Fall 23 Principles of Safe Autonomy: Lecture 10-12: State Estimation, Filtering and Localization

Sayan Mitra

Reference: Probabilistic Robotics by Sebastian Thrun, Wolfram Burgard, and Dieter Fox

Slides: From the book’s website

Announcements

Project sign-up https://forms.gle/vLas1iPogg7hr8uG6

- 2-4 people / group, could be different than MP groups
- By Sunday midnight (10/8)
- Only one from each group needs to submit form

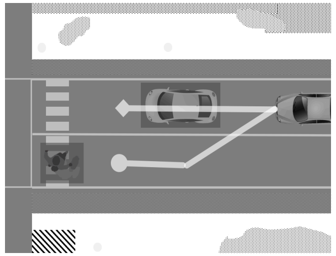
Midterm 1: Average: 66, Standard Deviation: 14, Max: 88

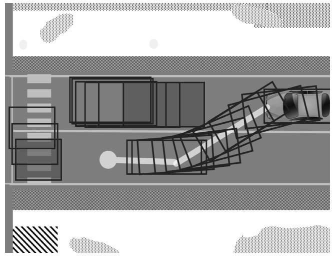
Optional: Due 11:59pm CT next Friday(10/13) in gradescope

- You can **re-solve** up to 12 points worth of MT1 problems and make it count towards your score
- You can request **regrade** for your current solutions for gross grading errors
- For each sub-problem (E.g. 1b), you can at most do one of the above

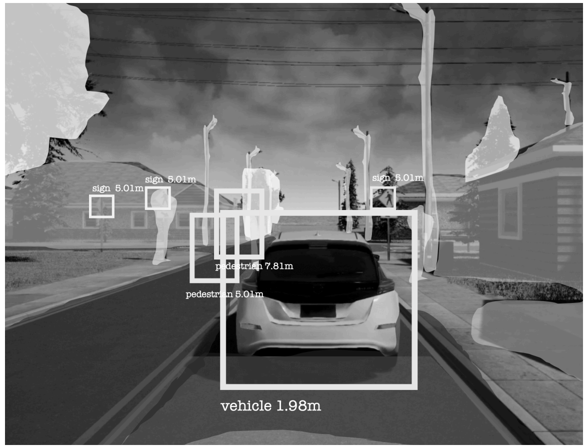
GEM platform

Autonomy pipeline





Sensing	Perception	Decisions and planning	Control
Physics-based models of camera, LIDAR, RADAR, GPS, etc.	Programs for object detection, lane tracking, scene understanding, etc.	Programs and multi-agent models of pedestrians, cars, etc.	Dynamical models of engine, powertrain, steering, tires, etc.



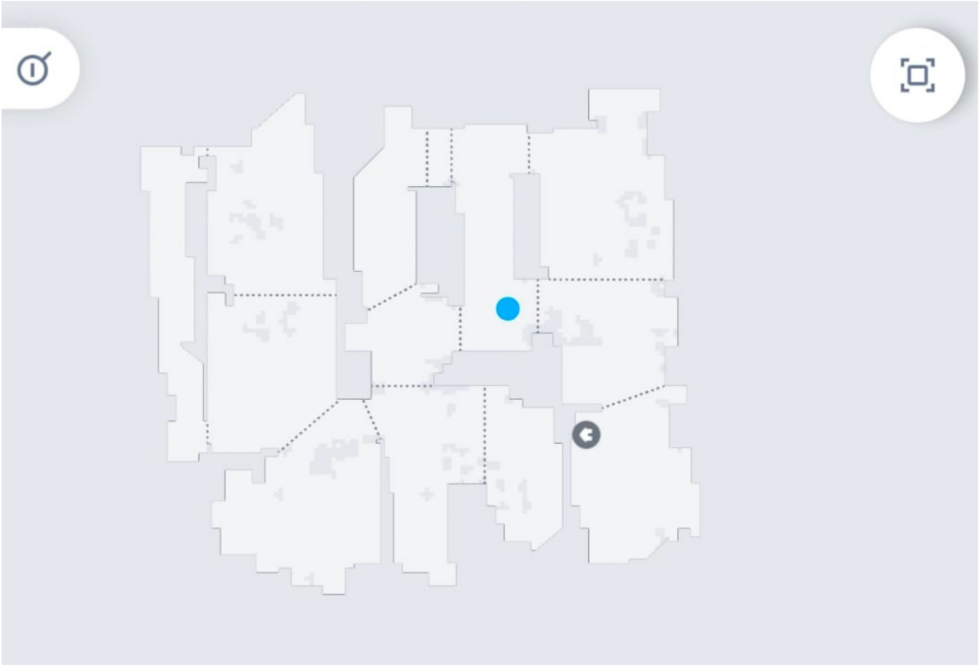
Perception

Programs for object detection, lane tracking, scene understanding, etc.

Outline of state estimation module

- Introduction: Localization problem, taxonomy
- Probabilistic models
- Discrete Bayes Filter
  - Review of Bayes rule and conditional probability
- Histogram filter
  - Grid localization
- Particle filter
  - Monte Carlo localization

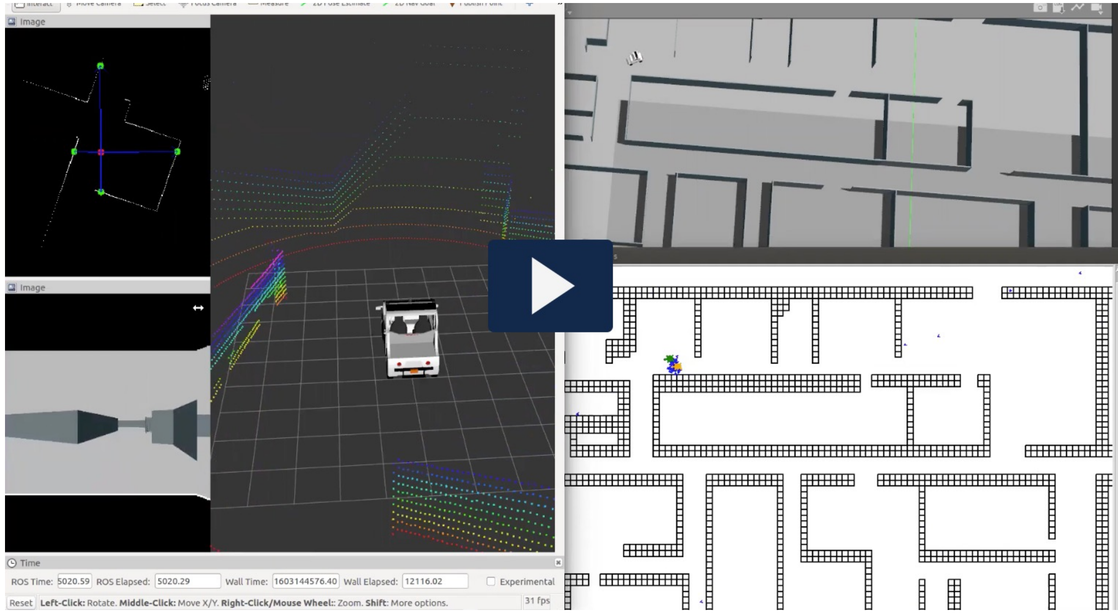
Roomba mapping



iRobot Roomba uses VSLAM algorithm to create maps for cleaning areas

State estimation and localization problem (MP3)

- For closed loop control, the controller needs to know the current state (position, attitude, pose)
  - x(t+1) = f(x(t), u(t)); u(t) = g(x(t))
- But, typically x(t) is not available directly. We have some other observables z(t) = h(x(t)) that are available. We have to get an estimate $\hat{x}(t)$ from observations of z(t)
- Examples of x(t) and z(t)
- Localization = Special case of state estimation. Determine the pose of the robot relative to the *given map* of the environment
- How does a robot know its position in ECEB (no GPS indoors)?



Setup: State evolution and measurement models

Familiar Deterministic model:

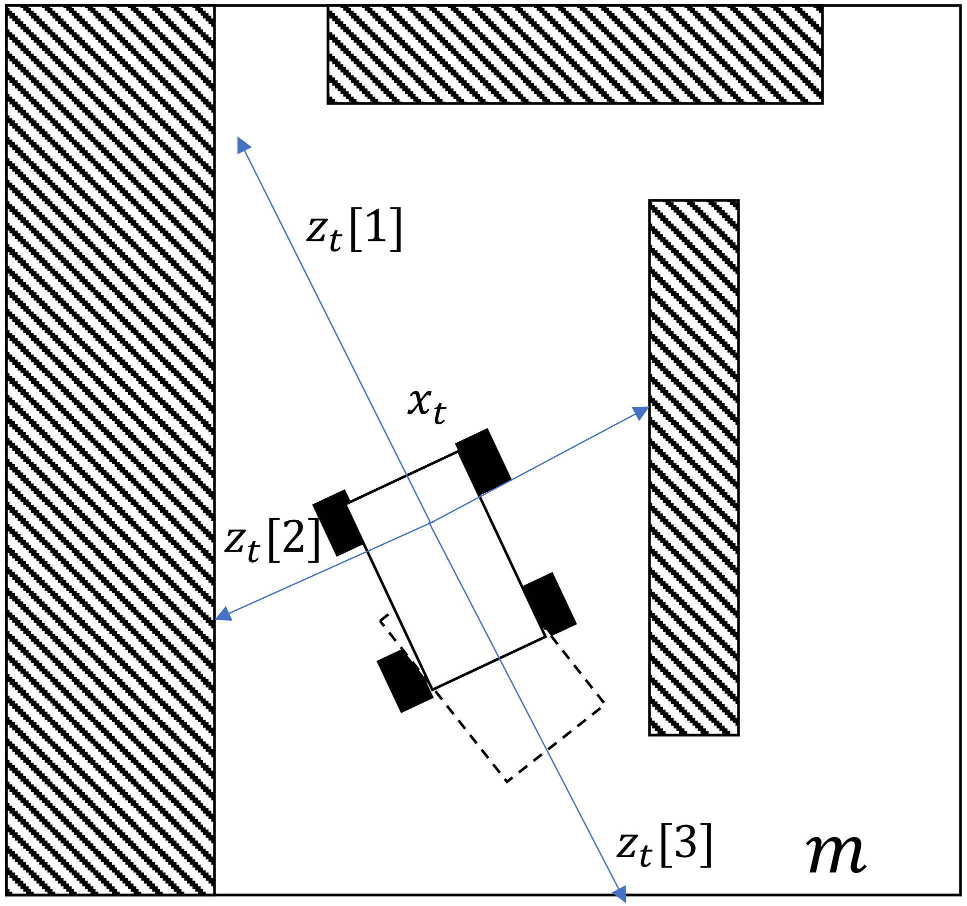
System evolution: $x_{t+1} = f(x_t, u_t)$

- x_t: unknown state of the system at time t
- u_t: known control input at time t
- f: known dynamic function, possibly stochastic

Measurement: $z_t = g(x_t, m)$

- z_t: known measurement of state x_t at time t
- m: unknown underlying map
- g: known measurement function

We will work with probabilistic models going forward



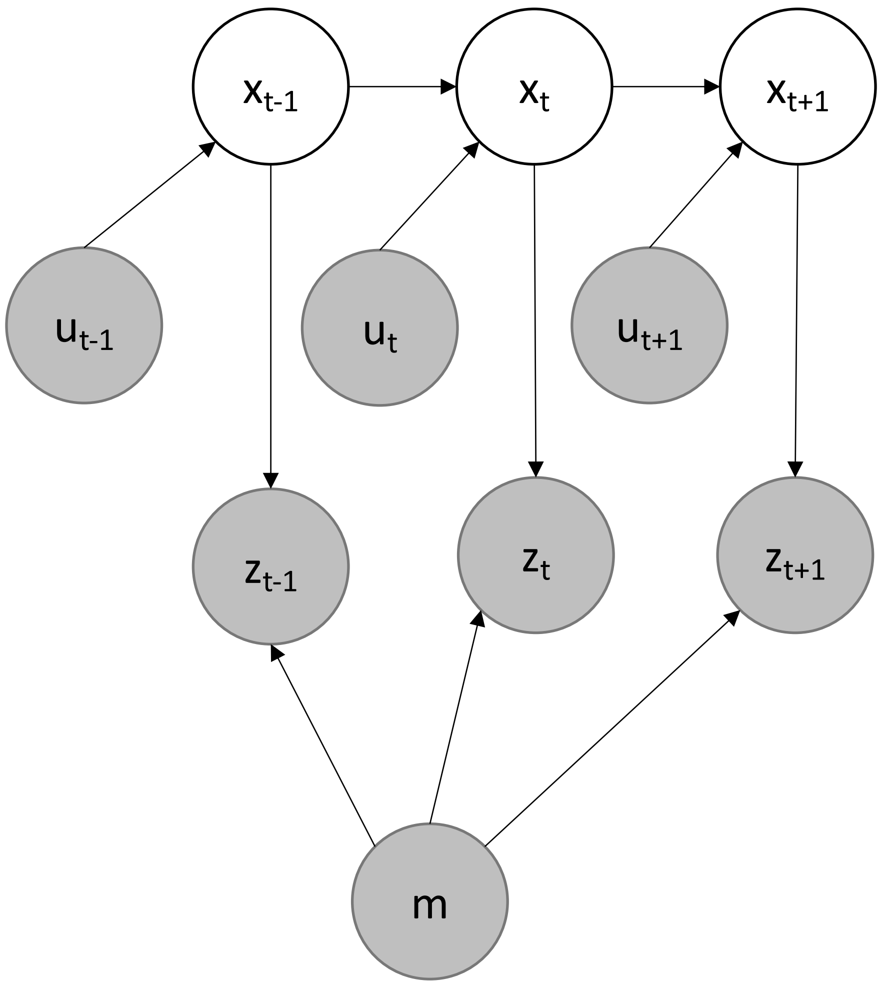
This is not exactly the measurement model of MP3

Localization as coordinate transformation

Shaded known:
map (m), control inputs (u), measurements(z). White nodes to be determined (x)

maps (m) are described in global coordinates. Localization = establish *coord transf.* between m and robot's local coordinates

Transformation used for objects of interest (obstacles, pedestrians) for decision, planning and control



Localization taxonomy

Global vs Local

- Local: assumes initial pose is known, has to only account for the uncertainty coming from robot motion (*position tracking problem)*
- **Global**: initial pose unknown; harder and subsumes position tracking
- Kidnapped robot problem: during operation the robot can get teleported to a new unknown location (models failures)

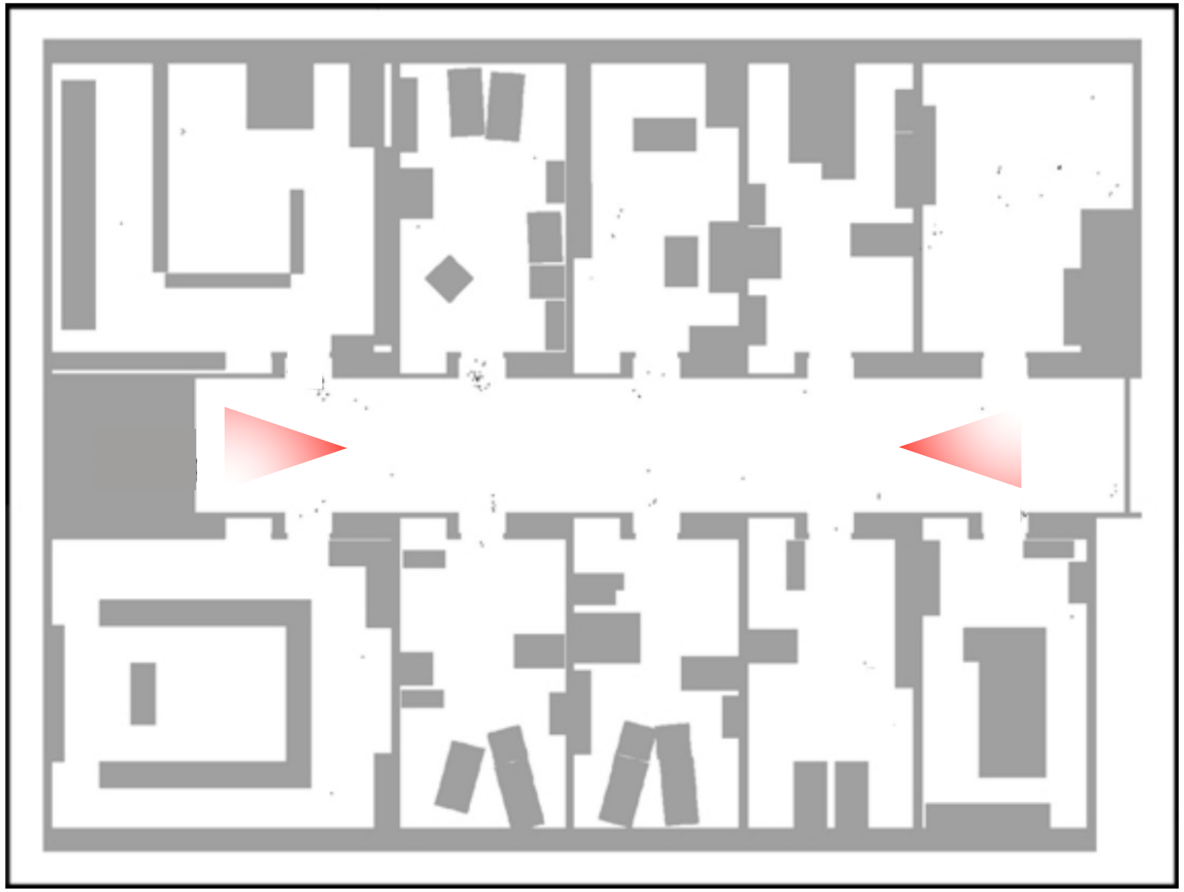
Static vs Dynamic Environments

Single vs Multi-robot localization

Passive vs Active Approaches

- **Passive**: localization module only observes and is controlled by other means; motion not designed to help localization (Filtering problem)
- Active: controls robot to improve localization

Ambiguity in global localization arising from locally symmetric environment



Discrete Bayes Filter Algorithm

- System evolution: $x_{t+1} = f(x_t, u_t)$
  - x_t: state of the system at time t
  - u_t: control input at time t
- Measurement: $z_t = g(x_t, m)$
  - z_t: measurement of state x_t at time t
  - m: unknown underlying map

Setup, notations

- Discrete time model
- $x_{t_1:t_2} = x_{t_1}, x_{t_1+1}, x_{t_1+2}, \dots, x_{t_2}$ sequence of robot states $t_1$to t_2
- Robot takes one measurement at a time
  - $z_{t_1:t_2} = z_{t_1}, \dots, z_{t_2}$ sequence of all measurements from $t_1$to t_2
- Control also exercised at discrete steps
  - $u_{t_1:t_2} = u_{t_1}, u_{t_1+1}, u_{t_1+2}, \dots, u_{t_2}$ sequence control inputs

Review of conditional probabilities

Random variable X takes values $x_1, x_2, ..$

Example: Result of a dice roll (X) and $x_i = 1, \dots, 6$

$P(X = x)$ *is written as* $P(x)$

Conditional probability: $P(x|y) = P(X = x \,|Y = y) = \frac{P(x,y)}{P(y)}$ provided $P(y) > 0$

$P(x, y) = P(x|y)P(y)$

$\qquad = P(y|x)P(x)$

Substituting in the definition of Conditional Prob. we get Bayes Rule

$P(x|y) = \frac{P(y|x)P(x)}{P(y)}$, provided $P(y) > 0$

Using measurements to update state estimates

$P(x|y) = \frac{P(y|x)P(x)}{P(y)}$, provided $P(y) > 0$ ---- *Equation (*)*

X *: Robot position,* Y *: measurement,*

$P(x)$: *Prior distribution (before measurement)*

$P(x|y)$: *Posterior distribution (after measurement)*

$P(y|x)$*: Measurement model / inverse conditional / generative model*

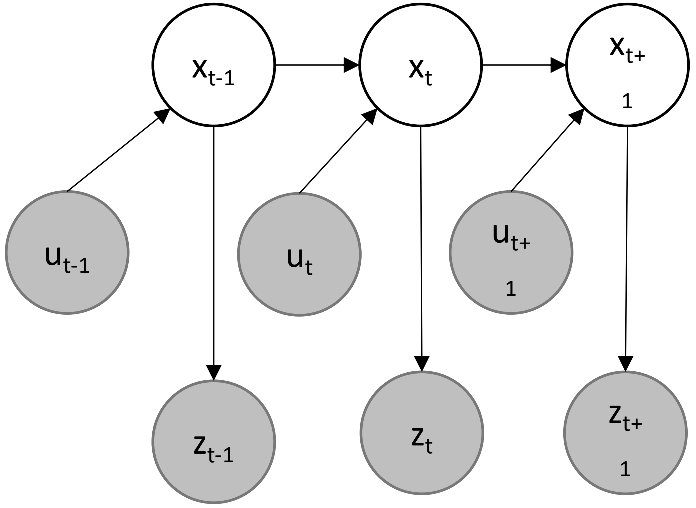
$P(y)$: does not depend on x; normalization constant

State evolution and measurement: probabilistic models

Evolution of state and measurements governed by probabilistic laws

$p(x_t \mid x_{0:t-1}, z_{1:t-1}, u_{1:t})$ describes motion/state evolution model

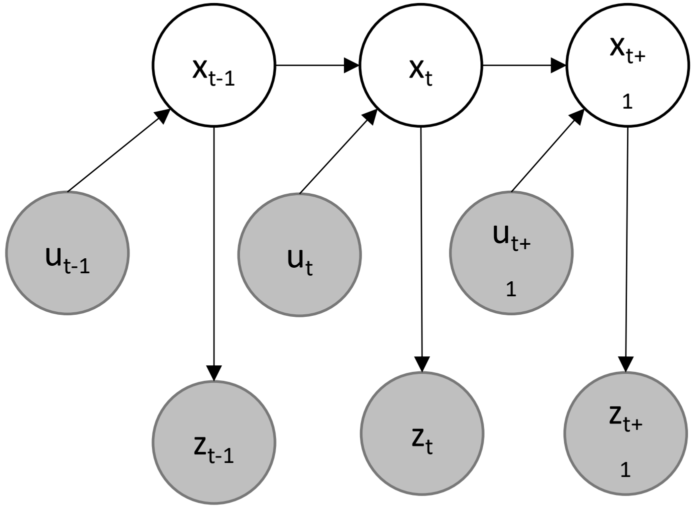
- If state is complete, sufficient summary of the history then
  - $p(x_t \mid x_{0:t-1}, z_{0:t-1}, u_{0:t-1}) = p(x_t \mid x_{t-1}, u_t)$ state transition prob.
  - $p(x' \mid x, u)$ if transition probabilities are time invariant



Measurement model

Measurement process $p(z_t \mid x_{0:t}, z_{1:t-1}, u_{0:t-1})$

- Again, if state is complete
- $p(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t \mid x_t)$
- $p(z_t \mid x_t)$: measurement probability
- $p(z \mid x)$: time invariant measurement probability



Beliefs

Belief: Robot's knowledge about the state of the environment

True state is unknowable / measurable typically, so, robot must infer state from data and we have to distinguish this inferred/estimated state from the actual state x_t

$$bel(x_t) = p(x_t | z_{1:t}, u_{1:t})$$

Posterior distribution over state at time t given all past measurements and control. This will be calculated in two steps:

1. Prediction: $\overline{bel}(x_t) = p(x_t | z_{1:t-1}, u_{1:t})$
2. Correction: Calculating $bel(x_t)$ from $\overline{bel}(x_t)$ a.k.a measurement update (will use Equation (*) from earlier)

Recursive Bayes Filter

Algorithm Bayes_filter($bel(x_{t-1}), u_t, z_t$)

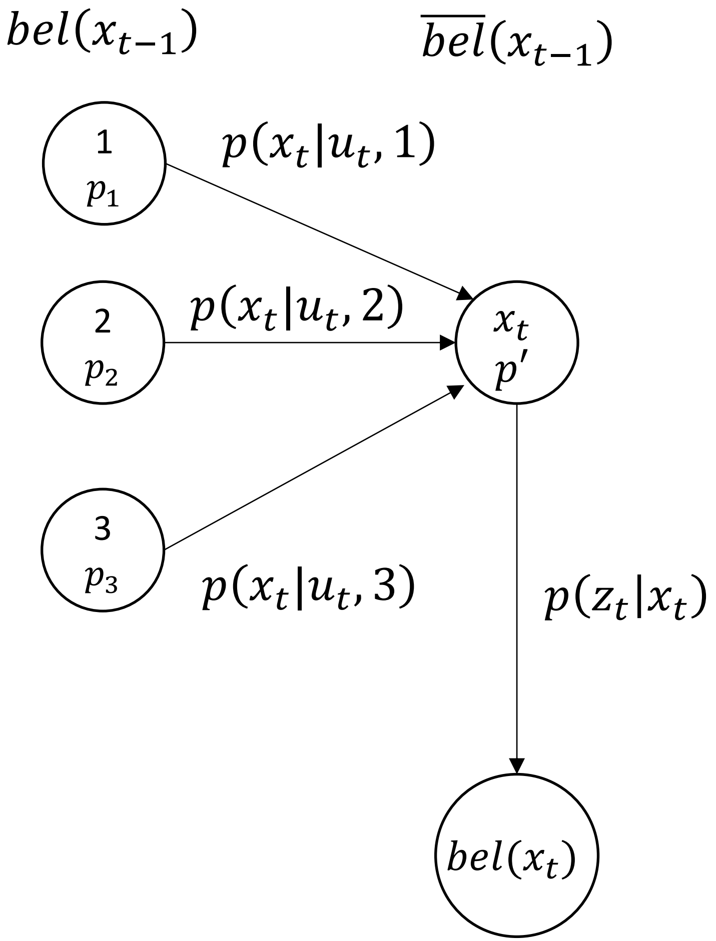
for all x_t do:

$$\overline{bel}(x_t) = \int p(x_t|u_{t,}x_{t-1})bel(x_{t-1})dx_{t-1}$$

$$bel(x_t) = \eta\, p(z_t|x_t)\, \overline{bel}(x_t)$$

end for

return $bel(x_t)$



Histogram Filter or Discrete Bayes Filter

Finitely many states x_i, x_k, etc. Random state vector X_t

$p_{k,t}$: belief at time t for state x_k; discrete probability distribution

Algorithm Discrete_Bayes_filter($\{p_{k,t-1}\}, u_t, z_t$**):**

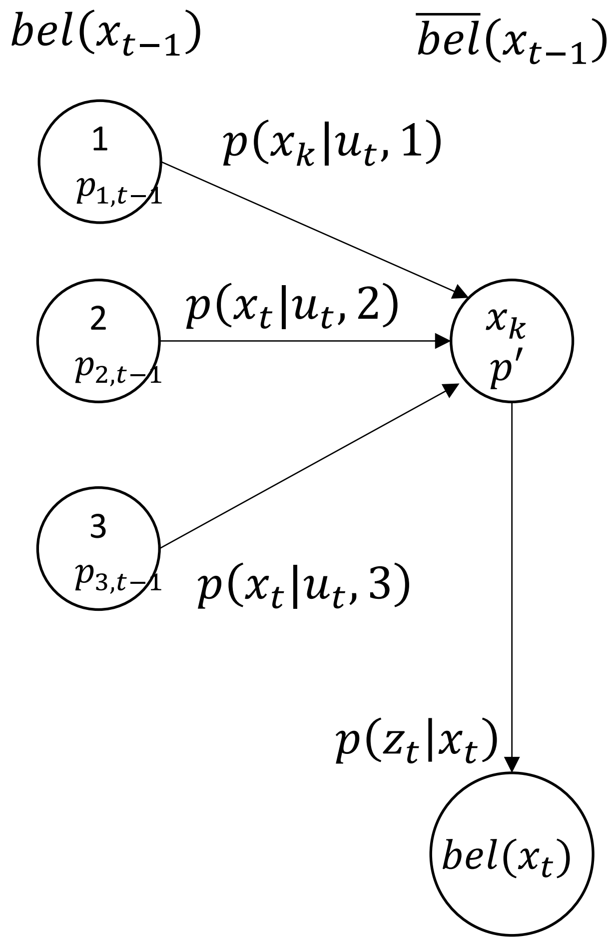
for all k do:

$$\bar{p}_{k,t} = \sum_i p(X_t = x_k | u_t, X_{t-1} = x_i) p_{i,t-1}$$

$$p_{k,t} = \eta \, p(z_t \, | X_t = x_k) \bar{p}_{k,t}$$

end for

return $\{p_{k,t}\}$



Grid Localization

- Solves global localization in some cases kidnapped robot problem
- Can process raw sensor data
  - No need for feature extraction
- Non-parametric
  - In particular, not bound to unimodal distributions (unlike Extended Kalman Filter)

Grid localization

Algorithm Grid_localization $(\{p_{k,t-1}\}, u_t, z_t, m)$

for all k do:

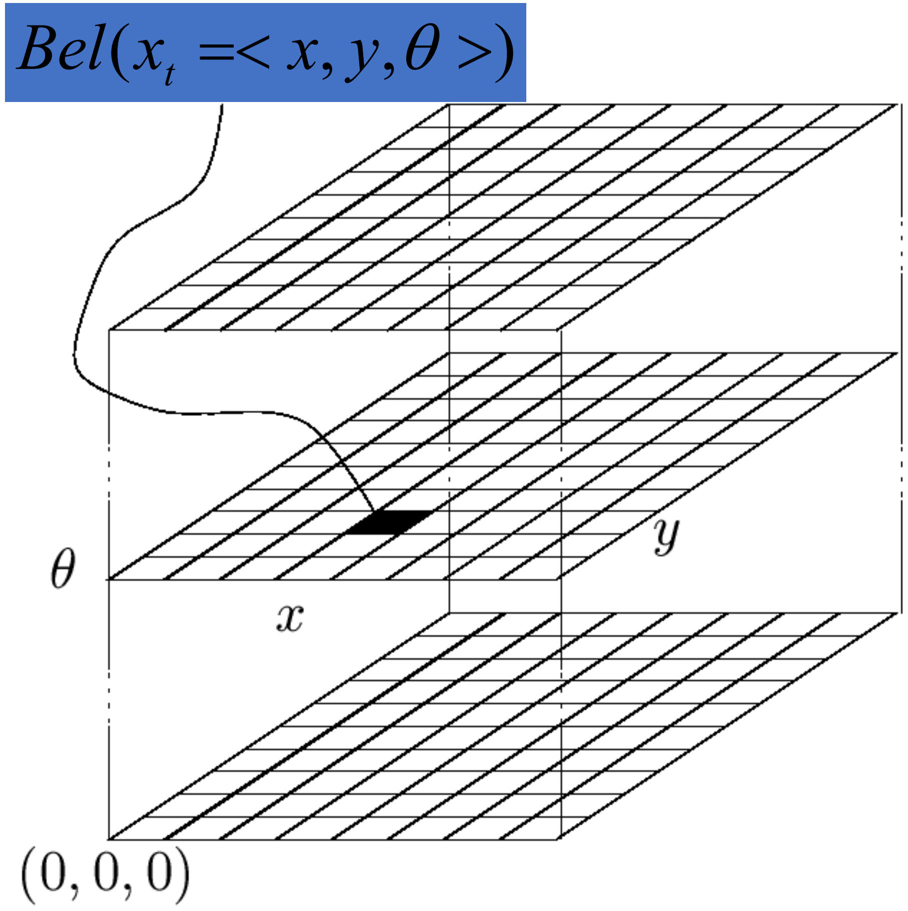
$\bar{p}_{k,t} = \sum_i p_{i,t-1}\, \boldsymbol{motion\_model}(mean(x_k), u_t, mean(x_i))$

$p_{k,t} = \eta\, \bar{p}_{k,t} \boldsymbol{measurement\_model}(z_t, mean(x_k), m)$

