Lecture 3: Safety II

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January 23, 2024

ECE484: Principles of Safe Autonomy

Administrivia

- Schedule is now online
  - Slides are posted before (blank) and after lecture (filled) 🤞
  - TBDs will likely be guest lectures
- Office Hours and HW party info posted on website
  - No OH today!
- Lab starts this week – will introduce MP0
  - Attendance may be taken!
- If you have DRES accommodations, please send me your letter

Example: Emergency Braking System

Executions and Behaviors

Definition: an execution is a particular behavior or trajectory of an automaton A

$$\alpha = q_0 q_1 q_2 \ldots$$

such that:

1. $q_0 \in Q$
2. $(q_i, q_{i+1}) \in D, \forall i$

Note that nondeterministic A will have <u>*many*</u> executions!

Safety Requirements

We want to express our safety requirements as:

1. A formula involving state variables
2. A subset of Q

The Safety Verification Problem

Reachability and the Post operator

Partial Summary

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  - However: computing all executions for unbounded time can be ***<u>hard</u>***

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  - To reason about all executions, we must work with infinite sets of states ☹
- One way to compute infinite sets is using the Post operator
  - However: computing all executions for unbounded time can be ***<u>hard</u>***
- We will now introduce a potential shortcut: invariants!

Inductive Invariants!

Inductive Invariants to Prove Safety

Proof by Induction

- For any execution of A, $\alpha = q_0, q_1, \ldots, q_k$, we will prove by induction on k that $\forall i\ q_i \in I$

1. Base case: $k = 0$, $\alpha = q_0 \in Q_0 \subseteq I$ by (1)
2. Inductive Step: Given $\alpha = q_0, q_1, \ldots, q_{k-1}, q_k$ and $q_{k-1} \in I$, show that $q_k \in I$

Simple requirement and candidate invariant (1)

Simple requirement and candidate invariant (2)

Another requirement

Adding more information

$\text{timer} := 0$

if $x_2 - x_1 < d_s$

    if $v_1 > a_b$

        $v_1 := v_1 - a_b$

        $\text{timer} := \text{timer} + 1$

    else $v_1 = 0$

else $v_1 := v_1$

$x_1 := x_1 + v_1$

$x_2 := x_2 + v_2$

Three Cases to Consider: (1)

Three Cases to Consider: (2)

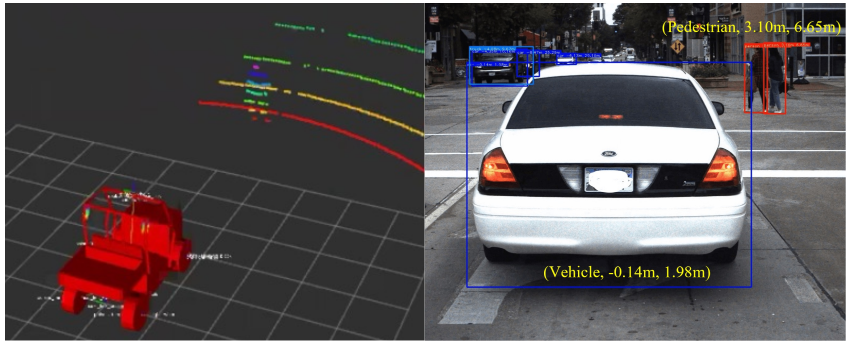
Three Cases to Consider: (3)

Showing Safety with a Timer

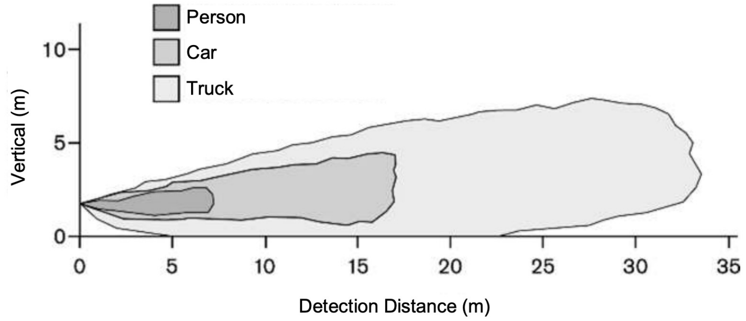
- Goal: show $x_2 - x_1 > 0$
- Maximum distance traveled by car 1 after detection:

Baked-in Assumptions (1)

- Perception.
  - Sensor detects obstacle **iff** distance $d \leq D_{sense}$
  - How to model vision errors?



