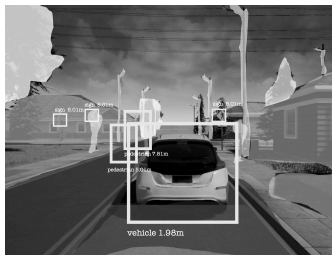
Fall 2023 Principles of Safe Autonomy
Lecture 4:
Basic Perception -> Edge Detection

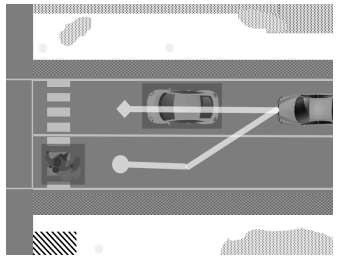
Sayan Mitra

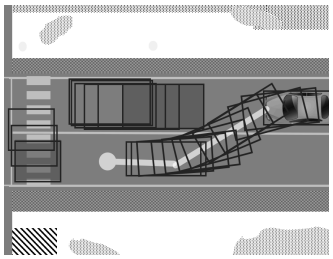
slides adapted from Svetlana Lazebnik

GEM platform

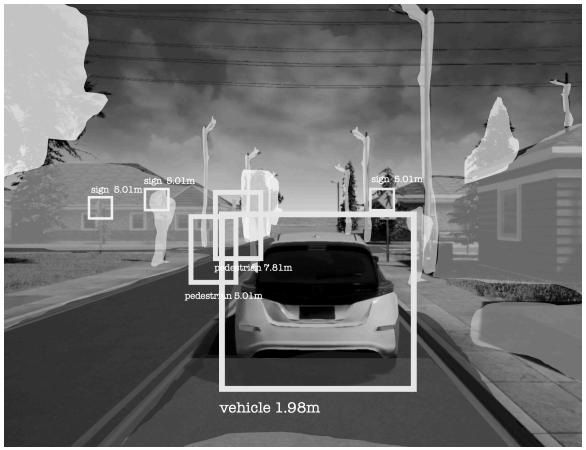
Autonomy pipeline





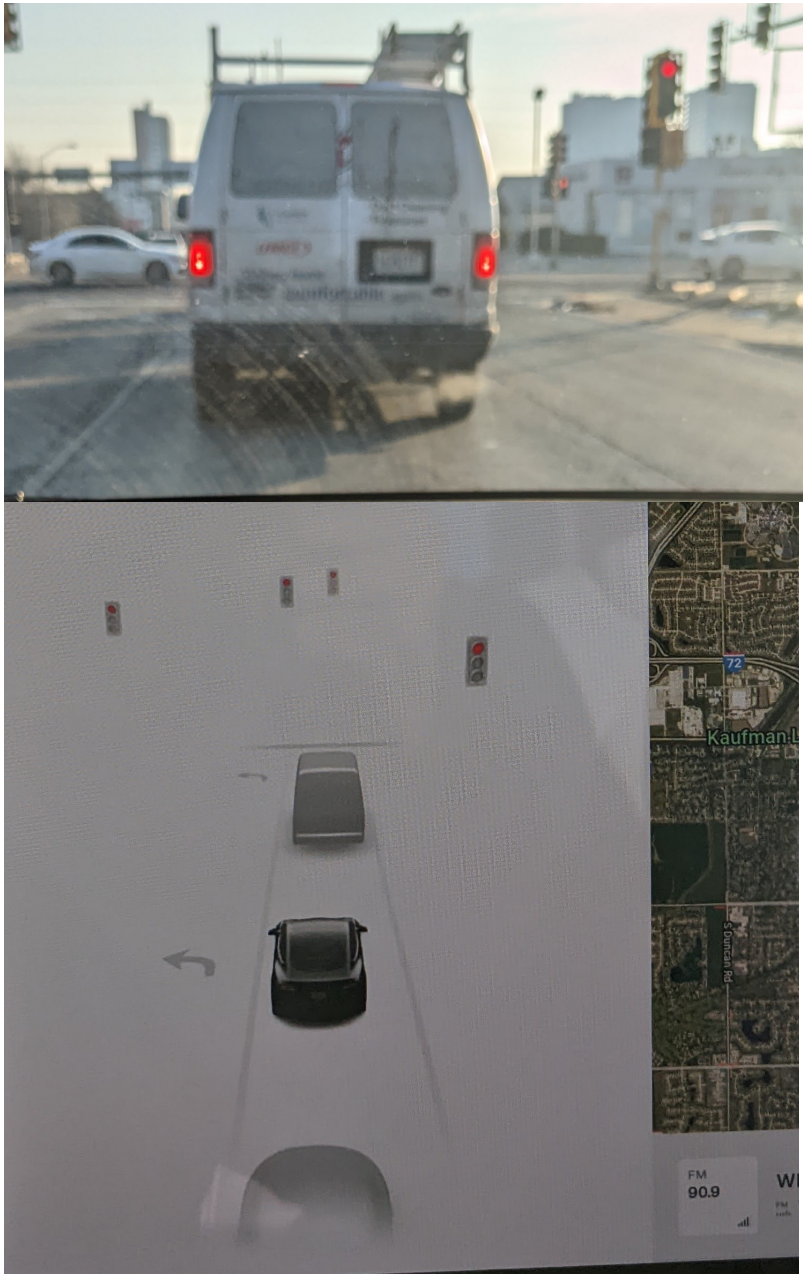


Sensing	Perception	Decisions and planning	Control
Physics-based models of camera, LIDAR, RADAR, GPS, etc.	Programs for object detection, lane tracking, scene understanding, etc.	Programs and multi-agent models of pedestrians, cars, etc.	Dynamical models of engine, powertrain, steering, tires, etc.



Perception

Programs for object detection, lane tracking, scene understanding, etc.



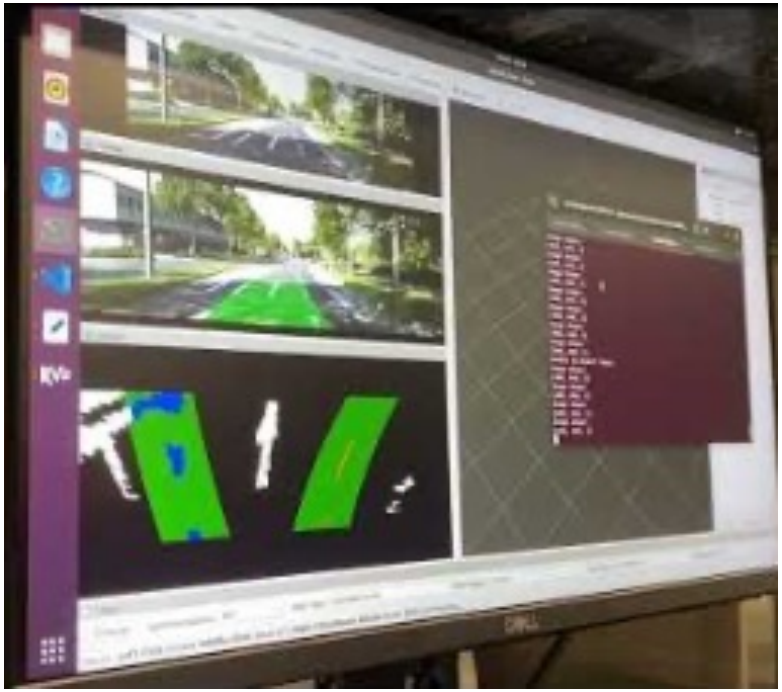
Perception subsystem converts signals from the environment into meaningful bits

Leaderboard

CI testing pipeline MP1

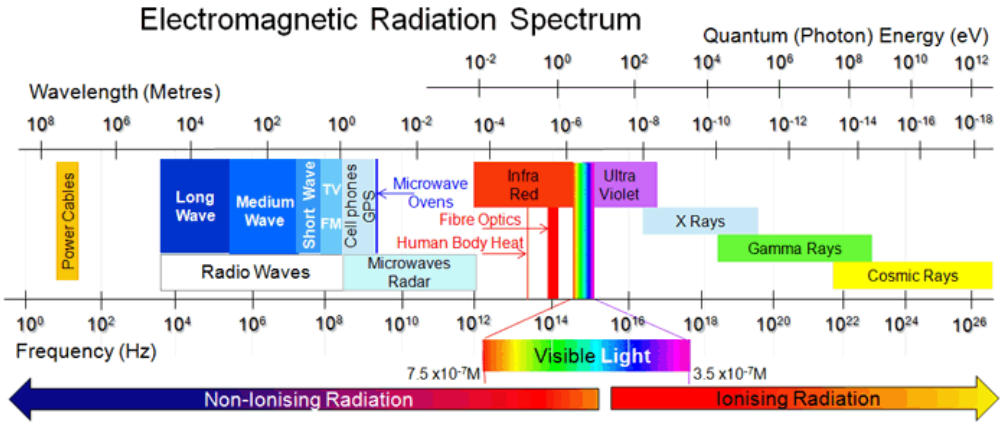
https://uiuc-ece-484-baf42ccc3426.herokuapp.com/

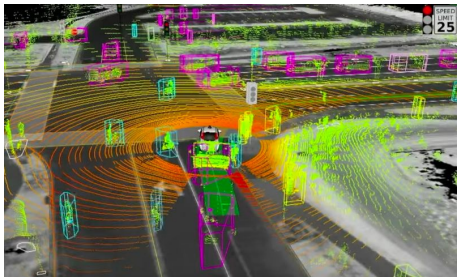
MP1: https://www.youtube.com/watch?v=V6sP6krS3AM

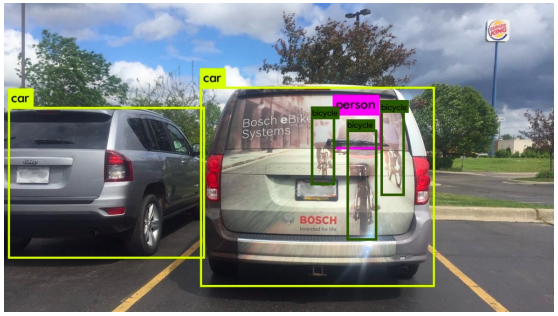


Perception: EM to objects

Problem: Process electromagnetic radiation from the environment to construct a *model* of the world, so that the constructed model is close to the real world





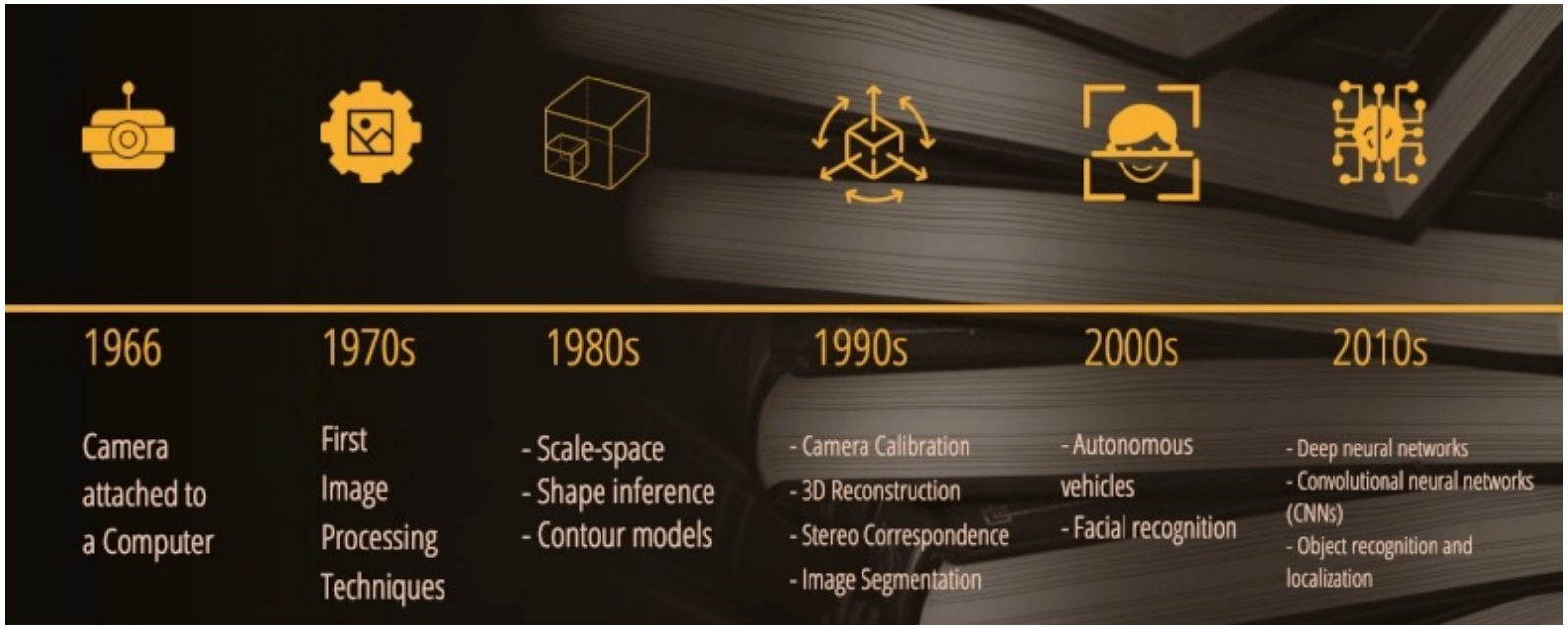


Challenging for computers: millions of years of evolution

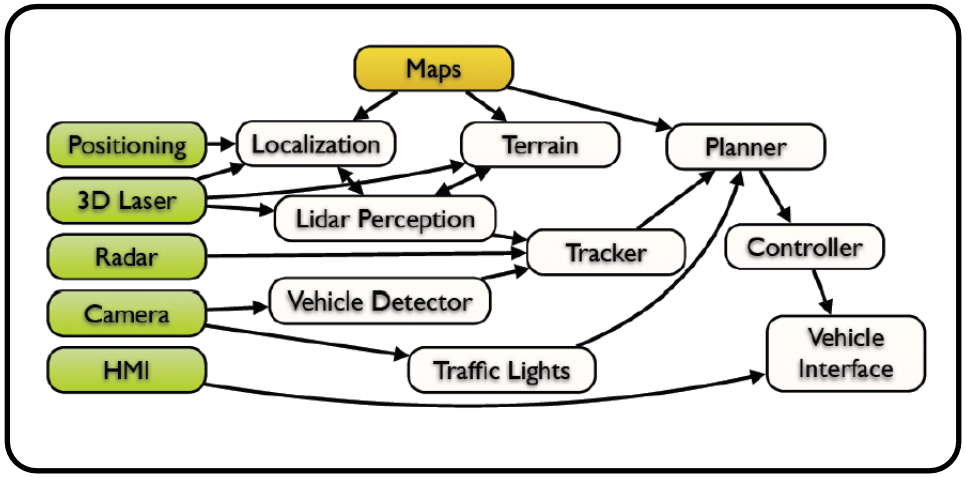
Moravec's paradox

Ill-defined problem: impossibility of defining meaning “car”, “bicycle”, etc.

