# Principles of Safe Autonomy: Lecture 12-13: Filtering and Robot Localization

Sayan Mitra

Reference: Probabilistic Robotics by Sebastian Thrun, Wolfram Burgard, and Dieter Fox

Slides: From the book's website

# Review from last time: Beliefs

*Belief*: Robot's knowledge about the state of the environment

True state is unknowable / measurable typically, so, robot must infer state from data and we have to distinguish this inferred/estimated state from the actual state $x_t$

$bel(x_t) = p(x_t | z_{1:t}, u_{1:t})$

Posterior distribution over state at time t given all past measurements and control.

This will be calculated in two steps:

1. Prediction: $\overline{bel}(x_t) = p(x_t | z_{1:t-1}, u_{1:t})$
2. Correction: Calculating $bel(x_t)$ from $\overline{bel}(x_t)$ a.k.a measurement update (will use Equation (*) from earlier)

# Recursive Bayes Filter

Algorithm Bayes_filter($bel(x_{t-1}), u_t, z_t$)

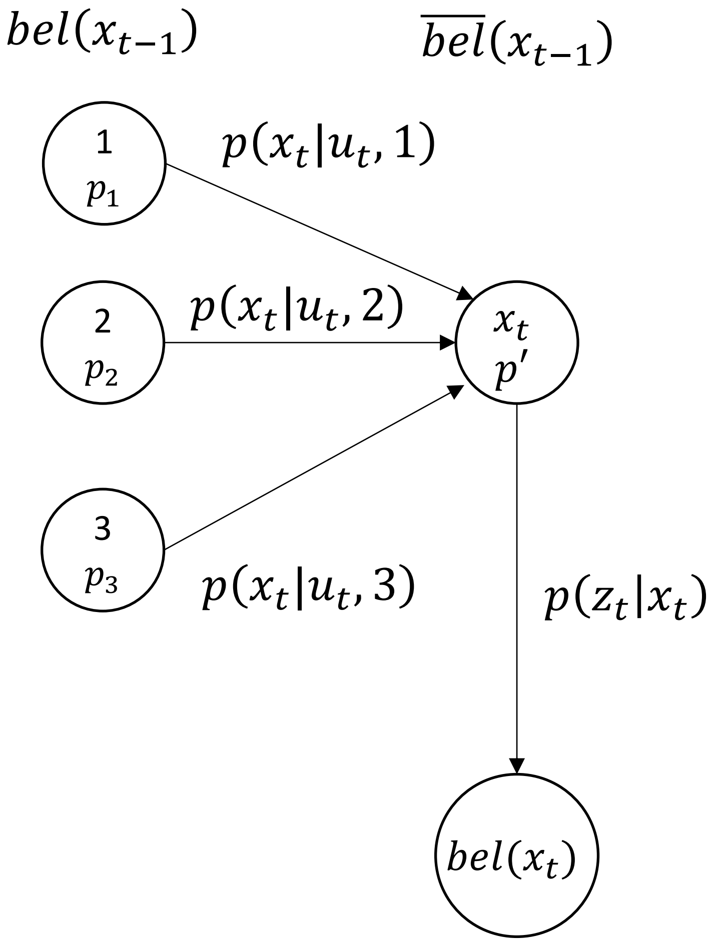
for all $x_t$ do:

