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DEVELOPMENT OF SLIDING MODE CONTROLLER FOR SMALL SATELLITE IN PLANETARY ORBITAL ENVIRONMENT FORMATION FLYING MISSIONS

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ABSTRACT

In the recent years the concept of Coulomb Spacecraft Formation (CSF) in Geostationary Orbits (GEO) and other high Earth orbits has gained a lot of attention in the space community. The advances in the field of Coulomb Spacecraft Formations prove that the electrostatic forces of the order of 10-1000 µN generated on the surface of the satellites due to ambient plasma and the photoelectric effect can be effectively utilized to propel satellites flying in close formations. There is an increasing interest to prove the feasibility of this new hybrid propulsion technology through harvesting the naturally available electrostatic forces to propel the satellites in a formation. Advances in propulsion technology will dramatically reduce the mission cost in addition to prolonging the mission lifetime.

Success of a formation flying missions depends not only on the efficiency of the propulsion system but also on the onboard Guidance, Navigation and Control strategies. The authors have done extensive research to develop novel Sliding Mode Controllers (SMC) for implementing Coulomb Spacecraft formations. The inherent robustness of the controller guarantees that the spacecraft is insensitive to a class of bounded disturbances due to solar radiation drag and other perturbation in high altitudes. The previous research utilized a generic, non-linear inertial model to describe the satellite dynamics. This paper describes further progress in modeling and control of CSF for circular orbits around the Earth. The dynamics of the satellites are governed by the Hill-Clohessy-Wiltshire (HCW) equations and a nonlinear, robust controller based on Sliding Mode Control is implemented for demonstrating how a group of satellites can aggregate from their initial positions to form a desired formation. The dynamics of the satellites in the formation is non-linear because it is a combination of the linear HCW equations and the non-linear Columbic forces. Since linearization is not recommended, this non-linearity increases the complexity in designing the sliding mode controller. Collision free navigation is achieved using the Artificial Potential Filed (APF) method **-** a popular path planning technique used in terrestrial robotics. The drift of the formation centre of mass can be prevented by integrating APF with SMC and by eliminating the reaching phase of the controller. The efficacy of the proposed algorithms for CSF will be presented through illustrating a few formation reconfiguration scenarios.

I. INTRODUCTION

Small satellites are highly suitable for formation flying missions, where they operate together as cluster or form multiple geometries and accomplish a task. They have low development cost when compared to large satellites, since commercial off the shelf components can be used. Similarly, their modular nature helps in replacement and up-gradation (1). However collision free navigation and control of satellite formations is a challenging problem which is dealt in this paper.

It has been observed for a long time that certain living beings like birds, animals, bacteria exhibit cooperative behaviour by living in swarms for avoiding predators and increasing the chance of finding food. Operational principles from such systems can be used in engineering for developing distributed cooperative control, coordination, and learning strategies for autonomous agent systems. The general understanding is that the swarming behaviour in living beings is a result of interplay between a long range attraction and short range repulsion between the individuals (2), (3), (4). A similar study on stable swarm aggregations using attraction/repulsion functions from an engineering perspective is done in (5).

Autonomous collision-free reconfiguration is a challenging task and needs to be accomplished with minimal use of on-board power, since it determines the cost and lifetime of the mission. Spacecraft propulsion utilising naturally available electrostatic forces is an emerging technology (6), (7). Another novel method of producing thrust called electromagnetic propulsion is to use electrostatic or electromagnetic forces to accelerate the reaction mass directly (8).

Formation flying requires an intelligent path planner to avoid collisions and reach their target locations. The computationally less expensive and collision-free efficient path planning Artificial Potential Field (APF) method was developed in (9). The robust Sliding Mode Controller (SMC) was first suggested in (10). In (11), the application of APF and SMC for aggregation of swarms is presented. In (5) and (12), various potential functions are studied and compared and their stability analysis is carried out. In the ESA report (13), the first results combining APF for navigation, SMC for controller design and Columb Forces for actuation for the formation of a satellite cluster are presented.

The dynamics of a satellite with respect to another nearby satellite inside a Planetary Orbital Environment (POE) is governed by the Hill-Clohessy-Wiltshire (HCW) equations (14). But novel control and navigation algorithms are first tested in Deep Space (DS); hence the dynamics reduces to a simple double integrator model (15). The control of a cluster of satellites in DS using a robust control algorithm using Sliding Mode Control (SMC) and an intelligent path planning algorithm designed using Artificial Potential Field (APF) method (16), which is widely used for collision avoidance of mobile robots; is discussed in (17). The major focus of this study is to extend this navigation and control strategy to POE.

