Lecture 9: Control I.5

Professor Katie Driggs-Campbell

February 15, 2024

ECE484: Principles of Safe Autonomy

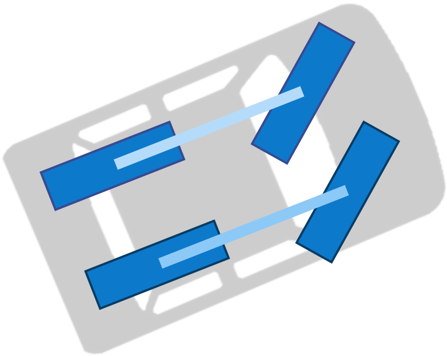
Administrivia

1232 CSL Studio

- Project Intros
  - ~~2/13 Field trip 1 to high bay to see the GEM~~
  - 2/15 Field trip 2 to see F1tenth cars
  - 2/20 Simulation project walkthrough
- MP2 released Friday 2/16
- Upcoming due dates:
  - HW1 and MP1 due Friday 2/23
  - HW2 and MP2 due Friday 3/01
  - Project Pitches in class 3/05 and 3/07
- Bonus MP in April

Simple vehicle model: Dubin's car

- Key assumptions
  - Front and rear wheel in the plane in a stationary coordinate system
  - Steering input, front wheel steering angle δ
  - No slip: wheels move only in the direction of the plane they reside in
- Zeroing out the accelerations perpendicular to the plane in which the wheels reside, we can derive simple equations



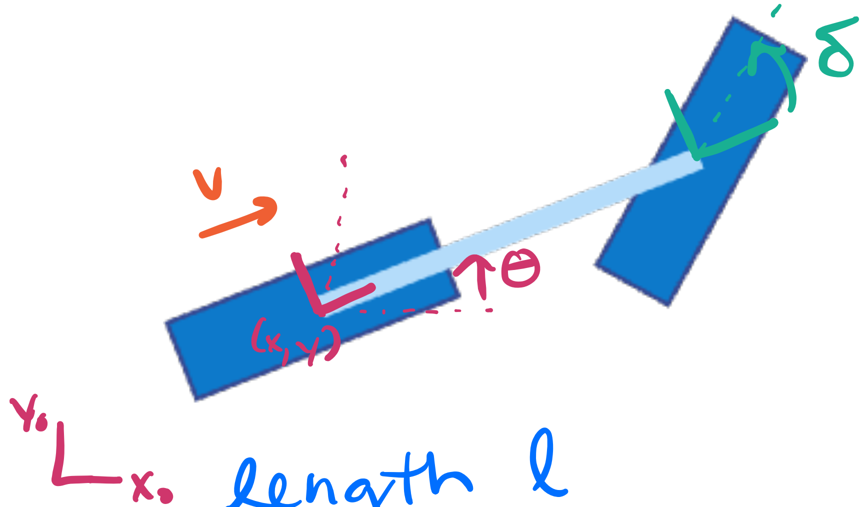
Reference: Paden, Brian, Michal Cap, Sze Zheng Yong, Dmitry S. Yershov, and Emilio Frazzoli. 2016. A survey of motion planning and control techniques for self-driving urban vehicles. IEEE Transactions on Intelligent Vehicles 1 (1): 33–55.