History and State of the art



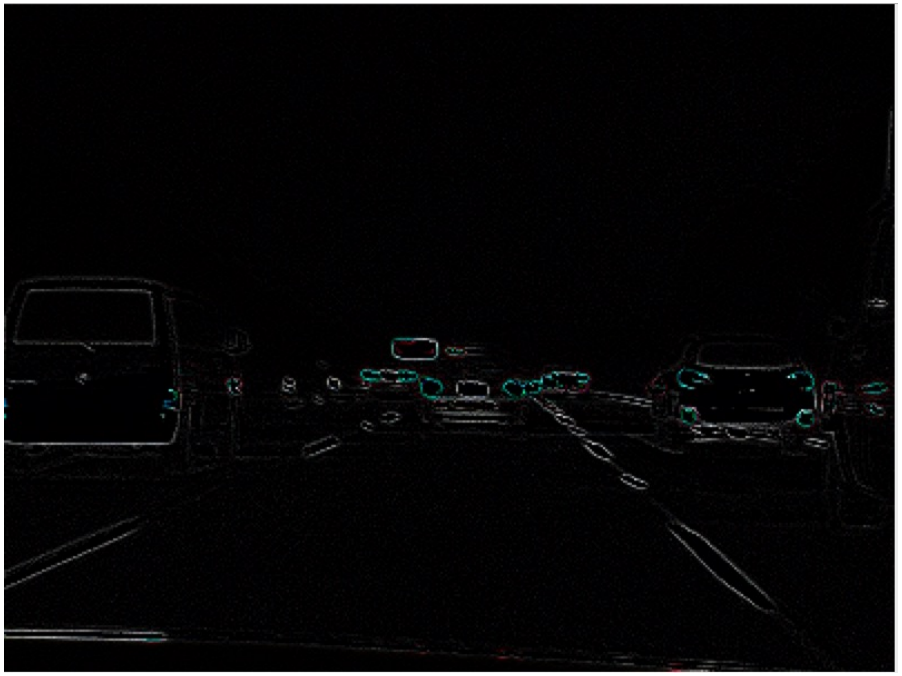
https://www.clickworker.com/ai-glossary/computer-vision/



This architecture from a slide from M. James of Toyota Research Institute, North America

Outline

- Linear filtering
- Edge detection



Motivation: Image denoising

- How can we reduce noise in a photograph?

Image representation

Images are represented as 2D arrays of pixels. Each pixel is represented by (array of) value(s) representing its color.

```
# read an image
img = cv2.imread('images/noguchi02.jpg')

# show image format (basically a 3-d array of pixel color info, in BGR format)
print(img)
```

```
[
    [[72 99 143]  [76 103 147]  [78 106 147]  ...,
    [[74 101 145]  [77 104 148]  [77 105 146]  ...,
    [[76 103 147]  [77 104 148]  [76 104 145]  ...,
    ...,
    [[39 78 130]  [39 78 130]  [40 79 131]  ...,   [
    [[32 71 123]  [32 71 123]  [32 71 123]  ...,   [
    [[39 78 130]  [39 78 130]  [39 78 130]  ...,   [
]
```

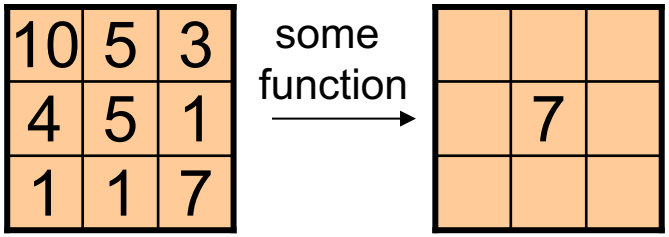
Where [72 99 143] is the blue, green, and red values of that pixel.

We will work with grayscale images

Denote by img[i,j] (or f[i,j]) the value of the i,j-th pixel

What is filtering?

Modify the pixels in an image based on some function of a local neighborhood of the pixels.



Bright(img,k): for all i,j

img'[i][j] = k*img[i][j]

Shifting right by s Shift(img,s):

img'[k] = img[k-s]; img'[0]…img'[s-1] is undefined

Simplest: Linear filtering

replace each pixel by a linear combination of neighbors

Moving average

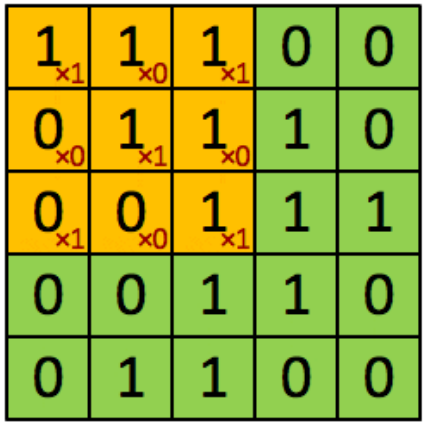
- Let's replace each pixel with a *weighted* average of its neighborhood
- The weights are called the *filter kernel*
- What are the weights for the average of a 3x3 neighborhood?

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

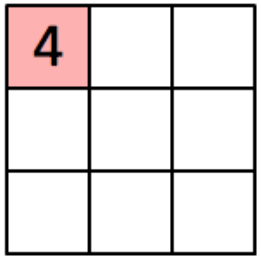
"box filter"

Source: D. Lowe

Convolution



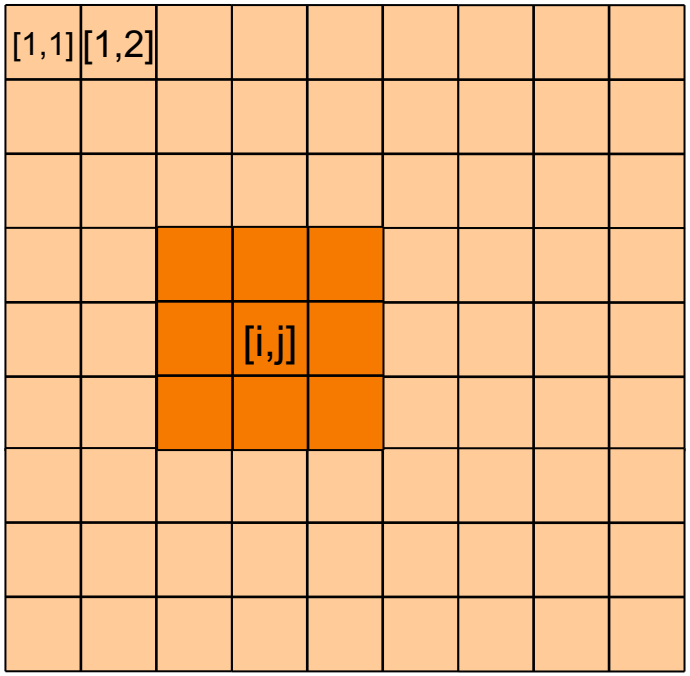
Image



Convolved Feature

Convolution

image[i,j]



convolution mask g[,]

1,1	1,2	1,3
2,1	2,2	2,3
3,1	3,2	3,3

Output or convolved image

f = g * img

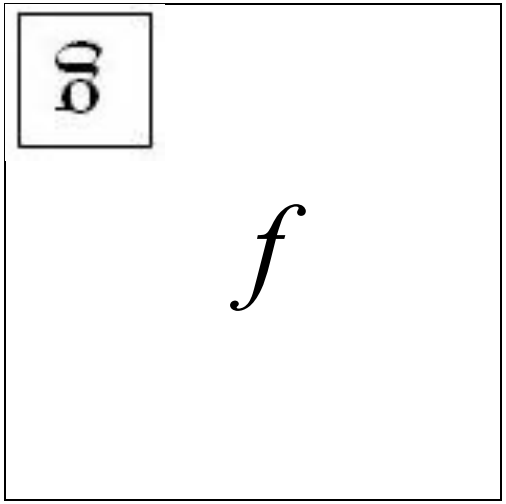
f[i,j] = g[1,1] img[i-1,j-1] + g[1,2] img[i-1,j] + g[1,3] img[i-1,j+1]
+ g[2,1] img[i,j-1] + g[2,2] img[i,j] + g[2,3] img[i,j+1]
+ g[3,1] img[i+1,j-1] + g[3,2] img[i+1,j] + g[3,3] img[i+1,j+1]

Defining convolution

Let *f* be the image and *g* be the kernel. The output of convolving *f* with *g* is denoted *f* * *g*.

$$(f * g)[m,n] = \sum_{k,l} f[m-k, n-l]\, g[k,l]$$

Convention:
kernel is “flipped”



For analysis we will work with 1D images

Let *f* be the image and *g* be the kernel. The output of convolving *f* with *g* is denoted *f* * *g*.

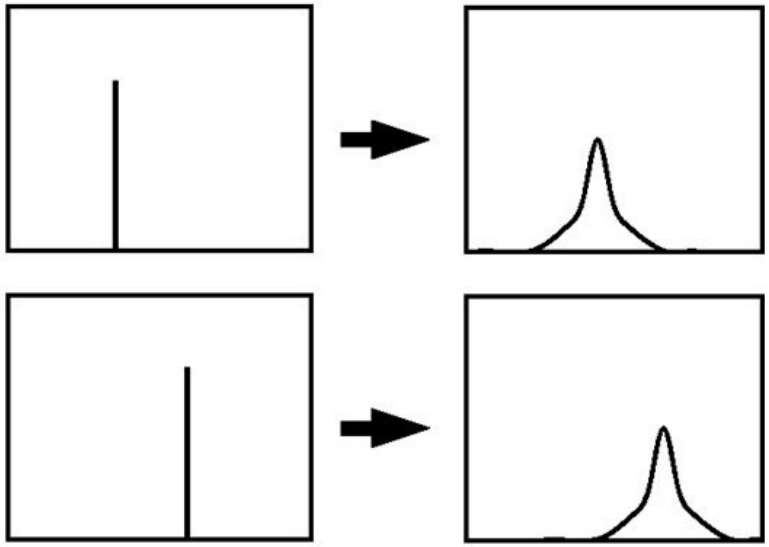
$$(f * g)[m] = \Sigma_k f[m-k]g[k]$$

Source: F. Durand

Key properties: Prove the first two

Shift invariance: same behavior regardless of pixel location:

filter(shift(f)) = shift(filter(f))



Linearity:

filter(f_1 + f_2) =
filter(f_1) + filter(f_2)

Theoretical result: any linear shift-invariant operator can be represented as a convolution

Properties in more detail

Commutative: $a * b = b * a$

- Conceptually no difference between filter and signal

Associative: $a * (b * c) = (a * b) * c$

- Often apply several filters one after another: $(((a * b_1) * b_2) * b_3)$
- This is equivalent to applying one filter: $a * (b_1 * b_2 * b_3)$

Distributes over addition: $a * (b + c) = (a * b) + (a * c)$

Scalars factor out: $ka * b = a * kb = k(a * b)$

Identity: unit impulse $e = [\ldots, 0, 0, 1, 0, 0, \ldots]$,
$a * e = a$

openCV: filter2D

Output image same size as input

Multi-channel: each channel is processed independently

Extrapolation of border

Examples

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

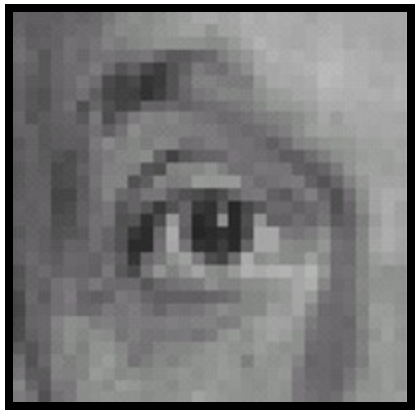
?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0



Filtered
(no change)

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

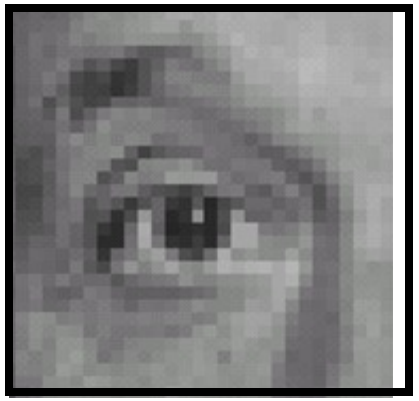
?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0



Shifted *left*
By 1 pixel

Source: D. Lowe

Practice with linear filters

Original

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

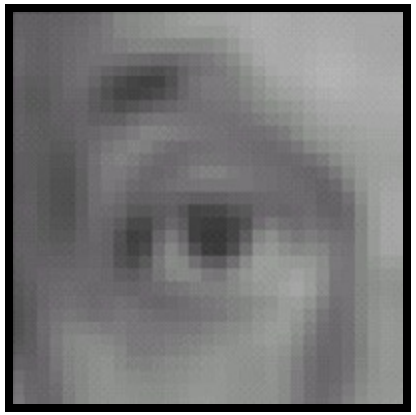
?