$$\overline{bel}(x_t) = \int p(x_t | u_t, x_{t-1}) bel(x_{t-1}) dx_{t-1}$$

$$bel(x_t) = \eta\, p(z_t | x_t)\, \overline{bel}(x_t)$$

end for

return $bel(x_t)$



# Histogram Filter or Discrete Bayes Filter

Finitely many states $x_i, x_k, etc.$ Random state vector $X_t$

$p_{k,t}$: belief at time t for state $x_k$; discrete probability distribution

**Algorithm Discrete_Bayes_filter(**$\{p_{k,t-1}\}, u_t, z_t$**):**

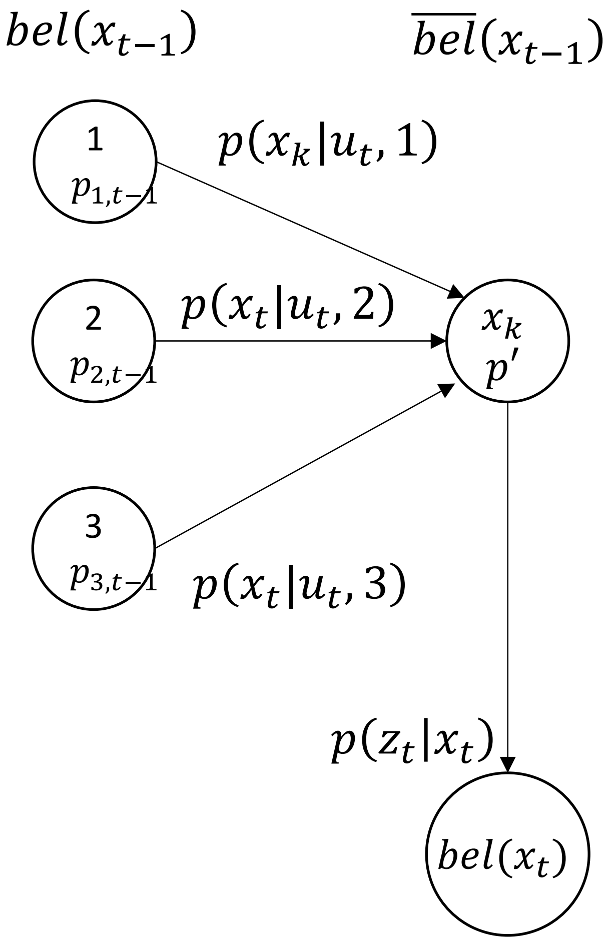
for all $k$ do:

$$\bar{p}_{k,t} = \sum_i p(X_t = x_k | u_t, X_{t-1} = x_i) p_{i,t-1}$$

$$p_{k,t} = \eta\, p(z_t \,| X_t = x_k) \bar{p}_{k,t}$$

end for

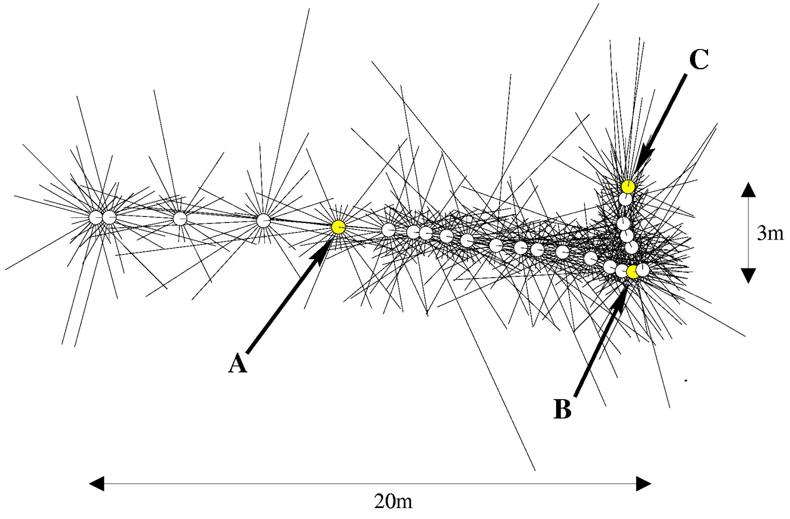
return $\{p_{k,t}\}$

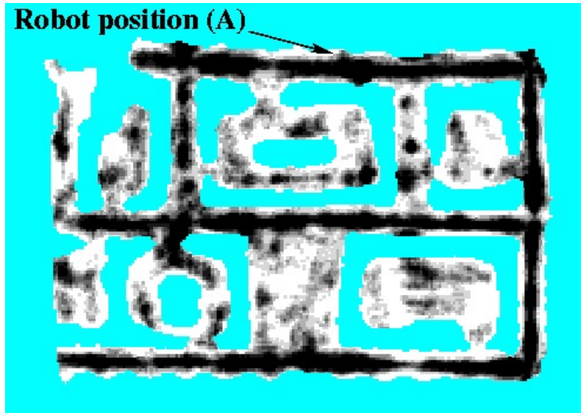


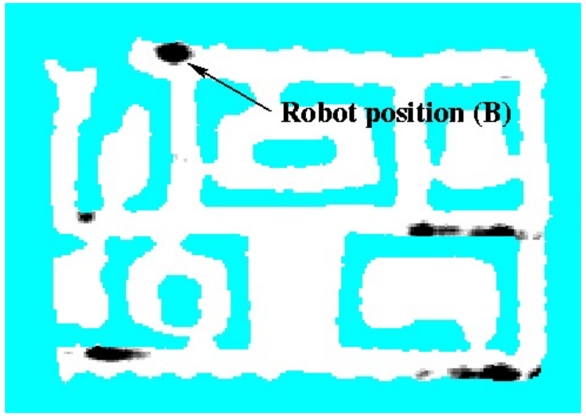
# Outline of filtering module

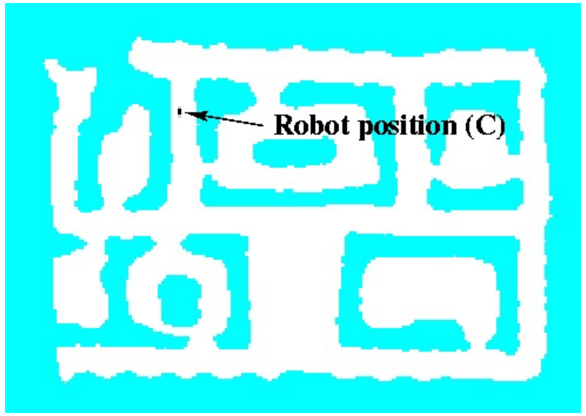
- Particle filter
  - Nonparametric representation of distributions with samples
  - Weighted particles
  - Importance sampling
- Monte Carlo localization
- Examples
- Conclusions

# Sonars and Occupancy Grid Map



# Monte Carlo Localization

- Represents beliefs by particles

# Particle Filters

- Represent belief by finite number of parameters (just like histogram filter)
- But, they differ in how the parameters (particles) are generated and populate the state space
- Key idea: represent belief $bel(x_t)$ by a random set of state samples
- Advantages
  - The representation is approximate and nonparametric and therefore can represent a broader set of distributions than e.g., Gaussian
  - Can handle nonlinear tranformations
- Related ideas: Monte Carlo filter, Survival of the fittest, Condensation, Bootstrap filter, Filtering: [Rubin, 88], [Gordon et al., 93], [Kitagawa 96], Dynamic Bayesian Networks: [Kanazawa et al., 95]d

# Particle filtering algorithm

$X_t = x_t^{[1]}, x_t^{[2]}, \dots x_t^{[M]}$ particles

Algorithm Particle_filter($X_{t-1}, u_t, z_t$):
$\bar{X}_t = X_t = \emptyset$
for all $m$ in [M] do:
  sample $x_t^{[m]} \sim p(x_t | u_t, x_{t-1}^{[m]})$
  $w_t^{[m]} = p\left(z_t \middle| x_t^{[m]}\right)$
  $\bar{X}_t = \bar{X}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$
end for
for all $m$ in [M] do:
  draw $i$ *with probability* $\propto w_t^{[i]}$
  add $x_t^{[i]}$ *to* $X_t$
end for
return $X_t$

ideally, $x_t^{[m]}$ is selected with probability prop. to $p(x_t \mid z_{1:t}, u_{1:t})$

$\bar{X}_{t-1}$ is the temporary particle set

// sampling from state transition dist.

// calculates *importance factor* $w_t$ or weight

// resampling or importance sampling; these are distributed according to $\eta\, p\left(z_t \middle| x_t^{[m]}\right) \overline{bel}(x_t)$

// survival of fittest: moves/adds particles to parts of the state space with higher probability

# Importance Sampling

suppose we want to compute $E_f[I(x \in A)]$ but we can only sample from density $g$