Dubin's Car

$$\dot{x} = v\cos\theta$$

$$\dot{y} = v\sin\theta$$

$$\dot{\theta} = v/\ell \tan\delta$$



length ℓ

pose $[x \; y \; \theta]^T$

speed v

steering δ

Many more advanced models…

[Kinematic] Bicycle Model

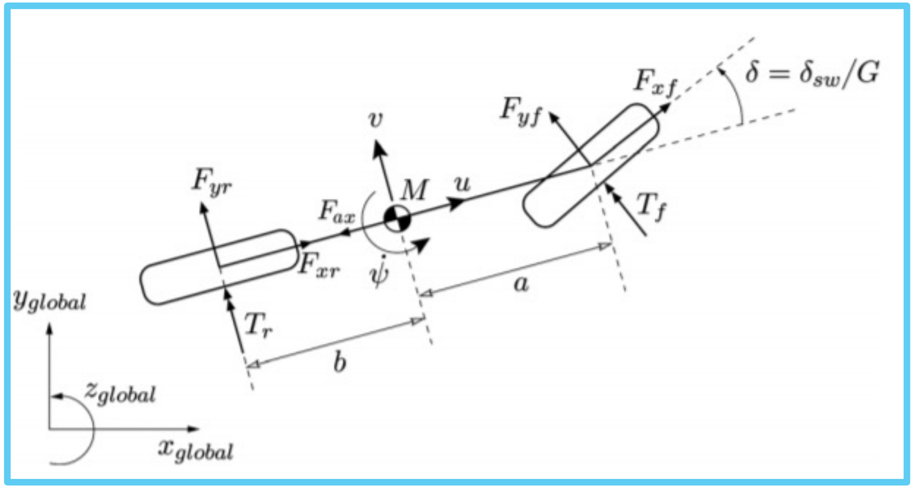


Image Credit and Reference: J.P. Timings and D.J. Cole. "Minimum maneuver time calculation using convex optimization." Journal of Dynamic Systems, Measurement, and Control 135.3 (2013).

[Dynamic] Tire Models

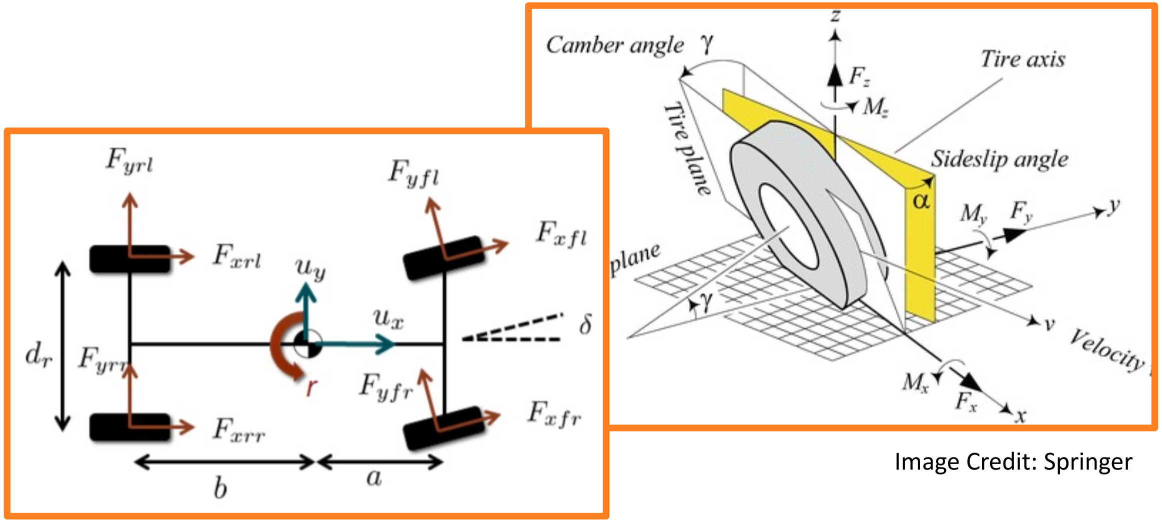
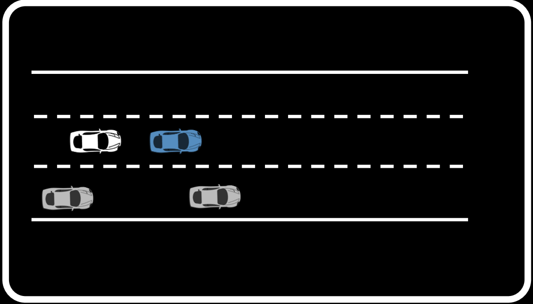


Image Credit: Springer

Image Credit and Reference: J.K. Subosits and J.C. Gerdes. "Impacts of Model Fidelity on Trajectory Optimization for Autonomous Vehicles in Extreme Maneuvers." IEEE Transactions on Intelligent Vehicles, 2021.

Dynamical system models



Nonlinear dynamics

Generally, nonlinear ODEs do not have closed form solutions!