Source: D. Lowe

Practice with linear filters

Original

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



Blur (with a box filter)

Source: D. Lowe

Practice with linear filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad ?$$

(Note that filter sums to 1)

Source: D. Lowe

Practice with linear filters

Original

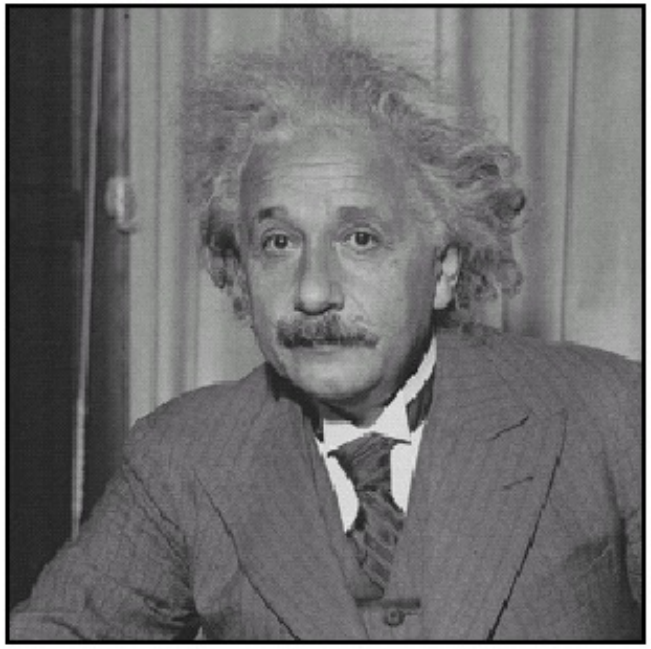
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Sharpening filter

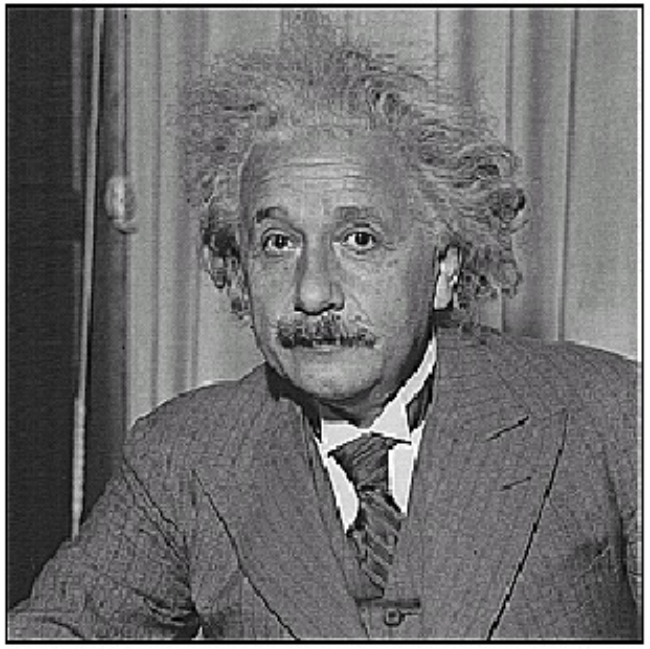
- Accentuates differences with local average

Source: D. Lowe

Sharpening



before



after

Source: D. Lowe

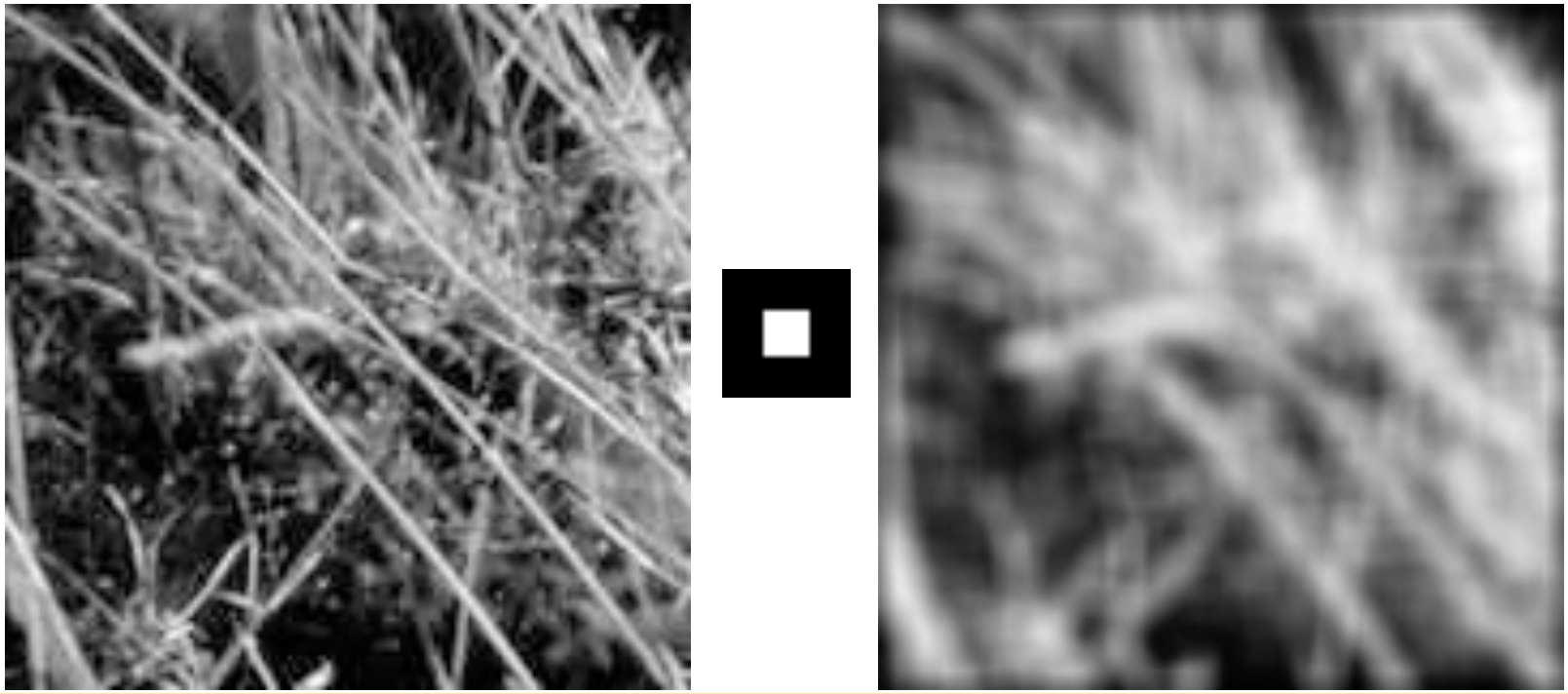
Sharpening

What does blurring take away?

Let's add it back:

Smoothing with box filter revisited

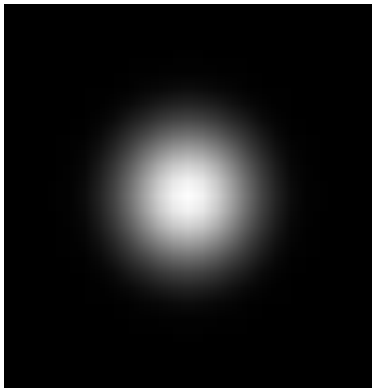
- What's wrong with this picture?
- What's the solution?



Source: D. Forsyth

Smoothing with box filter revisited

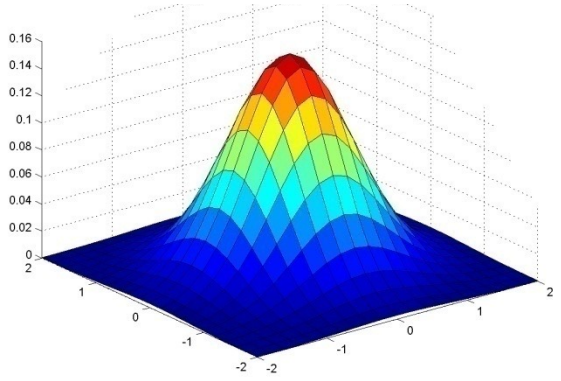
- What's wrong with this picture?
- What's the solution?
  - To eliminate edge effects, weight contribution of neighborhood pixels according to their closeness to the center

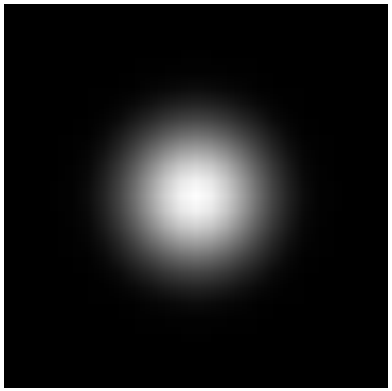


"fuzzy blob"

Gaussian Kernel

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$





0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

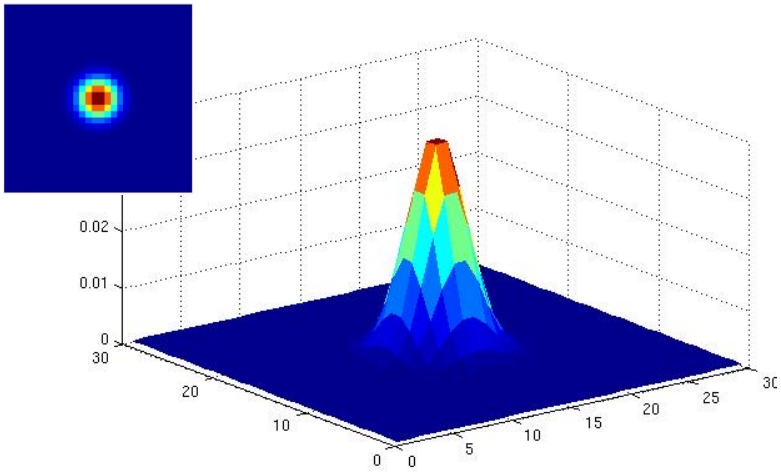
5 x 5, $\sigma = 1$

Constant factor at front makes volume sum to 1 (can be ignored when computing the filter values, as we should renormalize weights to sum to 1 in any case)

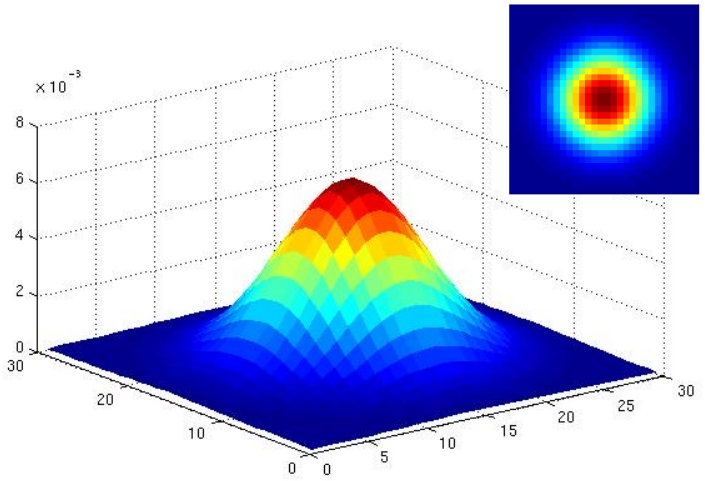
Source: C. Rasmussen

Gaussian Kernel

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$



σ = 2 with 30 x 30 kernel

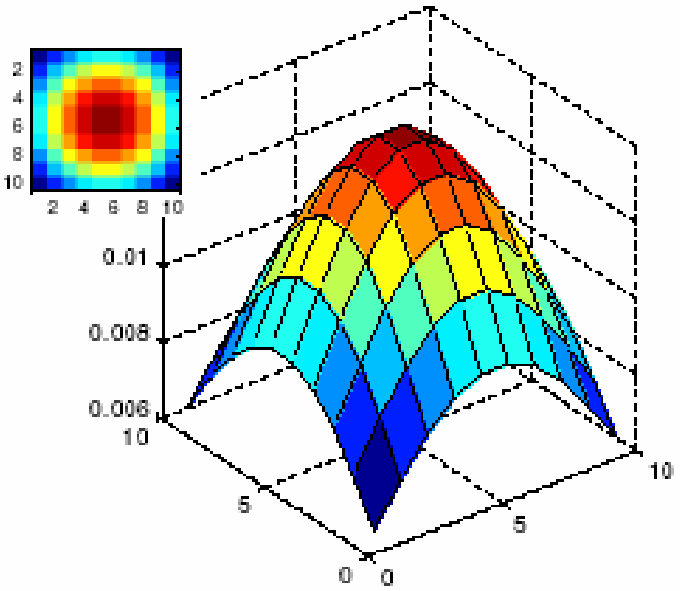


σ = 5 with 30 x 30 kernel

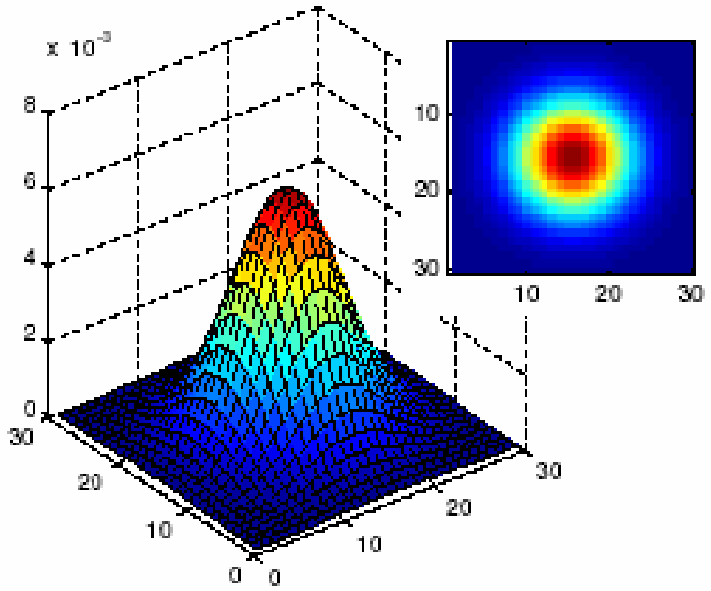
Standard deviation σ: determines extent of smoothing

