Lecture 16: Planning III (Decision-Making I)

Professor Katie Driggs-Campbell

April 9, 2024

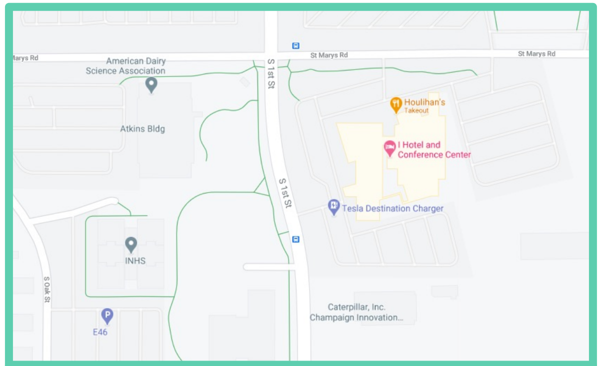
ECE484: Principles of Safe Autonomy

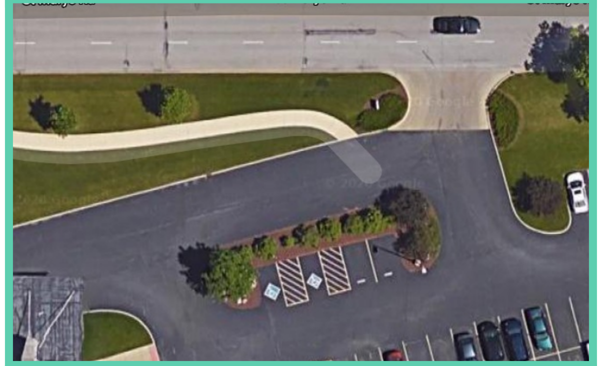
Administrivia

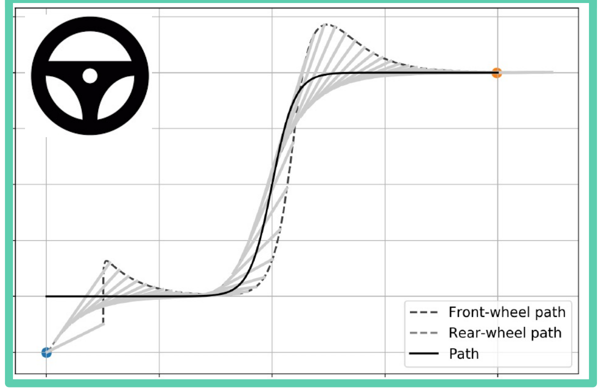
- Upcoming due dates:
  - Final Presentations in class on 4/23 and 4/25
  - Final Video due 5/3
- Exam on 4/18 at 7pm
  - Email me ASAP about conflict exams
  - Make reservation in testing center for DRES accommodations
  - Practice questions will be posted on CampusWire today
  - No cheatsheet will be needed
  - CA review session on Friday 4/19
  - In-class review session on Tuesday 4/16
- Prof. DC OH by appointment next week (4/16)
  - Otherwise in 260 CSL

Typical planning and control modules

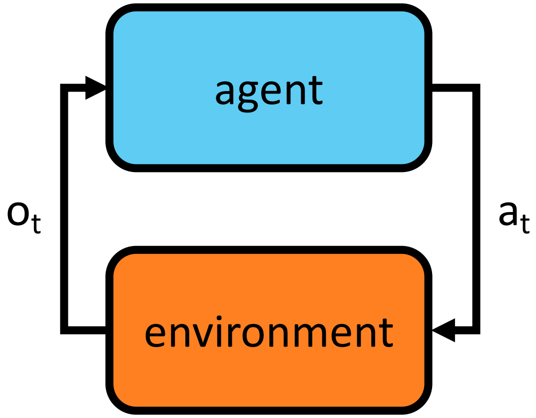
- Global navigation and planner
  - Find paths from source to destination with static obstacles
  - Algorithms: Graph search, Dijkstra, Sampling-based planning
  - Time scale: Minutes
  - Look ahead: Destination
  - Output: reference center line, semantic commands
- Local planner
  - Dynamically feasible trajectory generation
  - Dynamic planning w.r.t. obstacles
  - Time scales: 10 Hz
  - Look ahead: Seconds
  - Output: Waypoints, high-level actions, directions / velocities
- Controller
  - Waypoint follower using steering, throttle
  - Algorithms: PID control, MPC, Lyapunov-based controller
  - Lateral/longitudinal control
  - Time scale: 100 Hz
  - Look ahead: current state
  - Output: low-level control actions