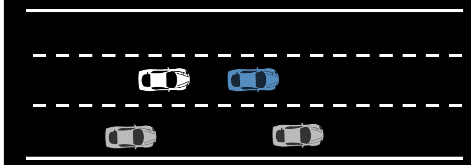
Dubin's car model

$\dot{v} = a$	Speed
$\frac{ds_x}{dt} = v\cos(\psi)$	Horizontal position
$\frac{ds_y}{dt} = v\sin(\psi)$	Vertical position
$\frac{d\delta}{dt} = v_\delta$	Steering angle
$\frac{d\psi}{dt} = \frac{v}{l}\tan(\delta)$	Heading angle

Physical plant

$\frac{dx}{dt} = f(x, u)$	System dynamics
$x[t+1] = f(x[t], u[t])$	
$x = [v, s_x, s_y, \delta, \psi]$	State variables
$u = [a, v_\delta]$	Control inputs

Nonlinear *hybrid* dynamics



Physical plant

$$\frac{dx}{dt} = f(x, u)$$

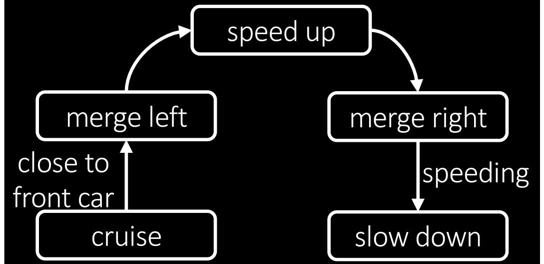
System dynamics

$$x[t+1] = f(x[t], u[t])$$

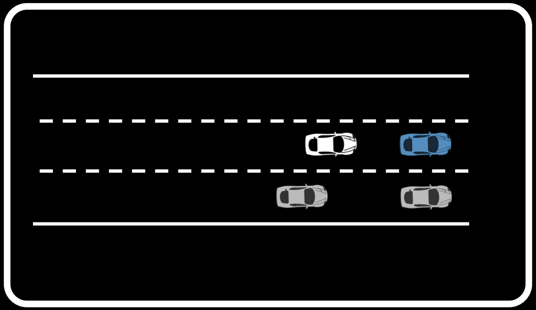
$x = [v, s_x, s_y, \delta, \psi]$ State variables

$u = [a, v_\delta]$ Control inputs

Decision and control software

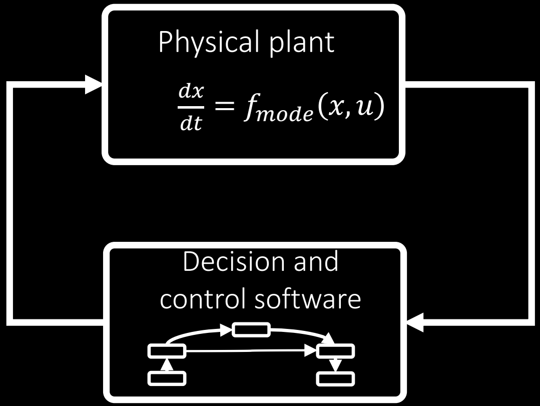


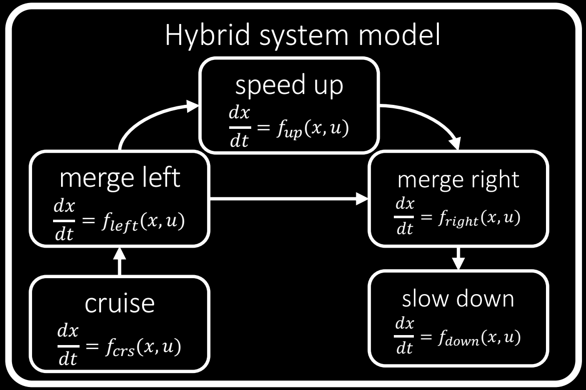
Typical system models



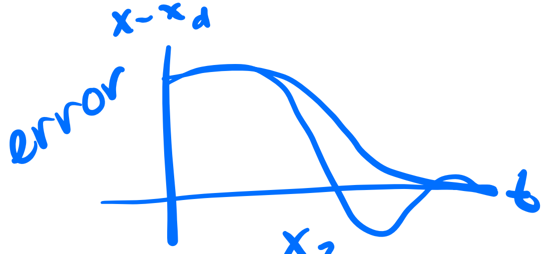
- Nonlinear *hybrid* dynamics

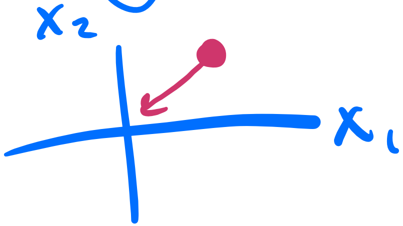
Interaction between computation and physics can lead to unexpected behaviors