Choosing kernel width

The Gaussian function has infinite support, but discrete filters use finite kernels



$\sigma = 5$ with 10x10 kernel

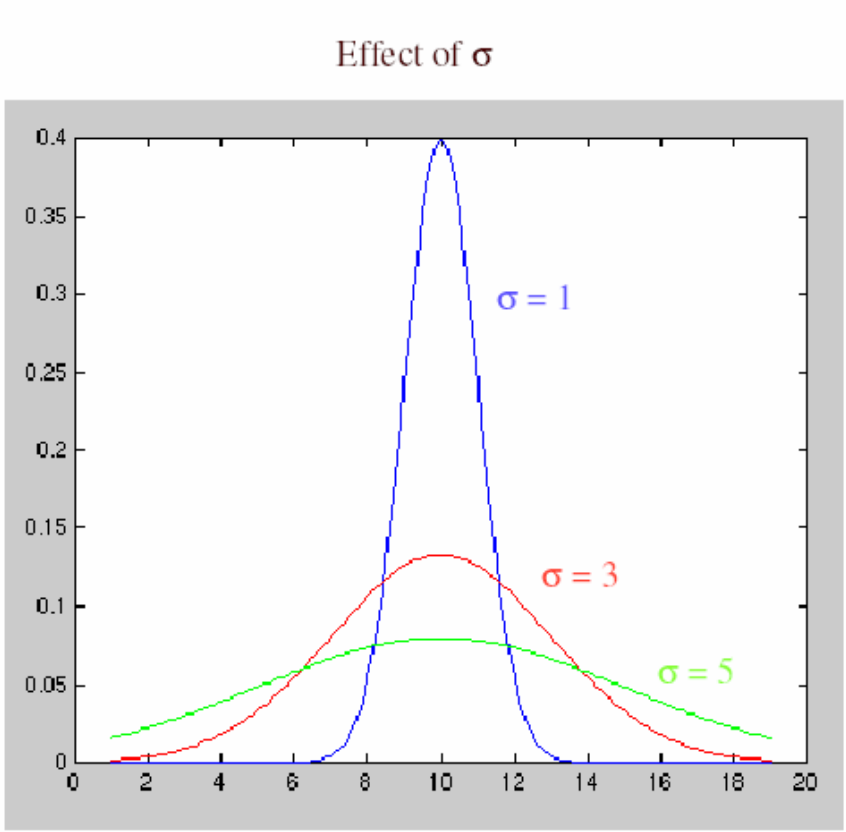


$\sigma = 5$ with 30x30 kernel

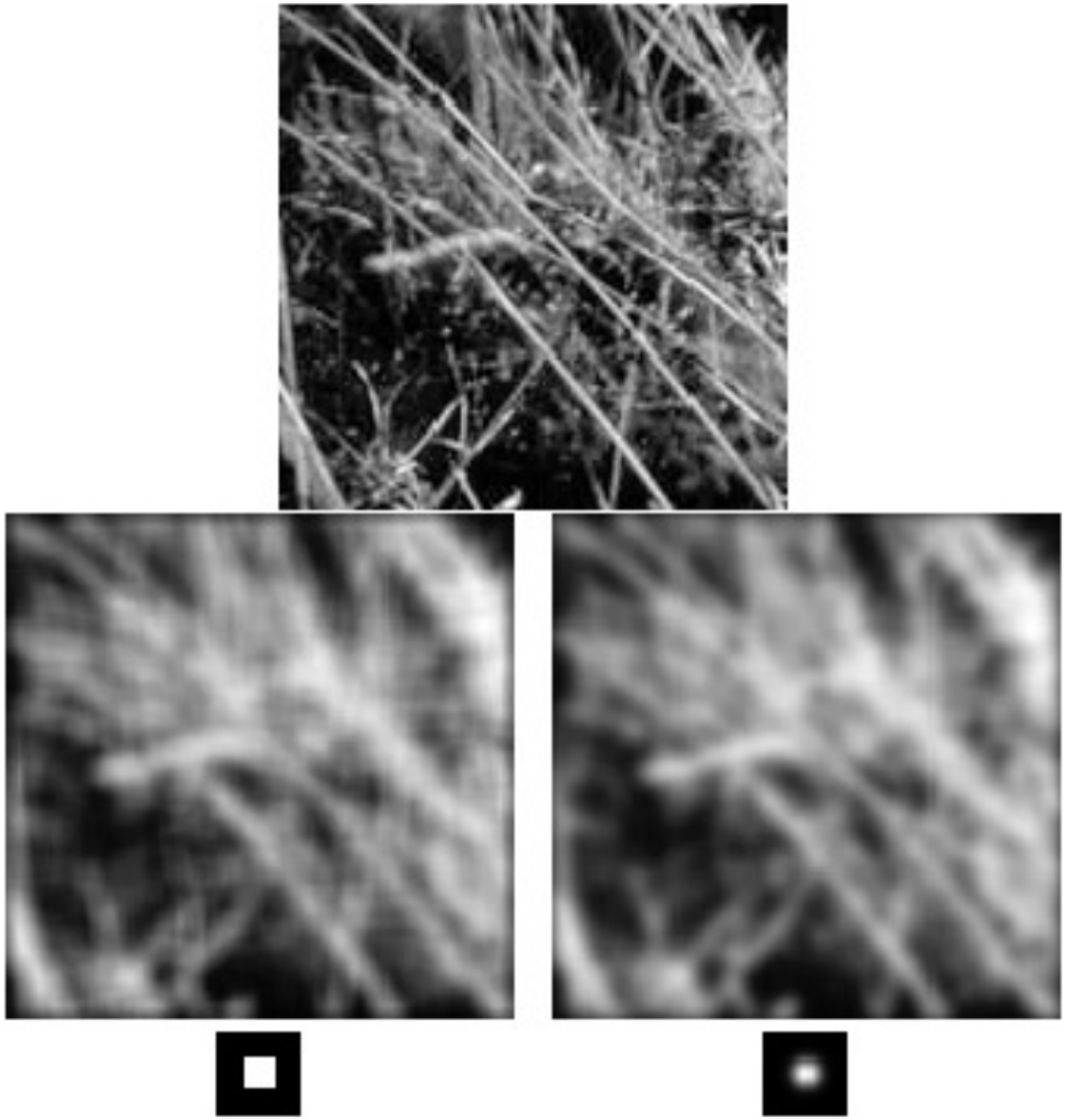
Source: K. Grauman

Choosing kernel width

Rule of thumb: set filter half-width to about 3σ



Gaussian vs. box filtering



Gaussian filters

- Remove high-frequency components from the image (*low-pass filter*)
- Convolution with self is another Gaussian
  - So can smooth with small-σ kernel, repeat, and get same result as larger-σ kernel would have
  - Convolving two times with Gaussian kernel with std. dev. σ is same as convolving once with kernel with std. dev. $\sigma\sqrt{2}$
- *Separable* kernel
  - Factors into product of two 1D Gaussians
  - Discrete example:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Source: K. Grauman

Separability of the Gaussian filter

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2}\exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{x^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{y^2}{2\sigma^2}}\right)$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

Source: D. Lowe

Why is separability useful?

- Separability means that a 2D convolution can be reduced to two 1D convolutions (one along rows and one along columns)
- What is the complexity of filtering an $n \times n$ image with an $m \times m$ kernel?
  - $O(n^2 m^2)$
- What if the kernel is separable?
  - $O(n^2 m)$

Noise

Original

Salt and pepper noise

Impulse noise

Gaussian noise

- **Salt and pepper noise**: contains random occurrences of black and white pixels
- **Impulse noise:** contains random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution

Source: S. Seitz

Reducing salt-and-pepper noise

3x3

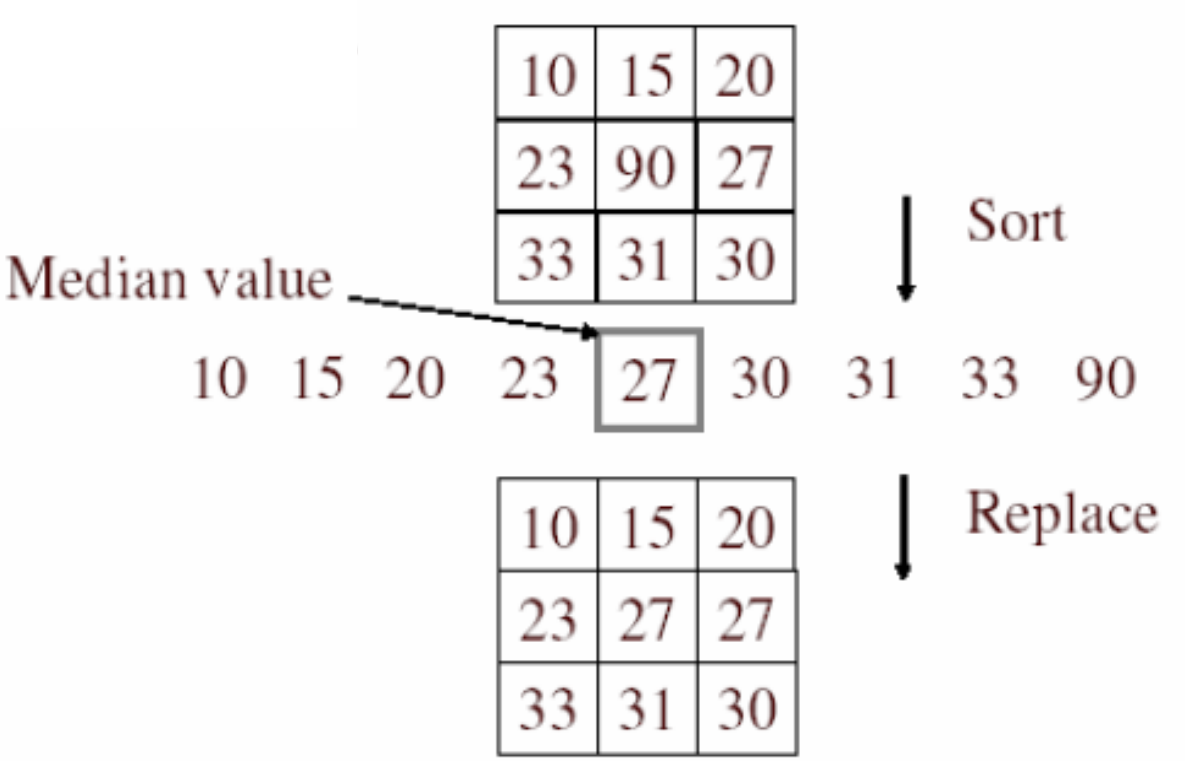
5x5

7x7

What's wrong with the results?

Alternative idea: Median filtering

- A **median filter** operates over a window by selecting the median intensity in the window



- Is median filtering linear?

