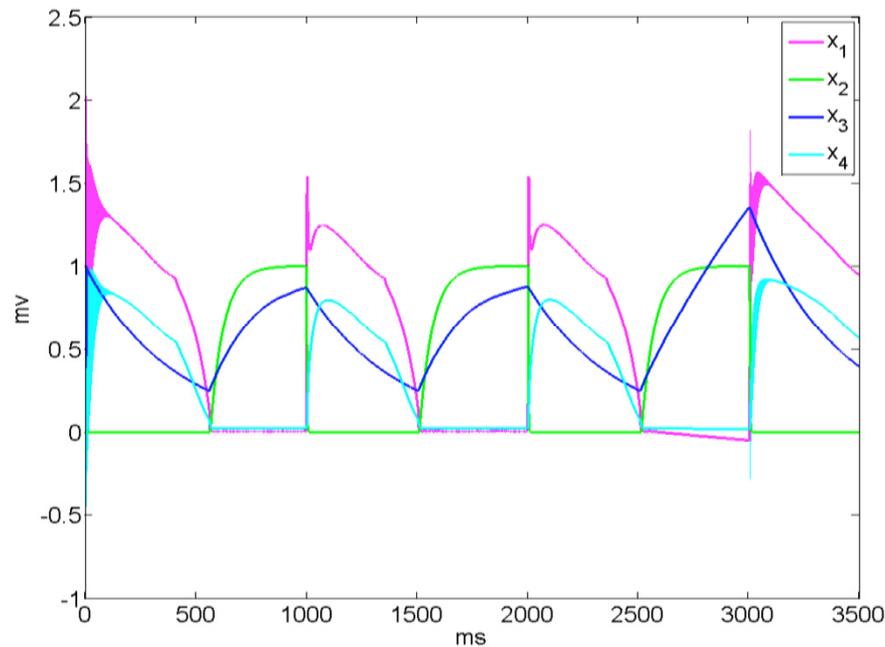
(a) Young patient



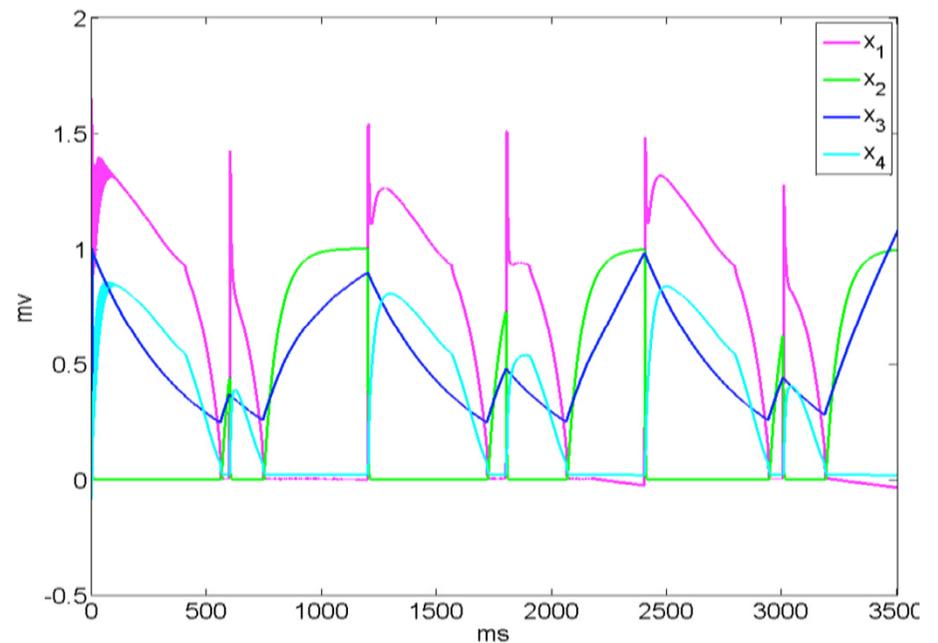
(b) Old patient

Rate modulation during exercise. Black dashed line indicates metabolic demand, and the green and red curves show rate-adaptive VVIR and fixed-rate VVI pacemakers.

# Alternans in the heart



Pacing rate: 1 s



Pacing rate: 0.6s

We plot the reach set from a set of initial states observe whether the AP durations alternate

[Invariant Verification of Nonlinear Hybrid Automata Networks of Cardiac Cells](#). Huang *et al*<sup>25</sup>  
In *CAV*, volume 8559 of LNCS, pages 373–390, Springer, 2014.