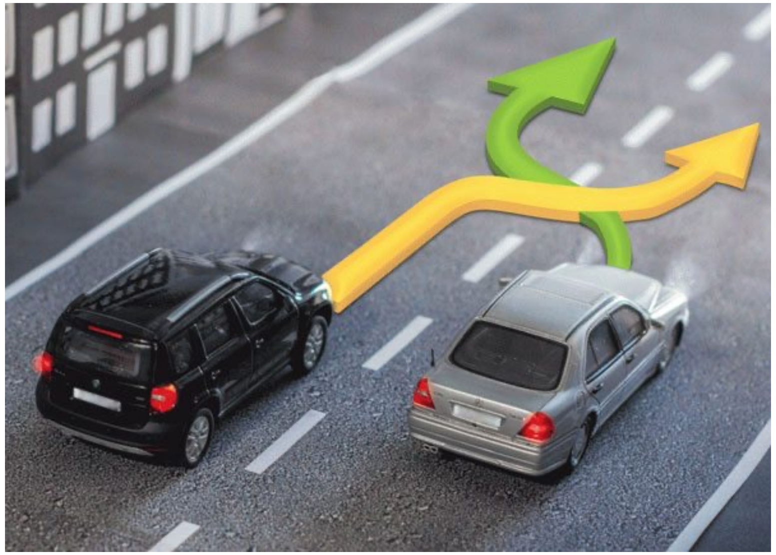


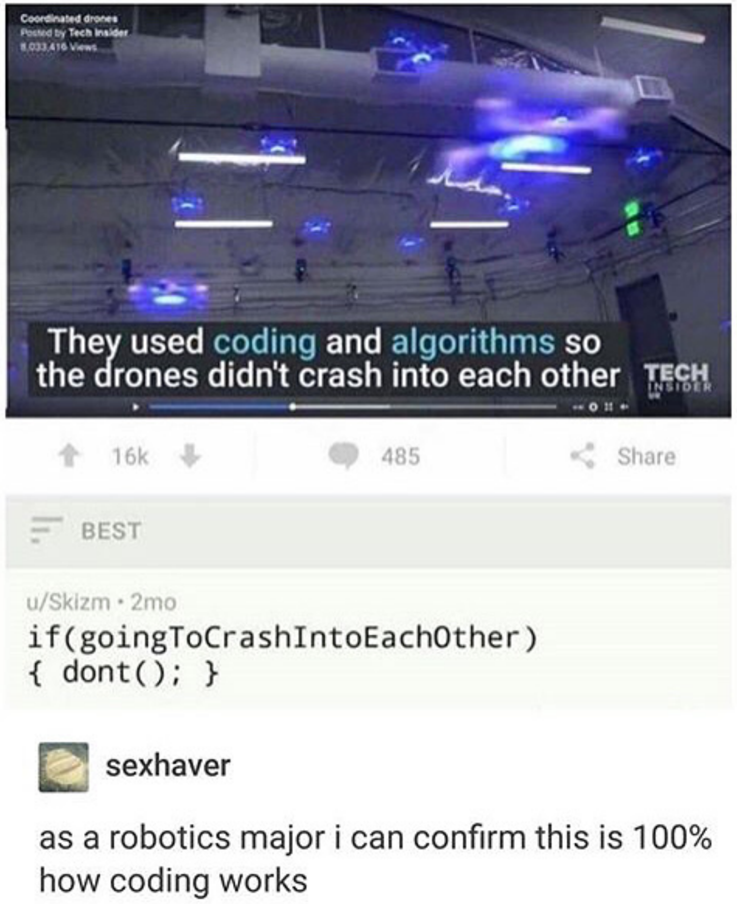




High-Level Decision-Making

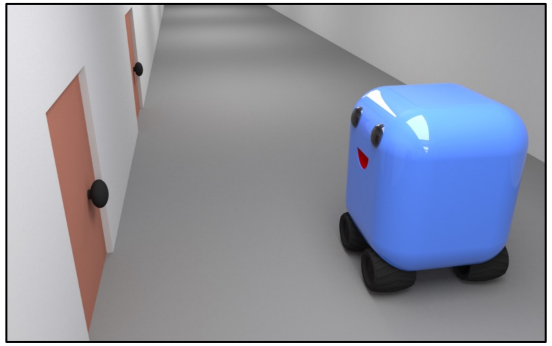


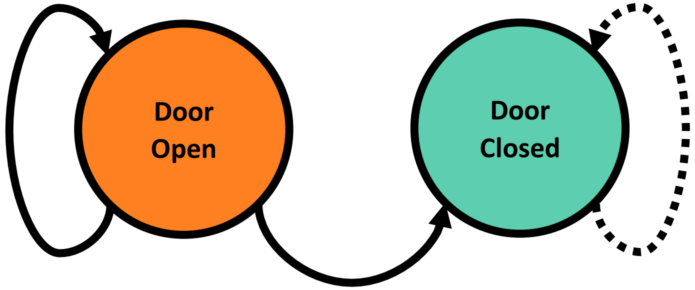




From Filtering to Decision-Making

Recall: Filtering allows us to recursively update our belief about some state





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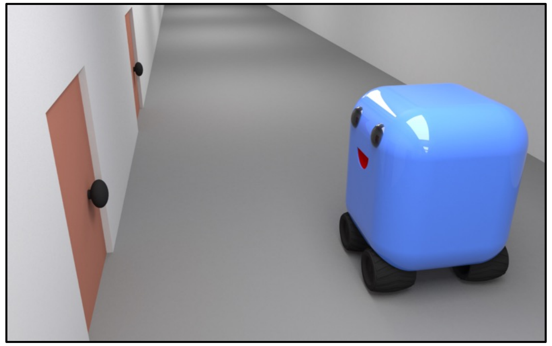

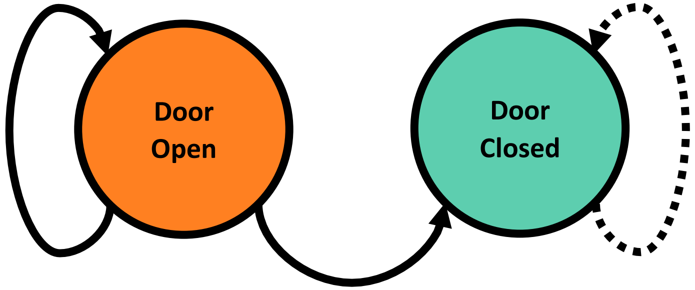