Median filter

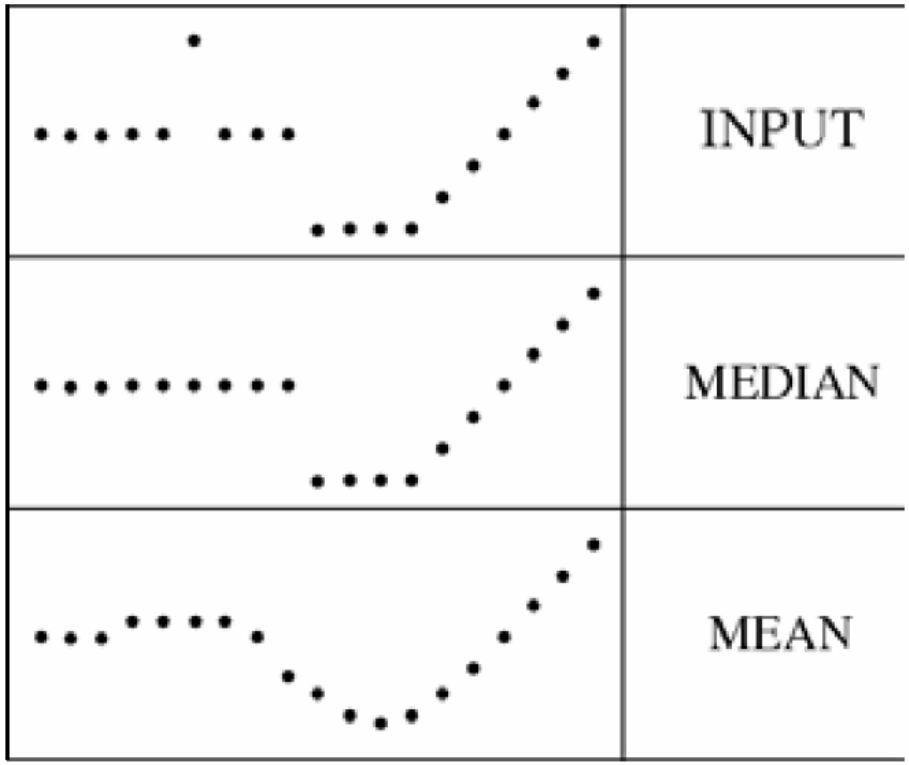
- Is median filtering linear?
- Let's try filtering

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Median filter

- What advantage does median filtering have over Gaussian filtering?
  - Robustness to outliers

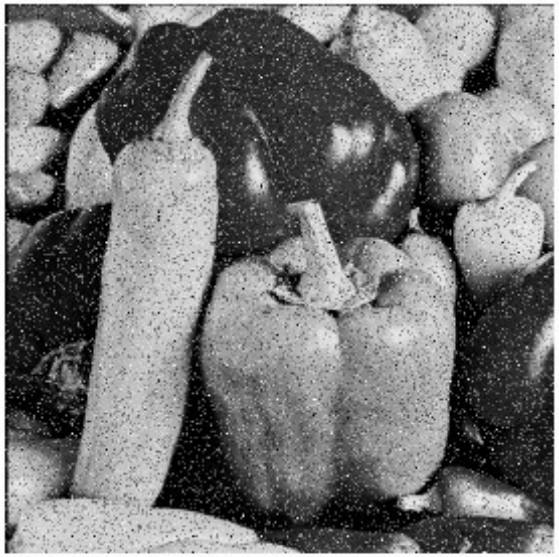
filters have width 5 :

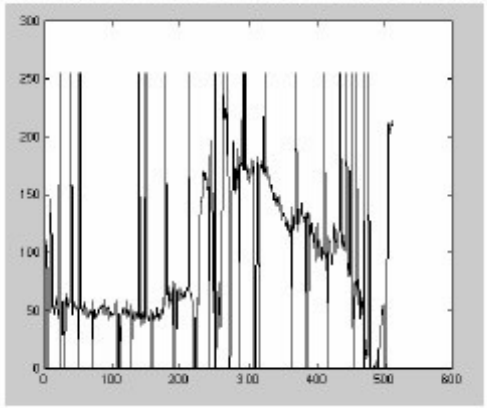


Source: K. Grauman

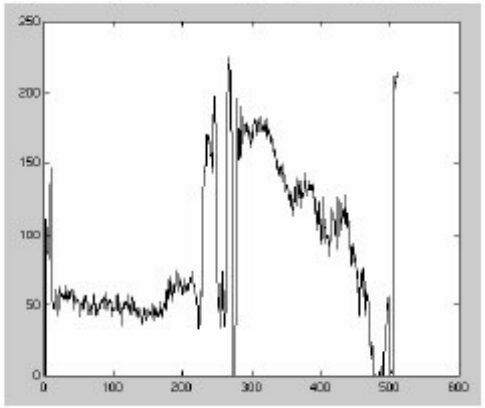
Median filter

Salt-and-pepper noise





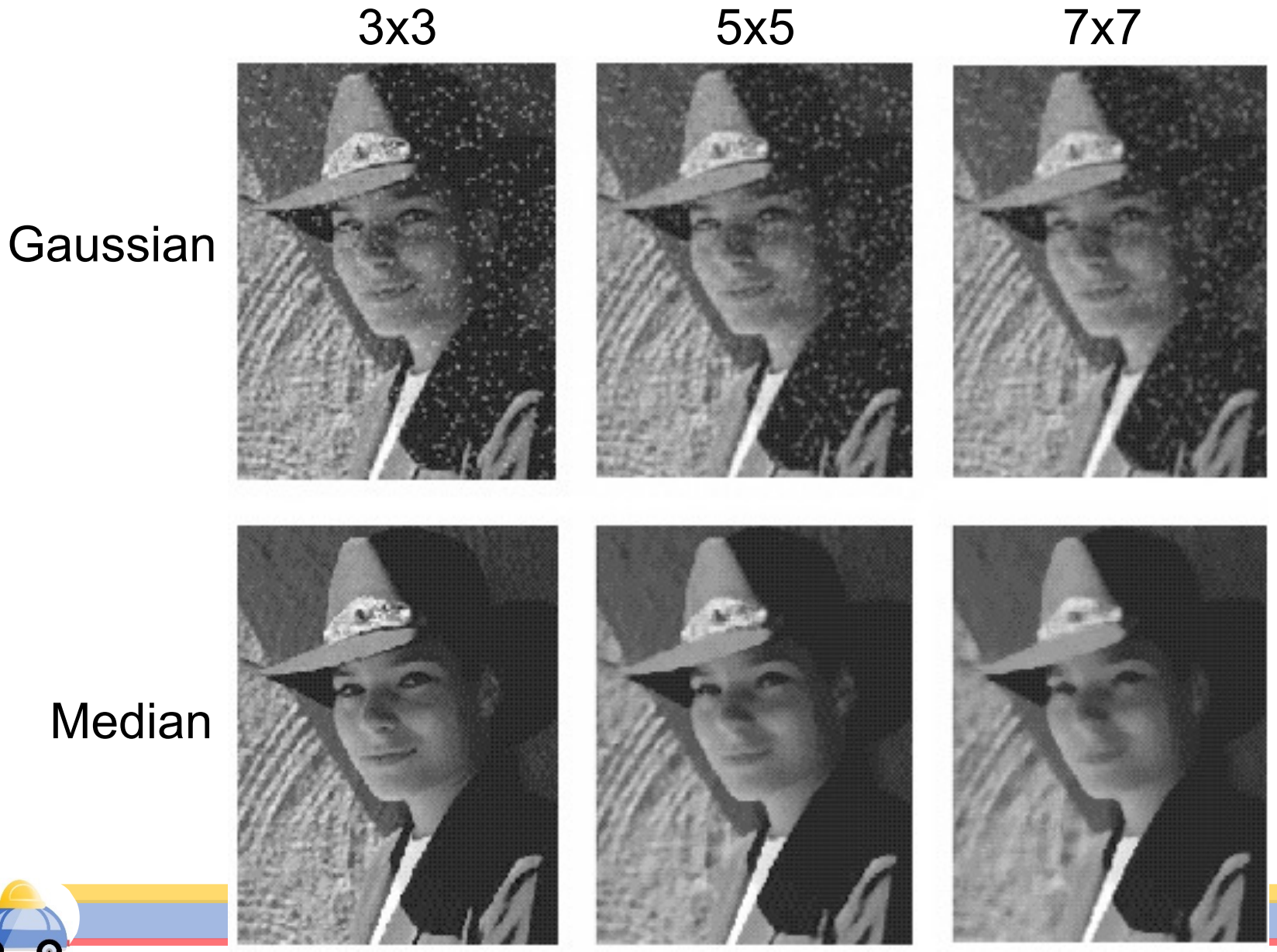
Median filtered



open cv: cv2.medianBlur (input, output,ksize)

Source: M. Hebert

Gaussian vs. median filtering

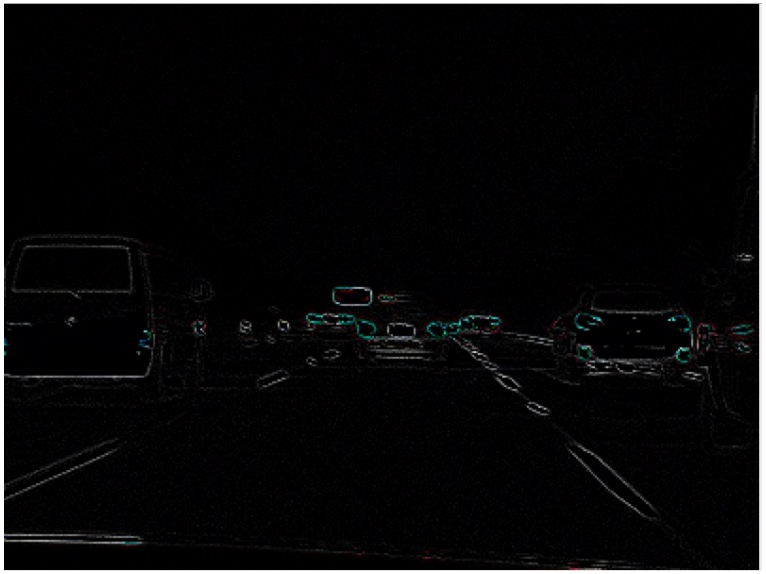


Review: Image filtering

- Convolution
- Box vs. Gaussian filter
- Separability
- Median filter

Outline

- Filtering
  - Convolution: Linearity, shift invariance, associativity, commutativity, …
  - Kernels: Gaussian, box, …
  - Separability of Gaussian
  - Median filter
- Today: Edge detection
- Object recognition: Classification