# Optimal parameter synthesis

- Automated verification aims to establish if a property holds for a given model
- Can we find a model so that a property is satisfied?
  - difficult...
- The **parameter synthesis problem** is
  - given a parametric network of timed I/O automata, set of controllable and uncontrollable parameters, CMTL property  $\phi$  and length of path  $n$
  - find the **optimal controllable** parameter values, for any uncontrollable parameter values, with respect to an **objective function**  $O$ , such that the property  $\phi$  is satisfied on paths of length  $n$ , if such values exist
- Objective function
  - maximise volume, or ensure robustness

# Optimal timing delays

- **Bi-level optimisation** problem
- Safe heart rhythm CMTL property (**inner** problem)

$$\phi = \square^{[0,T]} (vPeriod \in [500, 1000])$$

- at any time in  $[0,T]$  any two consecutive ventricular beats are between 500 and 1000 ms, i.e. heart rate of 60 and 120 BPM

- **Cost function** (**outer** problem)

$$2 \cdot \#_0^{60000} (act = AP) + 3 \cdot \#_0^{60000} (act = VP)$$

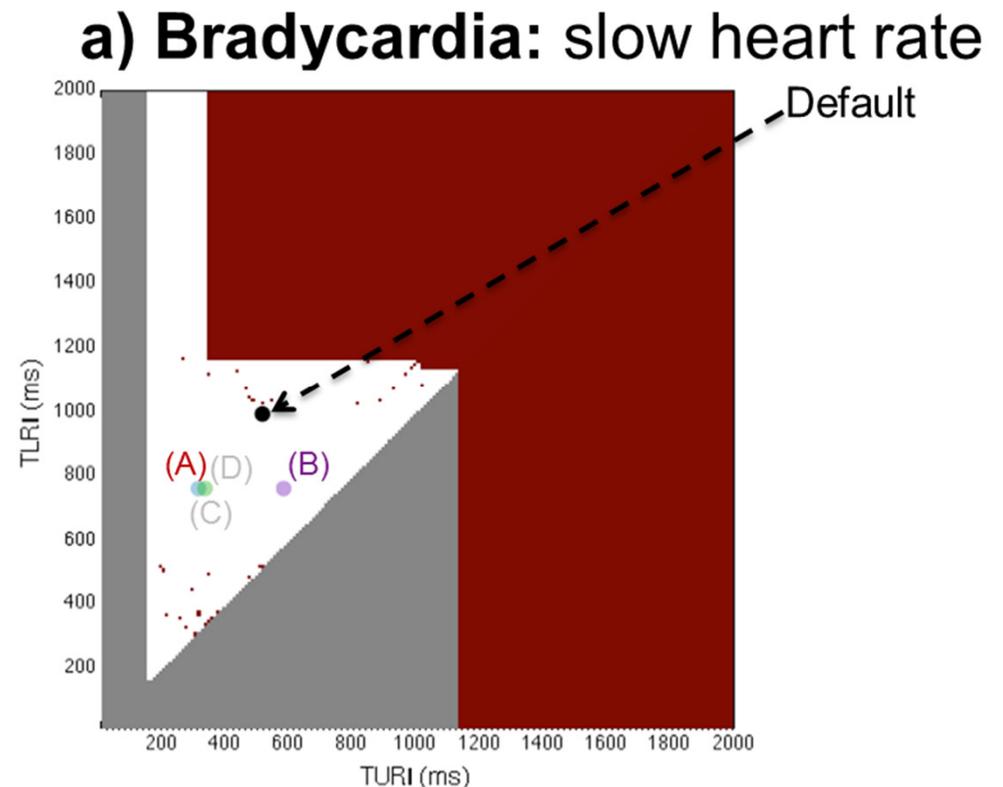
- energy consumption in 1 minute

$$\frac{\sum_{(\mathbf{q}, \eta) \in Vbeat(\rho')} |\eta(CO) - \overline{CO}|}{|Vbeat(\rho')|}$$

- mean difference between cardiac output and reference value

# Synthesis results

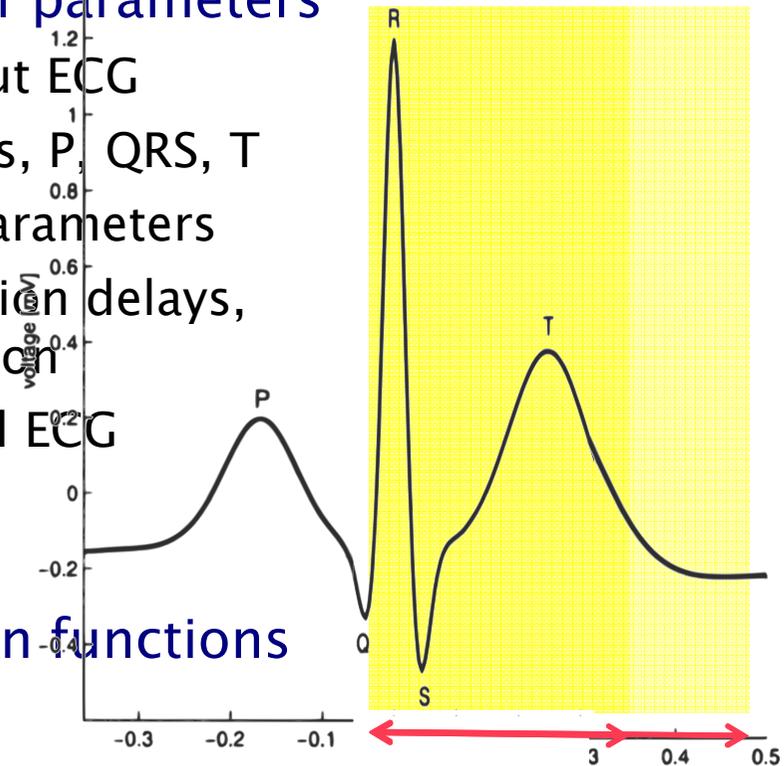
- Solved through **SMT encoding** (inner problem) combined with **evolutionary computation** (outer problem)
- Pacemaker parameters:
  - TLRI: time the PM waits before pacing atrium
  - TURI: time before pacing ventricle after atrial event
- Significant improvement (>50%) over default values
  - path 20
- A (exact), B (evo) energy
- C (exact), D (evo) CO
  - evo **faster**, less precise