Dynamical Systems Model





Describe behavior in terms of instantaneous laws:

$$\frac{dx(t)}{dt} = \dot{x}(t) = f\big(x(t), u(t)\big) = Ax + Bu$$

where $t \in \mathbb{R}, x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m$, and $f: \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ gives the dynamics / transition function

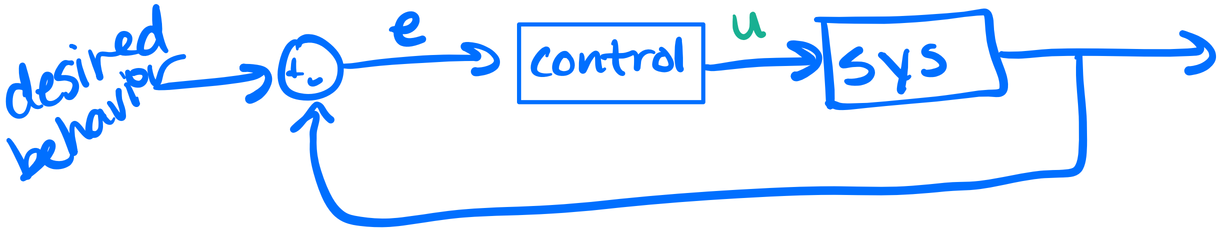
$$\dot{x} = Ax + Bu$$

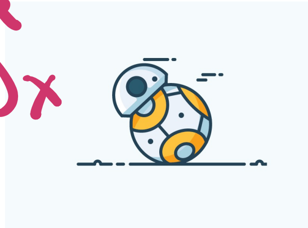
$$u = -kx$$

$$= Ax - kBx$$

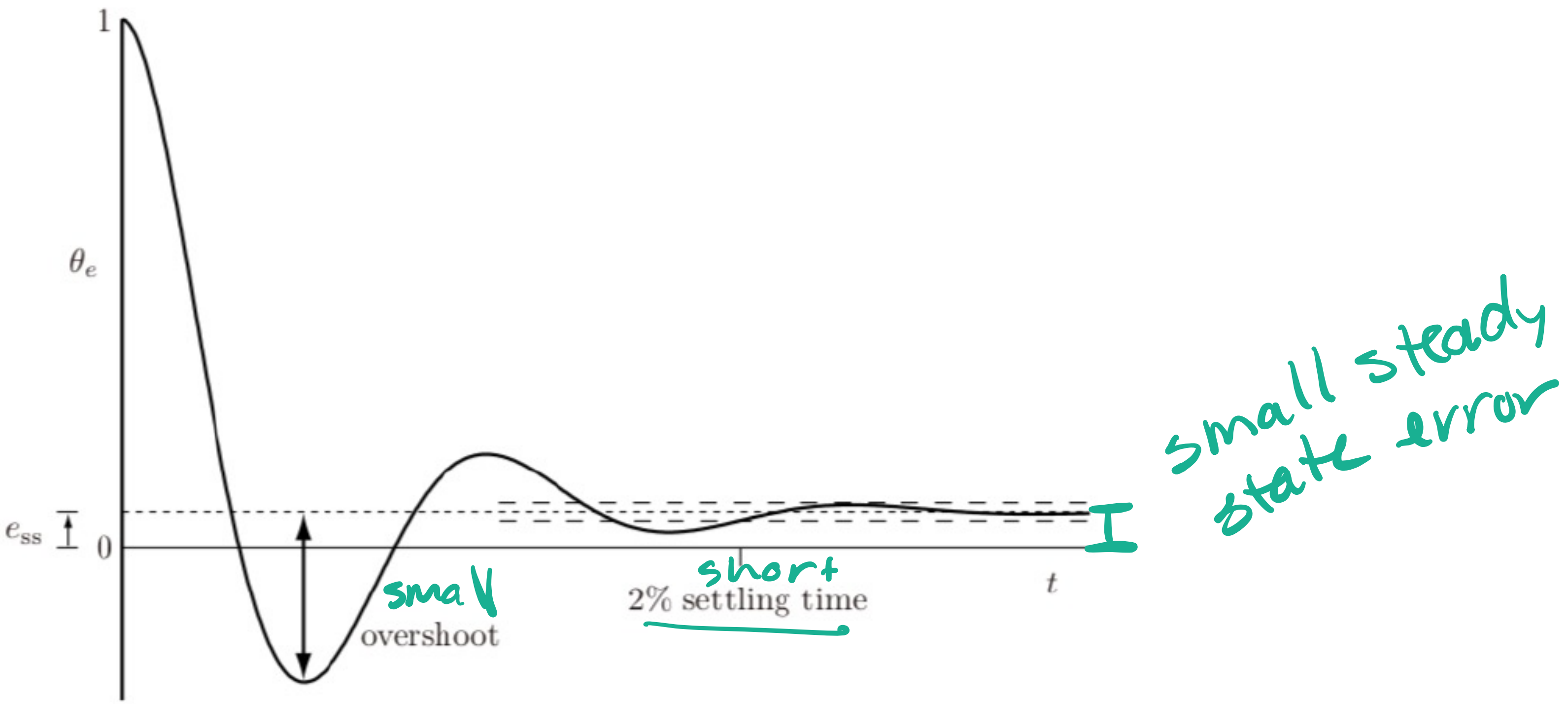