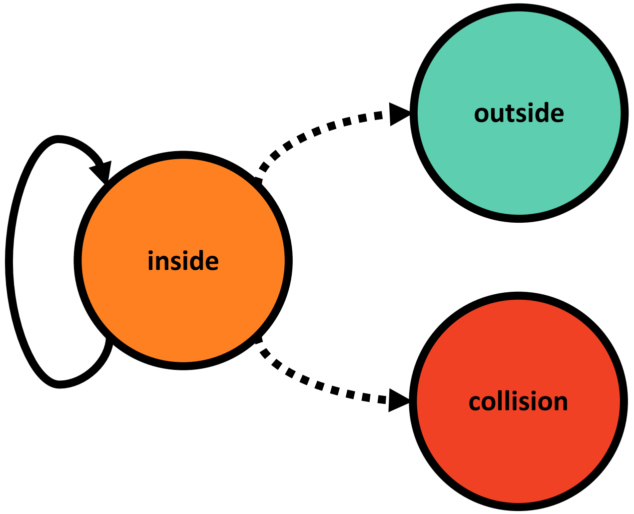

Decision-making helps us reason about what actions we should take

Today's Plan

- Possible solutions for decision-making
- Markov Decision Processes
- MDP Policies and Value Iteration

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Decision-Making Methods

1. Explicit programming
   - Ex: if/then statements
   - → Heavy burden on designer

Heuristic Method for Lane Changing: MOBIL

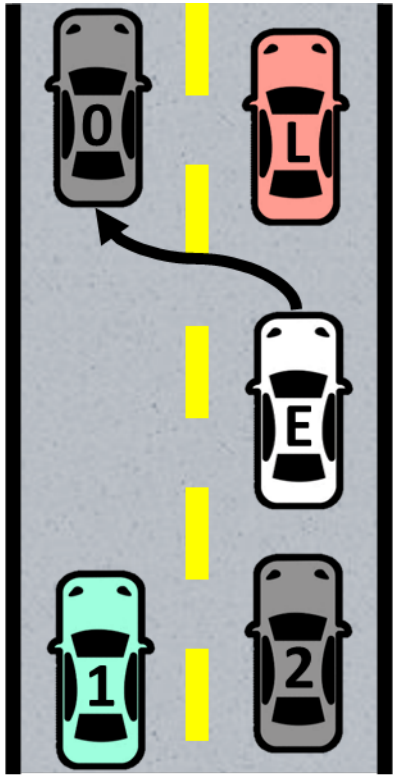
- Safety criterion:

$$\tilde{a}_E \geq -b_{safe}$$

- Decision rule:

$$\tilde{a}_E - a_E + p(\tilde{a}_1 - a_1 + \tilde{a}_2 - a_2) > \Delta a_{th}$$

- Politeness factor, p: 0.35
- Safe braking limit, b_{safe}: $2\ {}^{m}/_{s^2}$
- Acceleration threshold: $0.1\ {}^{m}/_{s^2}$
- Look-ahead horizon: $30m$



Decision-Making Methods

1. Explicit programming
   - Ex: if/then statements

   → Heavy burden on designer
2. Supervised learning
   - Ex: imitation learning

   → Generalizing is often a challenge
3. Optimization / optimal control
   - Ex: MPC

   → Requires a high-fidelity model and lots of computation
4. Planning
   - Given a stochastic model, how to algorithmically determine best policy?
5. Reinforcement Learning
   - If model is unknown (or very complex), learn policy through experience

Today's Plan

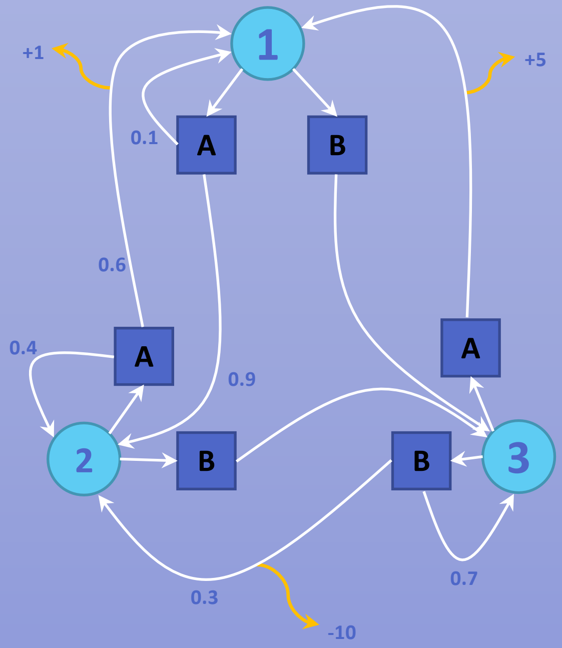
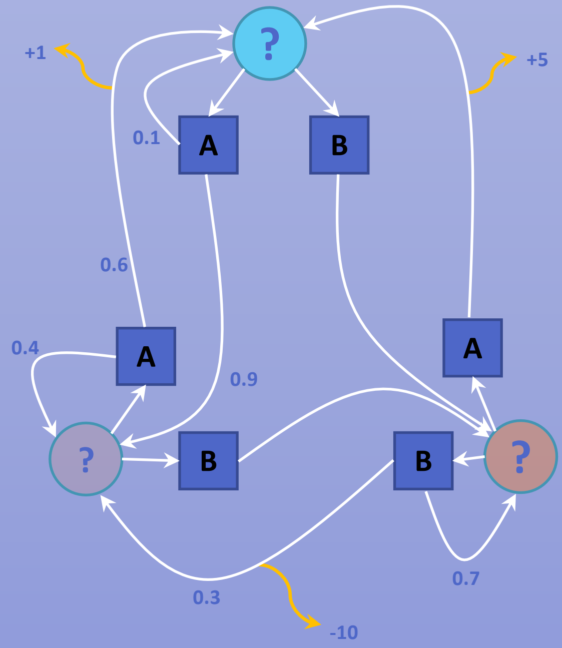
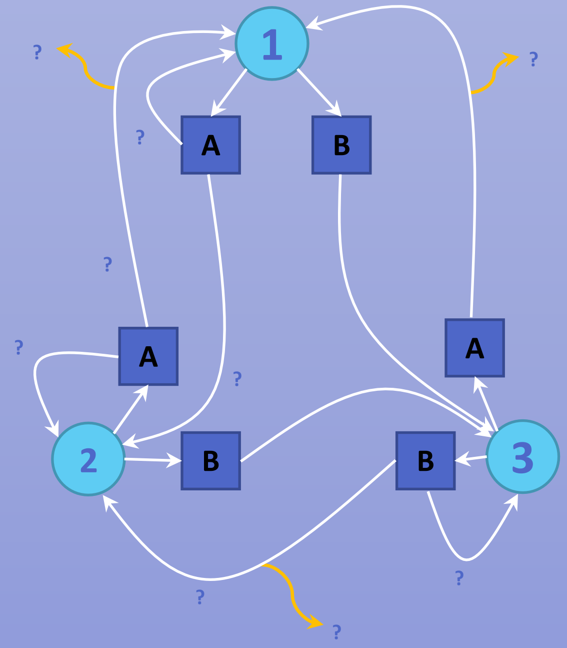
- Introduction to decision-making
- Markov Decision Processes
- MDP Policies and Value Iteration
- Simple Example