# Estimation from ECG data

- Steps towards **personalisation** of parameters

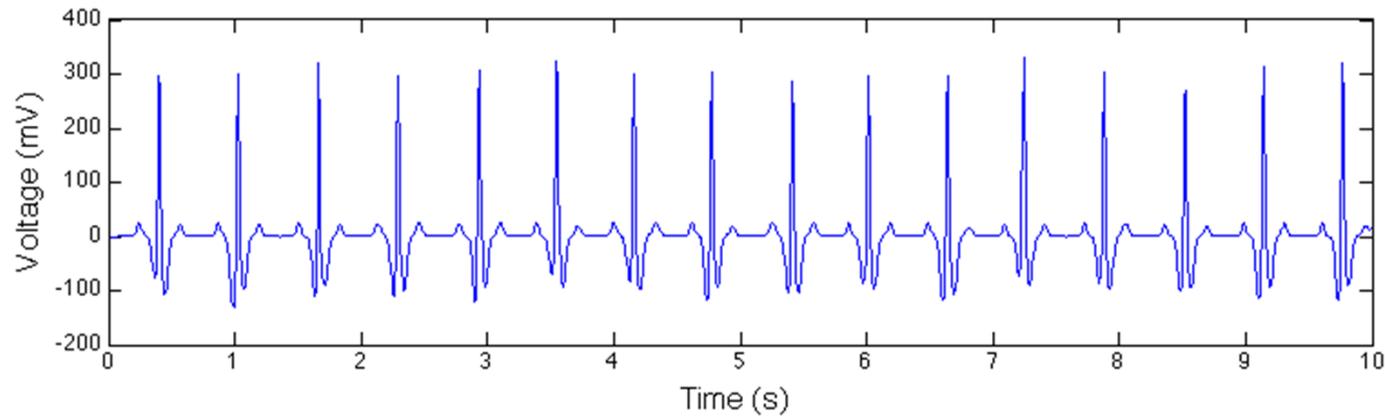
- filtering and analysis of the input ECG
- detection of characteristic waves, P, QRS, T
- mapping of intervals: **explicit** parameters
- **implicit** parameters, eg conduction delays, use Gaussian Process optimisation
- compare synthetic ECG with real ECG using statistical distance



- Synthetic ECG** = sum of Gaussian functions centred at each wave  $l_i$

$$\text{synthECG}(t) = \sum_{i \in \{P, Q, R, S, T\}} \sum_{l_i \in \text{Peaks}_i} a_i \cdot \exp\left(-\frac{(t - l_i)^2}{2c_i^2}\right)$$

# Case study: Personalisation



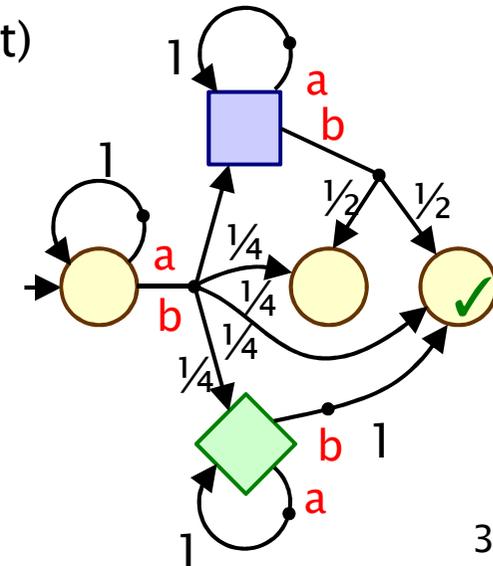
- Personalisation of wearable devices
  - estimate parameters for a heart model based on ECG data
  - generate **synthetic** ECG
  - useful for model-based development of personalised devices
- Developed HeartVerify based on Simulink/Stateflow
  - variety of tools and techniques
  - <http://www.veriware.org/pacemaker.php>

# Stochastic game modelling

- Control is playing a game
  - find a **controller** working under **all** environment conditions, including **adversarial**
  - some **uncertainty** is quantifiable: stochastic games
- System built from several components:
  - manage design/engineering complexity
  - need theory for **compositional** analysis
- Engineering is the art of making trade-offs:
  - designs have to meet **multiple quantitative** objectives
  - **contracts** (if I can assume A, I guarantee B)