end for

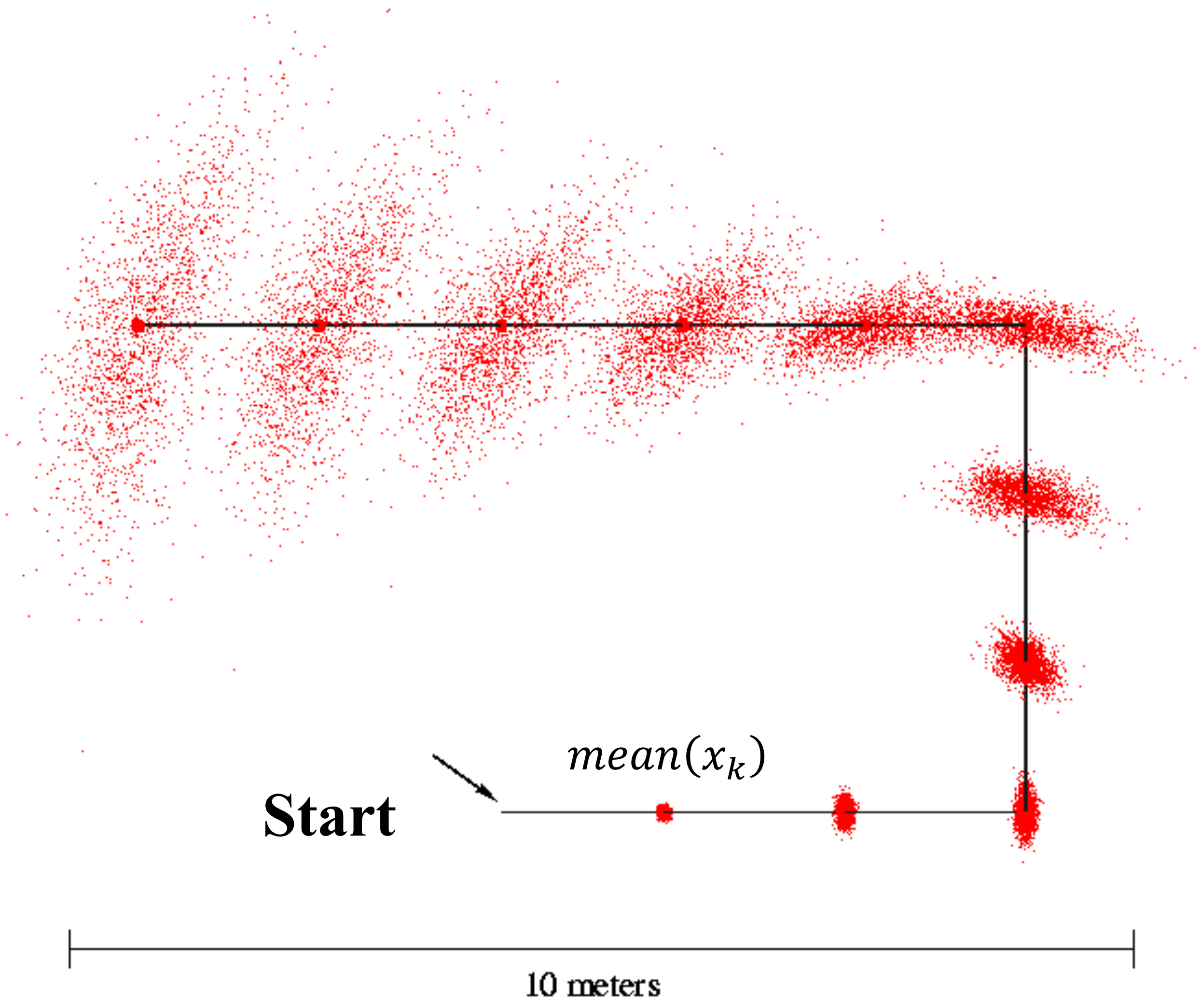
return $bel(x_t)$

Piecewise Constant Representation

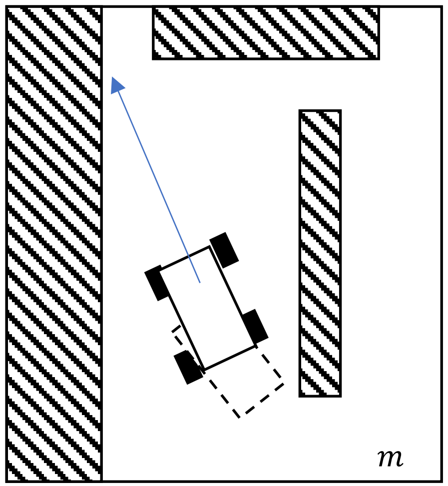


Fixing an input u_t we can compute the new belief

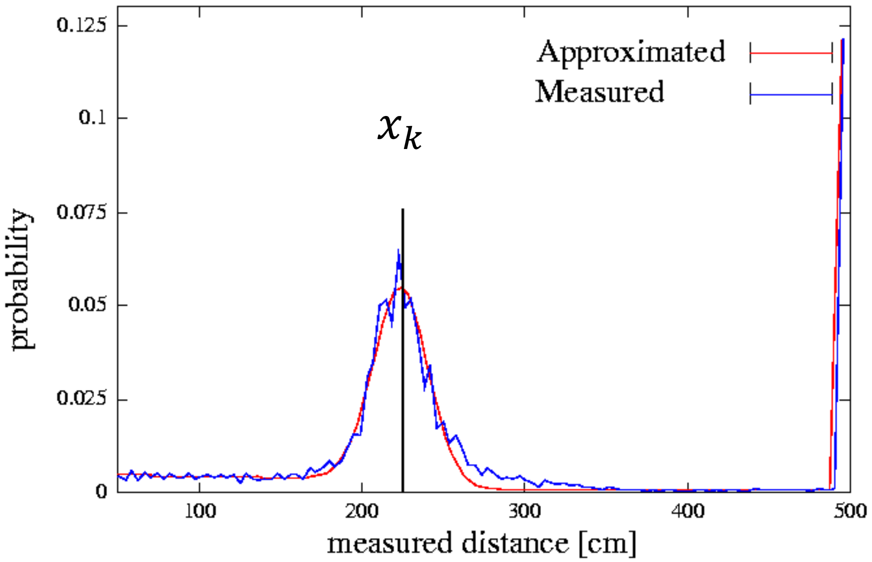
Motion Model without measurements



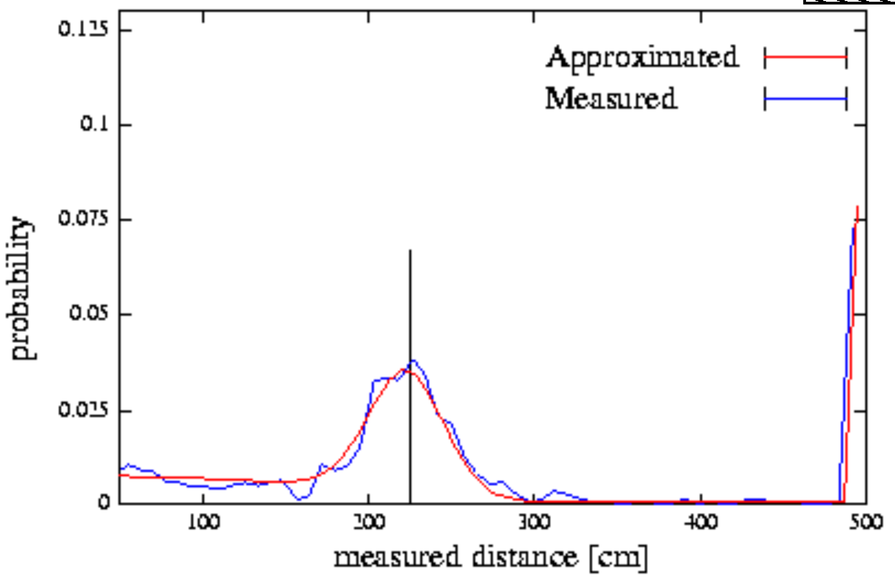
Proximity Sensor Model



$p(z_t \,|X_t = x_k)$

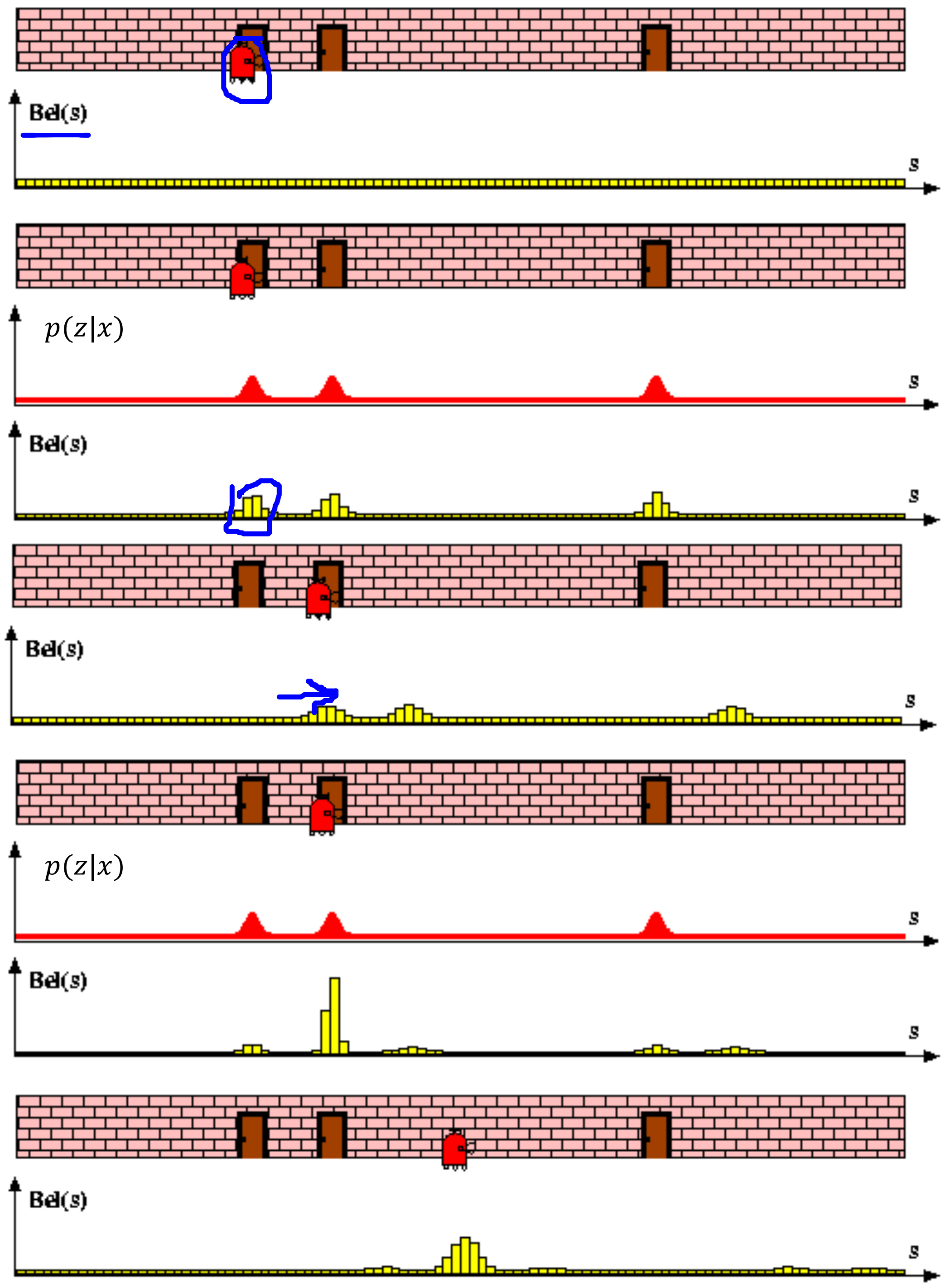


Laser sensor

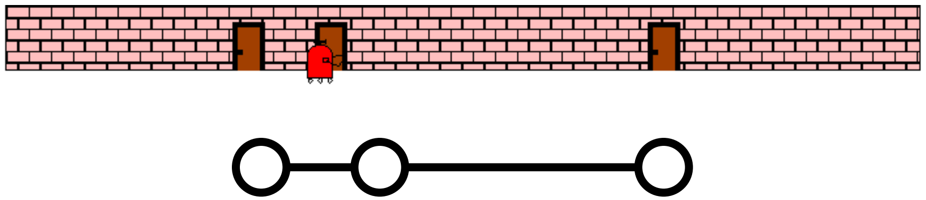


Sonar sensor

Grid localization, $bel(x_t)$represented by a histogram over grid

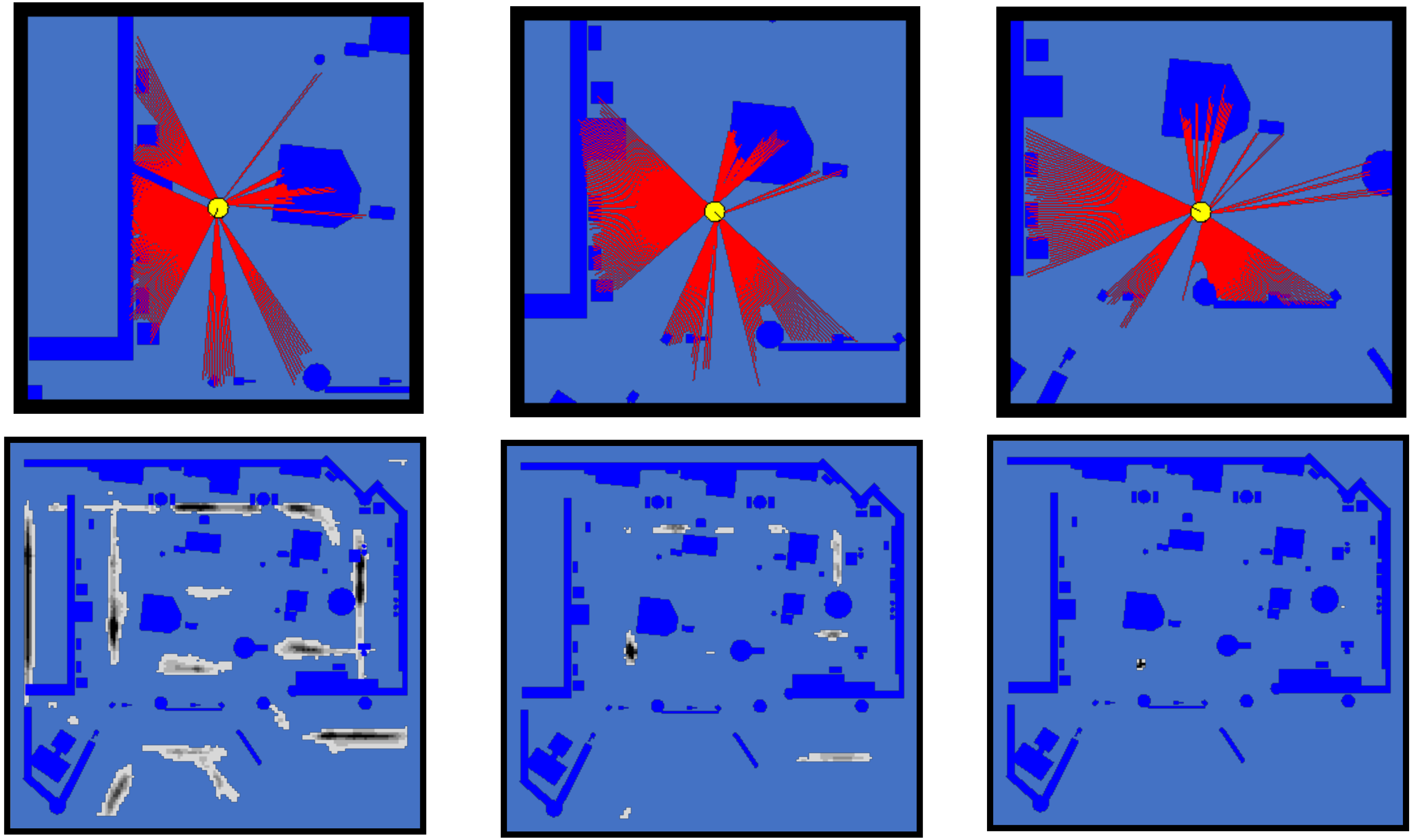


Summary

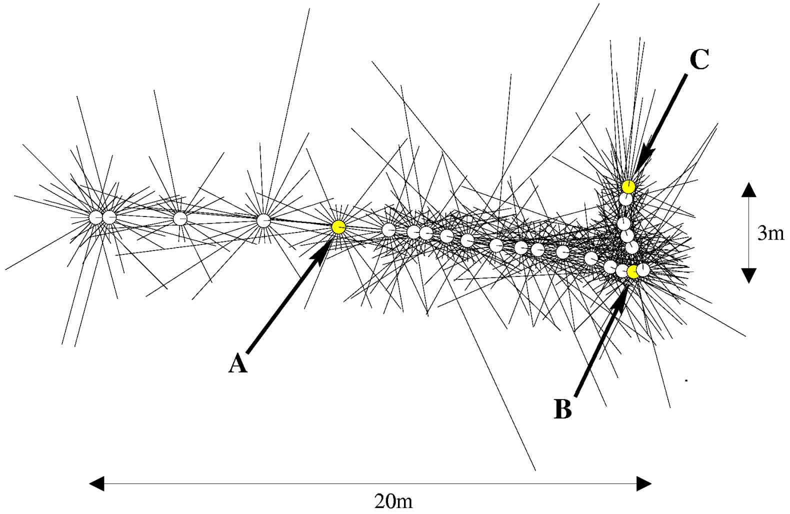


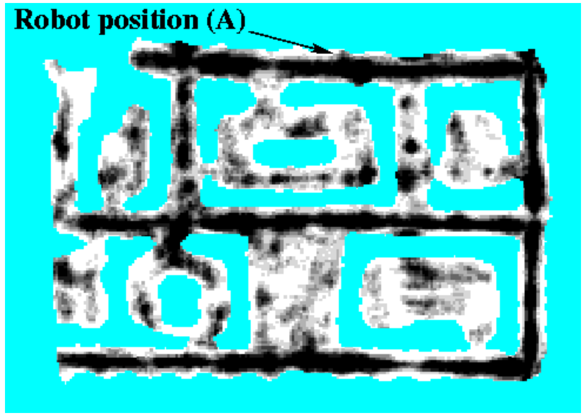
- Key variable: Grid resolution
- Two approaches
  - Topological: break-up pose space into regions of significance (landmarks)
  - Metric: fine-grained uniform partitioning; more accurate at the expense of higher computation costs
- Important to compensate for coarseness of resolution
  - Evaluating measurement/motion based on the center of the region may not be enough. *If motion is updated every 1s, robot moves at 10 cm/s, and the grid resolution is 1m, then naïve implementation will not have any state transition!*
- Computation
  - Motion model update for a 3D grid required a 6D operation, measurement update 3D
  - With fine-grained models, the algorithm cannot be run in real-time
  - Some calculations can be cached (ray-casting results)

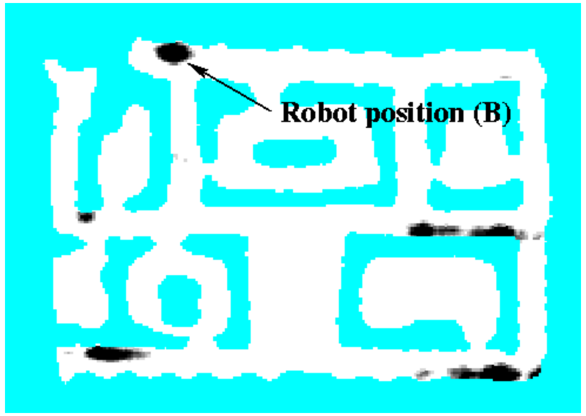
Grid-based Localization

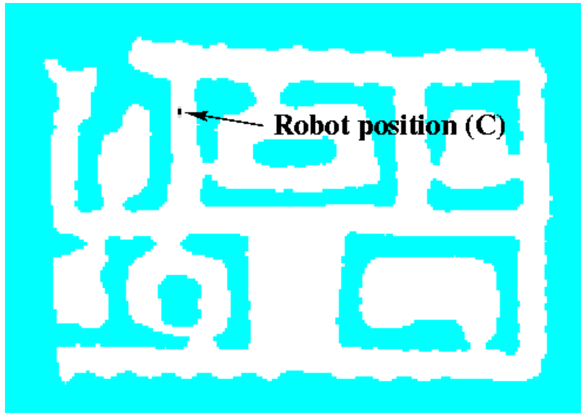


Sonars and Occupancy Grid Map









Monte Carlo Localization

- Represents beliefs by particles

Particle Filters

- Represent belief by finite number of parameters (just like histogram filter)
- But, they differ in how the parameters (particles) are generated and populate the state space
- Key idea: represent belief $bel(x_t)$ by a random set of state samples
- Advantages
  - The representation is approximate and nonparametric and therefore can represent a broader set of distributions than e.g., Gaussian
  - Can handle nonlinear tranformations
- Related ideas: Monte Carlo filter, Survival of the fittest, Condensation, Bootstrap filter, Filtering: [Rubin, 88], [Gordon et al., 93], [Kitagawa 96], Dynamic Bayesian Networks: [Kanazawa et al., 95]d

Particle filtering algorithm

$X_t = x_t^{[1]}, x_t^{[2]}, \dots x_t^{[M]}$ particles

Algorithm Particle_filter(X_{t-1}, u_t, z_t):
$\bar{X}_{t-1} = X_t = \emptyset$

for all m in [M] do:

sample $x_t^{[m]} \sim p(x_t | u_t, x_{t-1}^{[m]})$

$w_t^{[m]} = p\left(z_t \middle| x_t^{[m]}\right)$

$\bar{X}_t = \bar{X}_t + \langle\, x_t^{[m]}, w_t^{[m]} \rangle$

end for

for all m in [M] do:

draw $i\ with\ probability \propto w_t^{[i]}$

add $x_t^{[i]}\ to\ X_t$

end for

return X_t

ideally, $x_t^{[m]}$ is selected with probability prop. to $p(x_t \mid z_{1:t}, u_{1:t})$

$\bar{X}_{t-1}$ is the temporary particle set

// sampling from state transition dist.

// calculates *importance factor* w_t or weight

// resampling or importance sampling; these are distributed according to $\eta\, p\left(z_t \middle| x_t^{[m]}\right) \overline{bel}(x_t)$

// survival of fittest: moves/adds particles to parts of the state space with higher probability

Importance Sampling

suppose we want to compute $E_f[I(x \in A)]$ but we can only sample from density g

$E_f[I(x \in A)]$

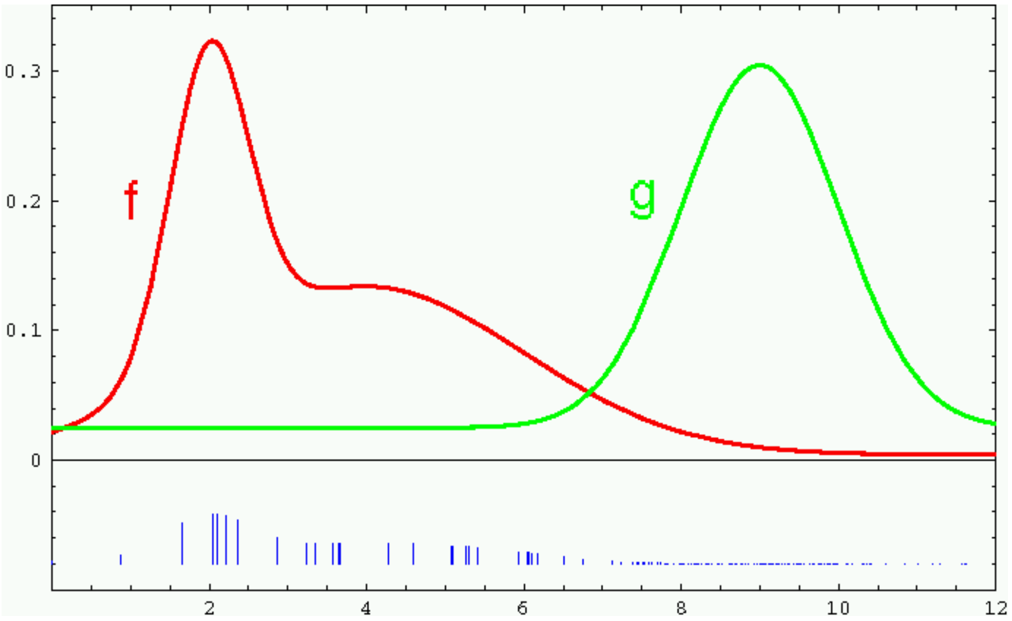
$= \int f(x) I(x \in A) dx$

$= \int \frac{f(x)}{g(x)} g(x) I(x \in A) dx$, provided $g(x) > 0$

$= \int w(x) g(x) I(x \in A) dx$

$= E_g[w(x) I(x \in A)]$

We need $f(x) > 0 \Rightarrow g(x) > 0$



Weight samples: $w = f / g$

Monte Carlo Localization (MCL)

$X_t = x_t^{[1]}, x_t^{[2]}, \ldots x_t^{[M]}$ particles

Algorithm MCL(X_{t-1}, u_t, z_t,m):

$\bar{X}_{t-1} = X_t = \emptyset$

for all m in [M] do:

$x_t^{[m]} = \boldsymbol{sample\_motion\_model}(u_t\ x_{t-1}^{[m]})$

$w_t^{[m]} = \boldsymbol{measurement\_model}(z_t, x_t^{[m],m})$

$\bar{X}_t = \bar{X}_t + \langle\, x_t^{[m]}, w_t^{[m]} \rangle$

end for

for all m in [M] do:

draw $i\ with\ probability\ \propto w_t^{[i]}$

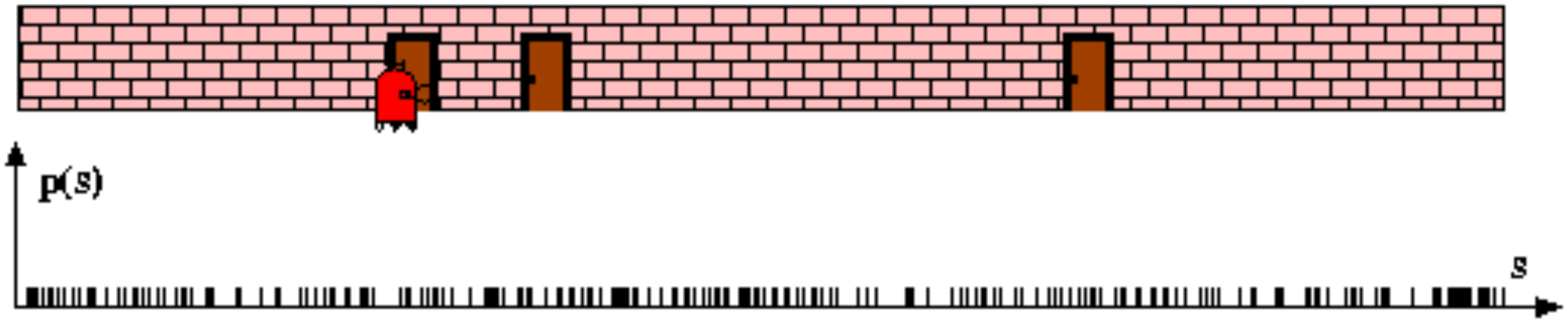
add $x_t^{[i]}\ to\ X_t$

end for

return X_t

Plug in motion and measurement models in the particle filter

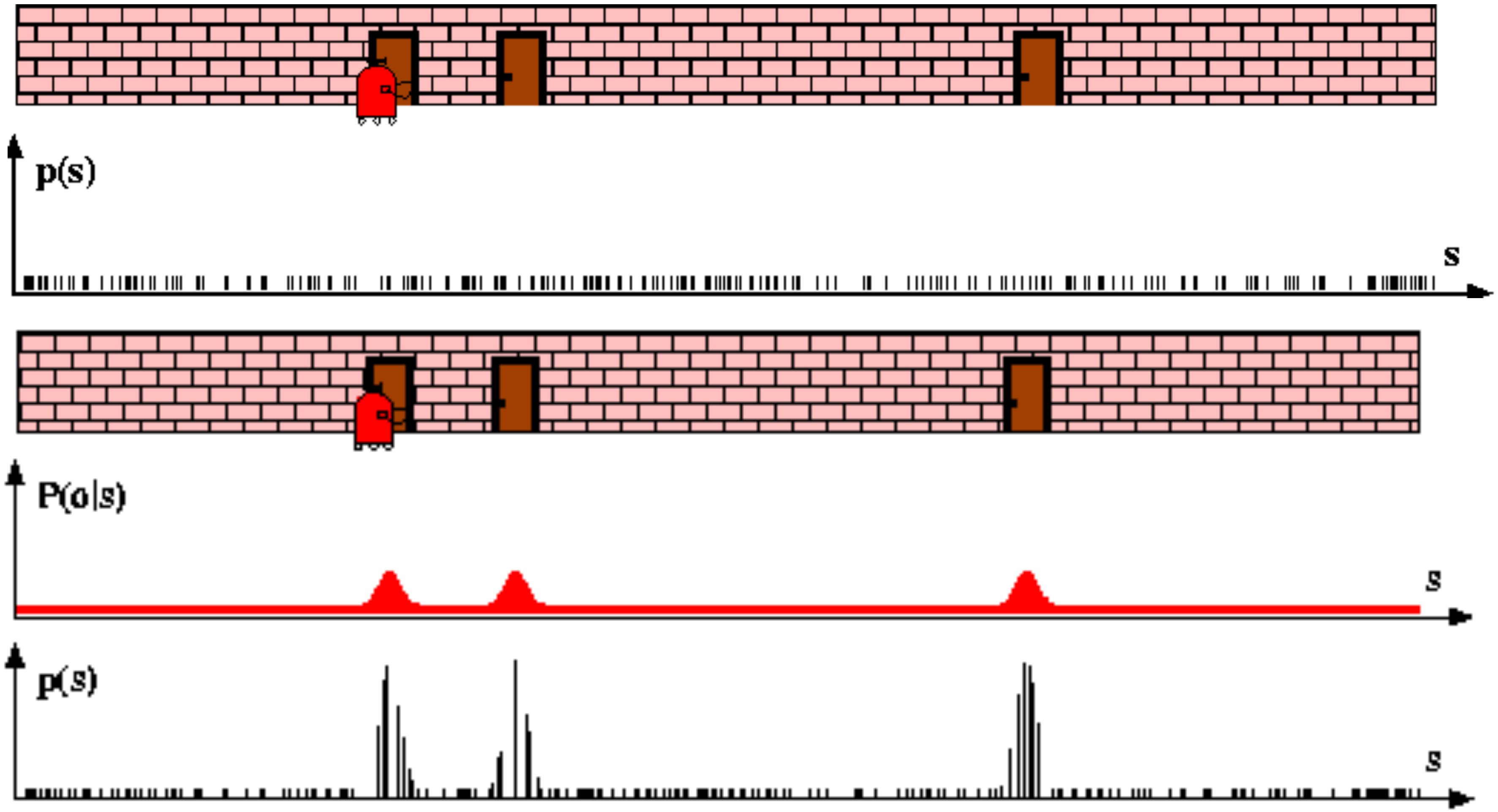
Particle Filters



Sensor Information: Importance Sampling

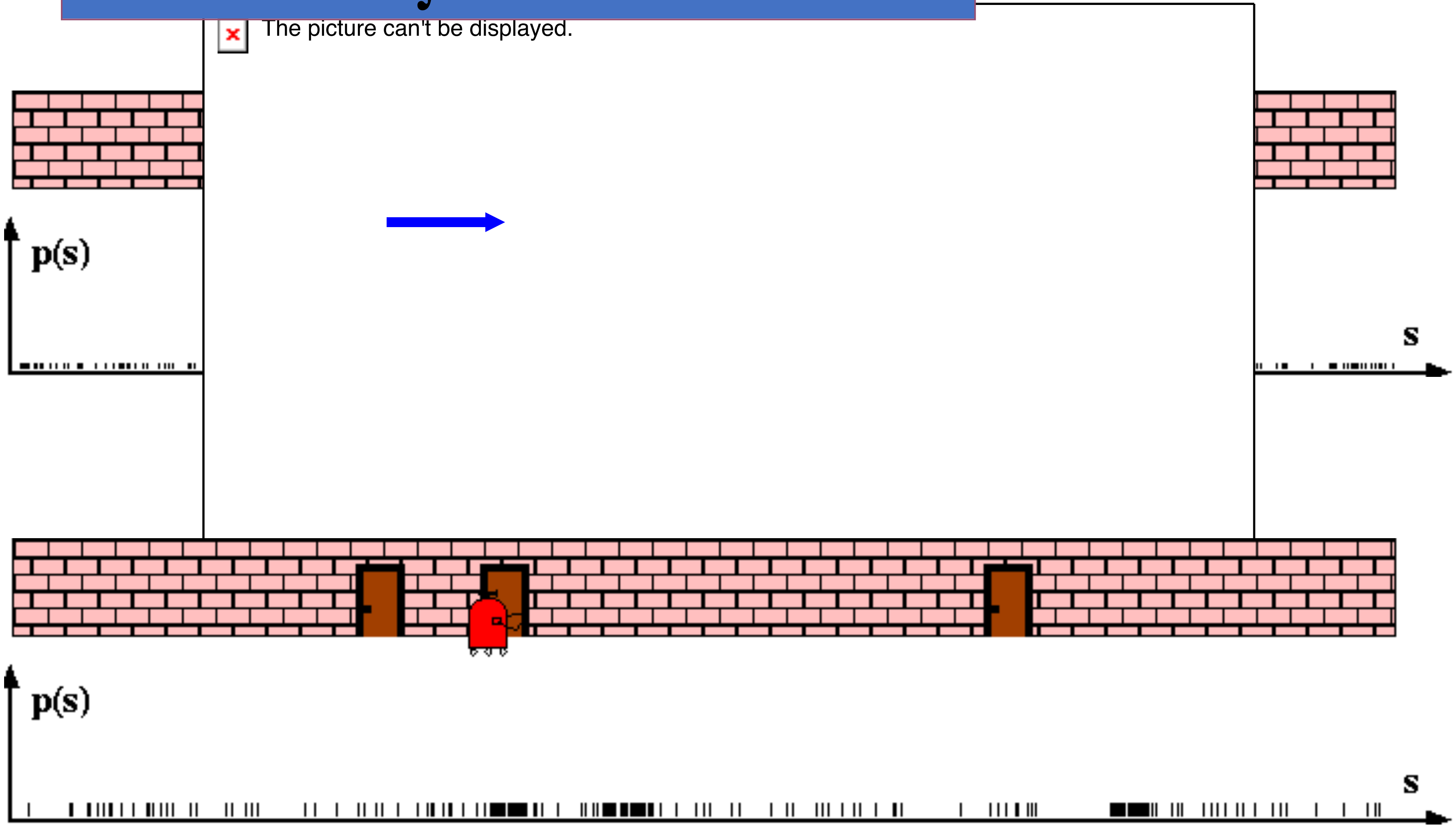
$$Bel(x) \leftarrow \alpha\ p(z \mid x)\ Bel^{-}(x)$$

$$w \leftarrow \frac{\alpha\ p(z \mid x)\ Bel^{-}(x)}{Bel^{-}(x)} = \alpha\ p(z \mid x)$$



Robot Motion

$$Bel^{-}(x) \quad \leftarrow \quad \int p(x \mid u, x')\, Bel(x') \;\mathrm{d}\, x'$$



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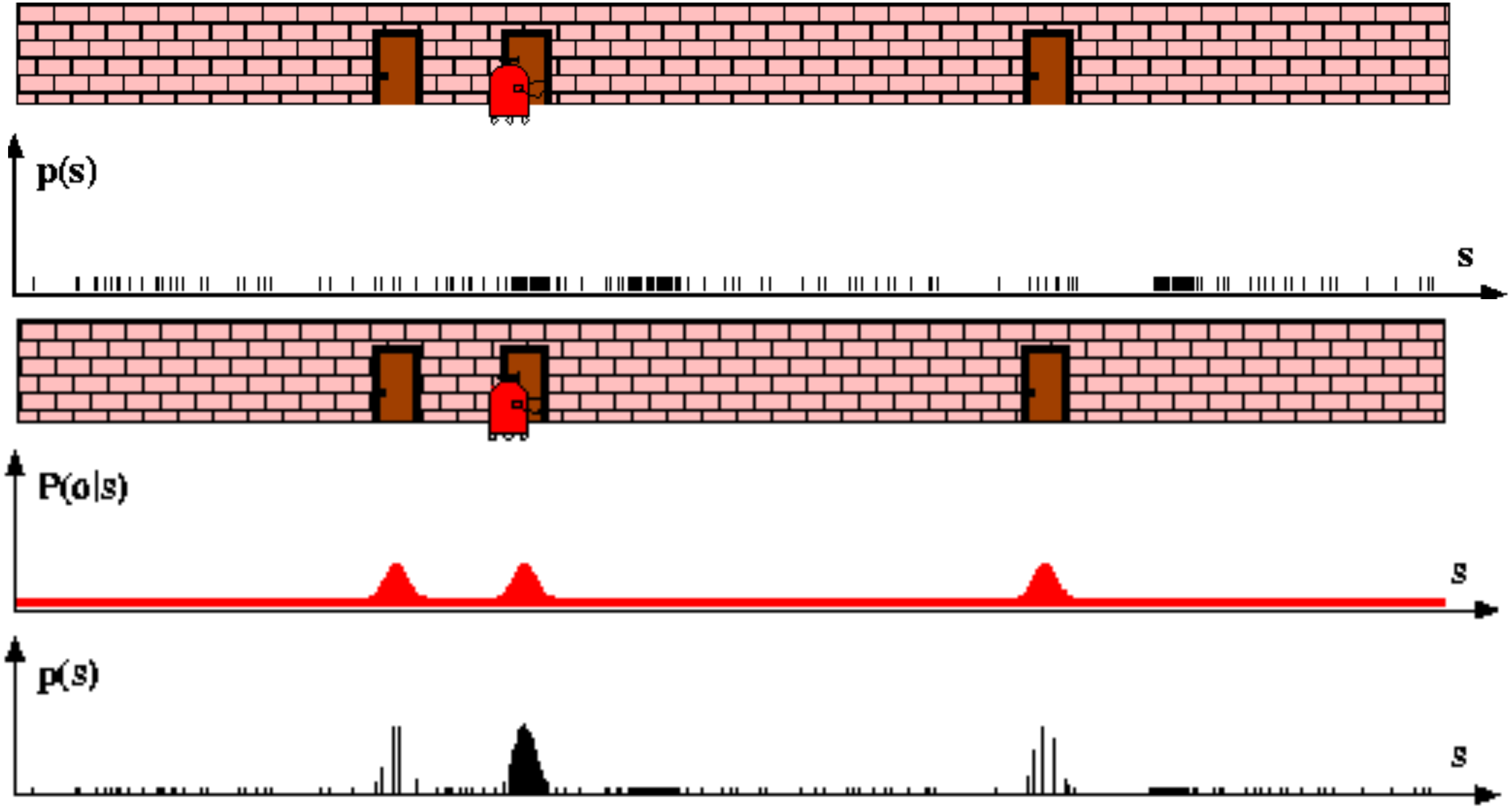