$$= (A - kB)x$$

$$= \tilde{A}x$$





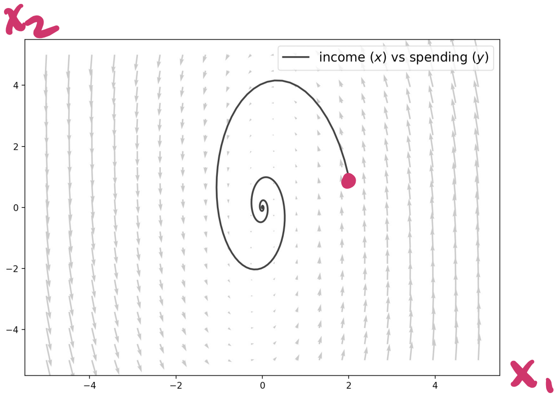
Error Dynamics

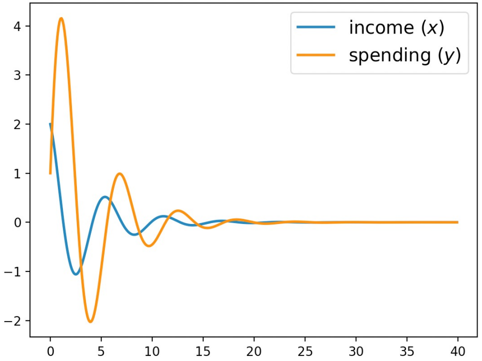


Examples $\hat{A}$

$$\begin{bmatrix}\dot{x}_2\\ \dot{x}_1\end{bmatrix}=\begin{bmatrix}-1/4 & -2/5\\ 3 & -1/4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$\lambda_1=-0.25-i1.10$
$\lambda_2=-0.25+i1.10$





Examples

$$\begin{bmatrix} \dot{x_2} \\ \dot{x_1} \end{bmatrix} = \begin{bmatrix} -1/4 & -2/5 \\ 3 & -1/4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