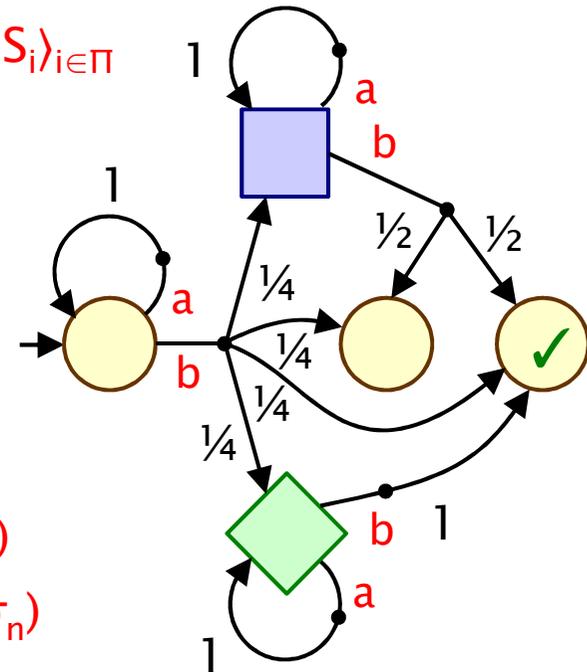
# Stochastic multi-player games (SMGs)

- **Key ingredients:**
  - probability (failures, noisy sensors, randomisation)
  - nondeterminism (concurrency, control, environment)
  - multiple players (competitive or collaborative behaviour)
- **Applications**
  - distributed coordination (selfish agents vs unselfish)
  - controller synthesis (system vs. environment)
  - security (defender vs. attacker)
- **Modelling of SMGs**
  - PRISM modelling language
  - extension of Reactive Modules
  - guarded command notation

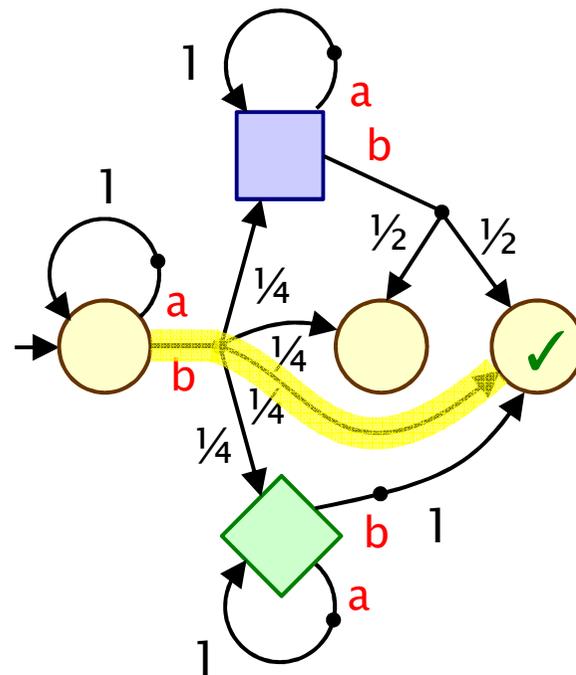


# Stochastic multi-player games

- Stochastic multi-player game (SMGs)
  - probability + nondeterminism + multiple players
- A (turn-based) SMG is a tuple  $(\Pi, S, \langle S_i \rangle_{i \in \Pi}, A, \Delta, L)$ :
  - $\Pi$  is a set of  $n$  players
  - $S$  is a (finite) set of states, with partition  $\langle S_i \rangle_{i \in \Pi}$
  - $A$  is a set of action labels
  - $\Delta : S \times A \rightarrow \text{Dist}(S)$  is a (partial) transition probability function
  - $L : S \rightarrow 2^{\text{AP}}$  labels states with atomic propositions from  $\text{AP}$
- Strategy for player  $i$ : choose in  $S_i$  states
  - based on history, i.e.  $\sigma_i : (SA)^* S_i \rightarrow \text{Dist}(A)$
  - strategy profile: for all players  $\sigma = (\sigma_1, \dots, \sigma_n)$
- Probability measure over paths:  $\Pr_s^\sigma$



# Example



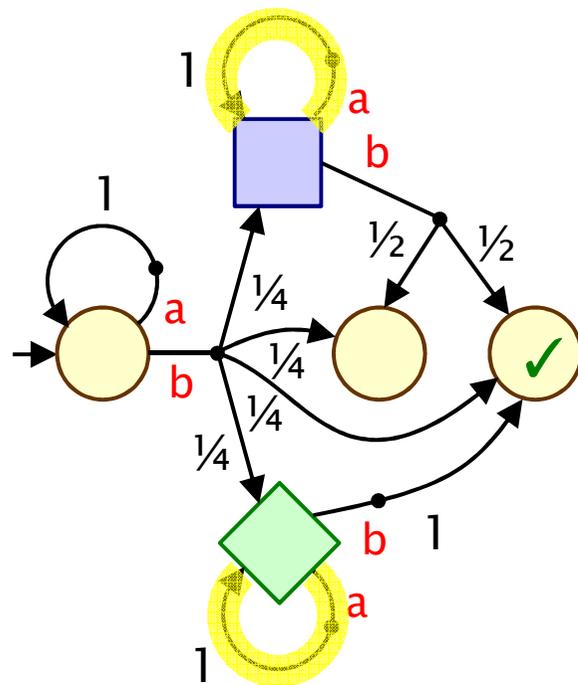
$\langle\langle \bigcirc \rangle\rangle P_{\geq 1/4} [ F \checkmark ]$