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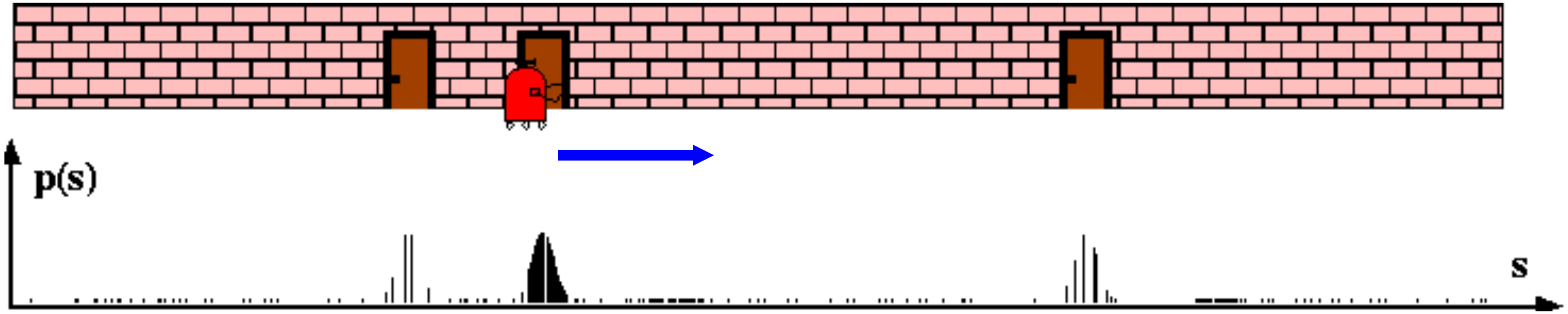
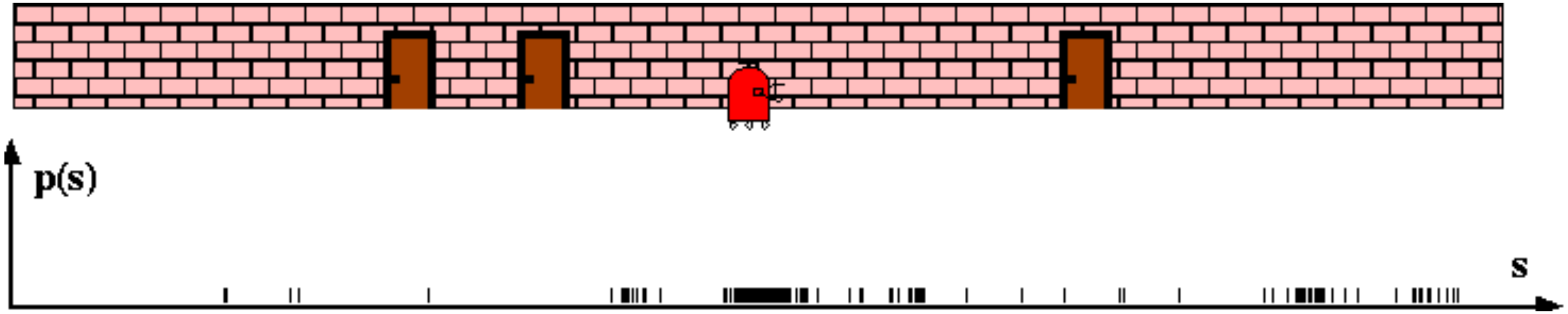
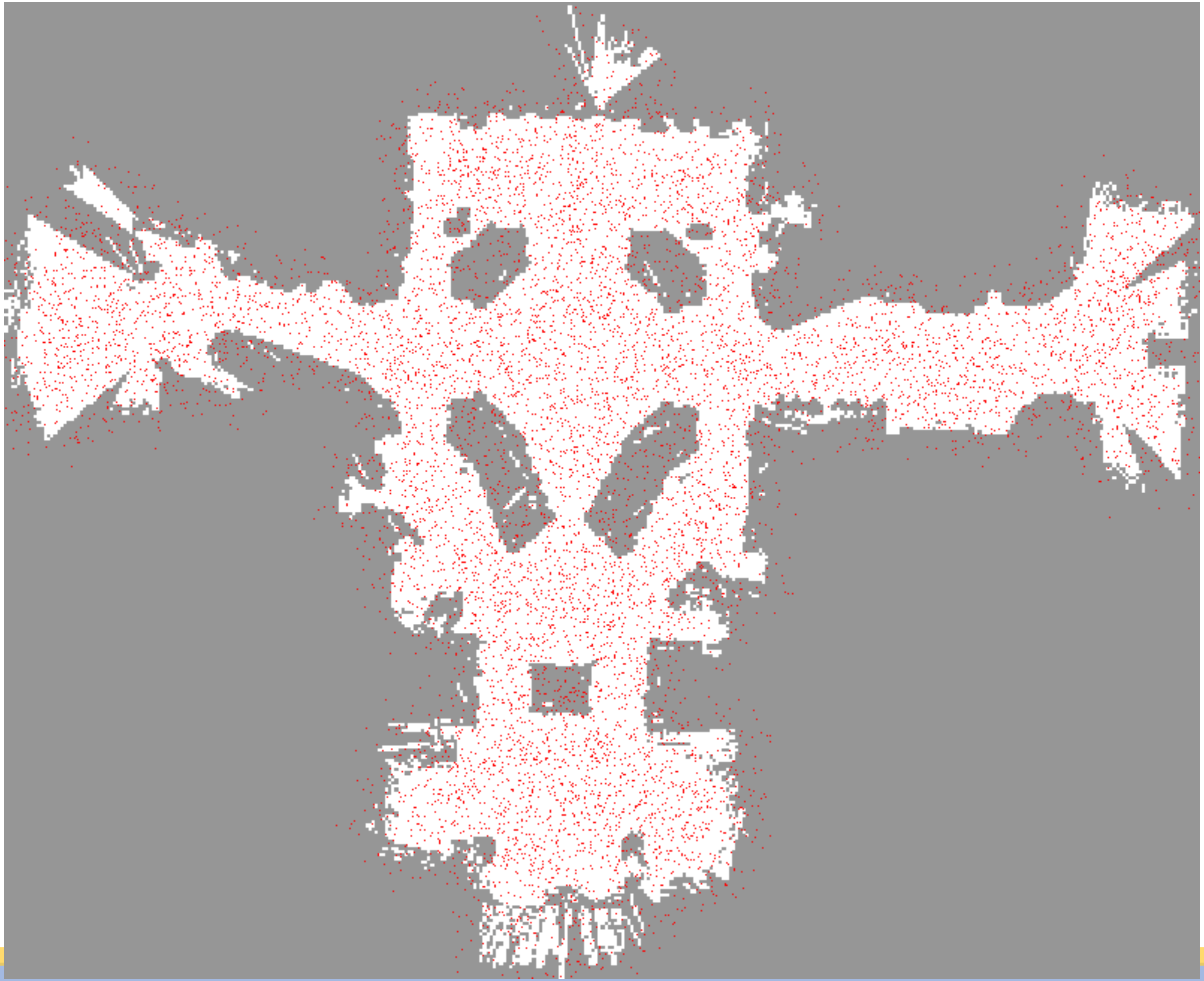
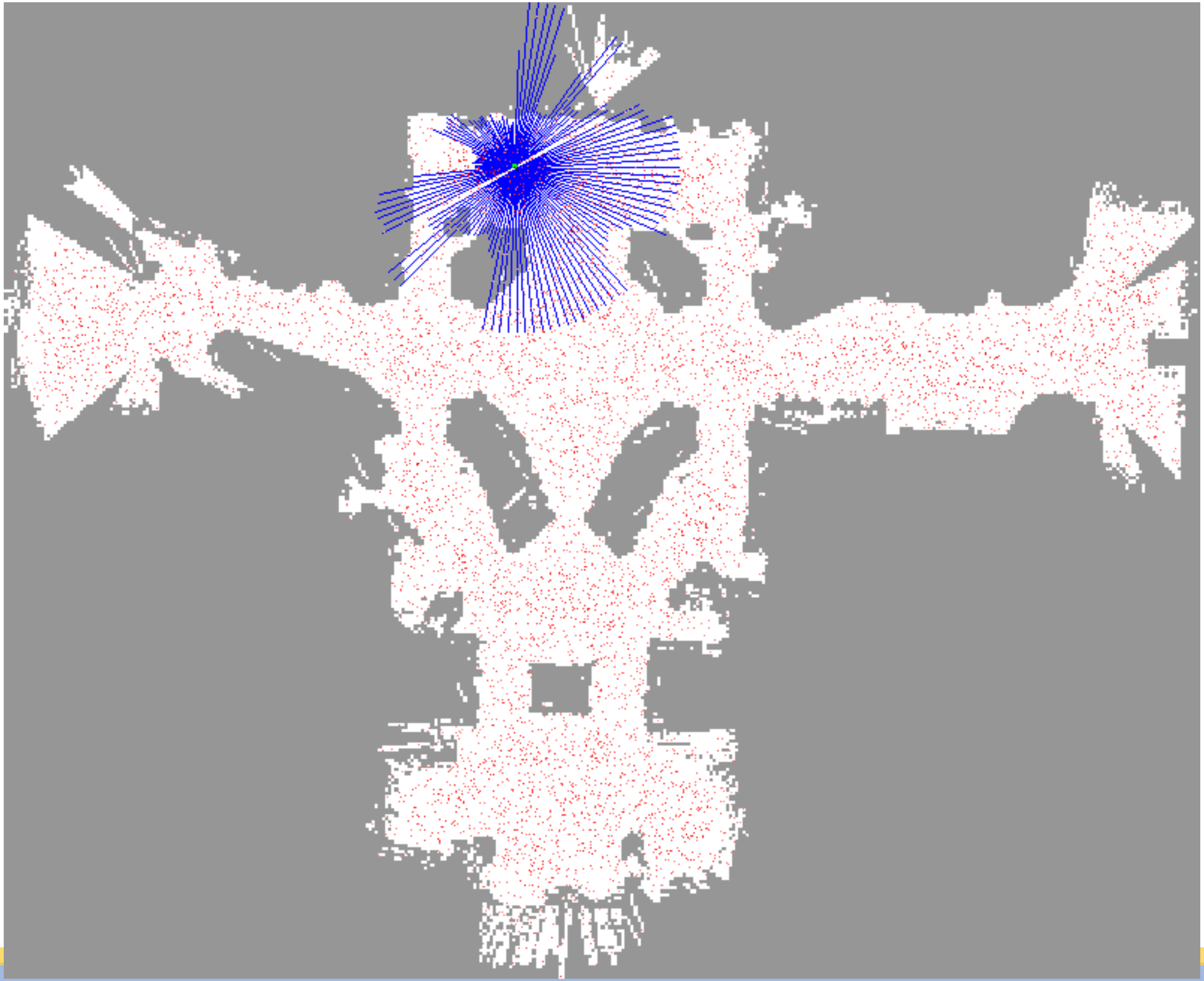
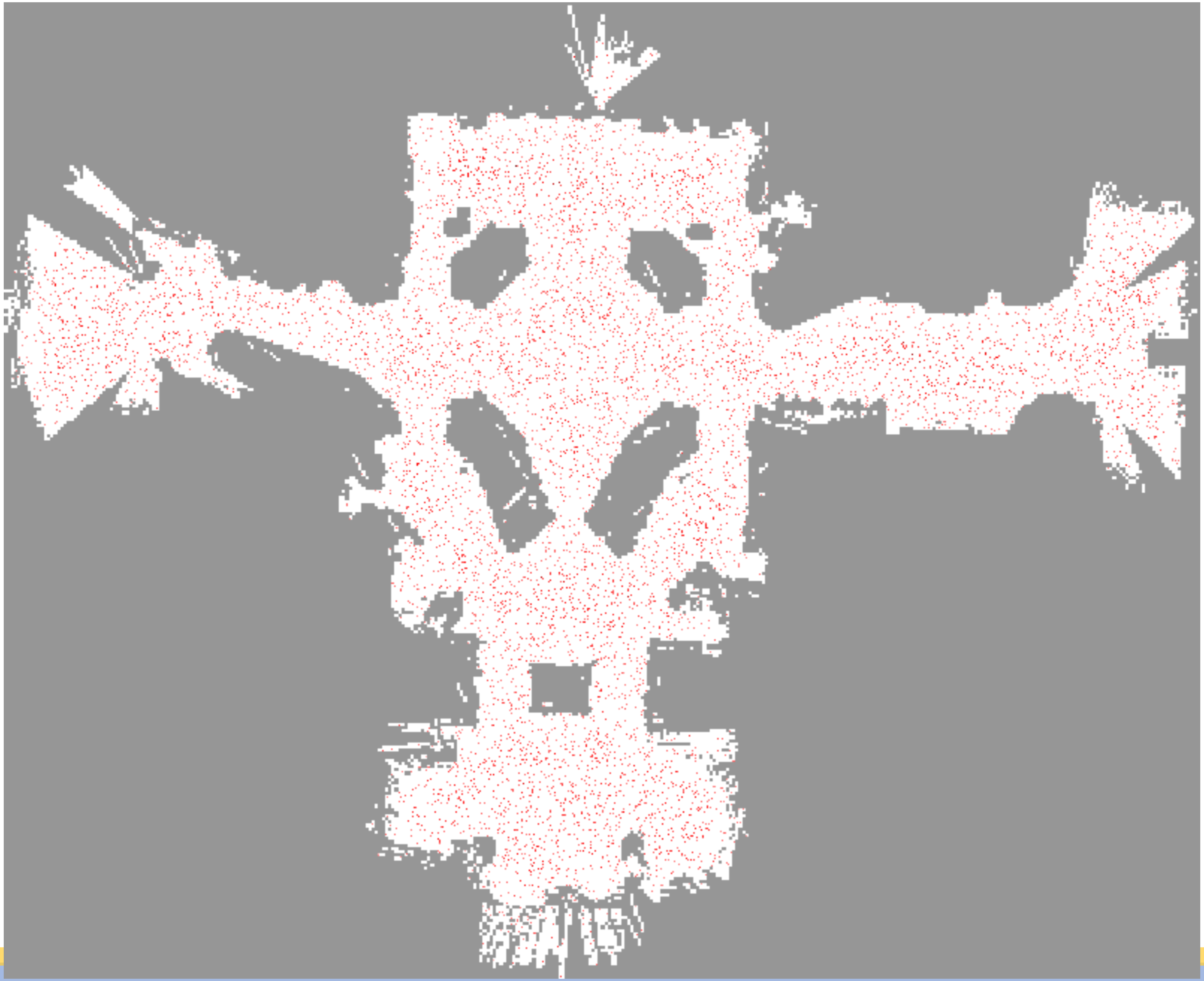
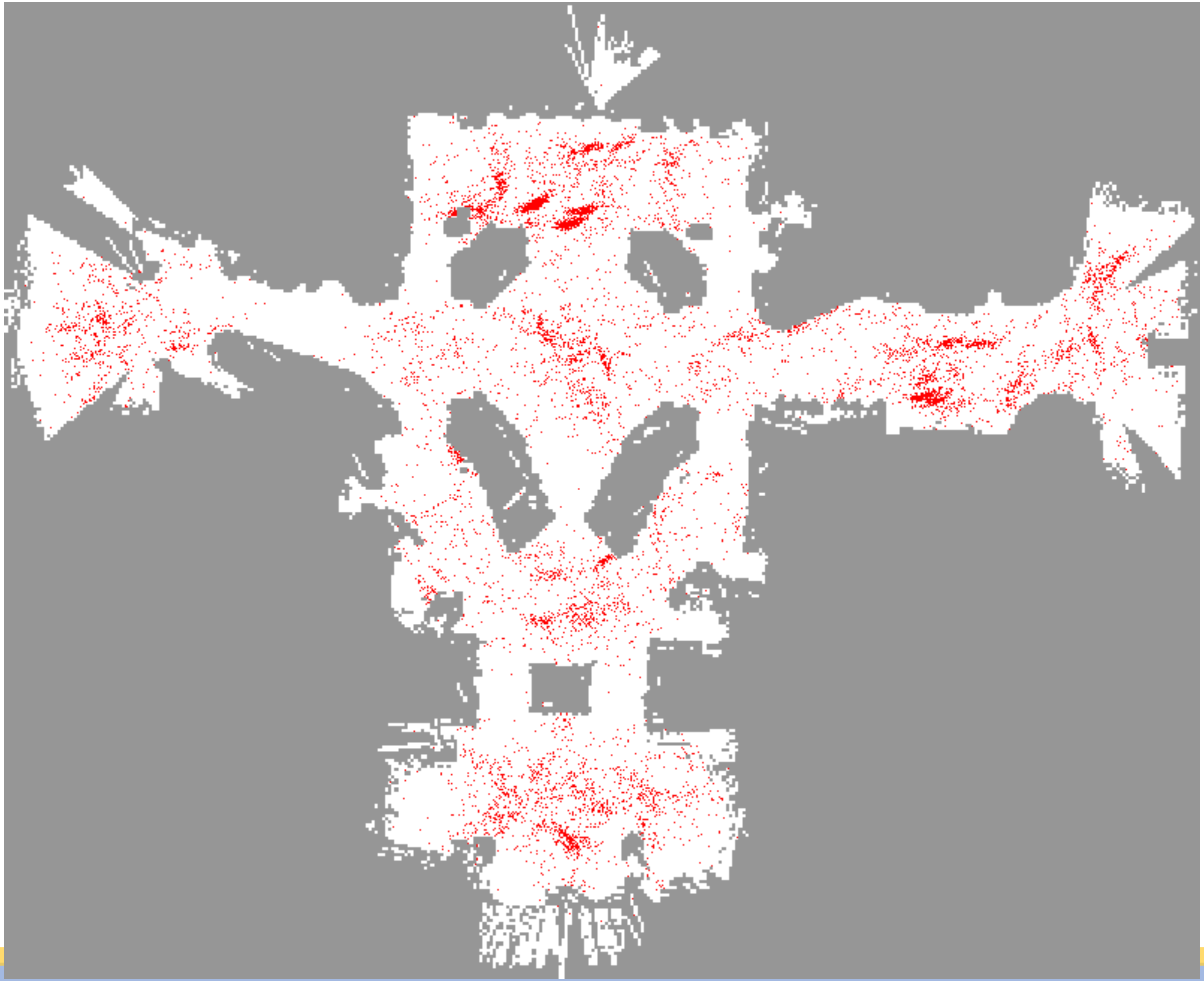