1.2.1.2 Vertical Detection Area



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- Pedestrian Behaviors.
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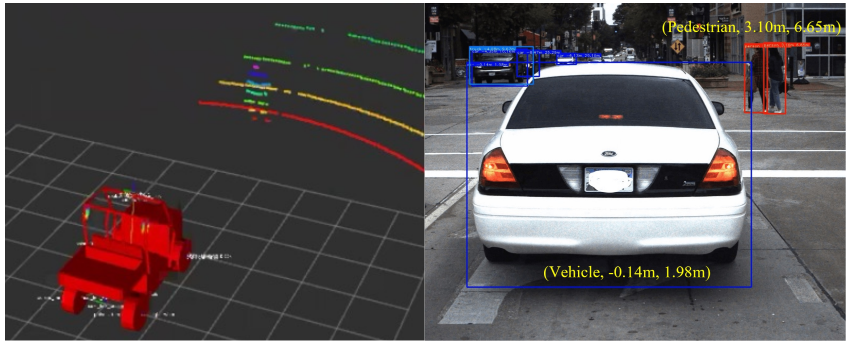

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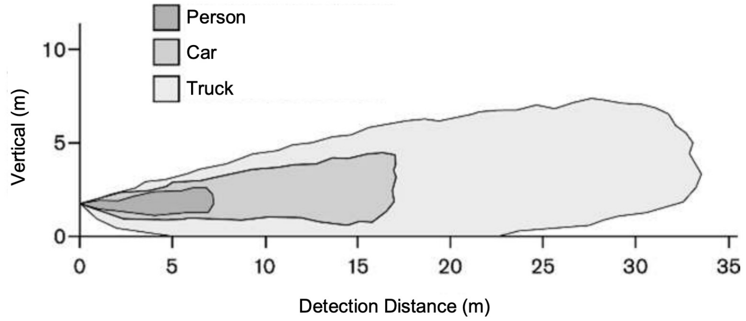

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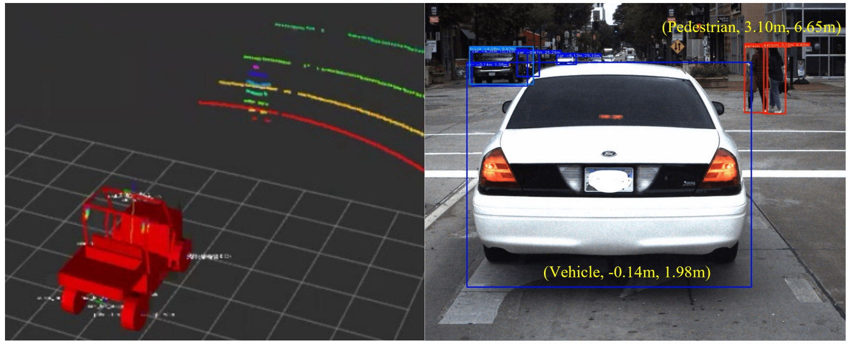
- Perception.
  - Sensor detects obstacle **iff** distance $d \leq D_{sense}$
  - How to model vision errors?
- Pedestrian Behaviors.
  - Pedestrian is assumed to be moving with constant velocity from initial position
- No sensing-computation-actuation delay.
  - The time step in which $d \leq D_{sense}$ is true is exactly when the velocity starts to decrease



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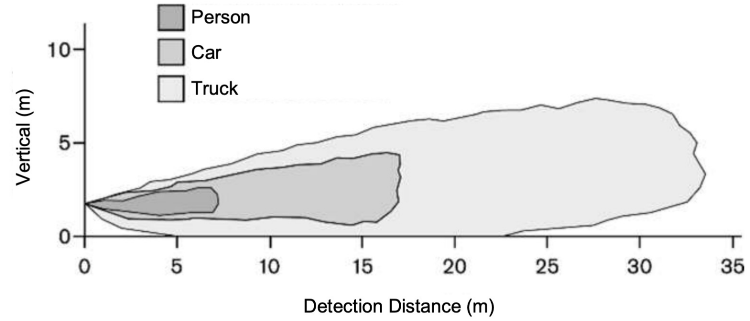
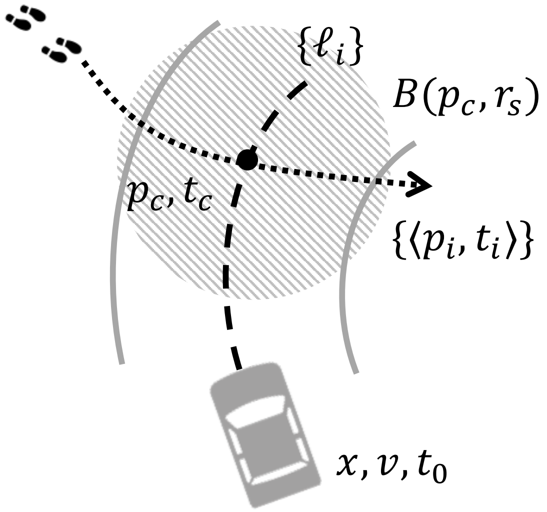
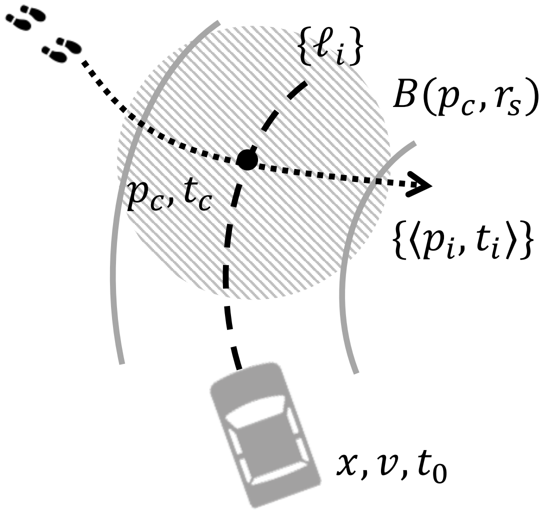