**true in initial state**

$\langle\langle \bigcirc \rangle\rangle P_{\geq 1/3} [ F \checkmark ]$

$\langle\langle \bigcirc, \square \rangle\rangle P_{\geq 1/3} [ F \checkmark ]$

# Example



$$\langle\langle \bigcirc \rangle\rangle P_{\geq 1/4} [ F \checkmark ]$$

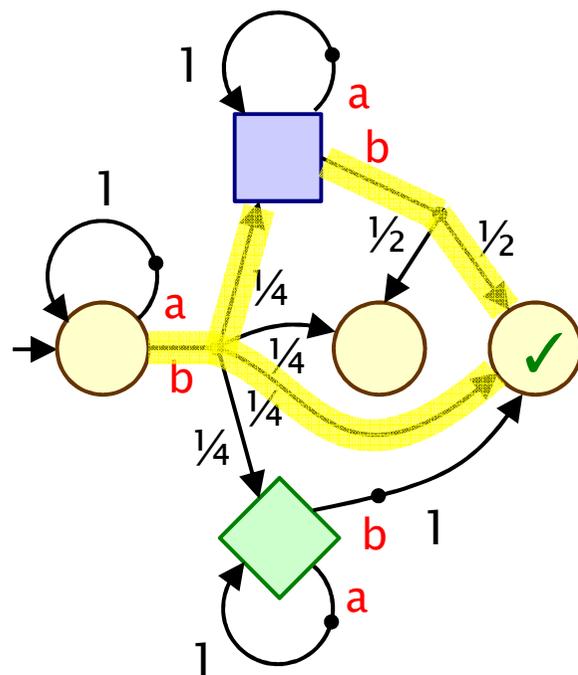
true in initial state

$$\langle\langle \bigcirc \rangle\rangle P_{\geq 1/3} [ F \checkmark ]$$

false in initial state

$$\langle\langle \bigcirc, \square \rangle\rangle P_{\geq 1/3} [ F \checkmark ]$$

# Example



$\langle\langle \bigcirc \rangle\rangle P_{\geq 1/4} [ F \checkmark ]$

**true** in initial state

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**false** in initial state

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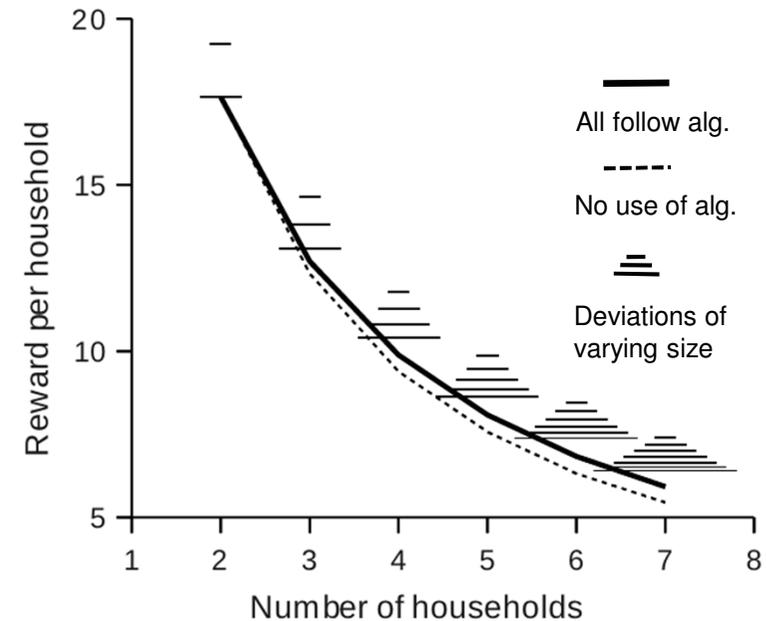
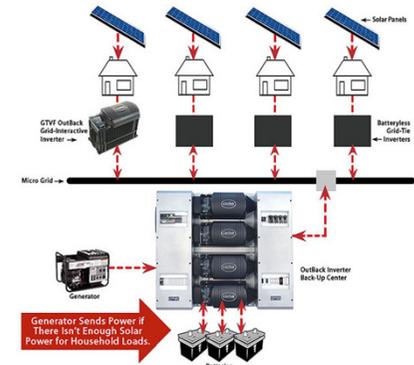
**true** in initial state

# Property specification

- **Basis: the temporal logic rPATL**
  - combines operators from PCTL with rewards (probabilistic, reward), ATL (strategies of player/coalitions) and
- **Examples**
  - $\langle\langle 4,5 \rangle\rangle P < 0.95 [ F \leq 100 \text{ "end"} ]$   
“Players 4 and 5 can ensure that the probability of reaching an “end”-state within 100 time-steps is  $< 0.95$ ”
  - $\langle\langle p1, p3 \rangle\rangle P_{\max}=? [ F \text{ "goal"} ]$   
“The maximum probability with which players p1 and p3 can guarantee that a “goal”-state is reached?”
  - $\langle\langle p1 \rangle\rangle R\{“r”\}_{\max}=? [ F \text{ "success"} ]$   
“Maximum expected value of reward “r” accumulated before reaching “success” that can be guaranteed by player p1?”