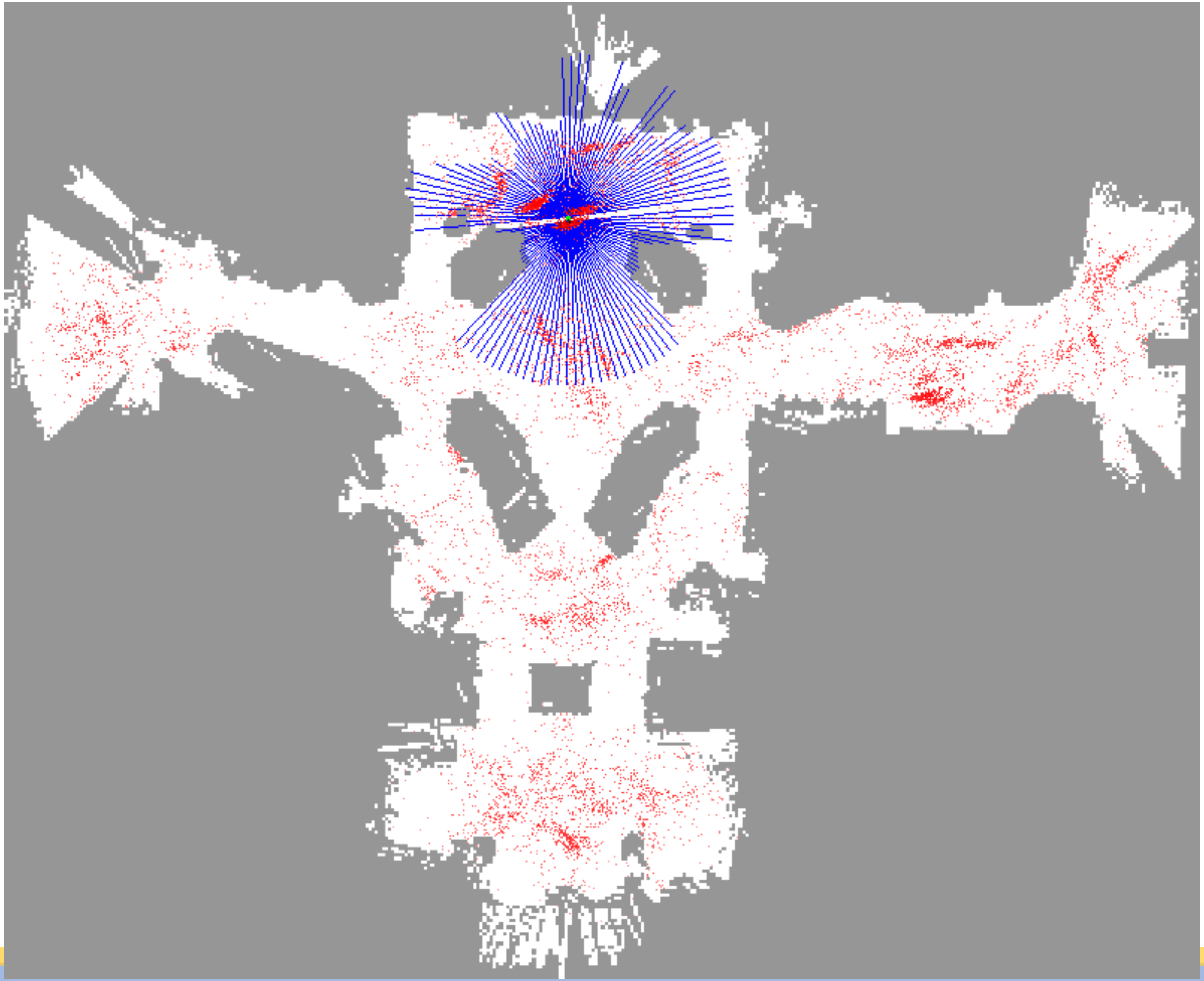


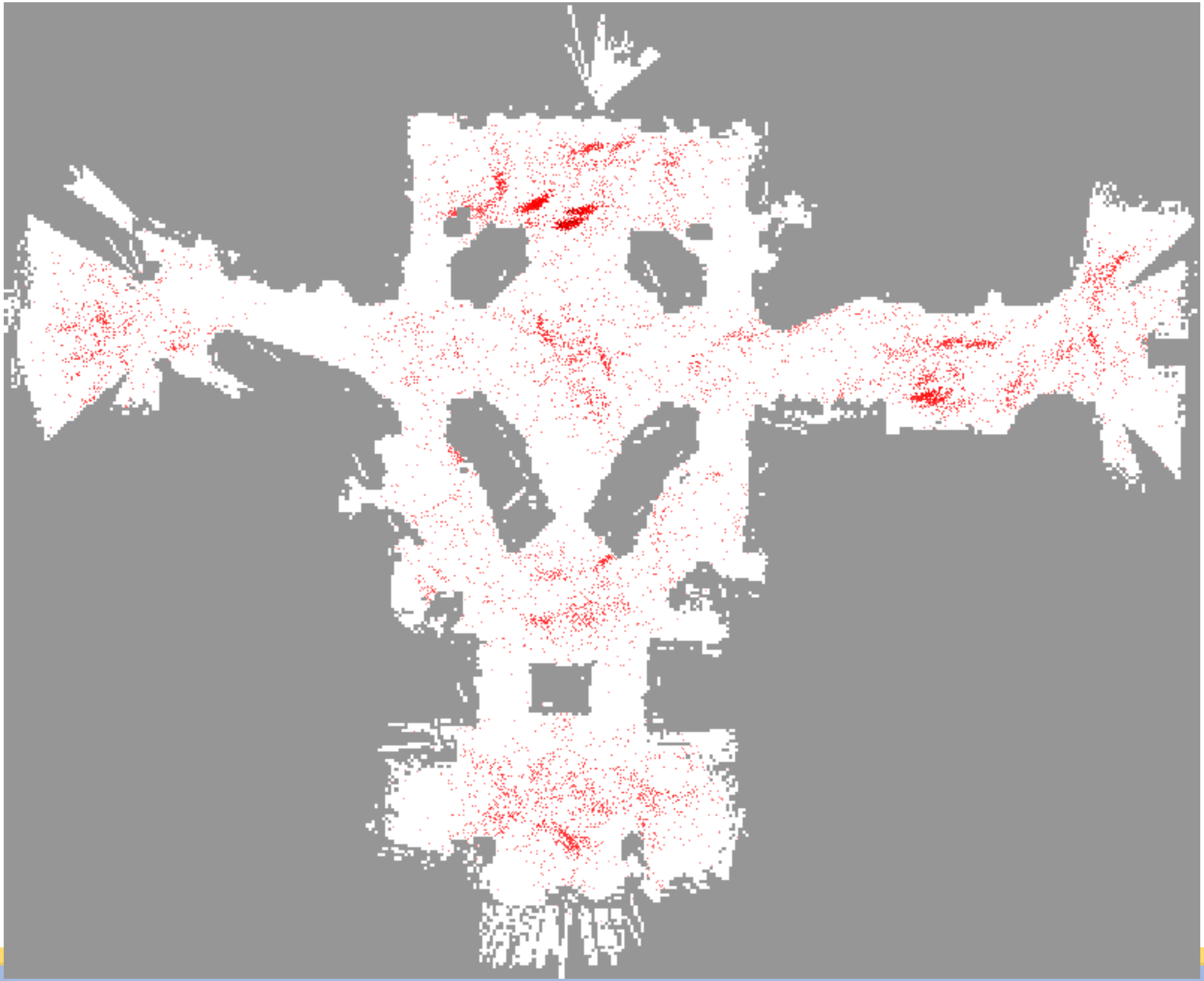


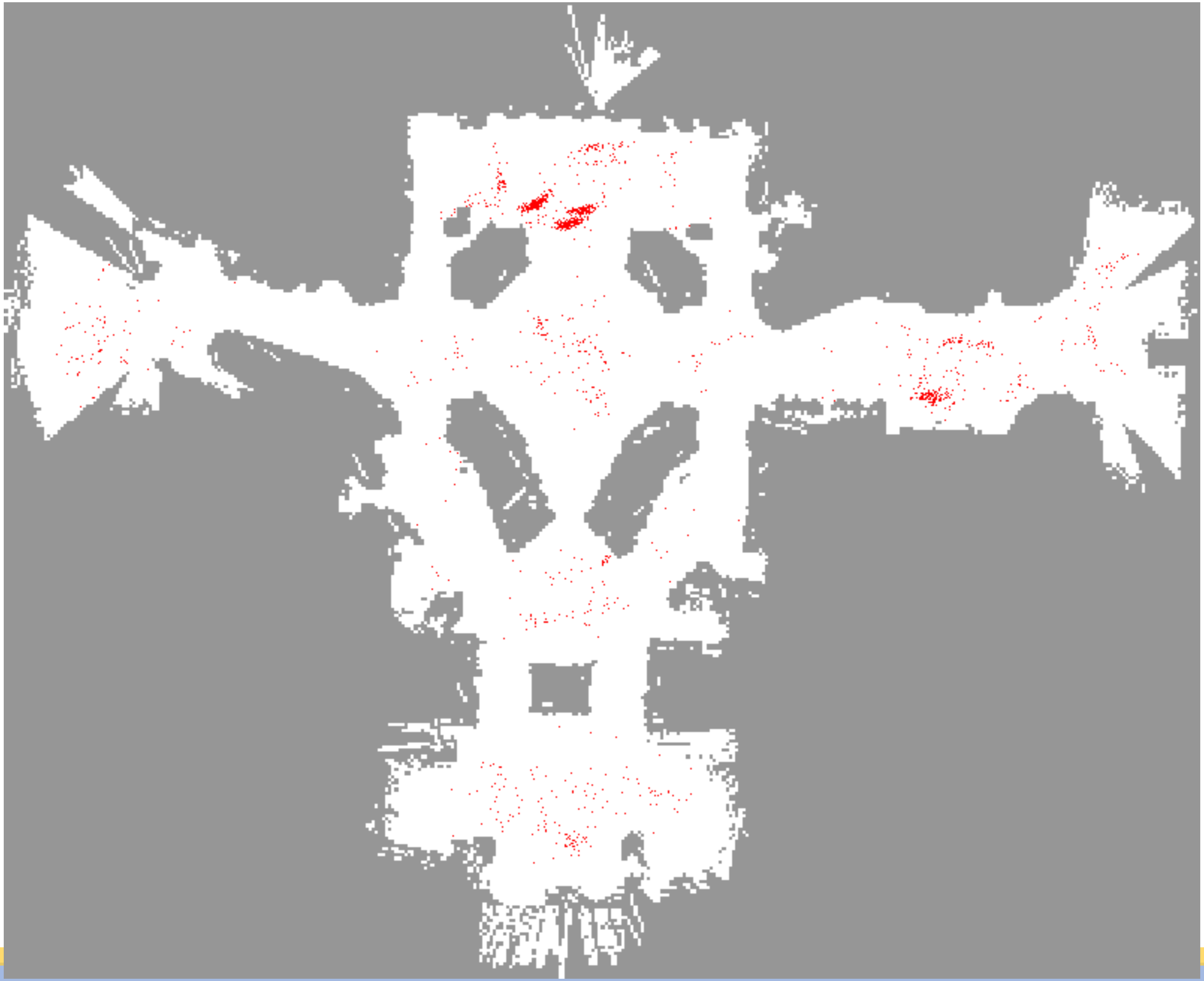


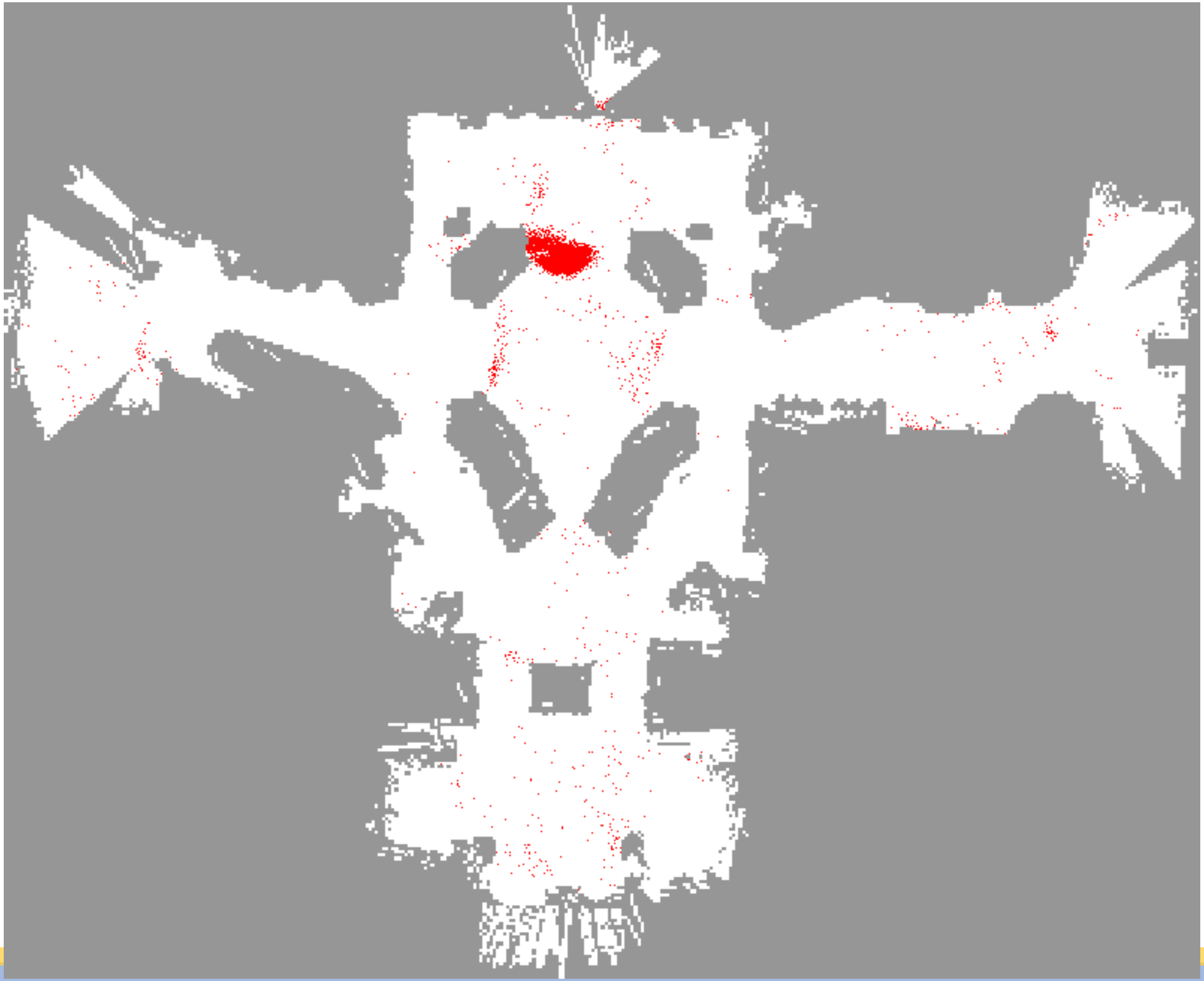


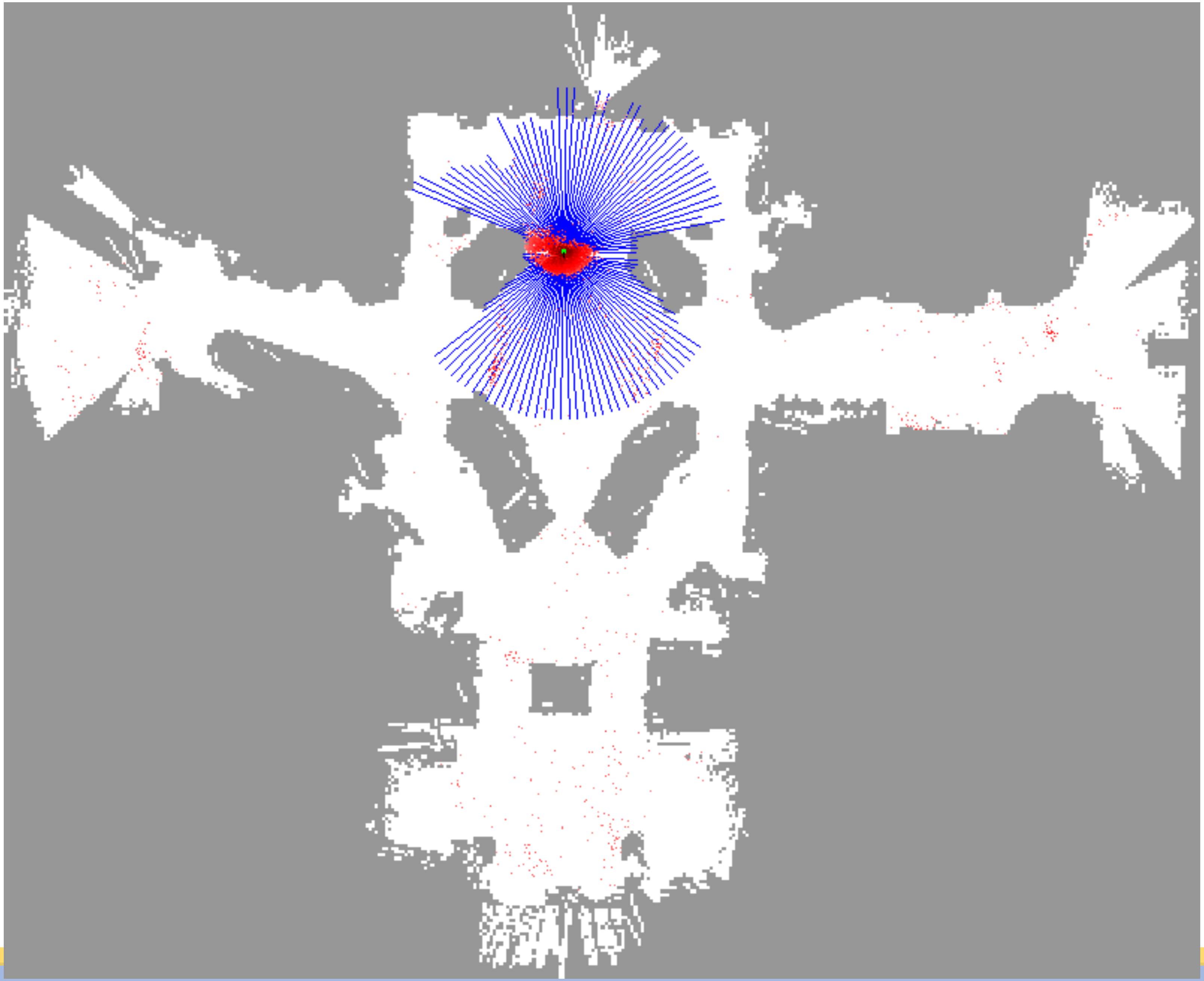


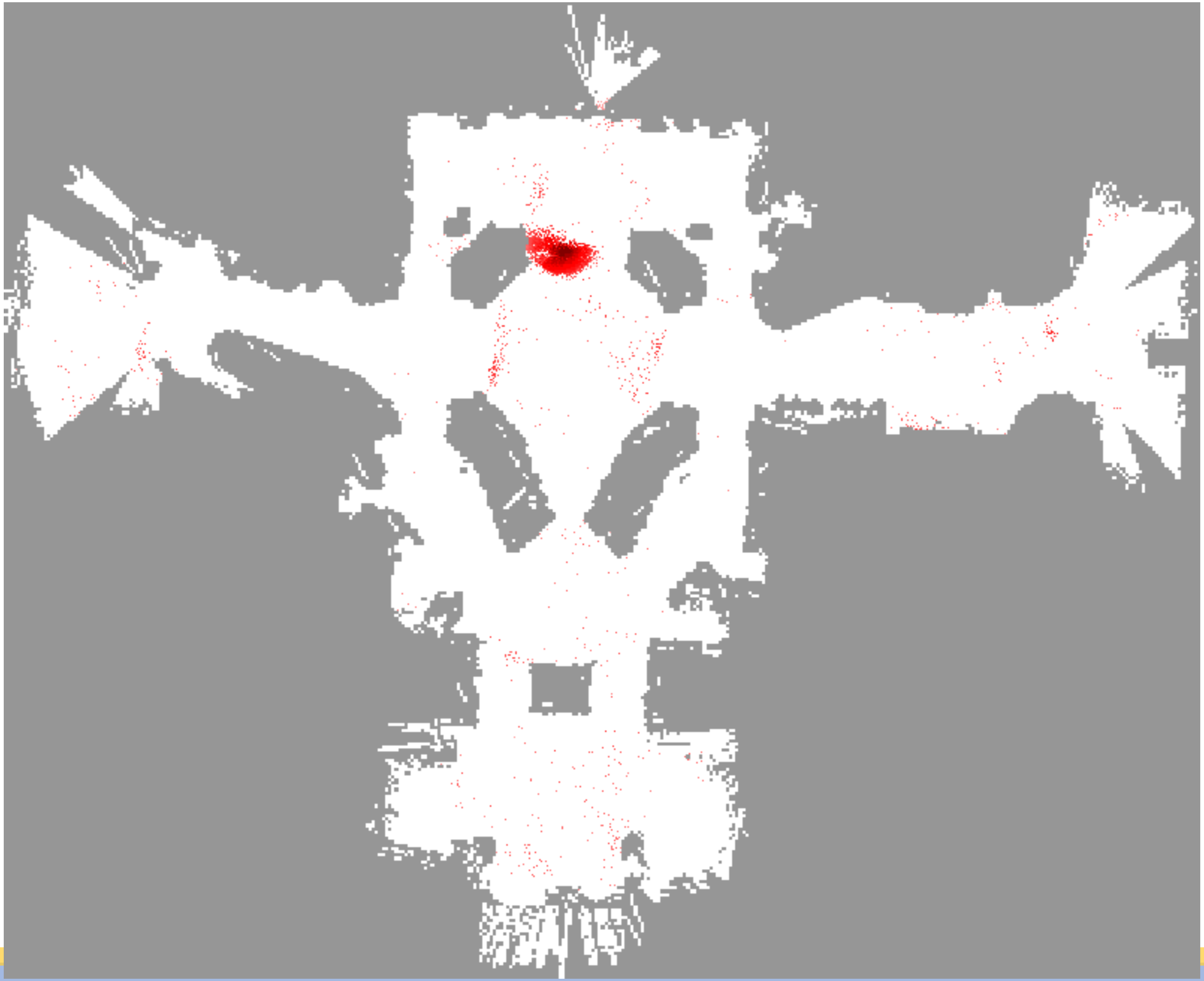


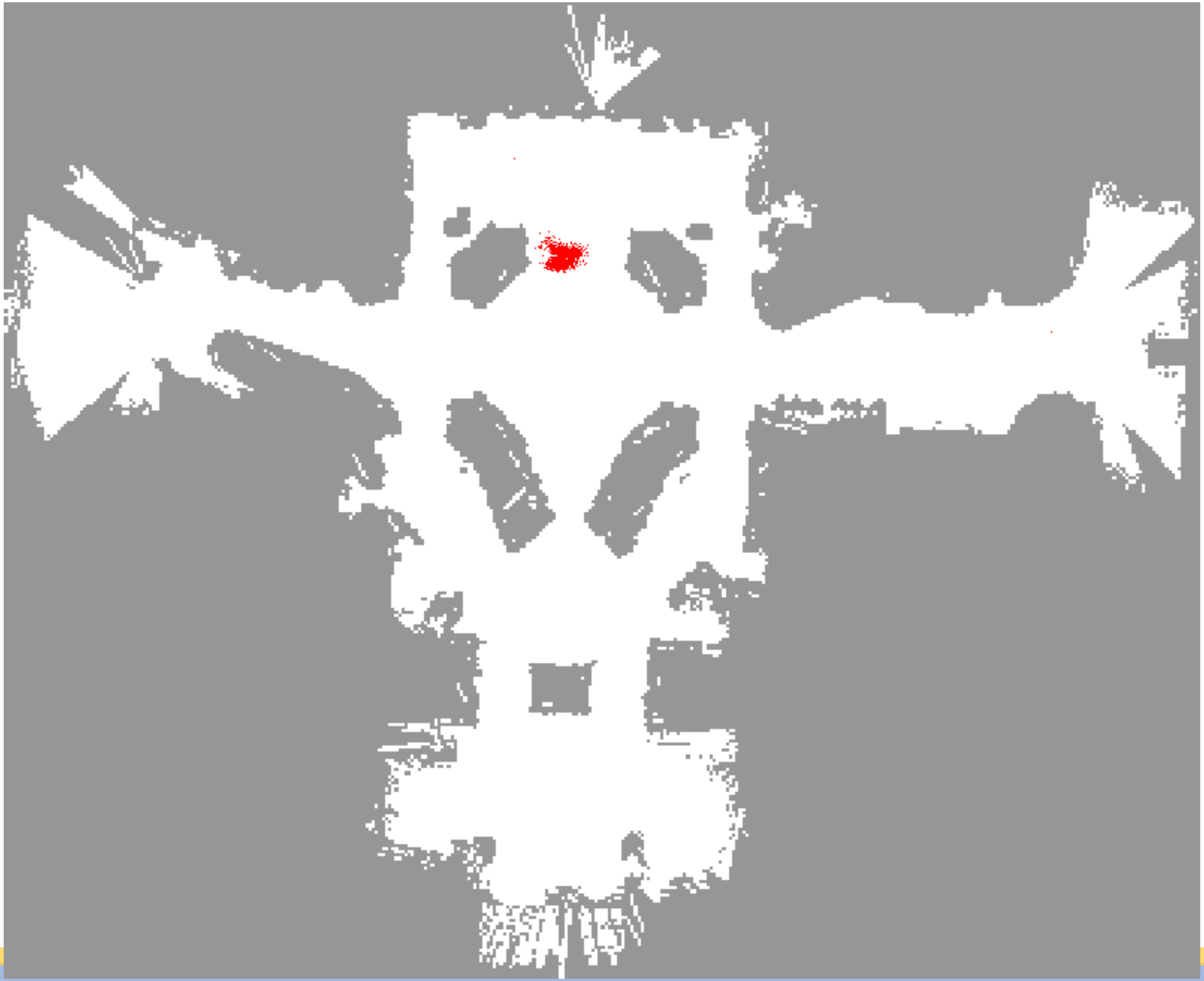