This paper presents how satellites, whose dynamics are governed by the planetary orbit, can aggregate towards a goal position to form a predefined formation. Based on the knowledge of current position, the APF method will optimise the trajectory to generate the next desired formation which will be achieved using the sliding mode controller. Since sliding mode control guarantees robust performance, the impact of external perturbations like solar wind and internal perturbations like change in mass due to fuel consumption will have negligible impact on the performance of the satellite. Simulation results for demonstrating the effectiveness of this control and navigation strategy have been shown. It is also seen that the combined Centre of Mass of the cluster of satellites does not move during the reconfiguration.

II. BACKGROUND RESULTS

A. Path Planning Using Artificial Potential Field

Let the formation consist of N individual agents, and the position of the i^{th} agent is described by $\boldsymbol{p_i}$. The motion of each agent in the formation is governed by the equation (11):

$$\dot{\boldsymbol{p}}_{\boldsymbol{i}} = -\sum_{j=1, j\neq i}^{N} g(\boldsymbol{p_i} - \boldsymbol{p_j}),\ i = 1, \dots, N, \quad (1)$$

where $g(.)$ is an artificial potential function (12). It is an odd function which represents the sum of the function of attraction and repulsion between the agents. It is needed for aggregation that the attraction term dominates on large distances and for avoiding collisions that the repulsion term dominates on short distances. There is a distance δ at which the attraction and the repulsion balance. The attraction/repulsion function that we consider in this paper is

$$g(\boldsymbol{p_i} - \boldsymbol{p_j}) = (\boldsymbol{p_i} - \boldsymbol{p_j})\left(a_{ij} - b_{ij}\exp\left(-\frac{\|\boldsymbol{p_i} - \boldsymbol{p_j}\|^2}{c_{ij}}\right)\right) \quad (2)$$

where a_{ij}, b_{ij} and c_{ij} are positive constants such that $b_{ij} > a_{ij}$. The term a_{ij} represents the attraction dominated for large distances, whereas the term $b_{ij}\exp\left(-\|\boldsymbol{p_i} - \boldsymbol{p_j}\|^2/c_{ij}\right)$ represents the repulsion and dominates for small distances. The main drawback with $g(.)$ is that it is strongly repulsive but not unbounded for infinitesimally small arguments, which may be needed to avoid collisions. Another drawback is that it has an infinite range, which is inconsistent with biology. The distance $\delta_{ij} = \sqrt{c_{ij}\ln(b_{ij}/a_{ij})}$ is the distance at which the attraction and repulsion balance. The potential function is symmetric and switches sign at a unique distance δ_{ij} (12).

B. Dynamics of Formation Flying Satellites in Planetary Orbit

The ability to accurately model the dynamic behaviour of separated spacecraft formations in orbit around a central body is critical to the success of the mission. In this section, the complete nonlinear equations of motion of a formation consisting of *n* point-mass spacecraft about a closed Keplerian reference orbit are derived. The resulting non-linear differential equations are then linearized about a bound Keplerian (i.e., circular) reference orbit. Under the assumption of a circular reference orbit, the linearized equations of motion reduce to the Hill-Clohessy-Wiltshire (HCW) equations (14).

Let us assume that formation of *n* spacecrafts is located in Earth orbit where each spacecraft is modelled as a point-mass (18). An inertial frame of reference $\boldsymbol{F_N} = [\boldsymbol{n_1} \quad \boldsymbol{n_2} \quad \boldsymbol{n_3}]^T$ is attached to the center of the Earth. The unit vector $\boldsymbol{n_1}$ points toward the vernal equinox, $\boldsymbol{n_3}$ points toward the geographic North Pole, and $\boldsymbol{n_2}$ completes the right-handed triad. The motion of the formation is described with respect to a bound, pure-Keplerian reference orbit, which is a solution of the following differential equation:

$$\ddot{\boldsymbol{R}}_{\boldsymbol{o}} = -\frac{\mu \boldsymbol{R_o}}{||\boldsymbol{R_o}||^3} \tag{3}$$

where ⍰ $= GM.\boldsymbol{R_o}$ is the distance of the from the Centre of Earth to the origin of the orbit frame, which is usually the Centre of Mass of the cluster of satellites.