# Case study: Energy management

- Energy management protocol for Microgrid
  - Microgrid: local energy management
  - randomised demand management protocol [Hildmann/Saffre'11]
  - probability: randomisation, demand model, ...
- Existing analysis
  - simulation-based
  - assumes all clients are unselfish
- Our analysis
  - stochastic multi-player game
  - clients can cheat (and cooperate)
  - exposes protocol weakness
  - propose/verify simple fix

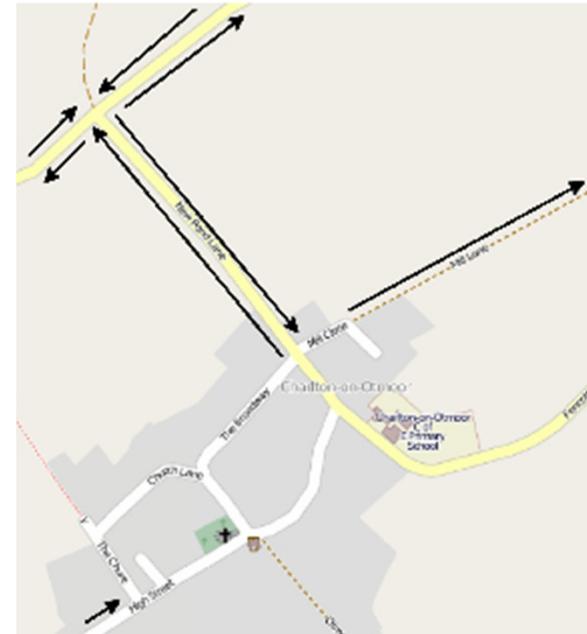


# Multi-objective strategy synthesis

- Explore trade-offs
  - e.g. between performance and resource usage
- Extension of rPATL: **Boolean** combinations of objectives
  - expected total rewards (for stopping games)
  - expected mean-payoffs or ratios (controllable multi-chain)
  - conjunctions of almost sure mean-payoffs/ratios (all games)
- Examples
  - “Player 1 can guarantee that the expected total reward values for reward structures "r1" and "r2" are at least v1 and v2, resp.”  
 $\langle\langle 1 \rangle\rangle ( R\{r1\} \geq v1 [ C ] \ \& \ R\{r2\} \geq v2 [ C ] )$
  - “Player 1 can guarantee that, whenever the expected ratio of long-run average values for "r1" and "c" is at most v1, then the ratio for "r2" and "c" is at least v2  
 $\langle\langle 1 \rangle\rangle ( R\{r1/"c"\} \leq v1 [ S ] \Rightarrow R\{r2/"c"\} \geq v2 [ S ] )$

# Case study: Autonomous urban driving

- Inspired by DARPA challenge
  - represent map data as a stochastic game, with environment **active**, able to select hazards
  - express goals as **conjunctions** of probabilistic and reward properties
  - e.g. “maximise probability of avoiding hazards **and** minimise time to reach destination”
- Solution (PRISM-games 2.0)
  - synthesise a **probabilistic** strategy to achieve the multi-objective goal
  - enable the exploration of **trade-offs** between subgoals
  - applied to synthesise driving strategies for English villages