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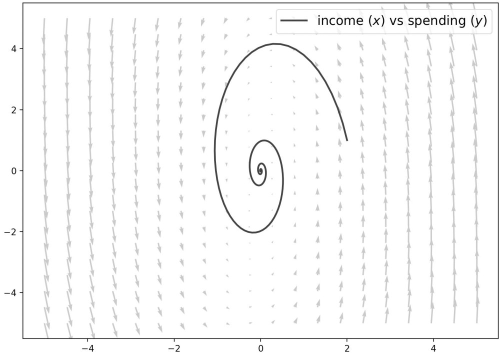
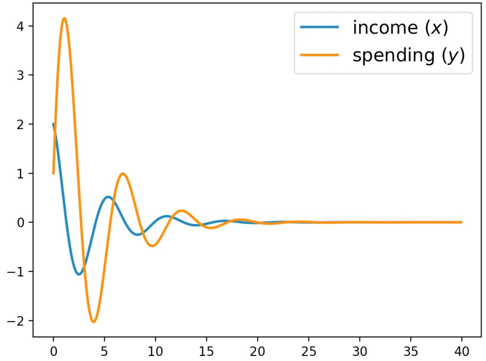
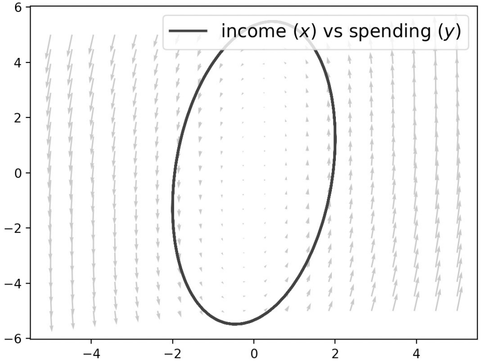
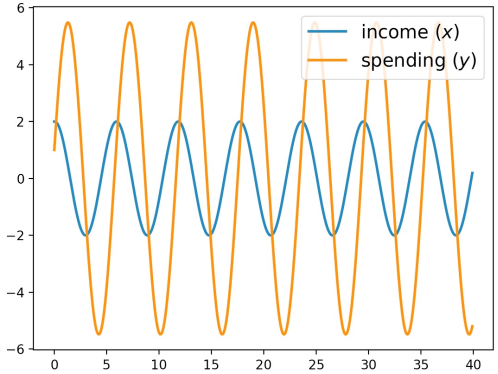




$$\begin{bmatrix} \dot{x_2} \\ \dot{x_1} \end{bmatrix} = \begin{bmatrix} 1/4 & -2/5 \\ 3 & -1/4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

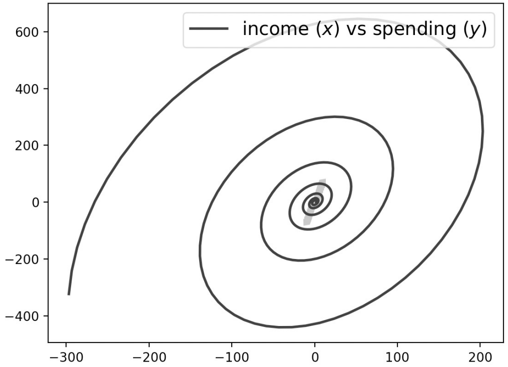
λ_1=+i0.1066
λ_2=-i0.1066

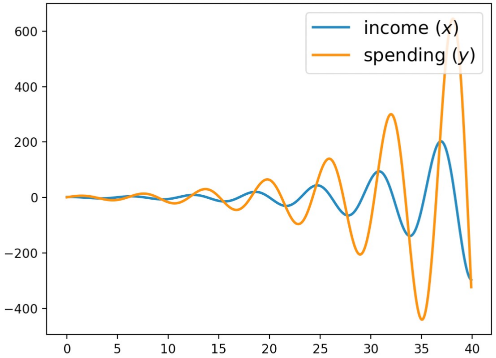




$$\begin{bmatrix} \dot{x_2} \\ \dot{x_1} \end{bmatrix} = \begin{bmatrix} 1/2 & -2/5 \\ 3 & -1/4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

λ_1=0.125+i1.029
λ_2=-0.125-i1.029





Examples

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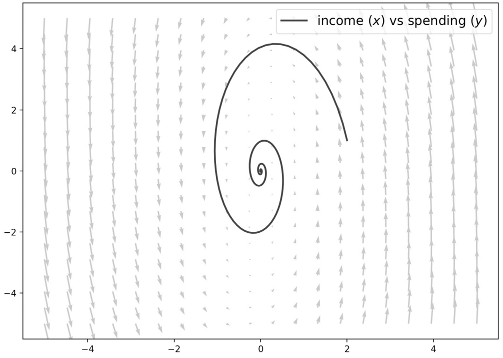