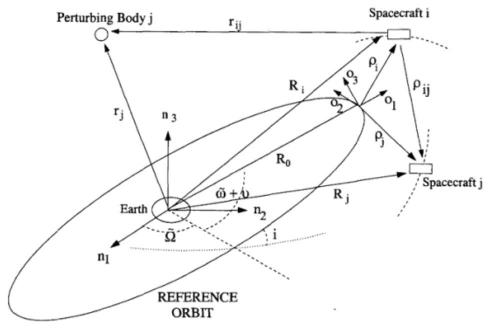


Figure 1: n Spacecrafts in Formation (adapted from (18))

The reference orbit can be described by the orbital elements: *a* (semi-major axis), *e* (eccentricity), *i* (inclination), Ω (longitude of the ascending node), *ω* (argument of perigee), ν (true anomaly), and T (time of perigee passage). $\boldsymbol{F_O} = [\boldsymbol{o_1} \quad \boldsymbol{o_2} \quad \boldsymbol{o_3}]^T$ is the orbit frame, where $\boldsymbol{o_1}$ points anti-nadir, the unit vector $\boldsymbol{o_3}$ points in the direction of the orbit normal, and $\boldsymbol{o_2}$ completes the right-handed triad. The equations of motion of the formation; linearized about the reference orbit, is given by (15):

$$\begin{aligned} &\ddot{x}_i - 2\boldsymbol{\omega_o}\dot{y}_i - \frac{2\mu x_i}{\boldsymbol{R_o^3}} - \boldsymbol{\omega_o^2}x_i - \dot{\boldsymbol{\omega}}_{\boldsymbol{o}}y_i + \ddot{\boldsymbol{R}}_{\boldsymbol{o}} - \boldsymbol{R_o}\boldsymbol{\omega_o^2} + \frac{\mu}{\boldsymbol{R_o^2}} = \frac{Q_{x_i}}{m_i} \\ &\ddot{y}_i + 2\boldsymbol{\omega_o}\dot{x}_i + \frac{\mu}{\boldsymbol{R_o^3}}y_i - \boldsymbol{\omega_o^2}y_i + 2\boldsymbol{\omega_o}\dot{\boldsymbol{R}}_{\boldsymbol{o}} + \dot{\boldsymbol{\omega}}_{\boldsymbol{o}}\boldsymbol{R_o} = \frac{Q_{y_i}}{m_i} \\ &\ddot{z}_i + \frac{\mu}{\boldsymbol{R_o^3}}z_i = \frac{Q_{z_i}}{m_i} \end{aligned} \tag{4}$$

where $\boldsymbol{R_o}$ and $\boldsymbol{\omega_o}$ are considered prescribed time-varying functions in the above equations. Assuming the reference orbit to be circular, the following relations hold:

$$\omega_o^2 = \frac{\mu}{\boldsymbol{R_o^3}}, \qquad \dot{\boldsymbol{R}}_{\boldsymbol{o}} = 0, \qquad \ddot{\boldsymbol{R}}_{\boldsymbol{o}} = 0 \tag{5}$$

Under the circular reference orbit assumption the linearized equations of motion reduce to the Hill-Clohessy-Wiltshire (HCW) equations.

$$\begin{aligned} &\ddot{x}_i - 2\boldsymbol{\omega_o}\dot{y}_i - 3\boldsymbol{\omega_o^2}x_i = \frac{Q_{x_i}}{m_i} \\ &\ddot{y}_i + 2\boldsymbol{\omega_o}\dot{x}_i = \frac{Q_{y_i}}{m_i} \\ &\ddot{z}_i + \boldsymbol{\omega_o^2}z_i = \frac{Q_{z_i}}{m_i} \end{aligned} \tag{6}$$

These HCW equations will be used henceforth in this paper as the dynamics model for the position control of satellites in formation flying around planetary orbital environment.

III. DESIGN OF SLIDING MODE CONTROLLER

The APF navigational algorithm and SMC control algorithm are implemented on a simple double integrator model since the satellites are in Deep Space (17). In this section, the same navigation and control algorithms are to be used for a swarm in Planetary Orbital Environment (POE).