Baked-in Assumptions (2)



- Mechanical or Dynamical assumptions
  - Vehicle and pedestrian moving in 1-D lane.
  - Does not go backwards.
  - Perfect discrete kinematic model for velocity and acceleration.

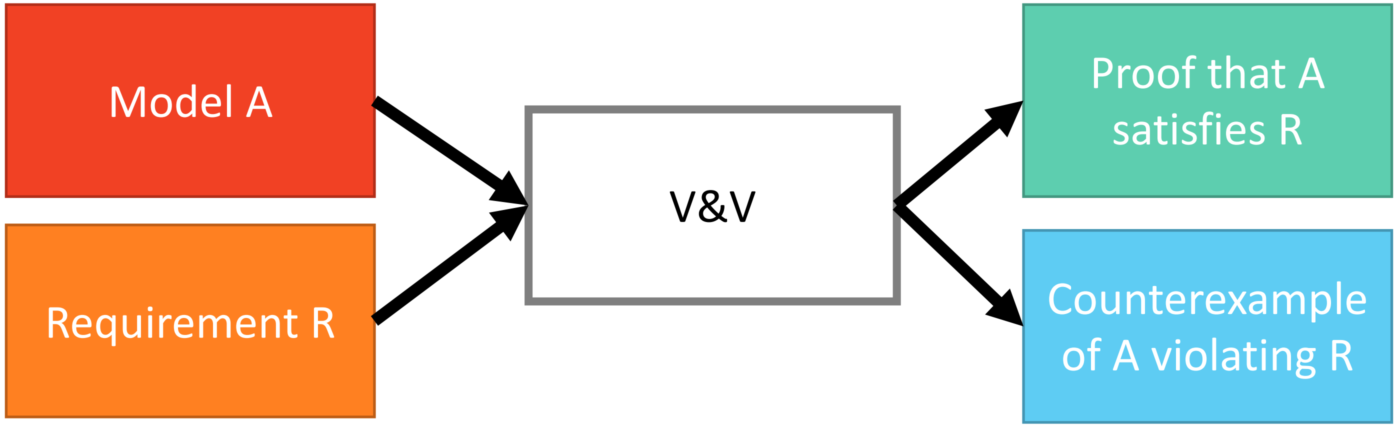
Baked-in Assumptions (2)



- Mechanical or Dynamical assumptions
  - Vehicle and pedestrian moving in 1-D lane.
  - Does not go backwards.
  - Perfect discrete kinematic model for velocity and acceleration.
- Nature of time
  - Discrete steps. Each execution of the above function models advancement of time by 1 step. If 1 step = 1 second, $x_1(t+1) = x_1(t) + v_1(t).1$
  - Atomic steps. 1 step = complete (atomic) execution of the program.
    - We cannot directly talk about the states visited after partial execution of program

Remarks and Takeaway

- The proof by induction shows a property of ***all behaviors of our model***
- The proof is conceptually simple, but can quickly get tedious and error prone
  - Verification and Validation tools like Z3, Dafny, PVS, CoQ, AST, MC2, automate this



Summary

- We must translate safety requirements into sets of states or formulas over state variables
- Reachability allows us to prove safety
- Invariant trick can give a shortcut for proving safety ☺
  - The invariant I may contain important information about conserved quantities and may also tell us why the system is safe
  - However, often requires guessing and checking and a lot of engineering effort
- Mind the gap between model and reality!
- Next: More safety (fun lecture)