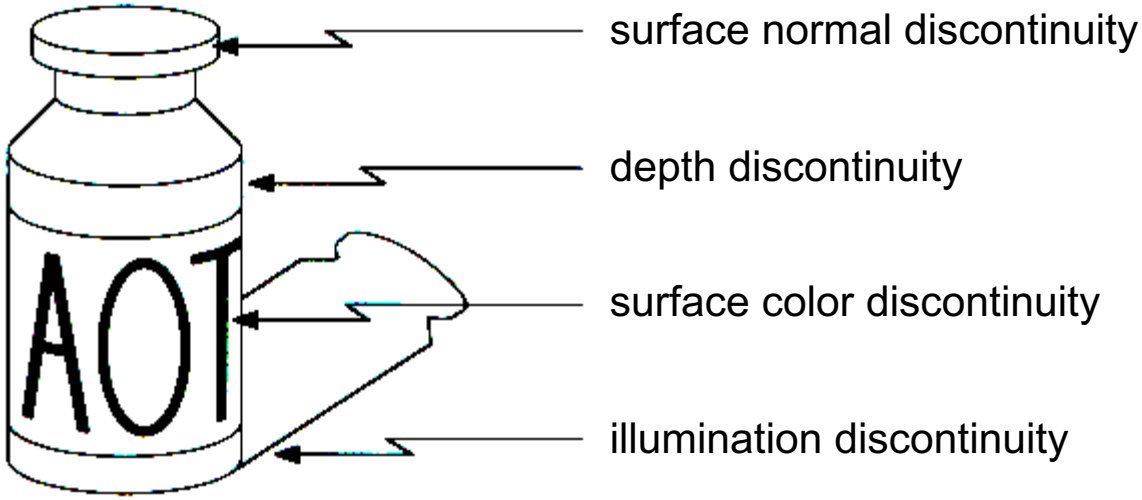


Edge detection

Winter in Kraków photographed by Marcin Ryczek

Edge detection

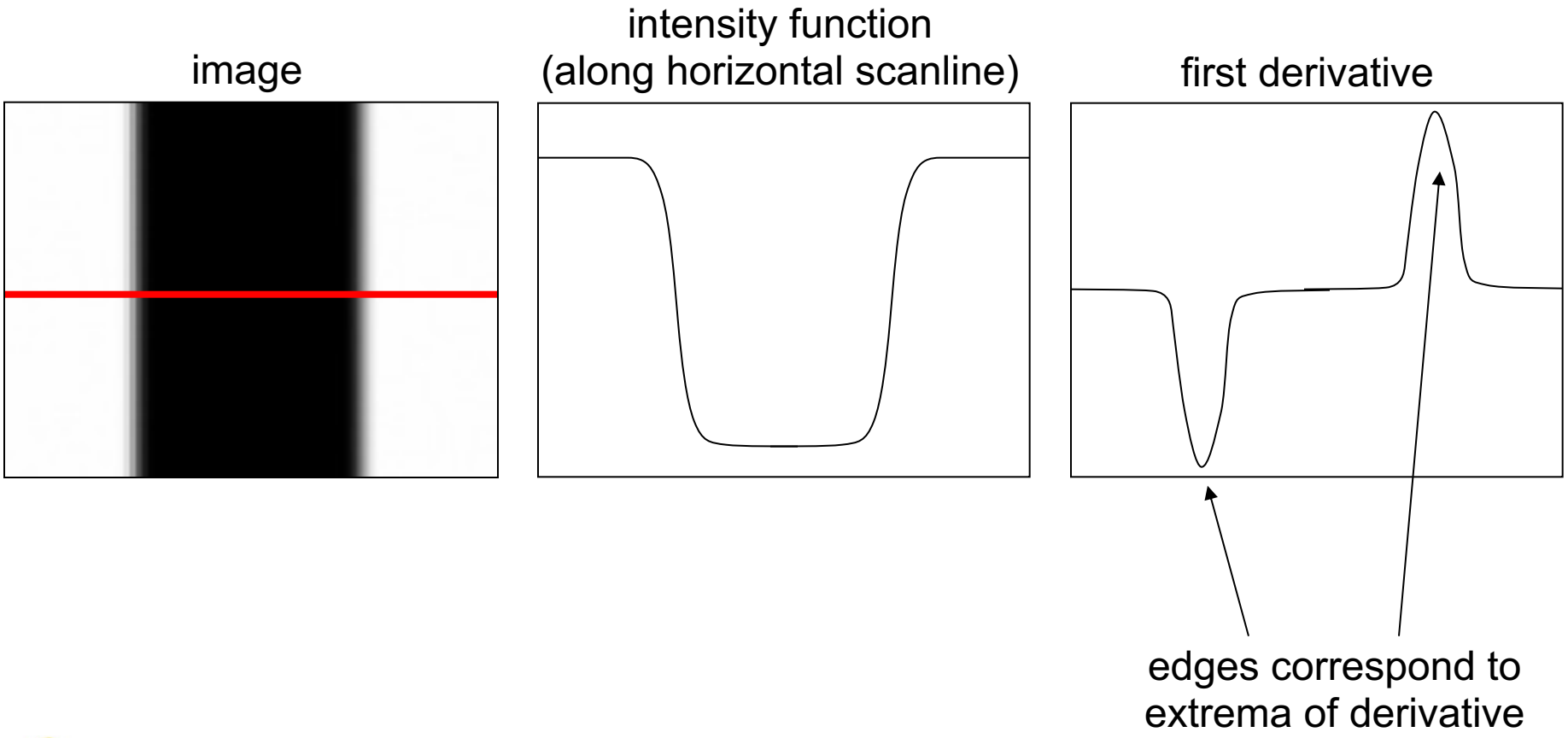
- **Goal:** Identify sudden changes (discontinuities) in an image
- Intuitively, edges carry most of the semantic and shape information from the image
  - E.g., Lanes, traffic signs, cars



Sources: D. Lowe and S. Seitz

Edge detection

An edge is a place of rapid change in the image intensity function



Derivatives with convolution

For 2D function f(x,y), the partial derivative w.r.t *x* is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

For discrete data, we can approximate using finite differences:

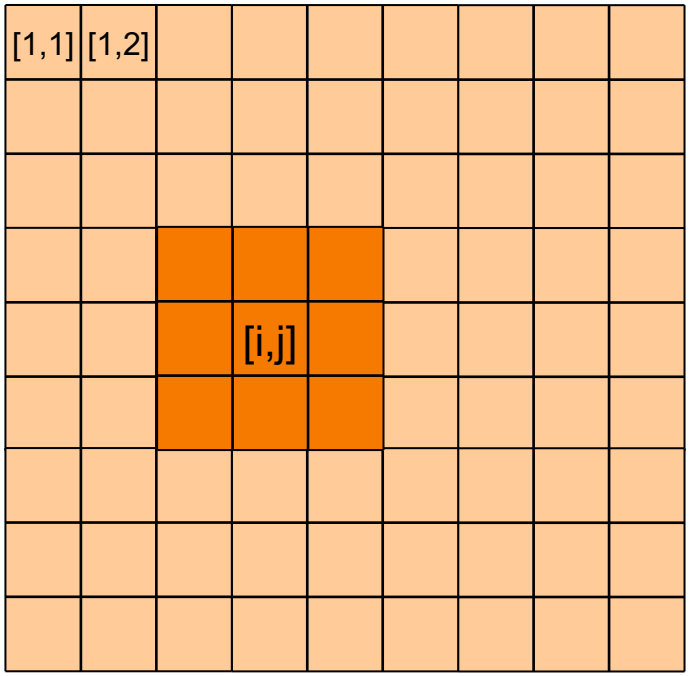
$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

To implement the above as convolution, what would be the associated filter?

Source: K. Grauman

Convolution

image[i,j]



convolution mask g[,]

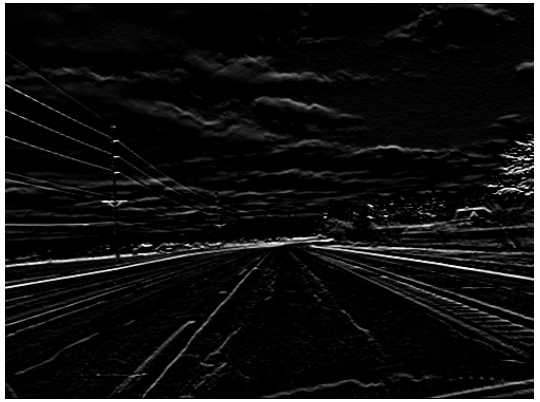
-1	1

Output or convolved image

f = g * img

f[i,j] = -1.img[i,j-1] + 1. img[i,j]

Partial derivatives of an image



$$\frac{\partial f(x,y)}{\partial x}$$

-1	1

$$\frac{\partial f(x,y)}{\partial y}$$

-1
1

1
-1

Which shows changes with respect to x?

Finite difference filters

Other approximations of derivative filters exist:

Prewitt: $M_x =$

-1	0	1
-1	0	1
-1	0	1

; $M_y =$

1	1	1
0	0	0
-1	-1	-1

Sobel: $M_x =$

-1	0	1
-2	0	2
-1	0	1

; $M_y =$

1	2	1
0	0	0
-1	-2	-1

Roberts: $M_x =$

0	1
-1	0

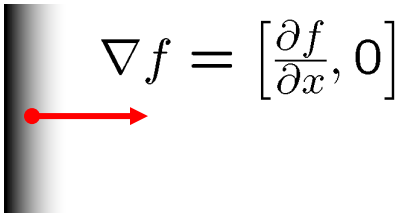
; $M_y =$

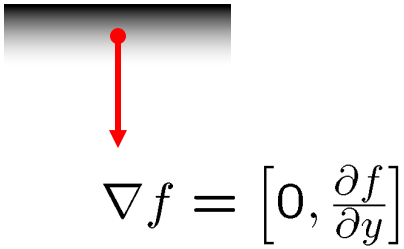
1	0
0	-1

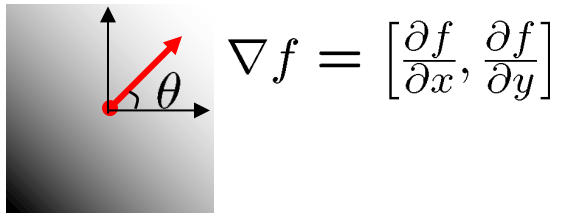
Source: K. Grauman

Image gradient

The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$







The gradient points in the direction of most rapid increase in intensity

- How does this direction relate to the direction of the edge?

The gradient direction is given by $\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$

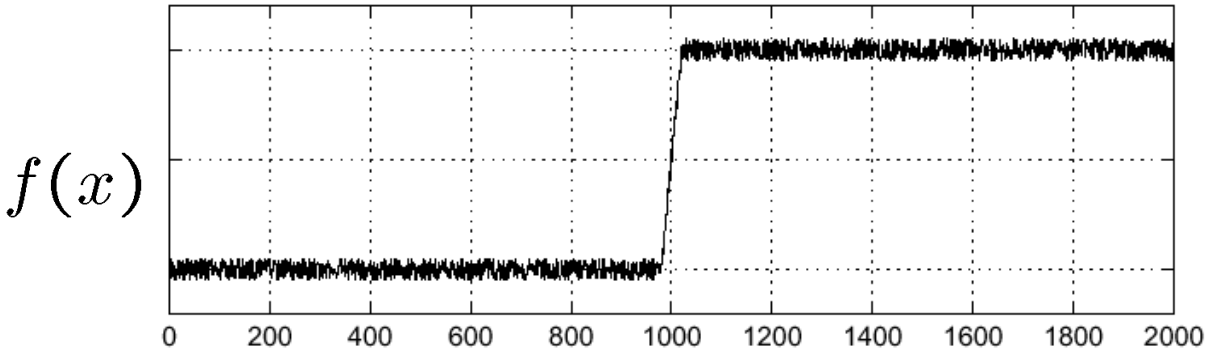
The edge strength is given by the gradient magnitude (norm)

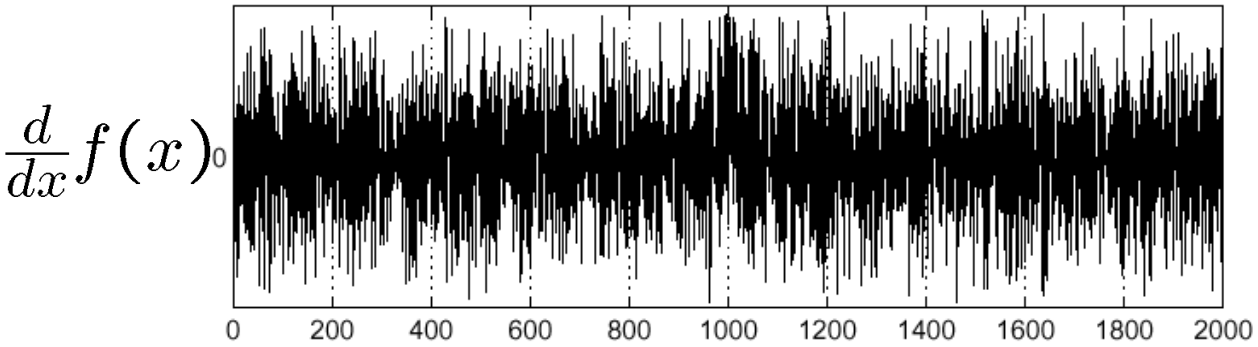
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Source: Steve Seitz

Effects of noise

Consider a single row or column of the image

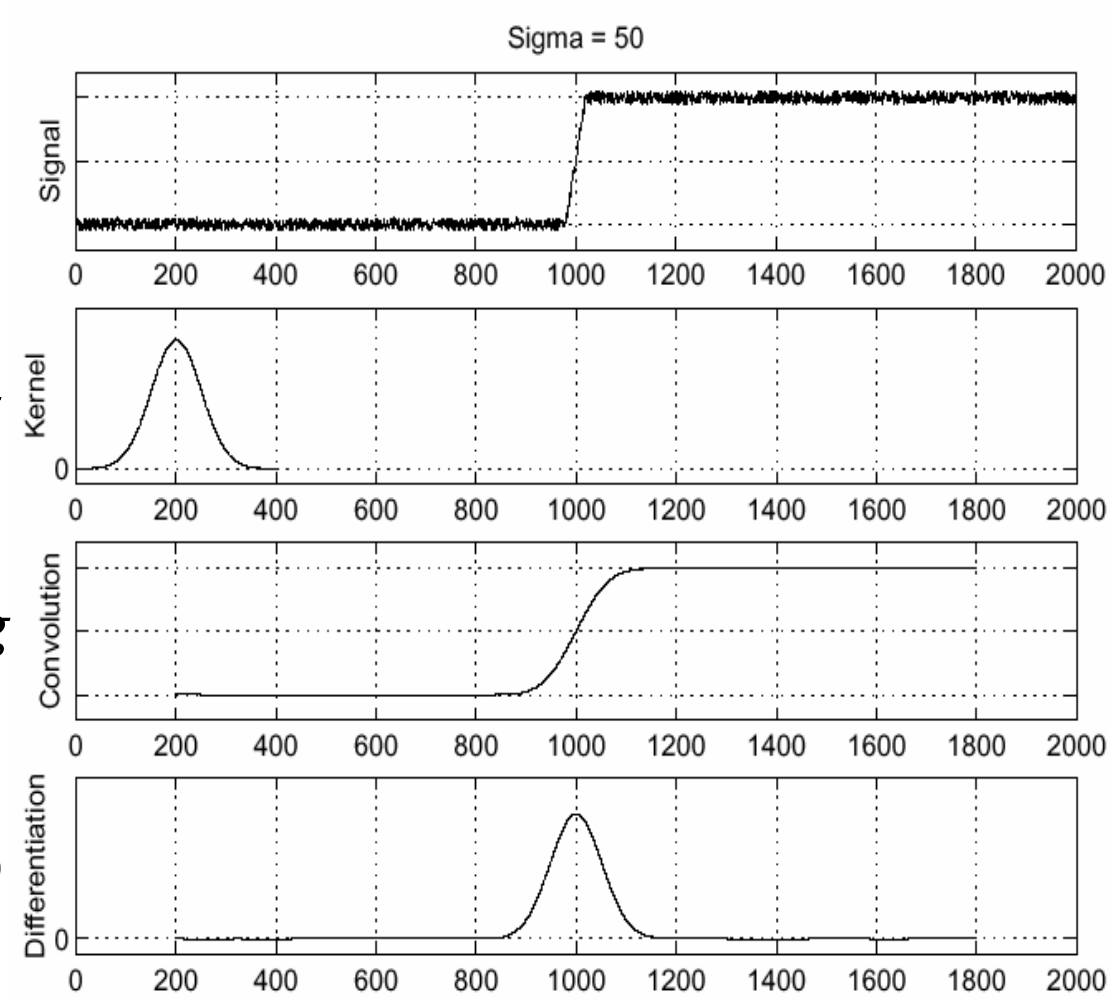




Where is the edge?