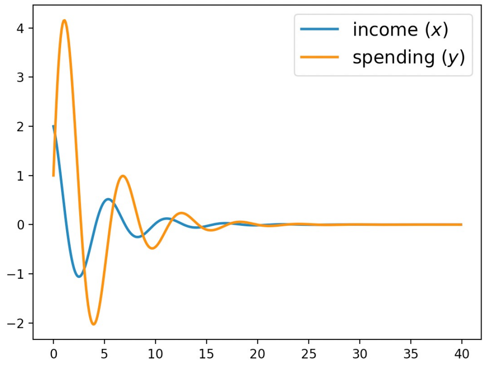

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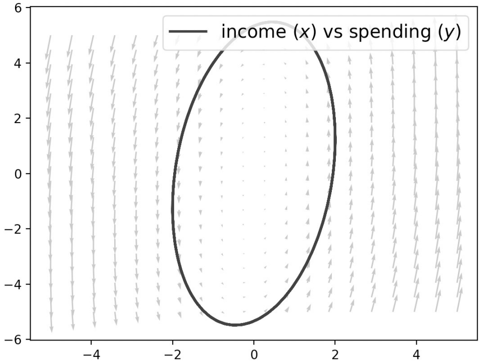

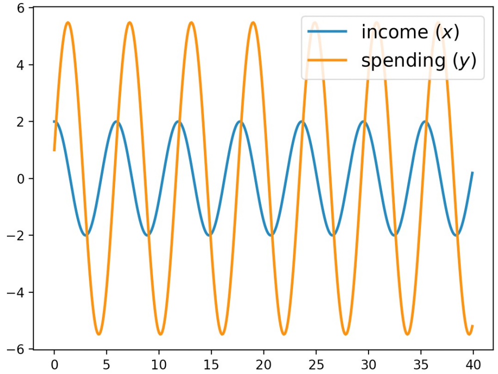

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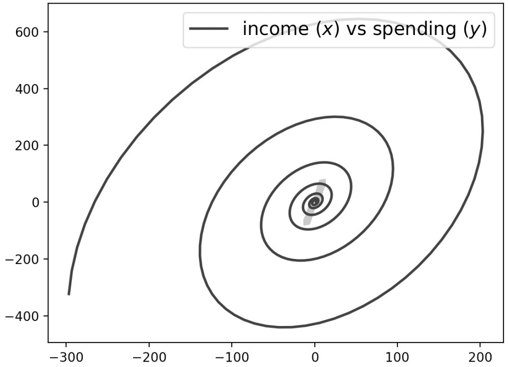
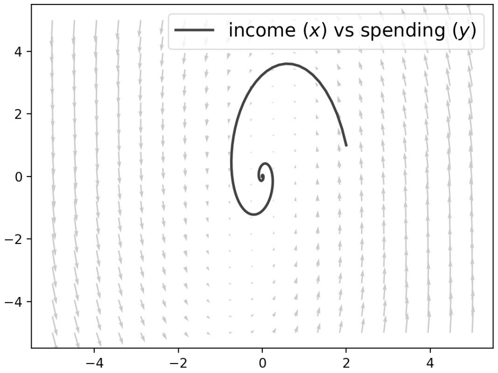
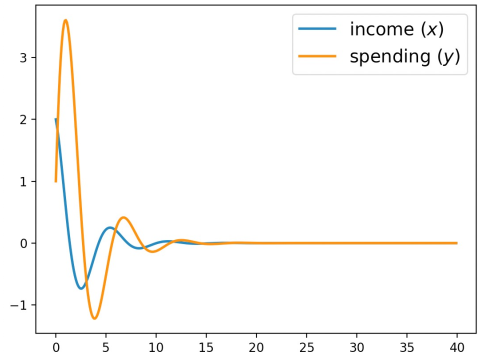


$$\begin{bmatrix}\dot{x_2}\\ \dot{x_1}\end{bmatrix}=\begin{bmatrix}-1/4 & -2/5\\ 3 & -1/2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

λ_1=−0.375−i1.088
λ_2=−0.375+i1.088





Summary

- Dynamical systems models allow us to reason about low-level behaviors of systems and determine what is and is not feasible
  - Typically required to design controllers!
- Discussed a few types of models from simple to complex
- Introduced error dynamics as a way to think about designing controllers
- *Next time:* Look at simple PID control design for trajectory following