Let $\boldsymbol{p_i} = [x_i \; y_i \; z_i]^T$, be the position vector of the i^{th} satellite from the origin, in orbit frame as defined in section II.B. Let $\boldsymbol{F_i} = [F_{xi} \quad F_{yi} \quad F_{zi}]^T$ be the forces produced onboard the i^{th} satellite in all three axes respectively. The open loop dynamic equation for the i^{th} satellite in the orbit frame is given by the HCW equations:

$$\ddot{\boldsymbol{p}}_{\boldsymbol{i}} = \begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \\ \ddot{z}_i \end{bmatrix} = \begin{bmatrix} 2\Omega\dot{y}_i + 3\Omega^2 x_i + F_{xi} \\ -2\Omega\dot{x}_i + F_{yi} \\ -\Omega^2 z_i + F_{zi} \end{bmatrix} \tag{7}$$

The Artificial Potential Field (APF) equation gives the switching hyperplane $\boldsymbol{s_i}$ (3X1 vector) for the i^{th} satellite is given by (13):

$$\boldsymbol{s_i} = \dot{\boldsymbol{p}}_{\boldsymbol{i}} + g(\boldsymbol{p_i} - \boldsymbol{p_k}), \qquad k = 1, \dots, n \tag{8}$$

We will use the artificial potential field equation [2] described in section II.A. Hence we get:

$$\boldsymbol{s_i} = \dot{\boldsymbol{p}}_{\boldsymbol{i}} + \sum_{j=1}^{n} (\boldsymbol{p_i} - \boldsymbol{p_j}) * \left(a_{ij} - b_{ij} \exp\left(-\frac{||\boldsymbol{p_i} - \boldsymbol{p_j}||^2}{c_{ij}} \right) \right) \tag{9}$$

$$i = 1, \dots, n \text{ and } i \neq j$$

where a_{ij}, b_{ij} and c_{ij} are the tuning parameters for APF. If δ_{ij} is the final distance to be maintained between the two satellites, then $a_{ij} = b_{ij} \exp(-\delta_{ij}{}^2/c_{ij})$.

In SMC, the Sliding mode should start in finite time. Hence we need $s\dot{s} \leq -\eta|s|$ which will ensure the sliding mode is reached in finite time. This is ensured by the reaching law. The constant plus proportional rate reaching law as given in (19), is $\dot{s} = -\gamma s - \epsilon \text{ sgn}(s)$, where s is a scalar and γ and ε are tuning parameters. In order to reduce the chattering phenomenon, the $\mathrm{sgn}(s)$ term is replaced by a smooth approximation using $\mathrm{sat}(s)$. The constant plus proportional rate reaching law for $\boldsymbol{s_i}$ (3X1 vector) is given by (20):

$$\dot{\boldsymbol{s}}_i = -\gamma_i \boldsymbol{s_i} - \begin{bmatrix} \varepsilon_{i,x} & 0 & 0 \\ 0 & \varepsilon_{i,y} & 0 \\ 0 & 0 & \varepsilon_{i,z} \end{bmatrix} * \boldsymbol{f}sat(\boldsymbol{s_i}, \phi) \qquad (10)$$

$$\text{where } \boldsymbol{f}sat(\boldsymbol{s_i}, \phi) = \left[sat\left(\frac{\boldsymbol{s_{i,x}}}{\phi}\right) \quad sat\left(\frac{\boldsymbol{s_{i,y}}}{\phi}\right) \quad sat\left(\frac{\boldsymbol{s_{i,z}}}{\phi}\right) \right]^T$$

where $\gamma, \varepsilon_x, \varepsilon_y, \varepsilon_z$ are the tuning parameters and ϕ is the boundary layer around the sliding surface, within which a proportional based and not a switching based controller is used. Differentiating equation (9) and equating it to equation (10):

$$-\boldsymbol{\gamma_i s_i} - \begin{bmatrix} \varepsilon_{i,x} & 0 & 0 \\ 0 & \varepsilon_{i,y} & 0 \\ 0 & 0 & \varepsilon_{i,z} \end{bmatrix} * \boldsymbol{f}sat(\boldsymbol{s_i}, \phi)$$
$$= \ddot{\boldsymbol{p}}_i + \sum_{j=1}^{n} \left((\dot{\boldsymbol{p}}_i - \dot{\boldsymbol{p}}_j) * \left(a_{ij} - b_{ij} \exp\left(-\frac{\left|\left|\boldsymbol{p_i} - \boldsymbol{p_j}\right|\right|^2}{c_{ij}} \right) \right) \right)$$
$$+ \sum_{j=1}^{n} \left(\frac{b_{ij}}{c_{ij}} (\boldsymbol{p_i} - \boldsymbol{p_j}) * \left[\exp\left(-\frac{\left|\left|\boldsymbol{p_i} - \boldsymbol{p_j}\right|\right|^2}{c_{ij}} \right) * \frac{d}{dt} \left|\left|\boldsymbol{p_i} - \boldsymbol{p_j}\right|\right|^2 \right] \right) \qquad (11)$$