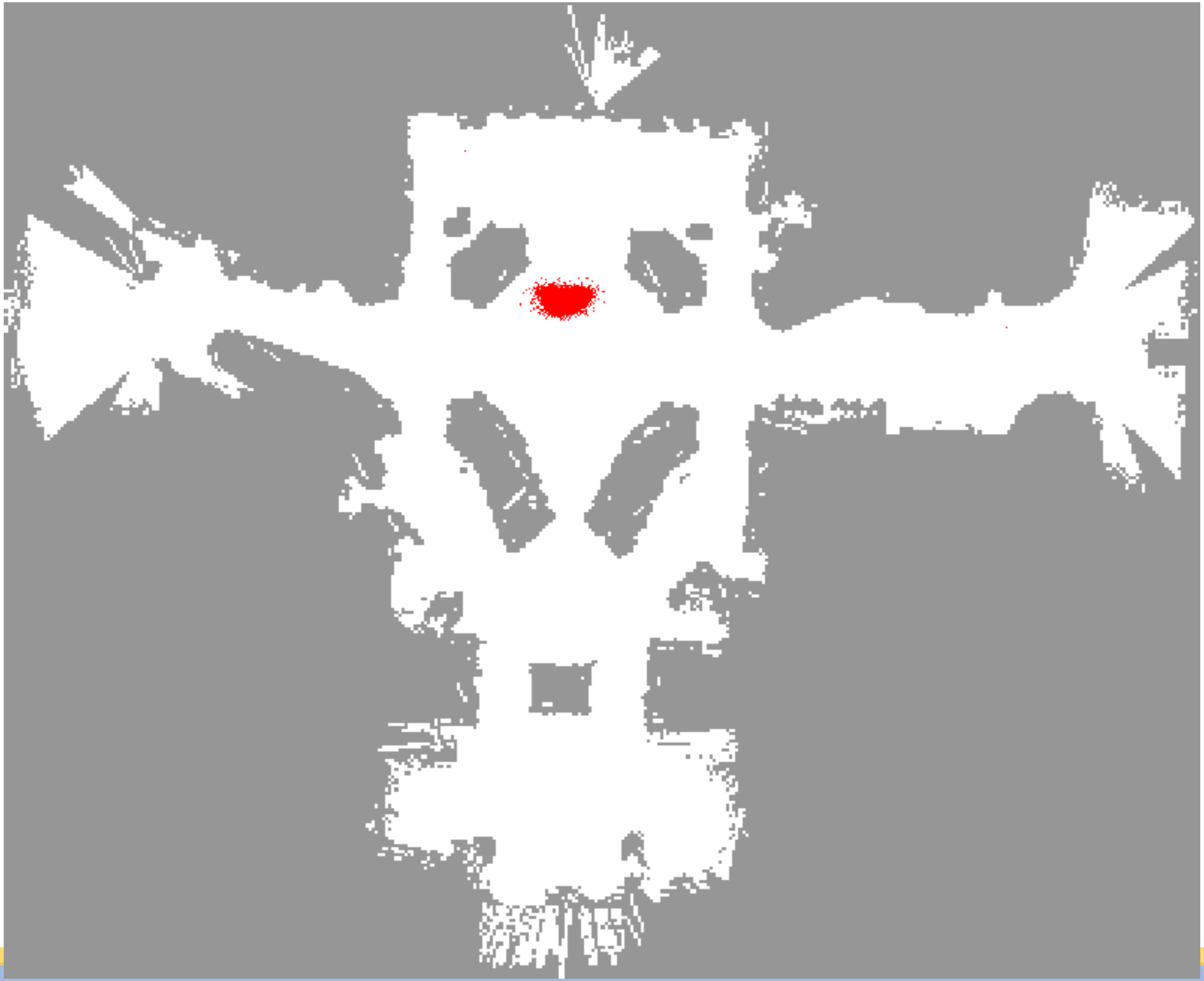


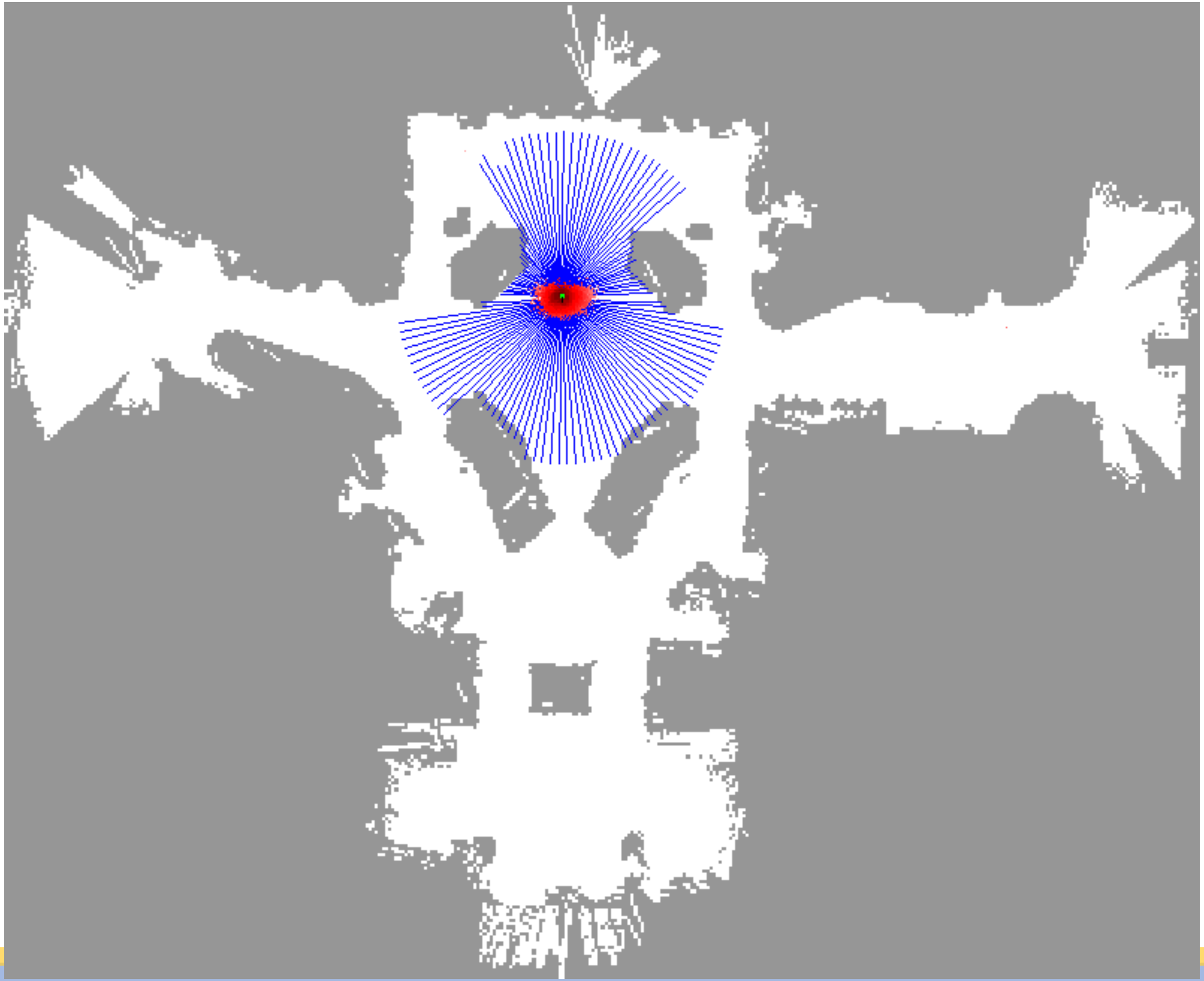


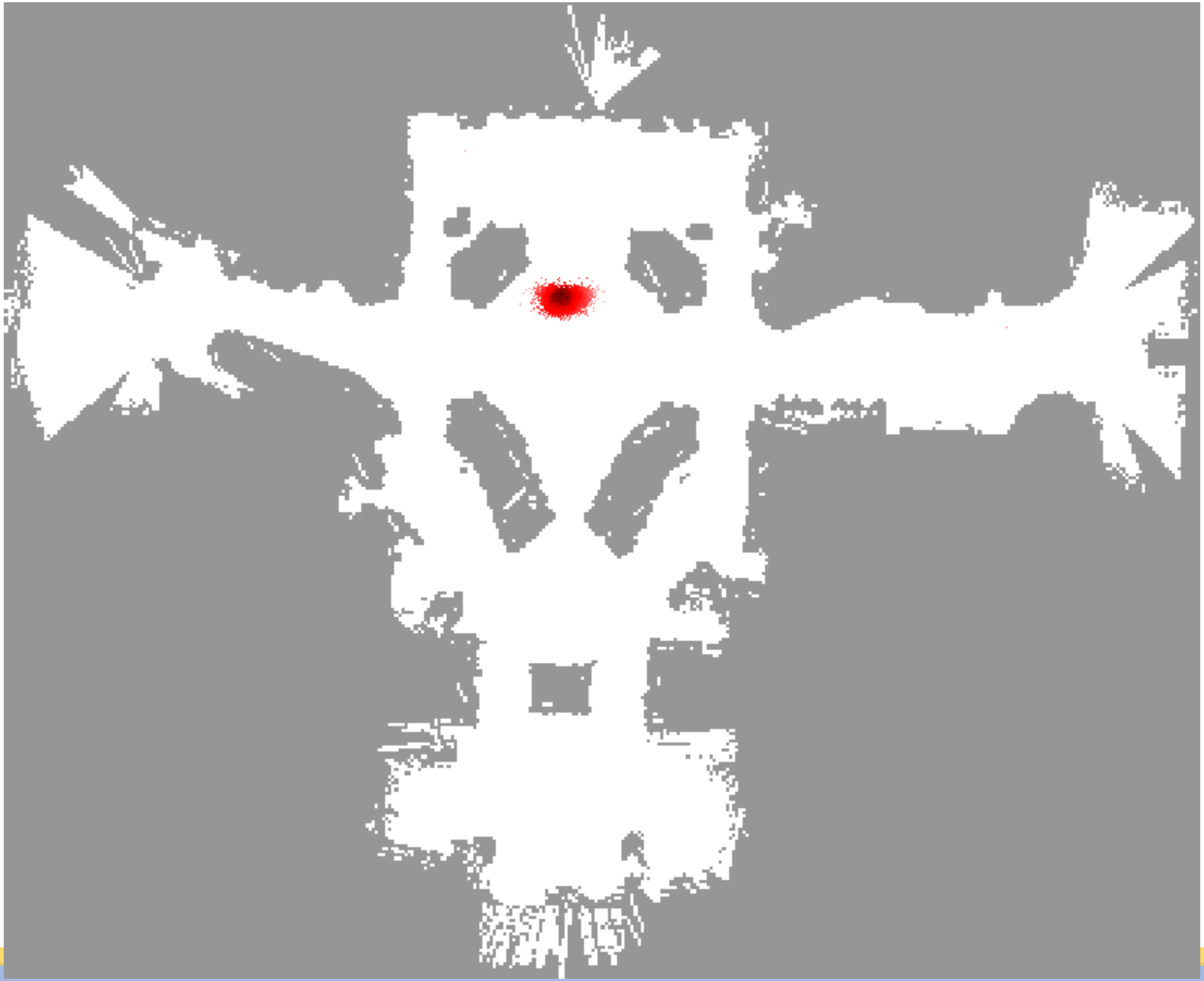


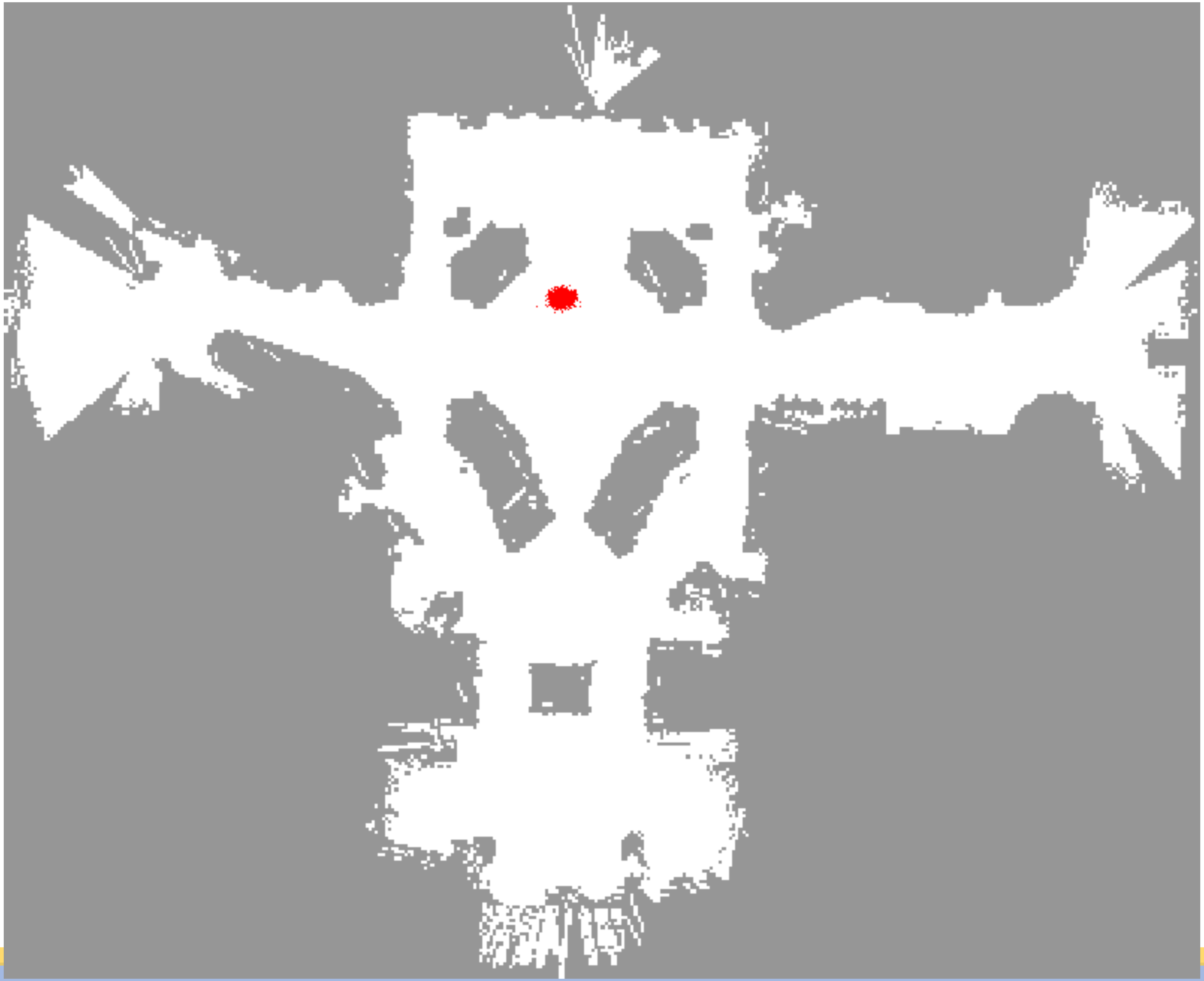


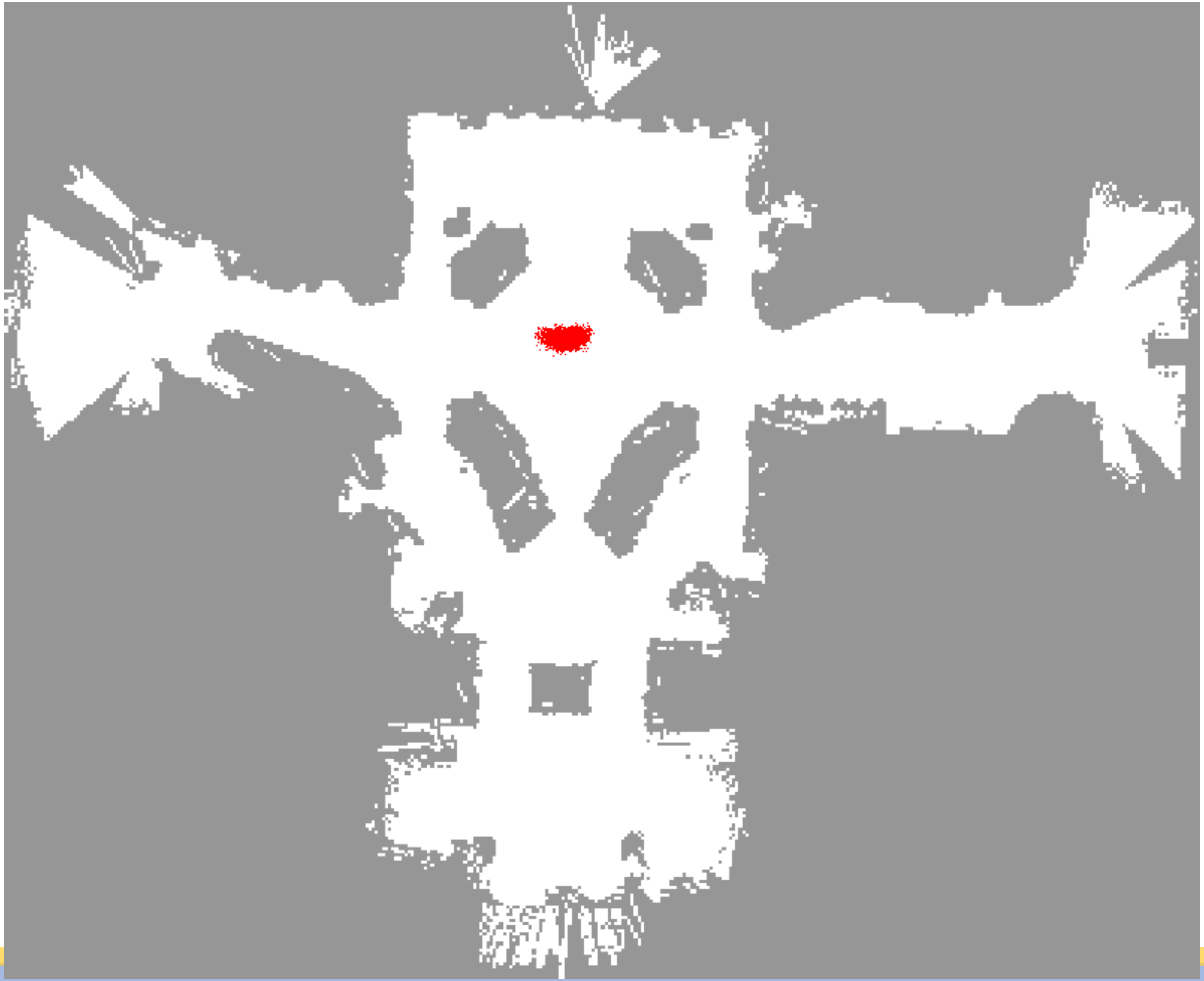


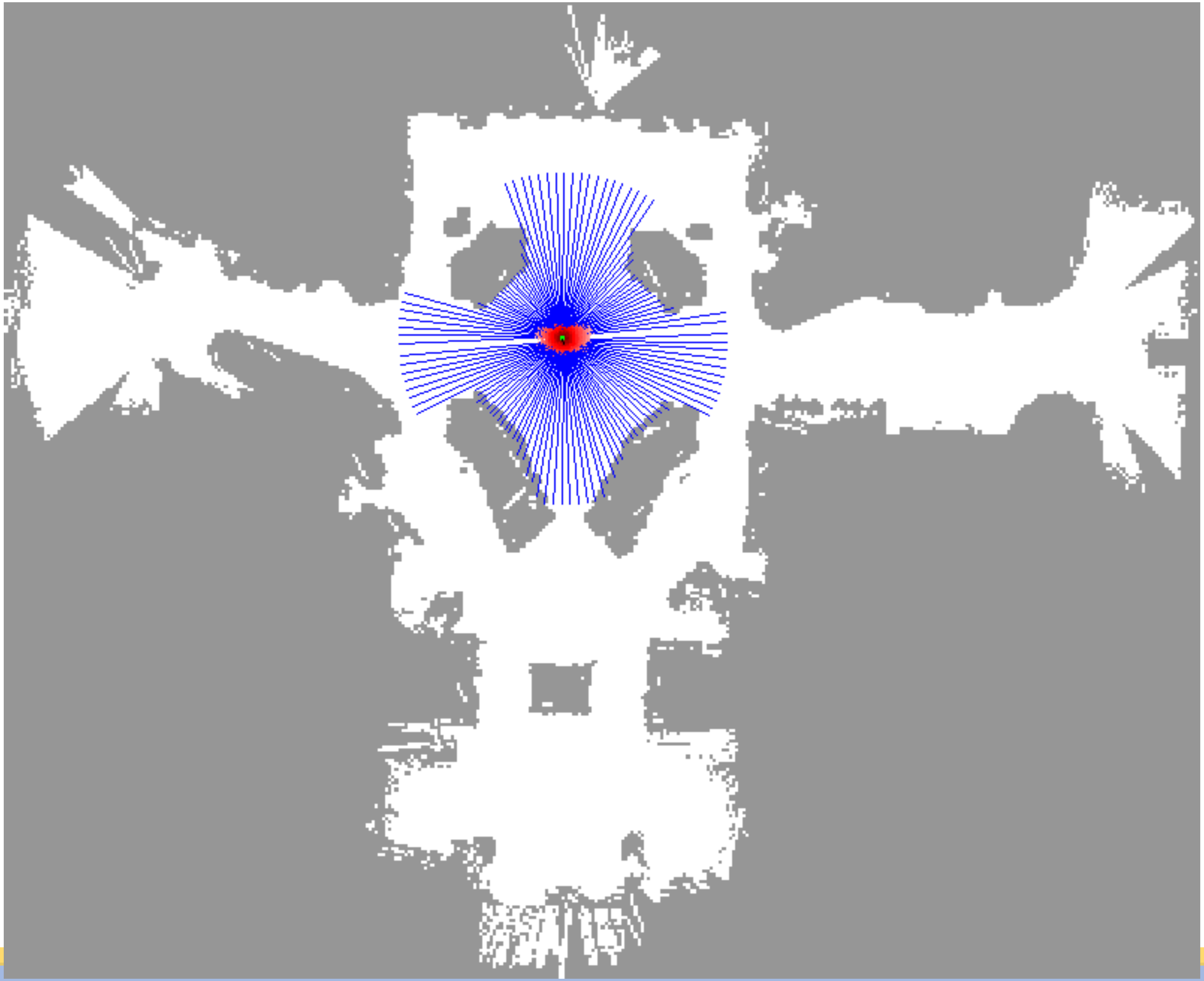


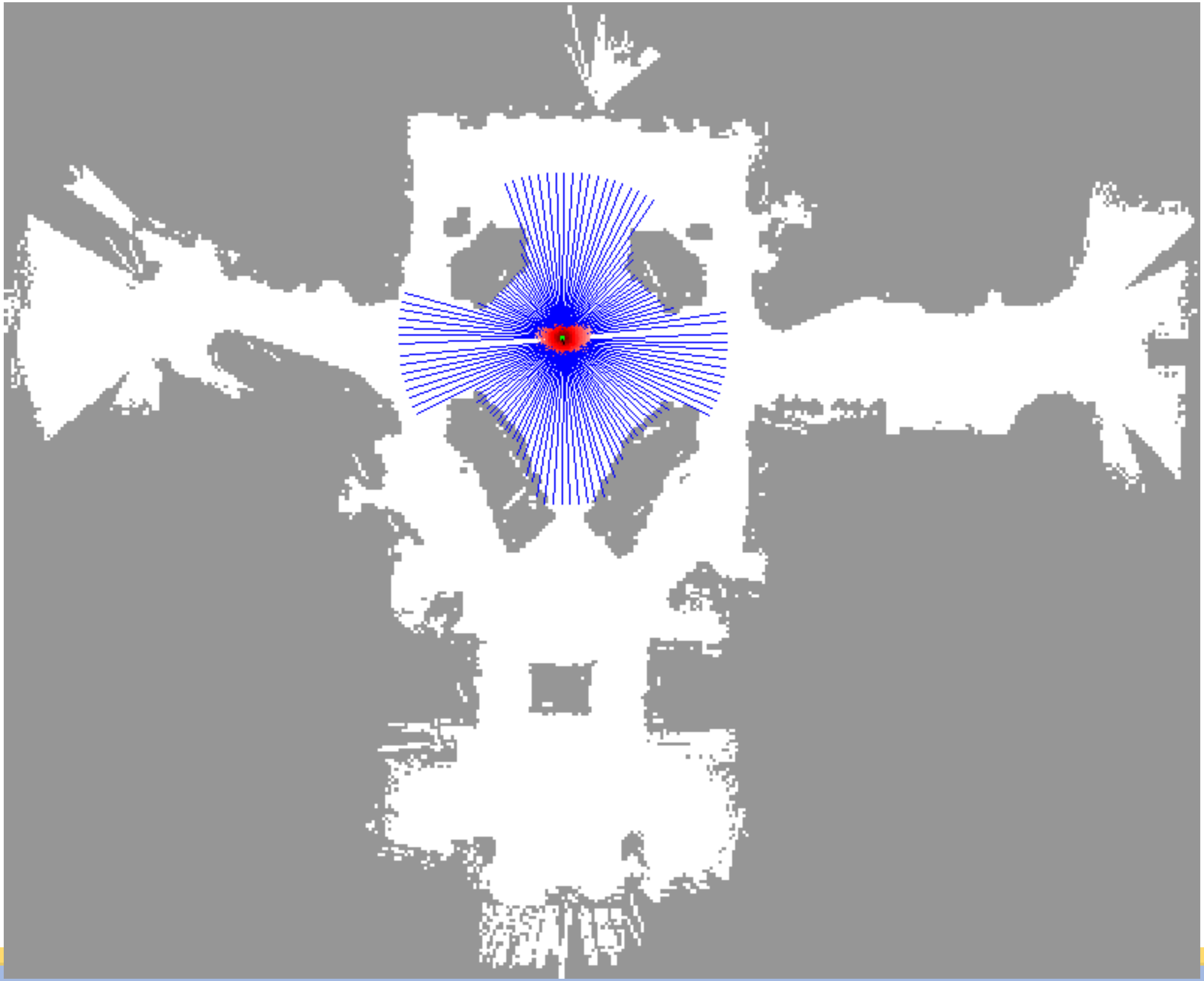