Markov Decision Processes (MDPs)

Uncertainty in Motion

- **Markov Decision Processes (MDPs)** model the AV and environment assuming full observability
  - $P(z|x)$: *deterministic* and bijective
  - $P(x'|x,u)$: may be nondeterministic
  - Must incorporate uncertainty into the planner and generate actions for each state
- A policy for action selection is defined for all states

Markov Models

<table>
<tr><th>Markov Decision Processes (MDP)</th><th>Partially Observable MDP</th><th>Reinforcement Learning</th></tr>
<tr><td>
</td><td>
</td><td>
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<tr><td>Uncertainty in effects of actions</td><td>Uncertainty in current state</td><td>Uncertainty in model</td></tr>
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Markov Assumptions and Common Violations

Markov Assumption postulates that past and future data are independent if you know the current state.

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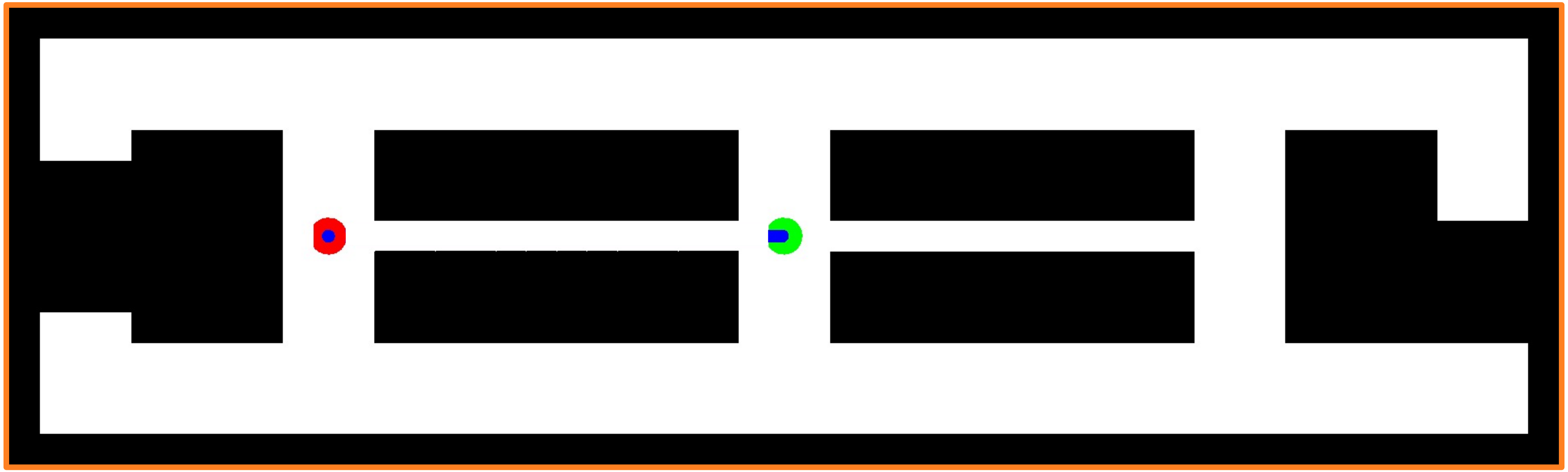
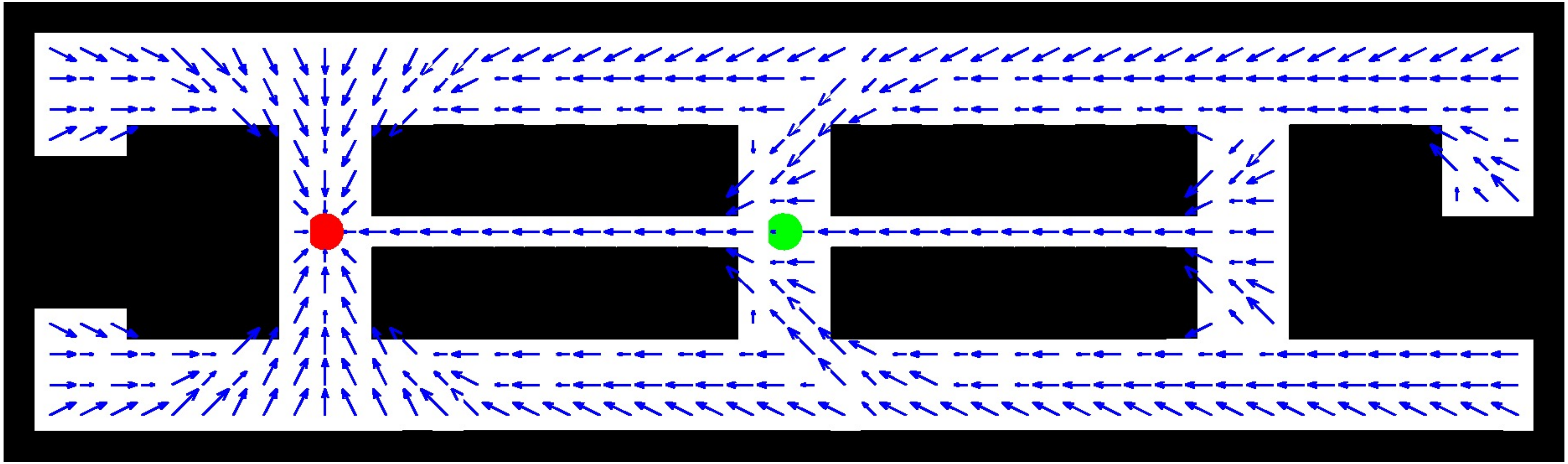
What are some common violations?

- Unmodeled dynamics in the environment not included in state
  - E.g., moving people and their effects on sensor measurements in localization
- Inaccuracies in the probabilistic model
  - E.g., error in the map of a localizing agent or incorrect model dynamics
- Approximation errors when using approximate representations
  - E.g., discretization errors from grids, Gaussian assumptions
- Variables in control scheme that influence multiple controls
  - E.g., the goal or target location will influence an entire sequence of control commands