Substituting for $\ddot{\boldsymbol{p}}_i$ from equation (7) into equation (11), to find the control input:

$$\begin{bmatrix} F_{xi} \\ F_{yi} \\ F_{zi} \end{bmatrix} = -\boldsymbol{\gamma_i s_i} - \begin{bmatrix} \varepsilon_{i,x} & 0 & 0 \\ 0 & \varepsilon_{i,y} & 0 \\ 0 & 0 & \varepsilon_{i,z} \end{bmatrix} * \boldsymbol{f}sat(\boldsymbol{s_i}, \phi) - \begin{bmatrix} 2\Omega \dot{y}_i + 3\Omega^2 x_i \\ -2\Omega \dot{x}_i \\ -\Omega^2 z_i \end{bmatrix}$$
$$- \sum_{j=1}^{n} \left(\frac{b_{ij}}{c_{ij}} (\boldsymbol{p_i} - \boldsymbol{p_j}) \left[\exp\left(-\frac{\left|\left|\boldsymbol{p_i} - \boldsymbol{p_j}\right|\right|^2}{c_{ij}} \right) * \frac{d}{dt} \left|\left|\boldsymbol{p_i} - \boldsymbol{p_j}\right|\right|^2 \right] \right)$$
$$- \sum_{j=1}^{n} \left((\dot{\boldsymbol{p}}_i - \dot{\boldsymbol{p}}_j) * \left(a_{ij} - b_{ij} \exp\left(-\frac{\left|\left|\boldsymbol{p_i} - \boldsymbol{p_j}\right|\right|^2}{c_{ij}} \right) \right) \right) \qquad (12)$$

The closed loop dynamics is obtained by substituting the control input (12) into the original system equations (7) and shown below:

$$\begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \\ \ddot{z}_i \end{bmatrix} = -\boldsymbol{\gamma_i s_i} - \begin{bmatrix} \varepsilon_{i,x} & 0 & 0 \\ 0 & \varepsilon_{i,y} & 0 \\ 0 & 0 & \varepsilon_{i,z} \end{bmatrix} * \boldsymbol{f}sat(\boldsymbol{s_i}, \phi)$$
$$- \sum_{j=1}^{n} \left(\frac{b_{ij}}{c_{ij}} (\boldsymbol{p_i} - \boldsymbol{p_j}) \left[exp\left(-\frac{\left|\left|\boldsymbol{p_i} - \boldsymbol{p_j}\right|\right|^2}{c_{ij}} \right) * \frac{d}{dt} \left|\left|\boldsymbol{p_i} - \boldsymbol{p_j}\right|\right|^2 \right] \right)$$
$$- \sum_{j=1}^{n} \left((\dot{\boldsymbol{p}}_i - \dot{\boldsymbol{p}}_j) * \left(a_{ij} - b_{ij} \exp\left(-\frac{\left|\left|\boldsymbol{p_i} - \boldsymbol{p_j}\right|\right|^2}{c_{ij}} \right) \right) \right) \qquad (13)$$

Hence the closed loop dynamics equation is independent of the original system dynamics. Hence the system will be robust when it is on the sliding surface, since its closed loop dynamics has no terms from the open loop dynamics and the external disturbances can't affect its closed loop dynamics.

IV. STABILITY OF THE FORMATION

It is essential to ensure that the sliding surface is stable, i.e. when the satellites are sliding on the sliding surface, they should move towards the formation. Hence, assuming $\boldsymbol{s_i} = 0$ in equation [7], and D_{ij} is the distance between satellite *i* and *j*, we get:

$$\dot{\boldsymbol{p}}_i = -\sum_{j=1}^{n} (\boldsymbol{p_i} - \boldsymbol{p_j}) * \left(a_{ij} - b_{ij} \exp\left(-\frac{D_{ij}^2}{c_{ij}} \right) \right) \qquad (14)$$