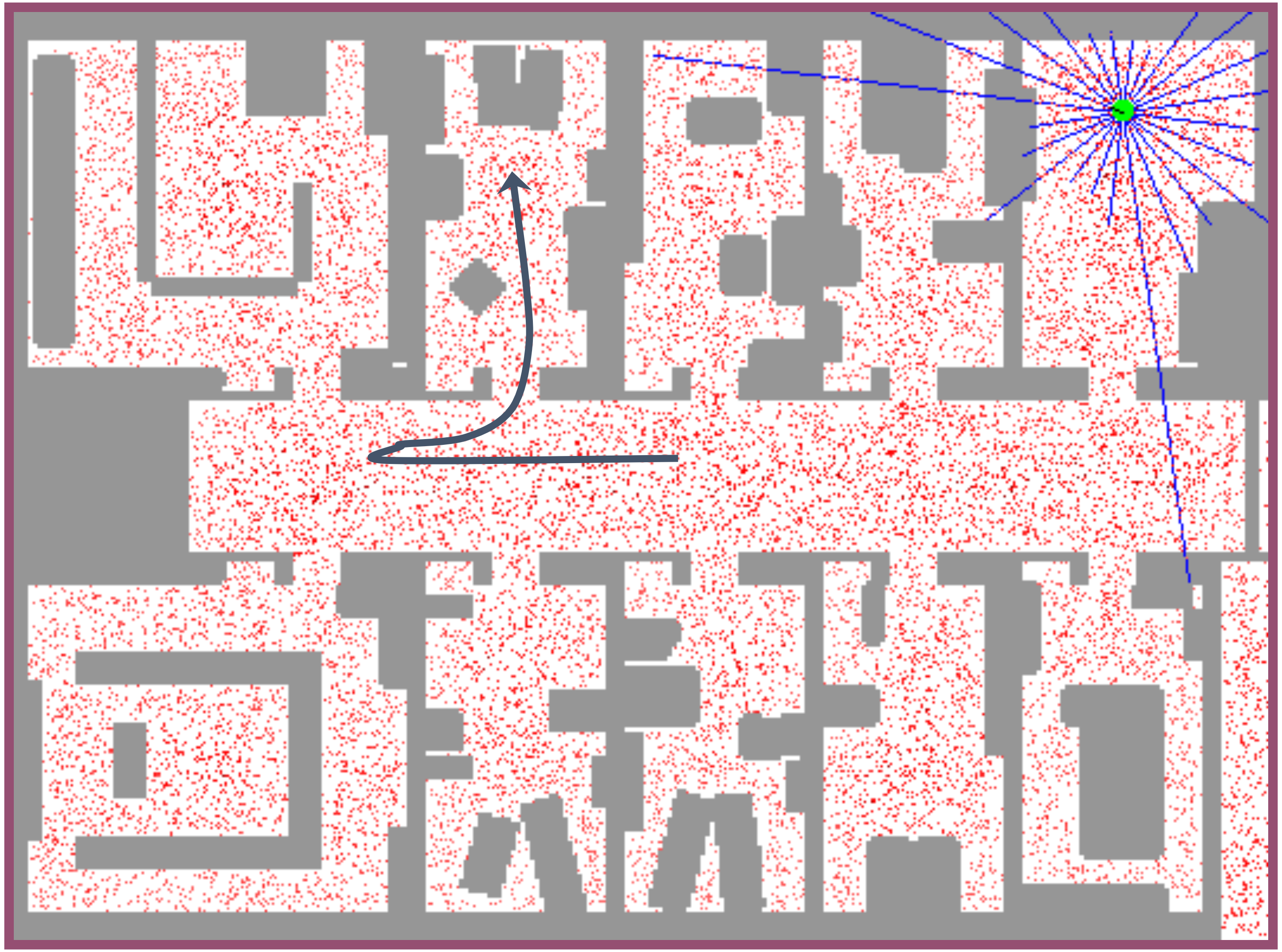




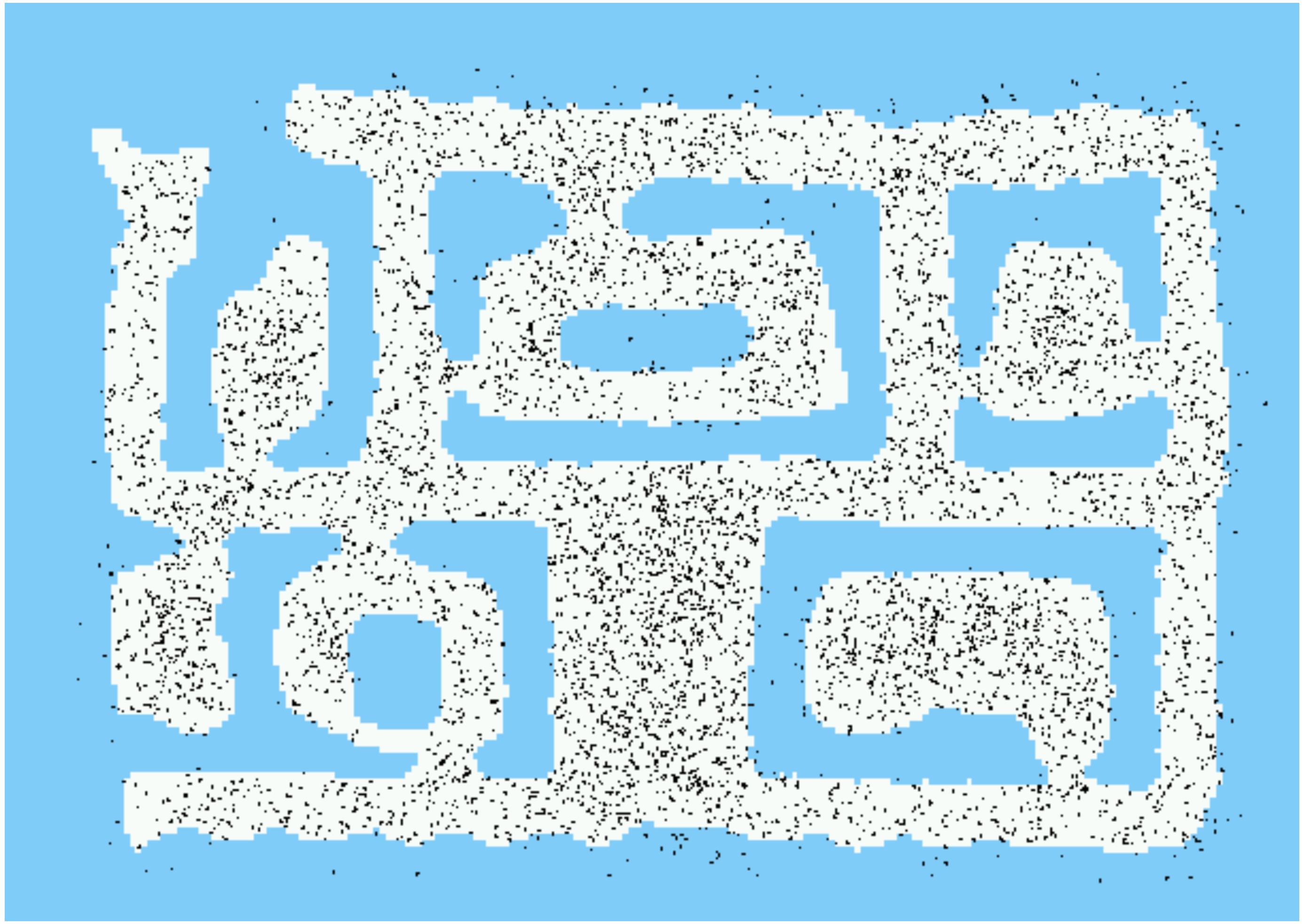




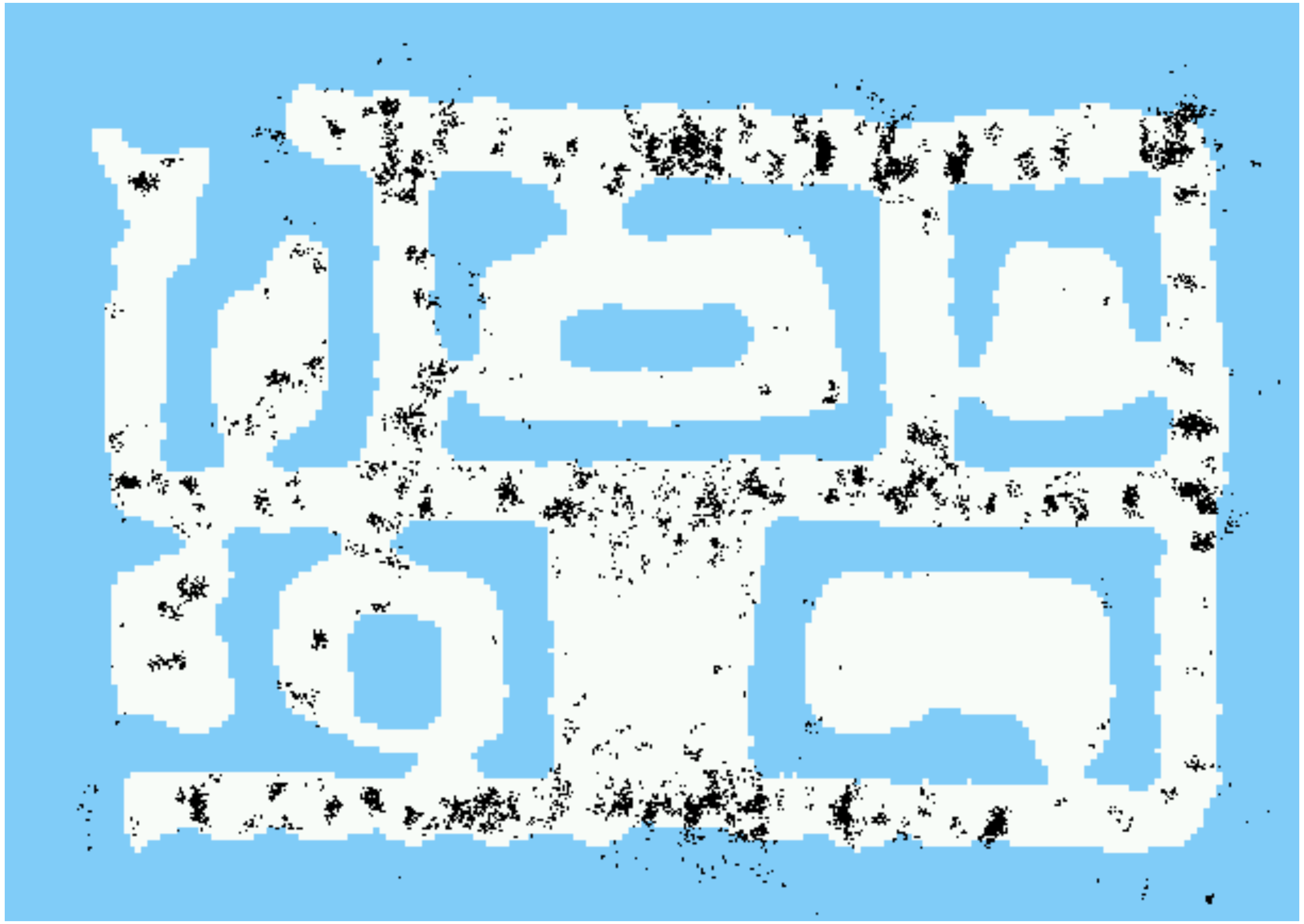
Sample-based Localization (sonar)



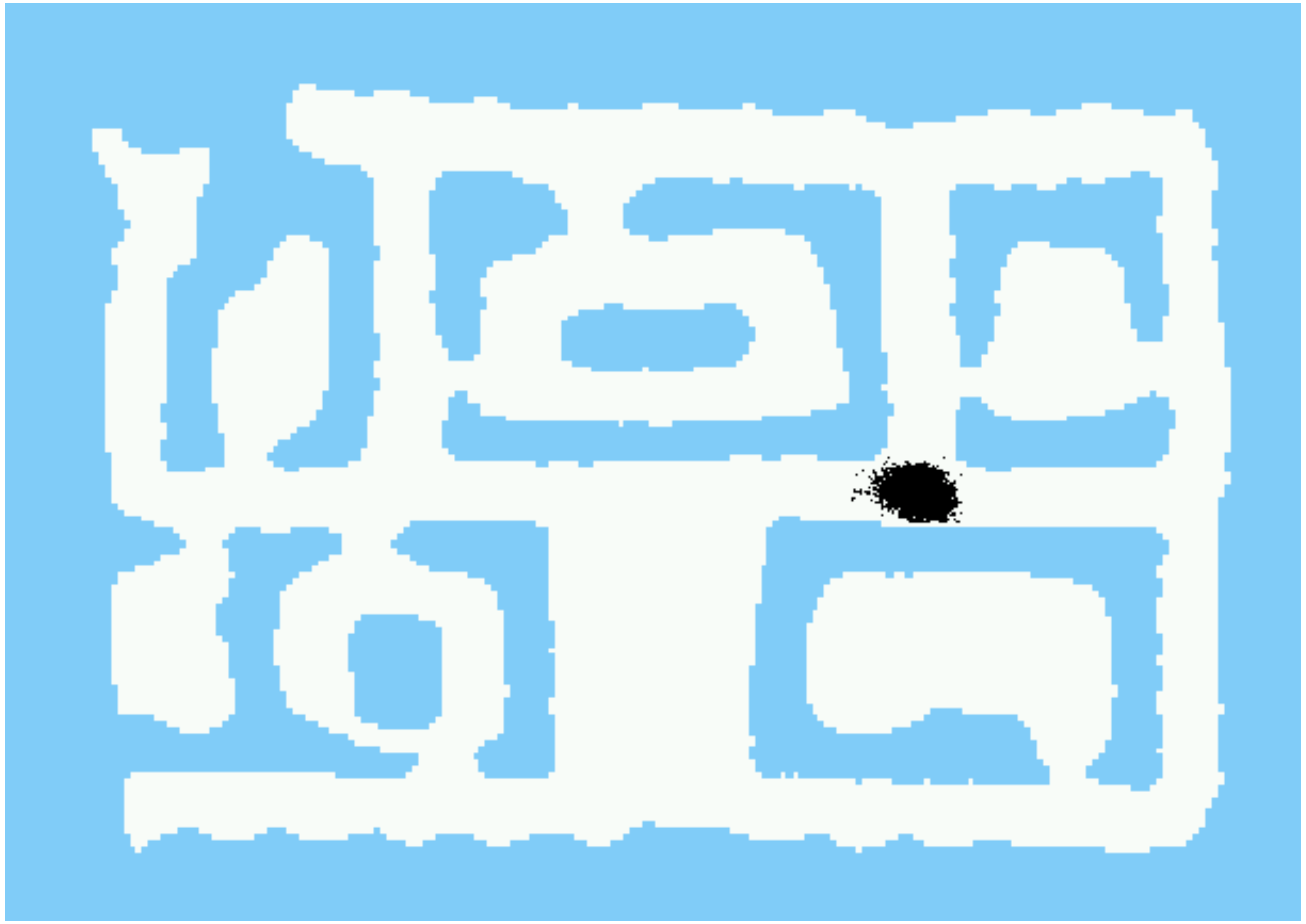
Initial Distribution



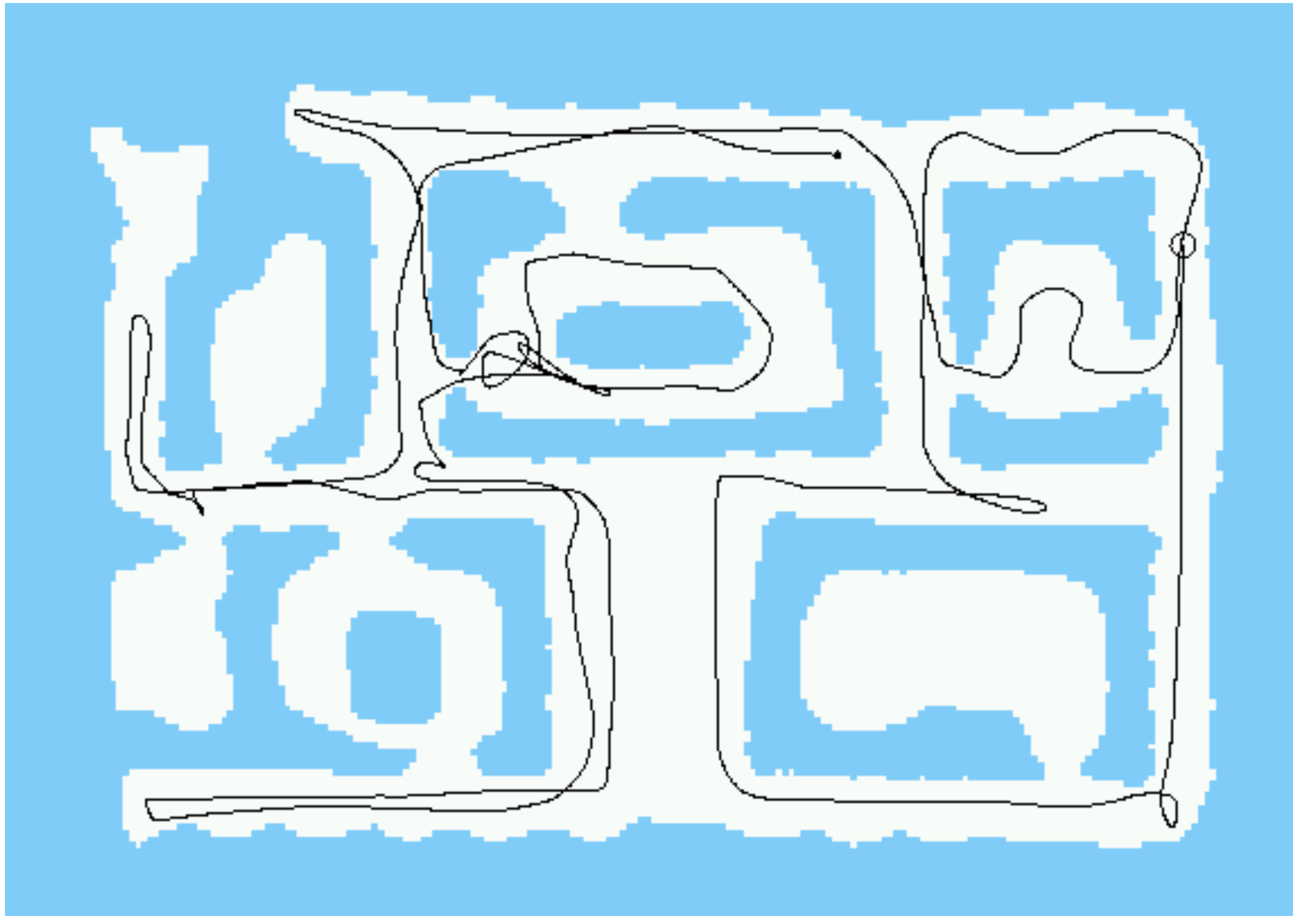
After Incorporating Ten Ultrasound Scans