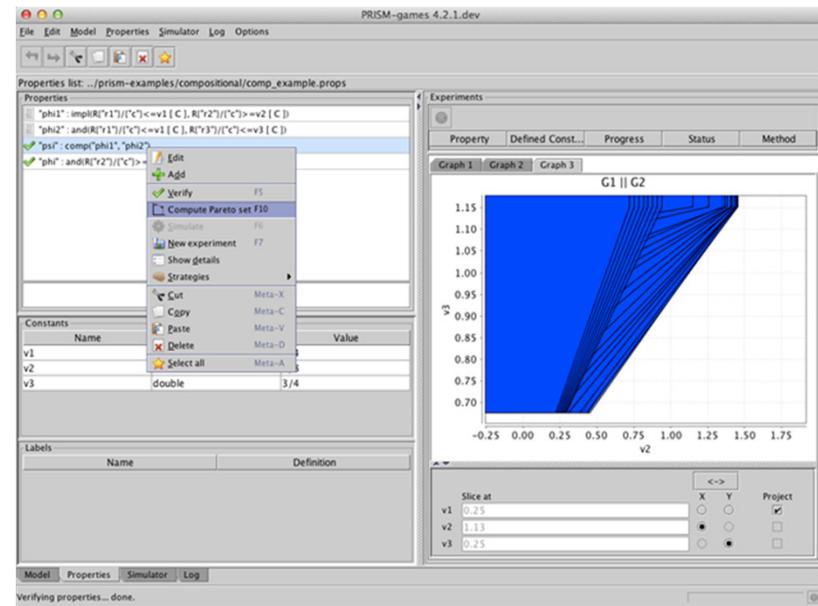


# Compositional strategy synthesis

- **Components**
  - reduce design complexity, increase reliability via redundancy
  - improve scalability of analysis, avoid product state space
- **Assume–guarantee synthesis:**
  - need a strategy for the full system satisfying a **global property**
  - synthesise one strategy per component, for **local properties**
  - use **assume–guarantee rules** to compose local strategies
- **Example:** local strategies for  $G_1 \models \phi^A$  and  $G_2 \models \phi^A \Rightarrow \phi^B$  compose to a global strategy for  $G_1 \parallel G_2 \models \phi^B$
- **Need to extend synthesis methods:**
  - **multi-objective properties** to use in local and global properties
  - long-run properties (e.g. ratios of rewards) to be compatible with **fairness** requirements for assume–guarantee rules

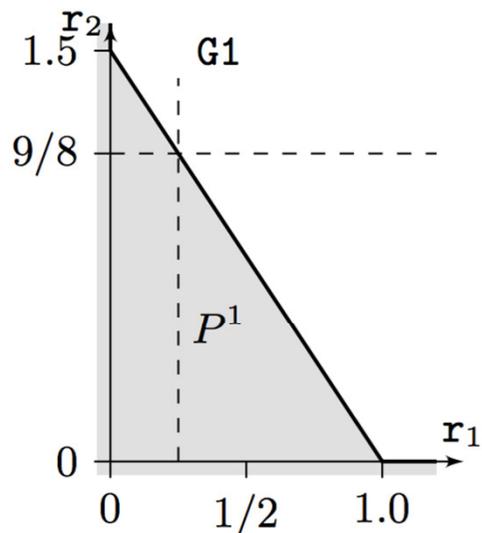
# Computation of Pareto sets

- **Multi-objective strategy synthesis**
  - epsilon-optimal strategies, randomised
  - value iteration over polytopic sets
  - stochastic memory update representation
- **Pareto sets**
  - optimal achievable trade-offs between objectives
- **Visualisation of high-dimensional Pareto sets**
  - projection
  - slicing

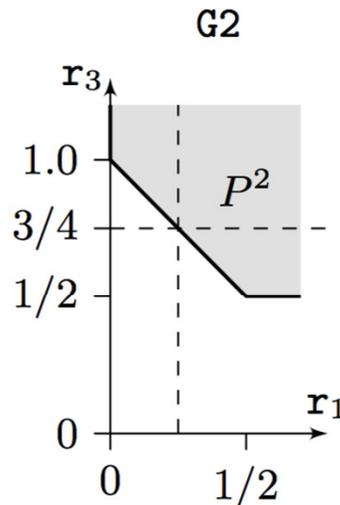


# Compositional strategy synthesis

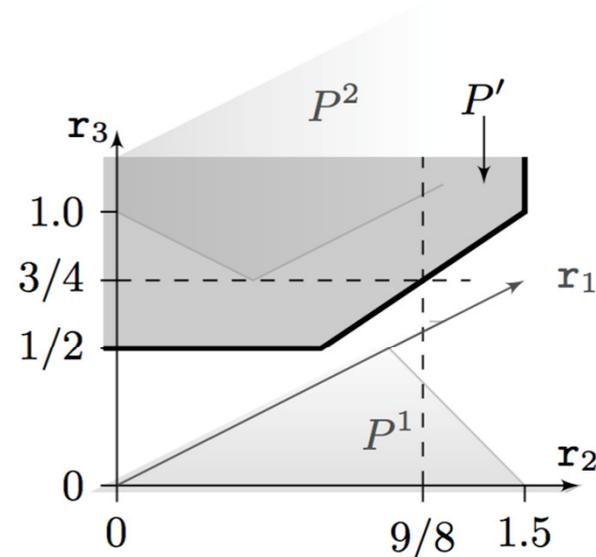
- Based on **assume-guarantee contracts** over component interfaces
- Synthesise **local** strategies for components, then compose into a **global** strategy using assume-guarantee rules
- Under-approximation of Pareto sets



$$\langle\langle 1 \rangle\rangle (\mathbb{R}\{r_1 / c\} \leq_{v_1} [S] \rightarrow \mathbb{R}\{r_2 / c\} \geq_{v_2} [S])$$