Source: S. Seitz

Solution: smooth first



f

g

$f * g$

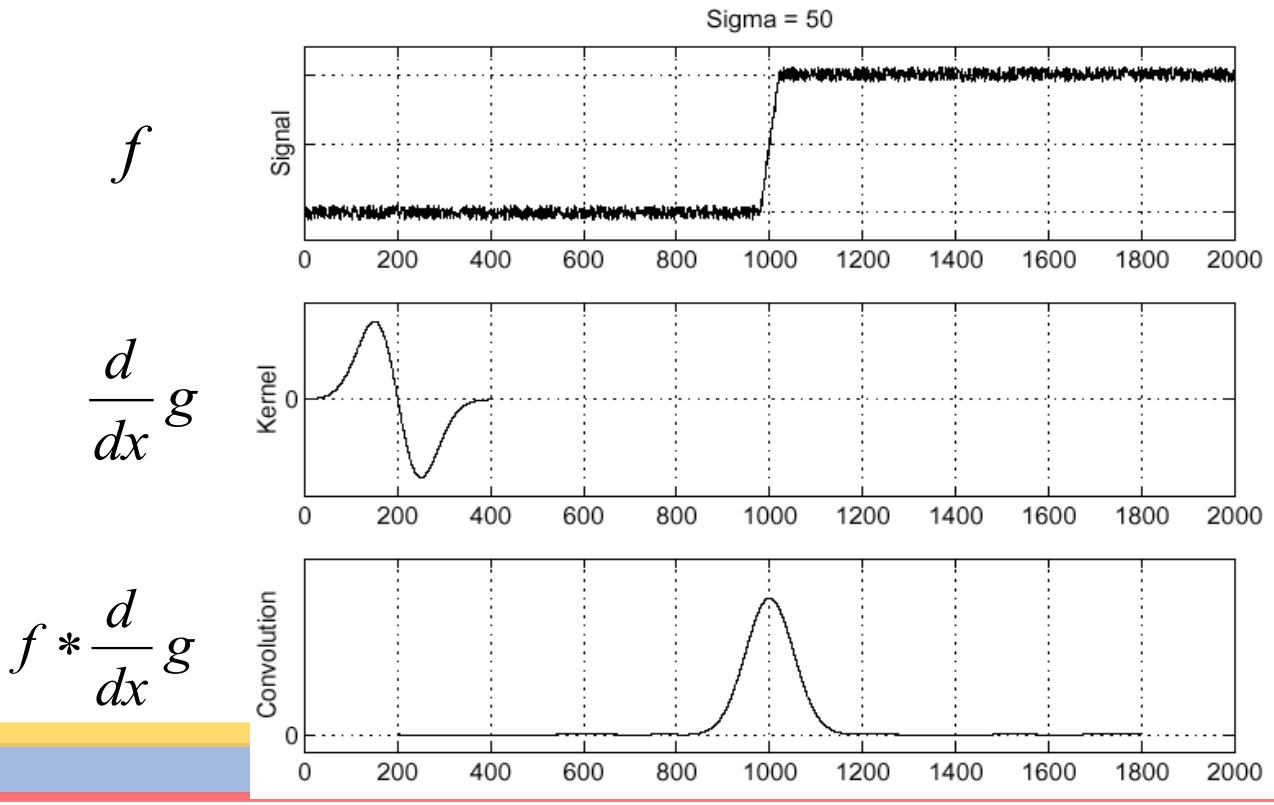
$\frac{d}{dx}(f * g)$

- To find edges, look for peaks in $\frac{d}{dx}(f * g)$

Source: S. Seitz

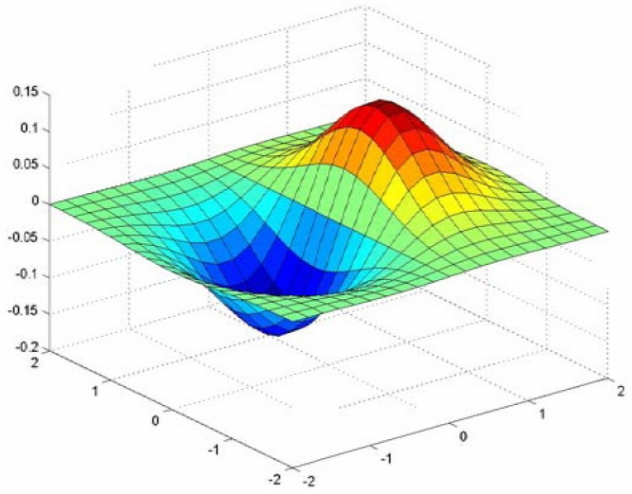
Derivative theorem of convolution

- Differentiation is convolution, and convolution is associative: $\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$
- This saves us one operation:

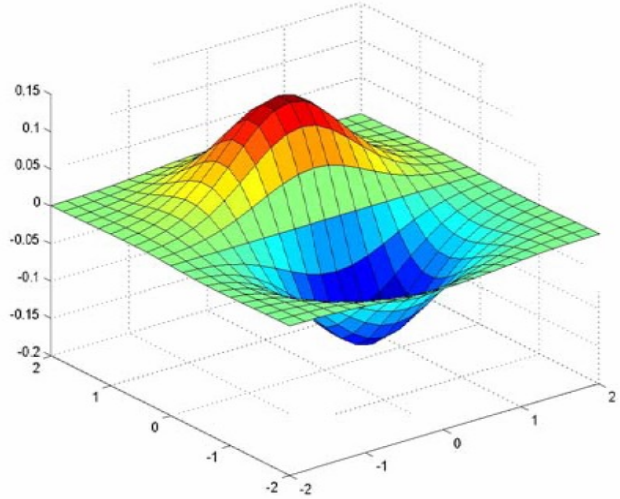


Source: S. Seitz

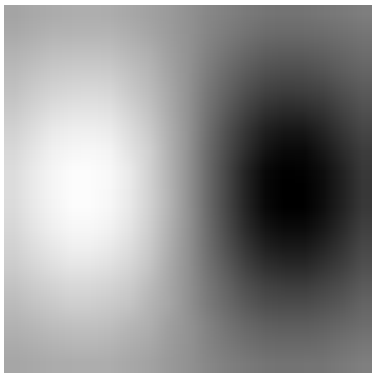
Derivative of Gaussian filters

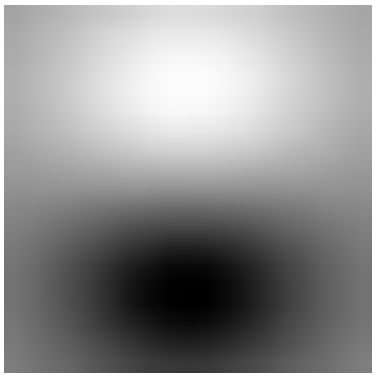


x-direction



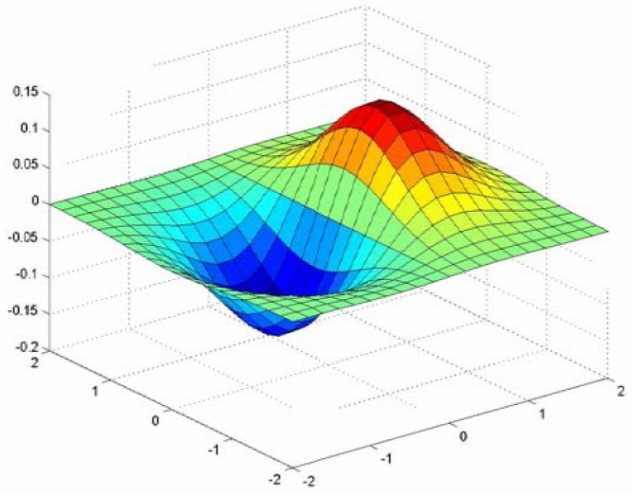
y-direction



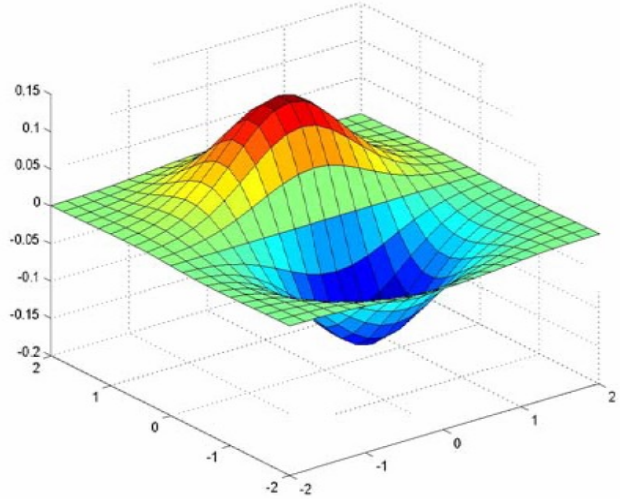


Which one finds horizontal/vertical edges?

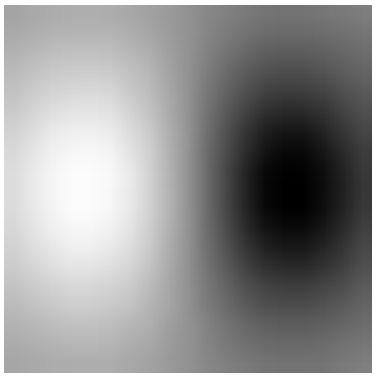
Derivative of Gaussian filters

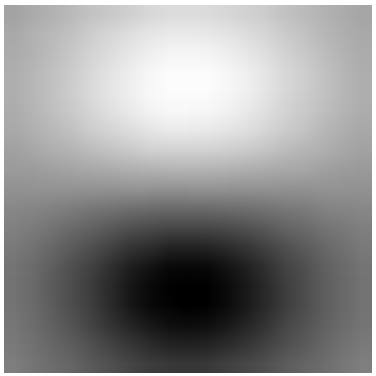


x-direction



y-direction





Are these filters separable?

Recall: Separability of the Gaussian filter

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2}\exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

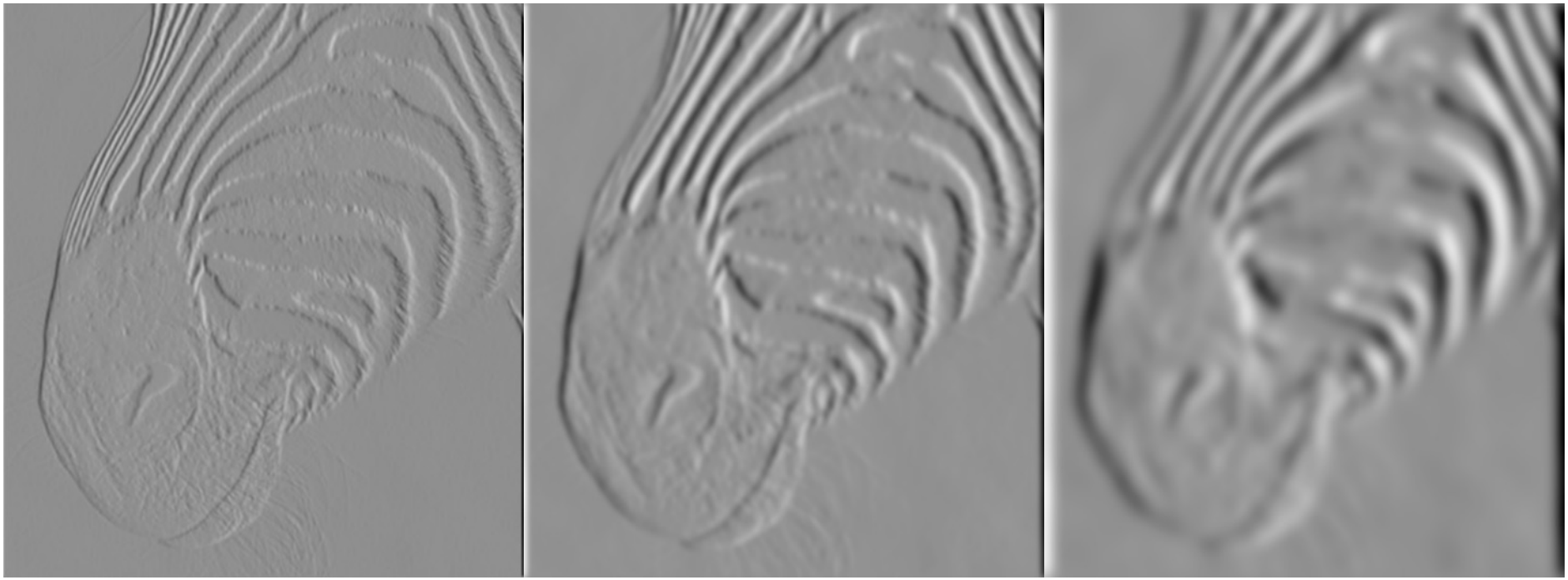
$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{x^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{y^2}{2\sigma^2}}\right)$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

Source: D. Lowe

Scale of Gaussian derivative filter



1 pixel 3 pixels 7 pixels

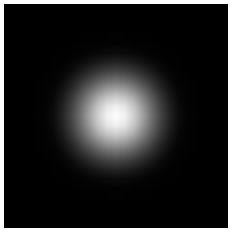
Smoothed derivative removes noise, but blurs edge