When we write equation [12] in state space form by substituting, $\boldsymbol{p_i} = [x_i \; y_i \; z_i]^T$, we get:

$$[\dot{x}_1 \quad \dot{y}_1 \quad \dot{z}_1 \quad \dot{x}_2 \quad \dot{y}_2 \quad . \quad . \quad \dot{y}_n \quad \dot{z}_n]^T$$
$$= \boldsymbol{A}[x_1 \quad y_1 \quad z_1 \quad x_2 \quad y_2 \quad . \quad . \quad y_n \quad z_n]^T \qquad (15)$$

where $\boldsymbol{A}$ is a matrix of 3nX3n size. The terms in $\boldsymbol{A}$ will include all the constant terms $\left(a_{ij} - b_{ij} \exp\left(-\frac{D_{ij}^2}{c_{ij}} \right) \right)$. The states should not asymptotically tend to zero; but tend towards the formation distance δ_{ij}. Hence the following two conditions must be satisfied:

- If all $D_{ij} > \delta_{ij}$, then all the eigenvalues of $\boldsymbol{A}$ are –ve, because all states should converge the origin's direction
- If all $D_{ij} < \delta_{ij}$, then all the eigenvalues of $\boldsymbol{A}$ are +ve, because all states should diverge from origin's direction

Hence we must choose the variables a_{ij}, b_{ij} and c_{ij} such that the $\boldsymbol{A}$ matrix will follow the above rules. There is another condition when some $D_{ij} > \delta_{ij}$ and some $D_{ij} < \delta_{ij}$, in this case some of the eigenvalues will be +ve and the rest will be –ve. Stability analysis for a select case is shown in the next section.

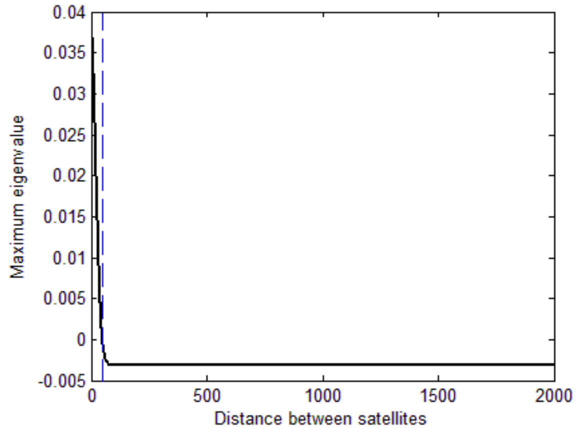
IV. SIMULATION RESULTS

In the above sections, the theory for sliding mode controller design using artificial potential field method for navigation is presented. The stability analysis for the sliding surface is also presented. In this section, simulation results shall be presented for a particular case, where four satellites, each weighing 10kg, are brought from a large distance to form a tetrahedron formation with inter-satellite separation of 50 meters and the formation is maintained. First the APF parameters are found such that they satisfy the stability criterion. Hence the *b* and *c* matrices used in this simulation are

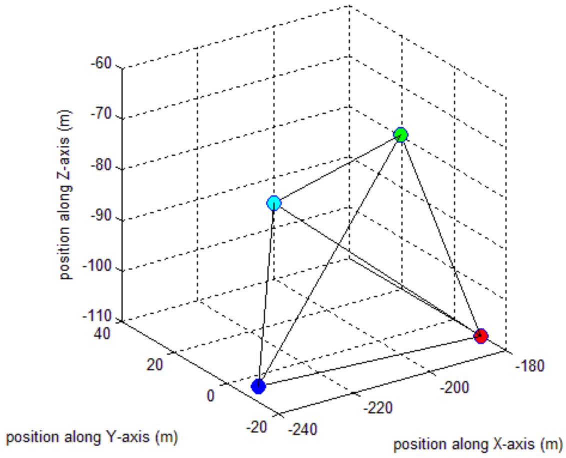
$$b = \begin{bmatrix} 0 & 0.01 & 0.01 & 0.01 \\ 0.01 & 0 & 0.01 & 0.01 \\ 0.01 & 0.01 & 0 & 0.01 \\ 0.01 & 0.01 & 0.01 & 0 \end{bmatrix},$$

$$c = \begin{bmatrix} 0 & 800 & 800 & 800 \\ 800 & 0 & 800 & 1000 \\ 1100 & 1100 & 0 & 900 \\ 1100 & 1000 & 900 & 0 \end{bmatrix}$$