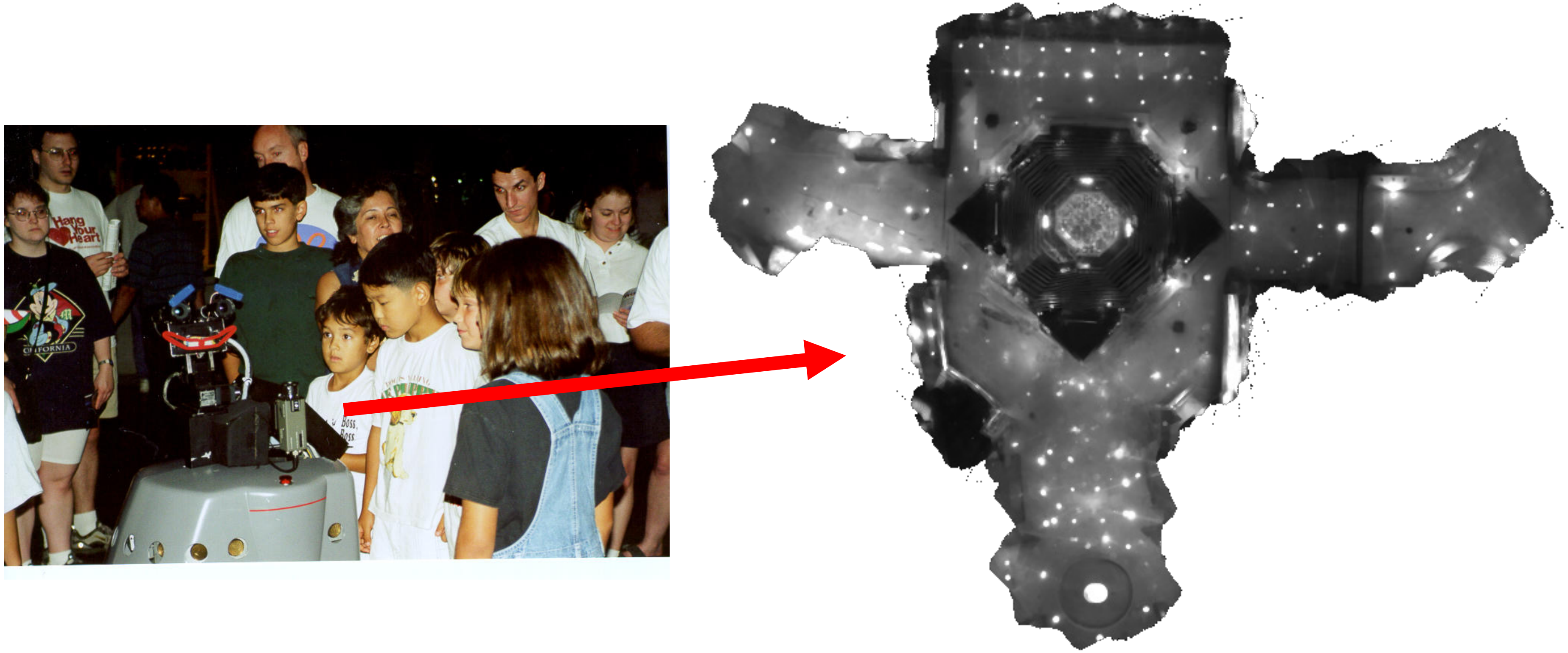
After Incorporating 65 Ultrasound Scans



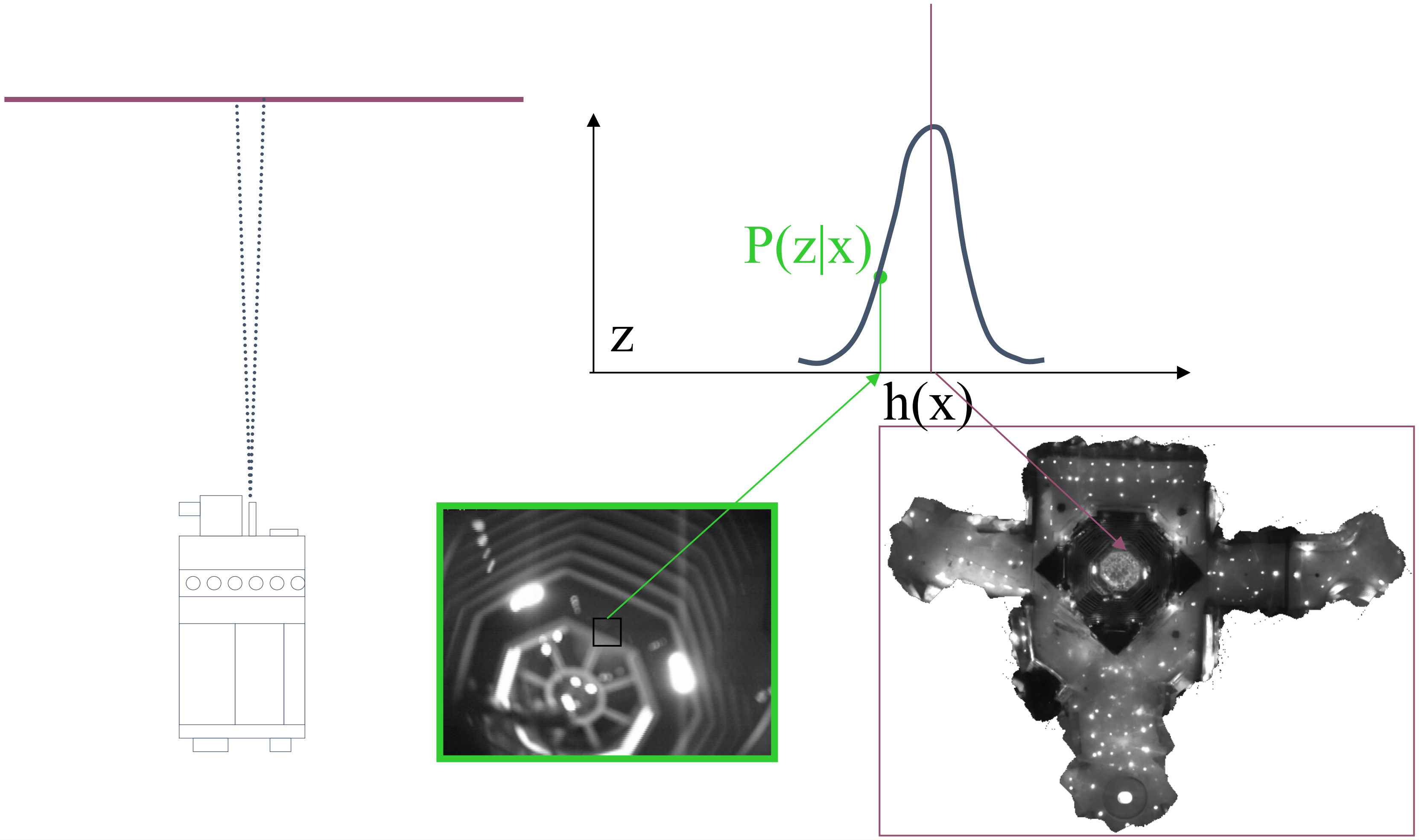
Estimated Path



Using Ceiling Maps for Localization

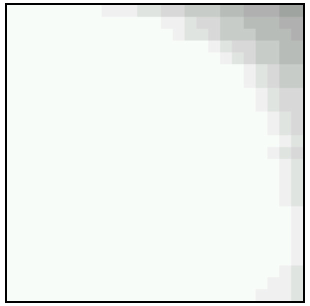


Vision-based Localization

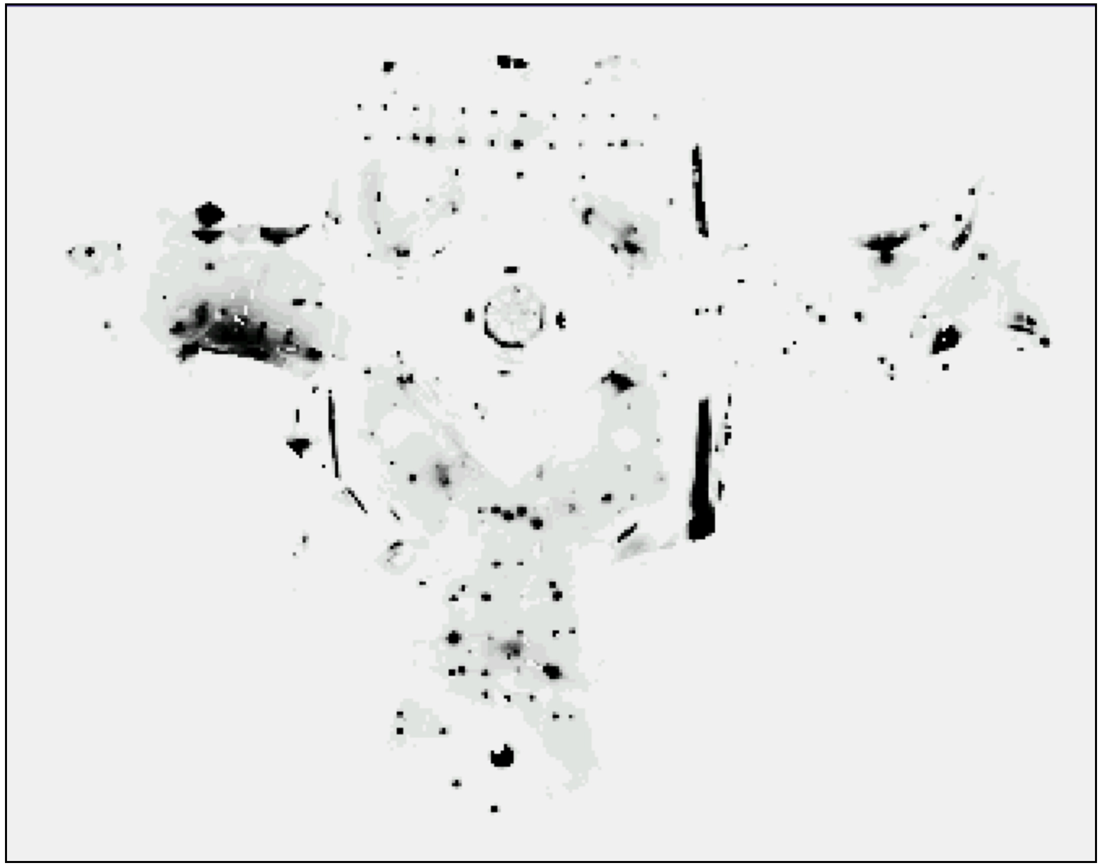


Under a Light

Measurement z:

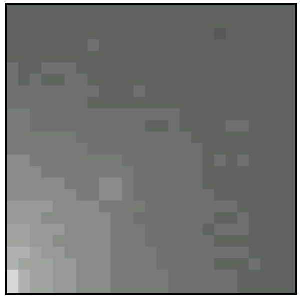


***P(z|x)*:**

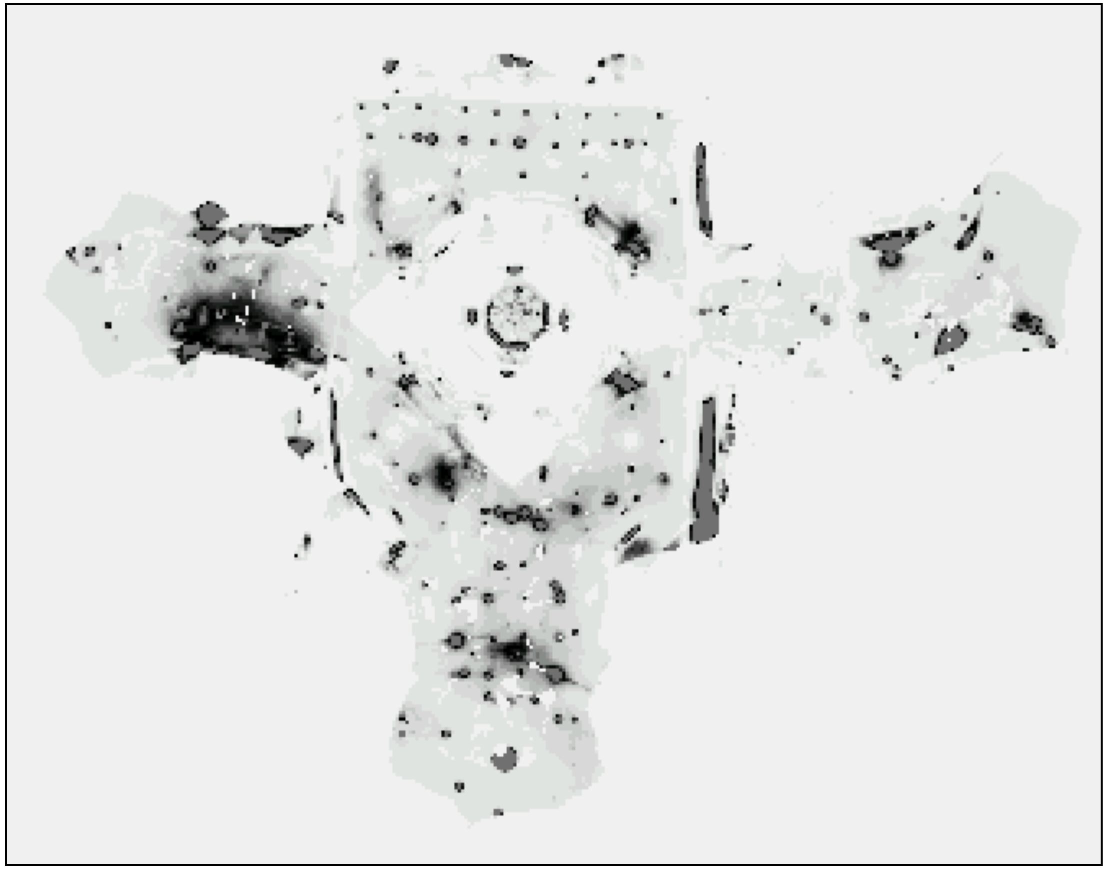


Next to a Light

Measurement z:



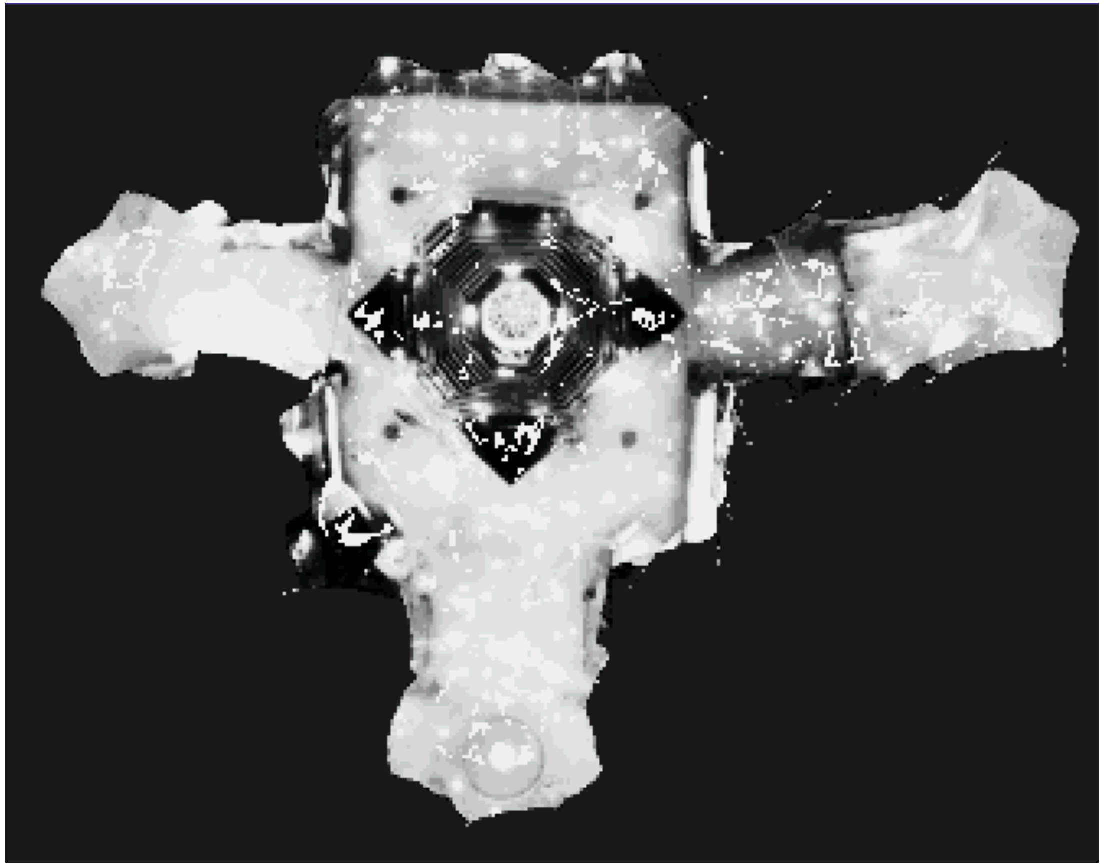
***P(z|x)*:**



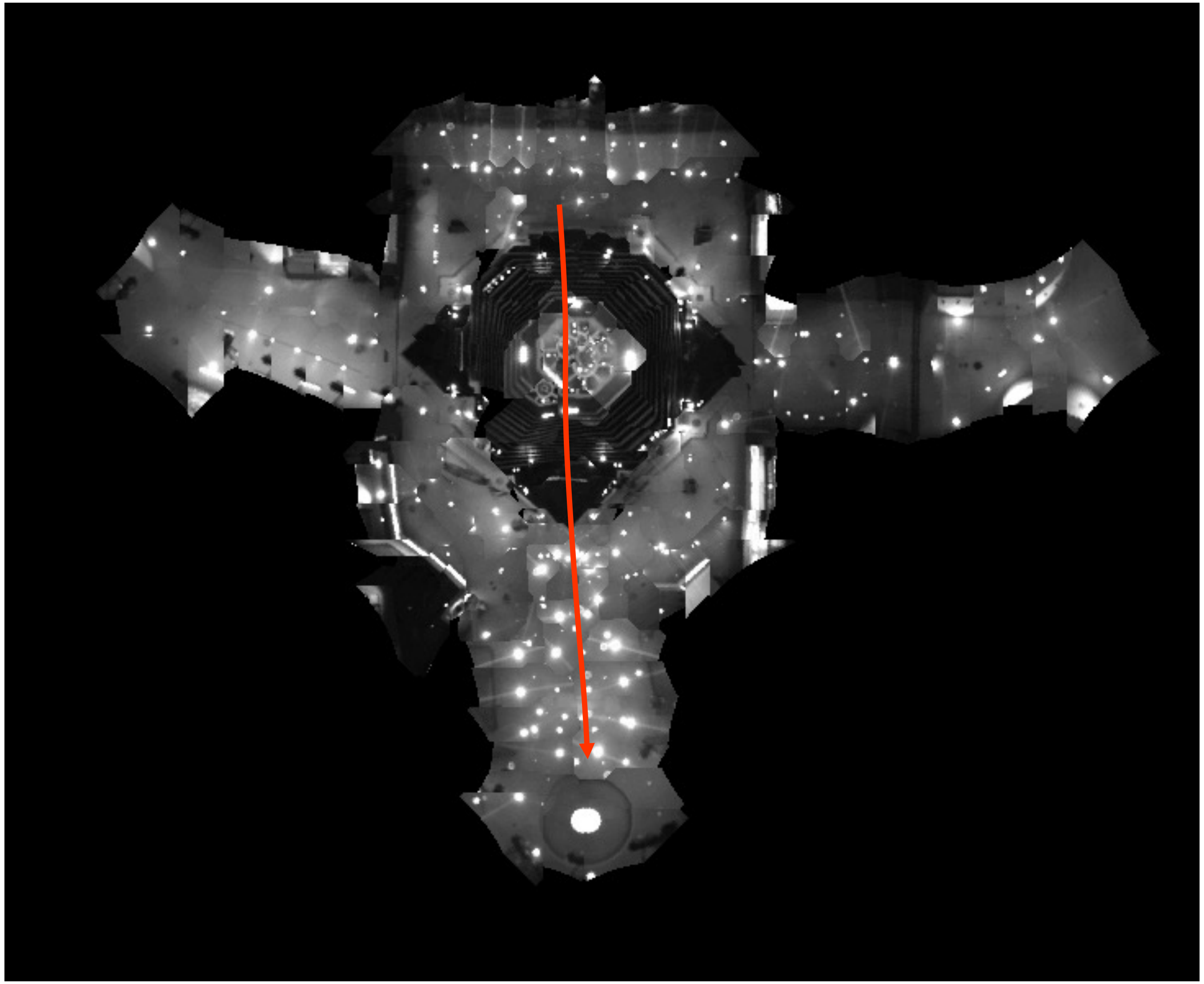
Elsewhere

Measurement z:

***P(z|x)*:**



Global Localization Using Vision



Limitations

- The approach described so far is able to
  - track the pose of a mobile robot and to
  - globally localize the robot.

- Can we deal with localization errors (i.e., the kidnapped robot problem)?
- How to handle localization errors/failures?
  - Particularly serious when the number of particles is small

Approaches

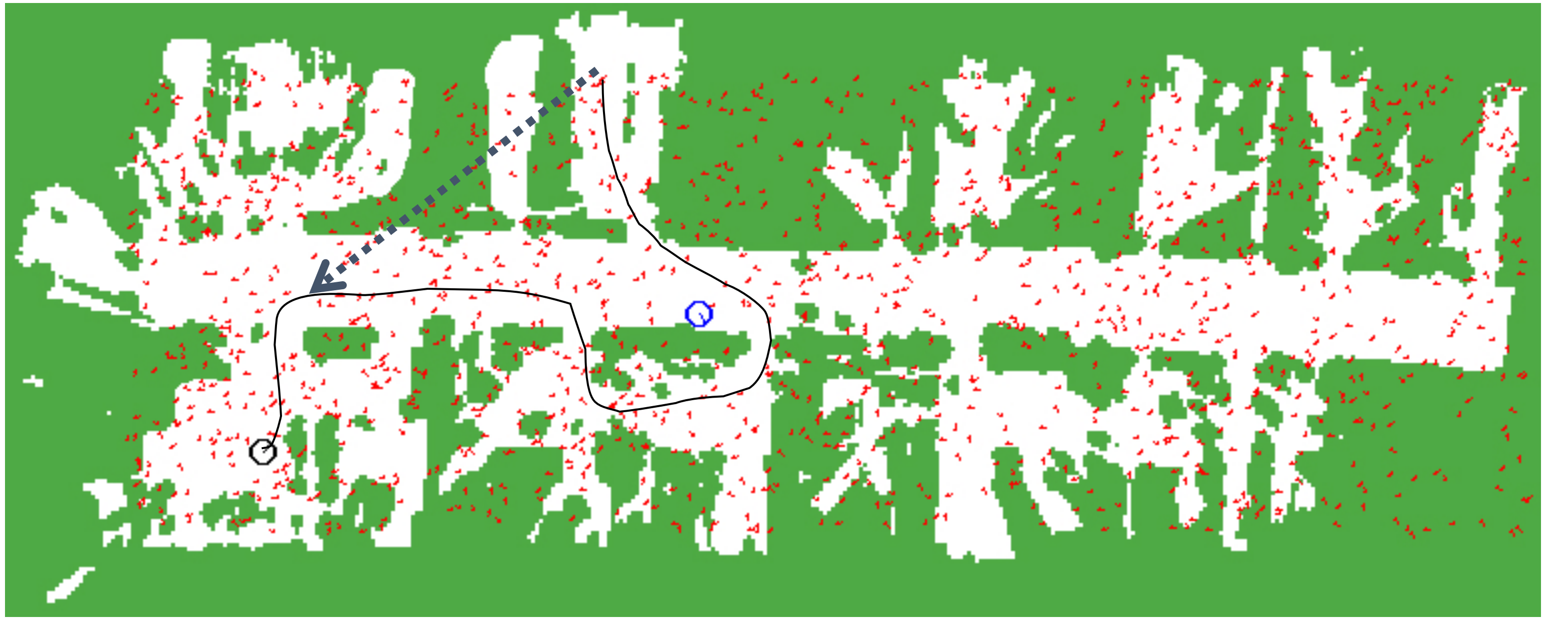
- Randomly insert samples
  - Why?
  - The robot can be teleported at any point in time
- How many particles to add? With what distribution?
  - Add particles according to localization performance
  - Monitor the probability of sensor measurements $p(z_t | z_{1:t-1}, u_{1:t}, m)$
  - For particle filters: $p(z_t | z_{1:t-1}, u_{1:t}, m) \approx \frac{1}{M} \sum w_t^{[m]}$
- Insert random samples proportional to the average likelihood of the particles (the robot has been teleported with higher probability when the likelihood of its observations drops).

Random Samples
Vision-Based Localization

936 Images, 4MB, .6secs/image

Trajectory of the robot:

Kidnapping the Robot



Summary

- Particle filters are an implementation of recursive Bayesian filtering
- They represent the posterior by a set of weighted samples.
- In the context of localization, the particles are propagated according to the motion model.
- They are then weighted according to the likelihood of the observations.
- In a re-sampling step, new particles are drawn with a probability proportional to the likelihood of the observation.