Also finds edges at different “scales”

Source: D. Forsyth

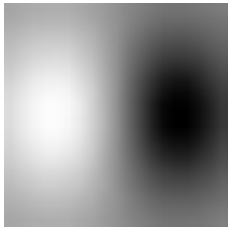
Review: Smoothing vs. derivative filters

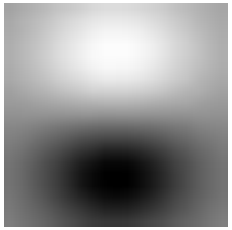
Smoothing filters



- Gaussian: remove “high-frequency” components; “low-pass” filter
- Can the values of a smoothing filter be negative?
- What should the values sum to?
  - **One:** constant regions are not affected by the filter

Derivative filters



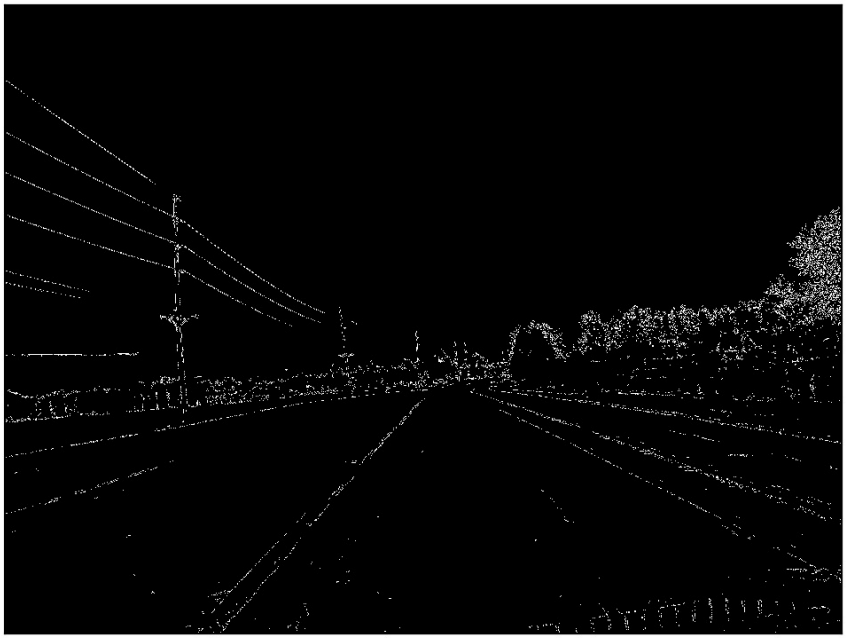


- Derivatives of Gaussian
- Can the values of a derivative filter be negative?
- What should the values sum to?
  - **Zero:** no response in constant regions

Building an edge detector

Original Image

Edge Image



original image

Grad output

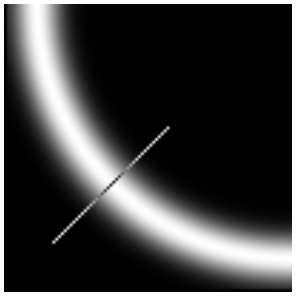
norm of the gradient $\|\nabla f\| = \sqrt{(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2}$

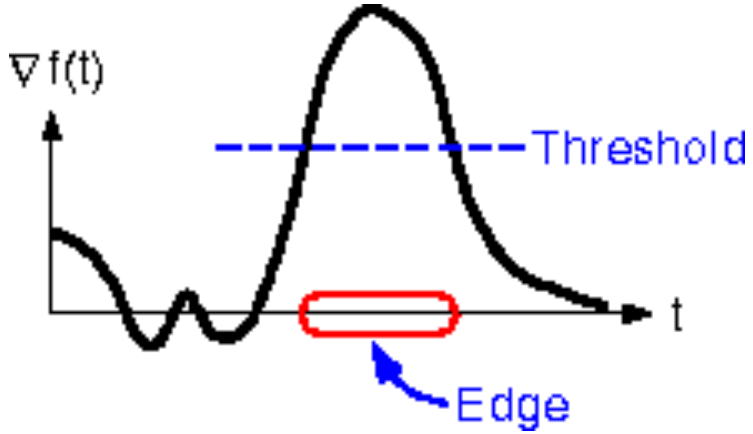
Building an edge detector

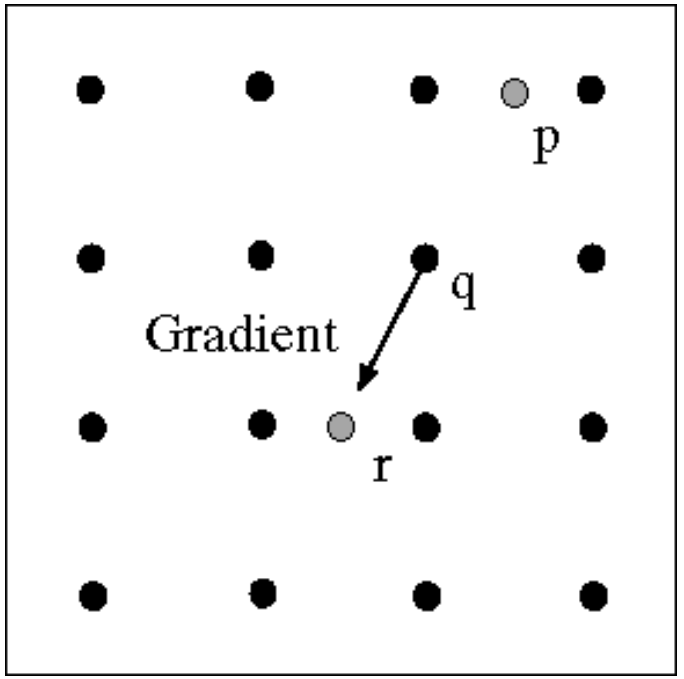
How to turn these thick regions of the gradient into curves?

Thresholded norm of the gradient

Non-maximum suppression







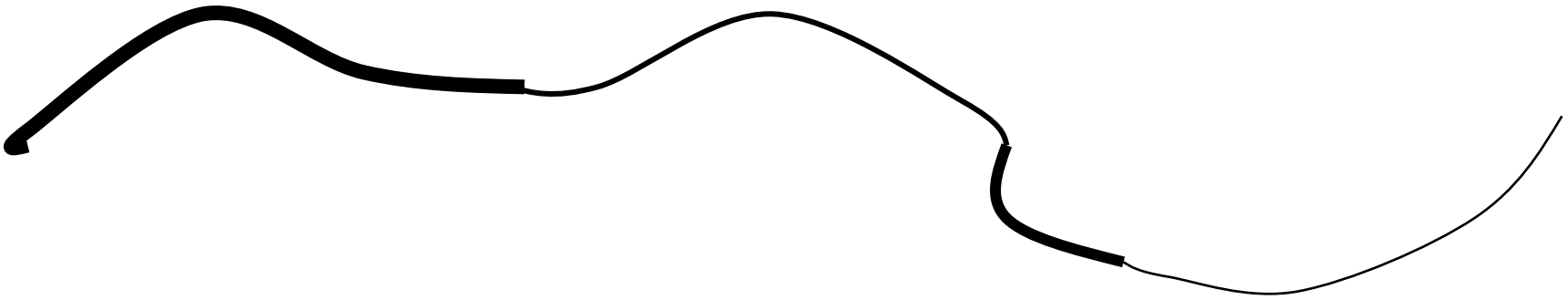
- For each location q above threshold, check that the gradient magnitude is higher than at neighbors p and r along the direction of the gradient
  - May need to interpolate to get the magnitudes at p and r

Non-maximum suppression

Another problem: pixels along this edge didn't survive thresholding

Hysteresis thresholding

Use a high threshold to start edge curves, and a low threshold to continue them.



Source: Steve Seitz

Hysteresis thresholding

original image

high threshold
(strong edges)

low threshold
(weak edges)

hysteresis threshold

Source: L. Fei-Fei

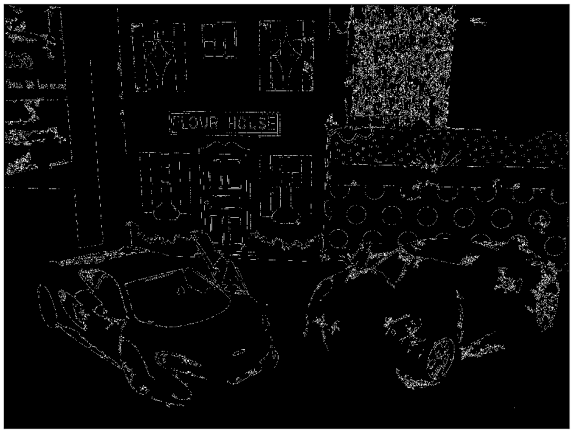
Recap: Canny edge detector

1. Compute x and y gradient images
2. Find magnitude and orientation of gradient
3. **Non-maximum suppression**:
   - Thin wide “ridges” down to single pixel width
4. **Linking and thresholding** (**hysteresis**):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

opencv: `canny(image,th1,th2)`

Original Image

Edge Image



J. Canny, A Computational Approach To Edge Detection, IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

Summary

- Convolution as translation invariant linear operations on signals and images
- Definition of convolution and its properties (associativity, commutativity, etc.)
- Artifacts of of hard-edge kernels
- Gaussian kernel, its definition and properties (separability)
- Median filter, sharpening
- Derivatives as convolution (Sobel, etc.)

Sharpening

What does blurring take away?

Let's add it back:

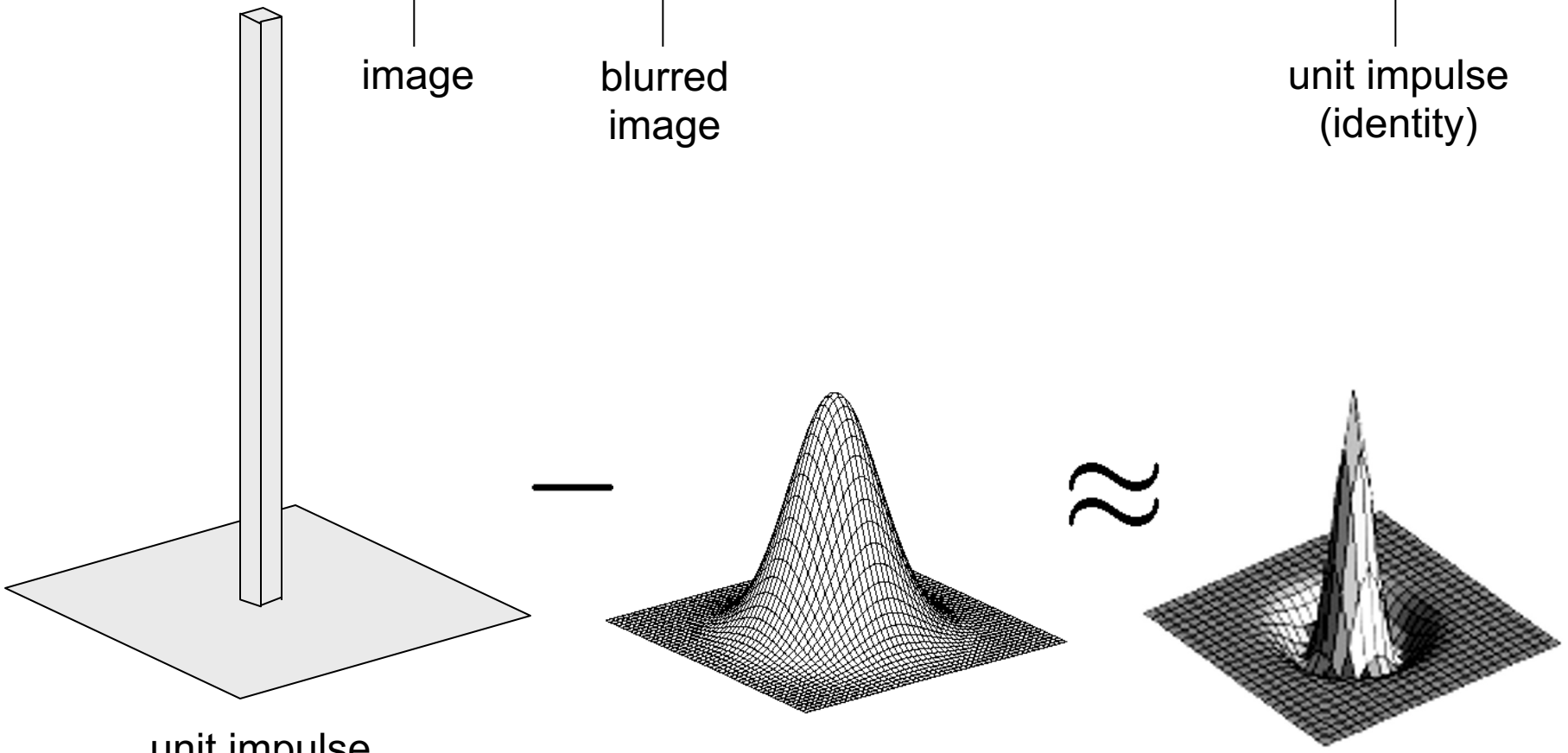
Unsharp mask filter

$$f + \alpha(f - f * g) = (1+\alpha)f - \alpha\, f * g = f * ((1+\alpha)e - g)$$

image

blurred image

unit impulse (identity)



unit impulse

Gaussian

Laplacian of Gaussian