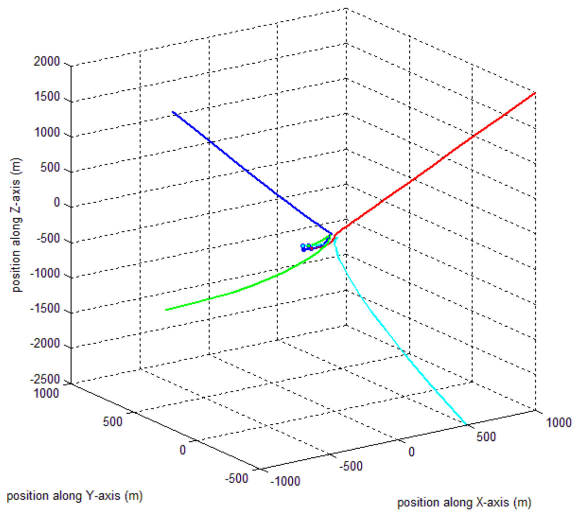
The a matrix is found using $a_{ij} = b_{ij} \exp(-\delta_{ij}{}^2 / c_{ij})$, where δ_{ij} is 50m for all i and j. Fig. 2 (a) shows the plot of the maximum eigenvalue of $\boldsymbol{A}$ matrix for these APF variables when the inter-satellite distance if varied from 1m to 1000m. In the plot the blue dotted line marks the 50m on X axis. As expected, for all inter-satellite distances less than 50m, the maximum eigenvalue of $\boldsymbol{A}$ is +ve and for all inter-satellite distances more than 50m, the maximum eigenvalue of $\boldsymbol{A}$ is –ve.

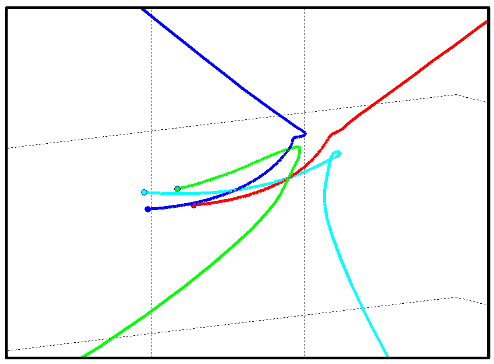


(a)



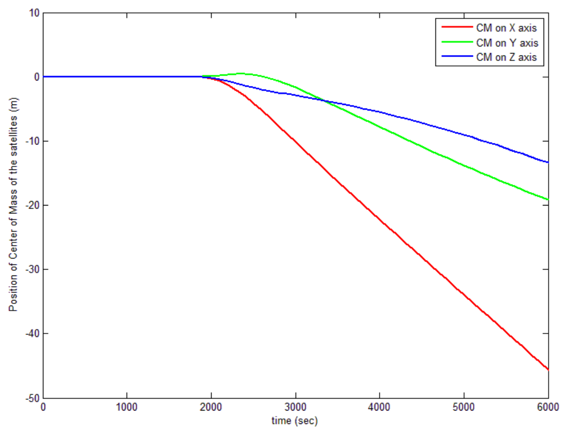
(b)



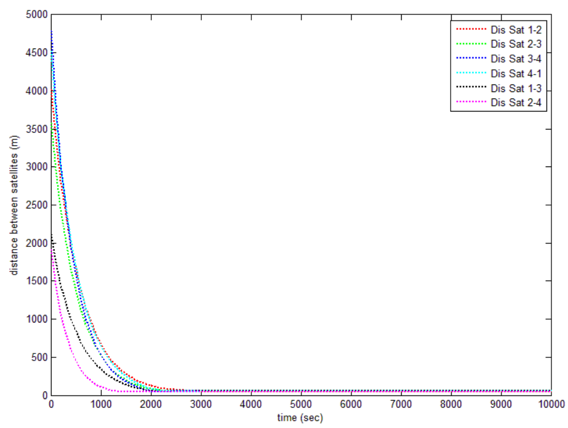


(close-up view)