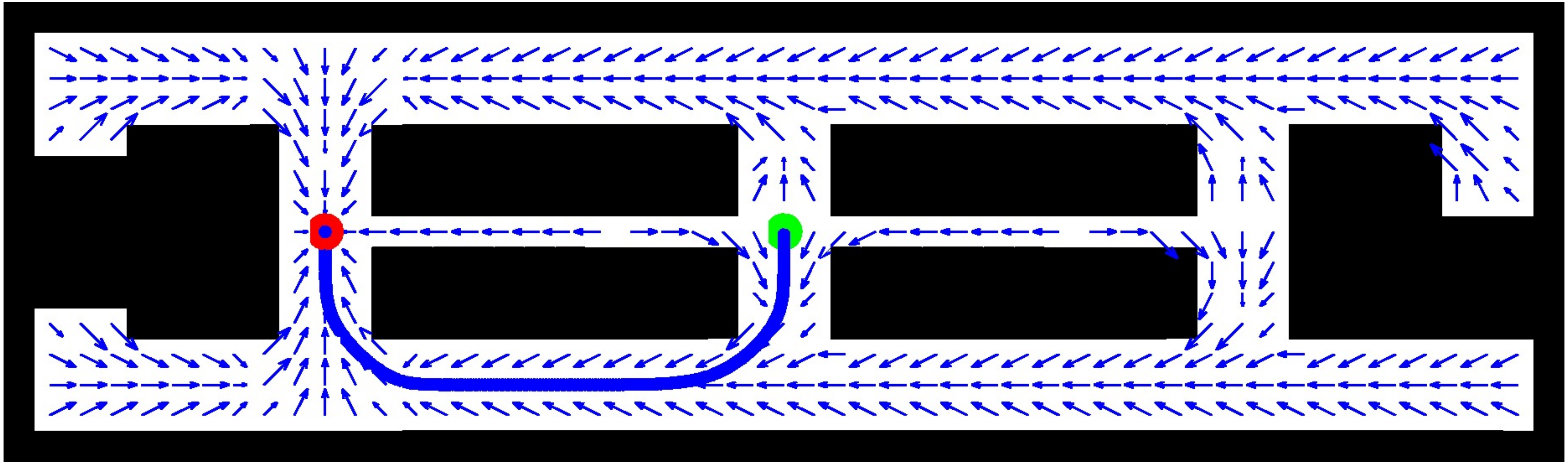
Grid World Example



Deterministic actions



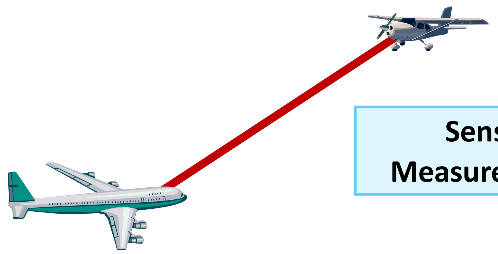
Nondeterministic actions



Defining Values

- Actions are driven by goals
  - E.g., reach destination, stay in lane
- Often, we want to reach goal while optimizing some cost
  - E.g., minimize time / energy consumption, obstacle avoidance
- We express both costs and goals in a single function, called the payoff function

Traffic Alert and Collision Avoidance System (TCAS)



Sensor Measurements

```
IF (ITF.A LT G.ZTHR)
 THEN IF(ABS(ITF.VMD) LT G.ZTHR)
  THEN SET ZHIT;
  ELSE CLEAR ZHIT;
 ELSE IF (ITF.ADOT GE P.ZDTHR)
  THEN CLEAR ZHIT
  ELSE
   ITF.TAUV = -ITF.A/ITF.ADOT;
   IF (ITF.TAUV LT TVTHR AND
    ((ABS(ITF.VMD) LT G.ZTHR) OR
    (ITF.TAUV LT ITF.TRTRU))
    THEN SET ZHIT
    ELSE CLEAR ZHIT
IF (ZHIT EQ $TRUE AND
 ABS(ITF.ZDINT) GT P.MAXZDINT
 THEN CLEAR ZHIT
```

Resolution Advisory

Surveillance

Advisory Logic

Display

Slide Credit: Mykel Kochenderfer

ACAS X: Simplified MDP

Own aircraft
ACAS X

Intruder Aircraft

State space	Action space
• Relative altitude • Own vertical rate • Intruder vertical rate • Time to lateral NMAC • State of advisory	• Clear of conflict • Climb > 1500 ft/min • Climb > 2500 ft/min • Descend > 1500 ft/min • Descend > 2500 ft/min
Dynamic model	**Reward model**
• Head-on, constant closure • Random vertical acceleration • Pilot response delay (5 s) • Pilot response strength (1/4 g) • State of advisory	• NMAC (-1) • Alert (-0.01) • Reversal (-0.01) • Strengthen (-0.009) • Clear of conflict (0.0001)