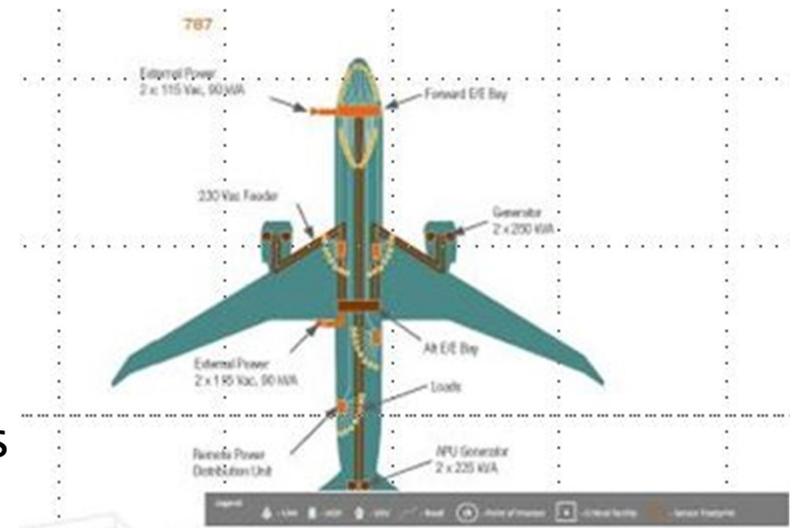
$$\langle\langle 1 \rangle\rangle (\mathbb{R}\{r_1 / c\} \leq_{v_1} [S] \wedge \mathbb{R}\{r_3 / c\} \leq_{v_3} [S])$$



$$\langle\langle 1 \rangle\rangle (\mathbb{R}\{r_2 / c\} \geq_{v_2} [S] \wedge \mathbb{R}\{r_3 / c\} \leq_{v_3} [S])$$

# Case study: Aircraft power distribution

- Consider Honeywell high-voltage AC (HVAC) subsystem
  - power routed from generators to buses through switches
  - represent as a stochastic game, modelling competition for buses, with stochasticity used to model failures
  - specify control objectives in LTL using **longrun average**
  - e.g. “maximise uptime of the buses **and** minimise failure rate”
- Solution (PRISM-games 2.0)
  - **compositional** strategy synthesis
  - enable the exploration of **trade-offs** between uptime of buses and failure rate



# Tool support: PRISM–games 2.0

- **Model checker for stochastic games**
  - integrated into PRISM model checker
  - using new explicit–state model checking engine
- **SMGs added to PRISM modelling language**
  - guarded command language, based on reactive modules
  - finite data types, parallel composition, proc. algebra op.s, ...
- **rPATL added to PRISM property specification language**
  - implemented value iteration based model checking
- **Supports strategy synthesis**
  - single and **multiple** objectives, **Pareto** curve
  - total expected reward, **longrun** average, **ratio rewards**
  - **compositional** strategy synthesis
- **Available now:**
  - <http://www.prismmodelchecker.org/games/>



# Case studies

- Evaluated on several case studies:
  - team formation protocol [CLIMA'11]
  - futures market investor model [McIver & Morgan]
  - collective decision making for sensor networks [TACAS'12]
  - energy management in microgrids [TACAS'12]
  - reputation protocol for user-centric networks [SR'13]
  - DNS bandwidth amplification attack [Deshpande et al]
  - self-adaptive software architectures [Camara, Garlan et al]
- Case studies using PRISM-games 2.0 functionality:
  - autonomous urban driving (multi-objective) [QEST'13]
  - UAV path planning with operator (multi-objective) [ICCPs'15]
  - aircraft electric power control (compositional) [TACAS'15]
  - temperature control (compositional) [Wiltsche PhD]

# Summing up...

- **CPSs take up is fast increasing**
  - implantable, closed-loop and wearable devices
  - software integrated and critical component
  - 24/7 health performance expectation
  - safety-critical context, software failures on the increase
- **Future work**
  - huge models!
  - more general models, more sensor types
  - need approximation, not discretisation
  - symbolic implementation
  - automatically inferring game decomposition and rules
  - synthesis for concurrent and non-zero sum games

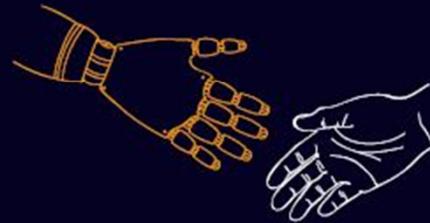
# Challenges

- **Model expressiveness**
  - full class of stochastic hybrid models
- **Partial information**
  - currently largely limited to complete information
- **Model learning and adaptation from data**
  - self-aware, self-adaptive systems
- **Model synthesis from specifications**
  - not just parameter synthesis
- **Scalability and efficiency**
- **Modelling social interactions**
  - e.g. trust
- **Certification**
  - incl. tools and standards

# Announcing...

## SOCIAL TRUST IN AUTONOMOUS ROBOTS

[http://qav.cs.ox.ac.uk/trust\\_in\\_autonomy/](http://qav.cs.ox.ac.uk/trust_in_autonomy/)



**How to** understand,  
formalise and express trust?

Open for poster submissions

Workshop of **Robotics: Science and Systems 2016**

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**EPSRC**  
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  - Oxford Martin School, Institute for the Future of Computing
- See also
  - **VERIWARE** [www.veriware.org](http://www.veriware.org)
  - PRISM [www.prismmodelchecker.org](http://www.prismmodelchecker.org)