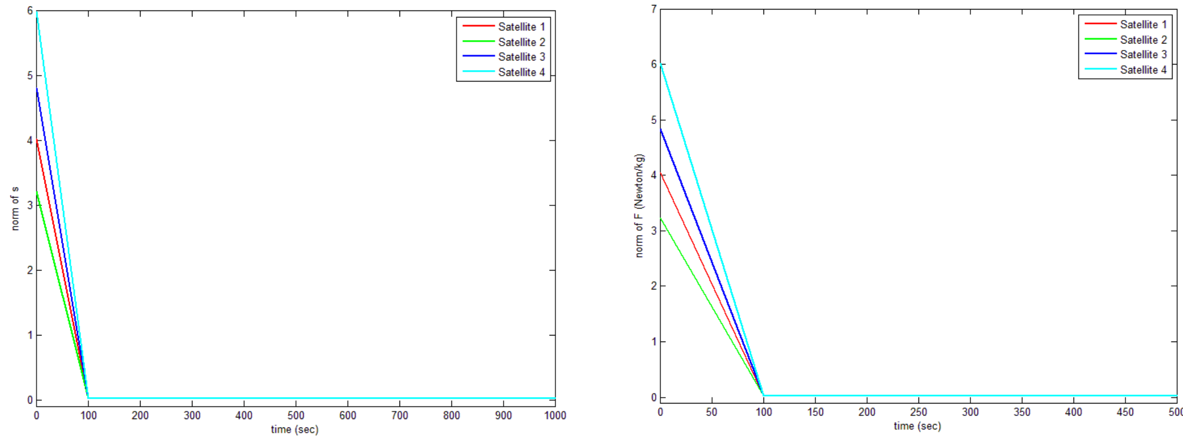
(c)



(d)



(e)



(f) **(g)**

Figure 2: Simulation Results for formation flying in POE: (a) Stability Analysis (b) Final position of the 4 satellites (c) Motion of Satellites from initial conditions to the formation with a close-up of the motion after the formation is attained (d) Motion of Centre of Mass (e) Distance between the satellites with time (f) Norm of the sliding surface s (g) Norm of the Force applied per unit mass

The four satellites start from an initial separation distance of approximately 1.5-2kms. The exact initial conditions are:

- [1000, -500, 2000] for satellite 1, marked in red
- [-500, 800, -1500] for satellite 2, marked in green
- [-1000, 200, 2000] for satellite 3, marked in blue
- [500, -500, -2500] for satellite 4, marked in cyan

Some of the other parameters used for the simulation are $\gamma = 1.0, \epsilon = 0.01$ and $\phi = 1.0$. The same values for γ, ϵ, ϕ are used for all satellites. The simulation was executed for 20,000 seconds and the results are shown in Fig 2.

Fig. 2 (b) shows the final position of the formation, which is tetrahedron. The final inter-satellite separation is:

	Sat 1	Sat 2	Sat 3	Sat 4
Sat 1	0	47.4	54.7	53.7
Sat 2	47.4	0	56.0	50.2
Sat 3	54.7	56.0	0	52.7
Sat 4	53.7	50.2	52.7	0

The final inter-satellite distances are well within tolerable limits. Fig. 2 (c) shows the trajectory of the four satellites as they head towards the formation. With the help of the close-up view, it can inferred that the satellite first form the formation and then slowly drift in one particular direction. Although the authors are not confident as to the cause of the drift, but it is probably caused by the unbalanced pseudo forces acting on the system since the orbit frame is a non-inertial frame. Fig. 2. (d) shows the Centre of Mass position changing very little when the satellites are aggregating towards the formation, but once the formation is achieved the CM drifts steadily. The formation is successfully achieved by 2500 seconds as can be inferred from the inter-satellite distances plotted in Fig 2 (e). In Fig. 2 (f), the reaching phase is visible the system hits the sliding surface within approximately 100 seconds. Thereafter the system continue in the sliding surface towards the formation. Fig. 2. (g) shows the plot of the Force per unit weight expended by the individual satellites for this control to be achieved. The authors have noted that the Force requirement is rather large and further research is in progress to reduce the initial Force requirement.

V. CONCLUSION

This paper presents the navigation and control algorithm for satellites in planetary orbital environment. The popular HCW equations are used for the satellite dynamics, APF is used for navigation and SMC is used for controller design. A novel method of stability analysis is also proposed. Simulation results show that the controller is indeed stable and efficient. Although the overall response is a bit aggressive and the formation is achieved within 2500 seconds, which is not practically possible due to actuator limitations. Moreover, state feed-back is assumed, which is usually not the case in practical systems. Hence further research is needed for developing these navigation and control algorithms for practical applications.

VI. ACKNOWLEDGEMENTS

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