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- Policy: $\pi : x_t \rightarrow u_t$
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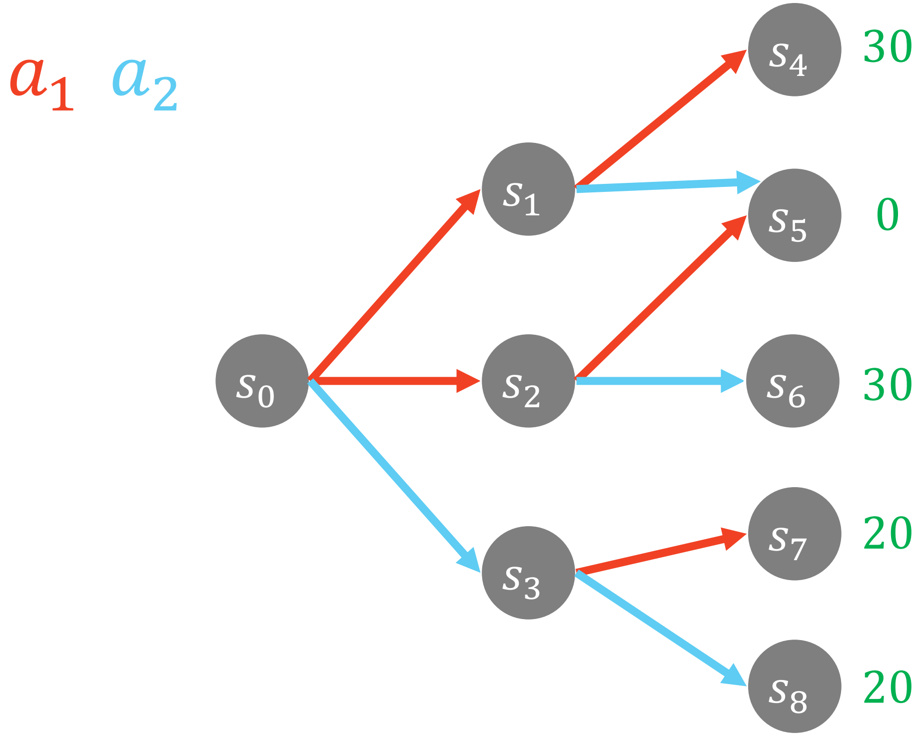
  - Can be low-level reactive algorithm or a long-term, high-level planner
  - May or may not be deterministic
- Typically, we want a policy that optimizes <u>future</u> payoff, considering optimal actions over a <u>planning (time) horizon</u>

Open vs. Closed Loop Planning

- Closed-Loop Planning: accounts for future information in planning. This creates a reactive plan (policy) that can react to different outcomes over time
- Open-Loop Planning: path panning algorithms develop a static sequence of actions

Open Loop vs. Closed Loop Planning



MDP Policies

- Policies map states to actions

$$\pi : x \rightarrow u$$

- We want to find a policy that maximizes future pay off
  - Suppose $T = 1$: $\pi_1(x) = \text{argmax}_u \; r(x, u)$

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  - Suppose $T = 1$: $\pi_1(x) = \text{argmax}_u \; r(x, u)$
- We write the Value Function for given π:

  $$V_1(x) = \gamma \max_u r(x, u)$$

- Generally, we want to find the sequence of actions that optimize the *expected cumulative discounted future payoff*

Expected Cumulative Payoff

$$R_T = \mathbb{E}\left[\sum_{\tau=0}^{T} \gamma^{\tau}\, r_{t+\tau}\right]$$

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2. Finite Horizon: $1 \leq T < \infty, (\gamma \leq 1)$
   → Optimize R_T for set time window
3. Infinite Horizon: $T = \infty, (\gamma < 1)$
   → Optimize R_∞ for all time
   If $|r| \leq r_{max}$, discounting guarantees R_∞ is finite

$$R_\infty \leq r_{max} + \gamma r_{max} + \gamma^2 r_{max} + \cdots = \frac{r_{max}}{1-\gamma}$$

Value Functions

For longer time horizons (T), we define V(x) recursively:

Recall: $V_1(x) = \gamma \max_u r(x, u)$

Value Functions

- In the infinite time horizon, we tend to reach equilibrium:

$$V_\infty(x) = \gamma \max_u \left[r(x,u) + \int V_\infty(x') p(x'|x,u)\, dx' \right]$$

- This is the *Bellman Equation*
  - Satisfying this is necessary and sufficient for an optimal policy

Computing the (Approximate) Value Function

- Initial guess for $\hat{V}$
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- Given estimate $\hat{V}(x)$, policy is found:
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- Often, we use the discrete version:
  - $\pi(x) = \mathrm{argmax}_u \left[r(x,u) + \sum_x' \hat{V}(x')p(x'|x,u) \right]$

Summary

- Discussed a different form of planning (often referred to as **decision-making**) schemes and how they fit into the AV stack
- Defined the **MDP** model for decision-making, including **goals, costs, payoff,** and **policies**
- Defined **Expected Cumulative Payoff,** which plays a key role in **optimizing actions over planning horizons**
- **Next time:**
  - Examples of computing policies for MDPs
  - Course wrap-up