$E_f[I(x \in A)]$

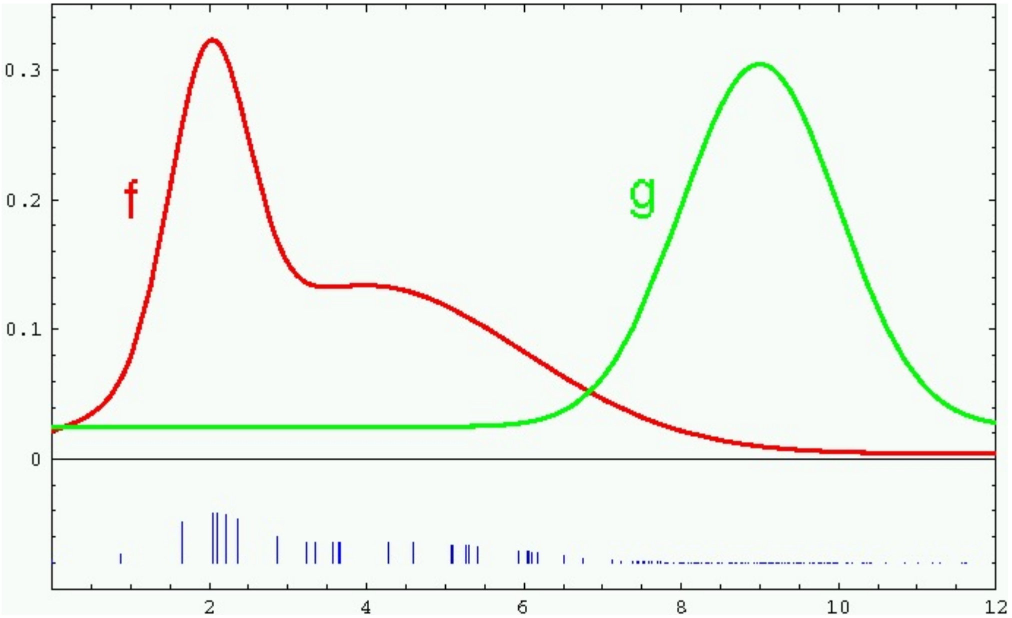
$= \int f(x) I(x \in A) dx$

$= \int \frac{f(x)}{g(x)} g(x) I(x \in A) dx$, provided $g(x) > 0$

$= \int w(x) g(x) I(x \in A) dx$

$= E_g[w(x) I(x \in A)]$

We need $f(x) > 0 \Rightarrow g(x) > 0$




**Weight samples:** $w = f / g$

# Monte Carlo Localization (MCL)

$X_t = x_t^{[1]}, x_t^{[2]}, \dots x_t^{[M]}$ particles

Algorithm MCL($X_{t-1}, u_t, z_t$,m):
$\bar{X}_{t-1} = X_t = \emptyset$
for all $m$ in [M] do:

$$x_t^{[m]} = \boldsymbol{sample\_motion\_model}(u_t\ x_{t-1}^{[m]})$$

$$w_t^{[m]} = \boldsymbol{measurement\_model}(z_t, x_t^{[m],m})$$

$$\bar{X}_t = \bar{X}_t + \langle\, x_t^{[m]}, w_t^{[m]} \rangle$$

end for

for all $m$ in [M] do:

draw $i\ with\ probability\ \propto w_t^{[i]}$

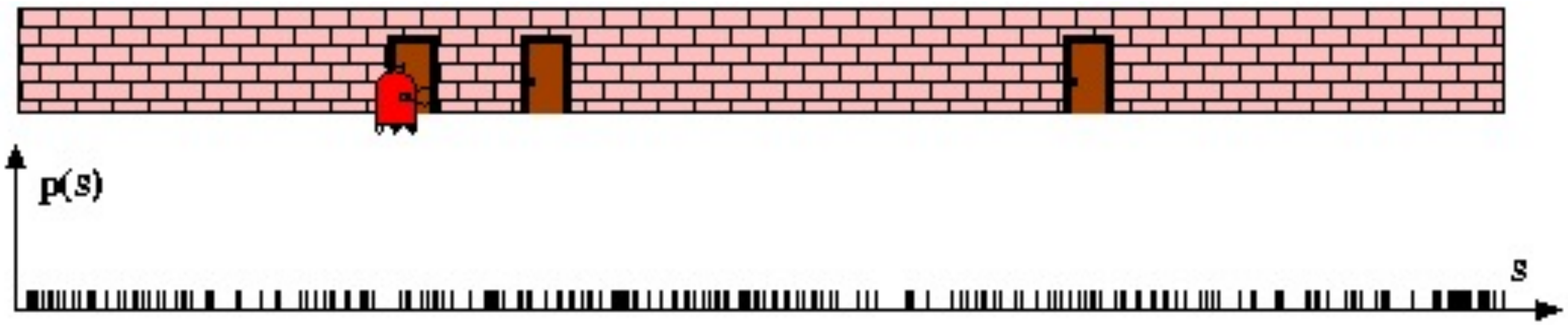
add $x_t^{[i]}\ to\ X_t$

end for

return $X_t$

Plug in motion and measurement models in the particle filter

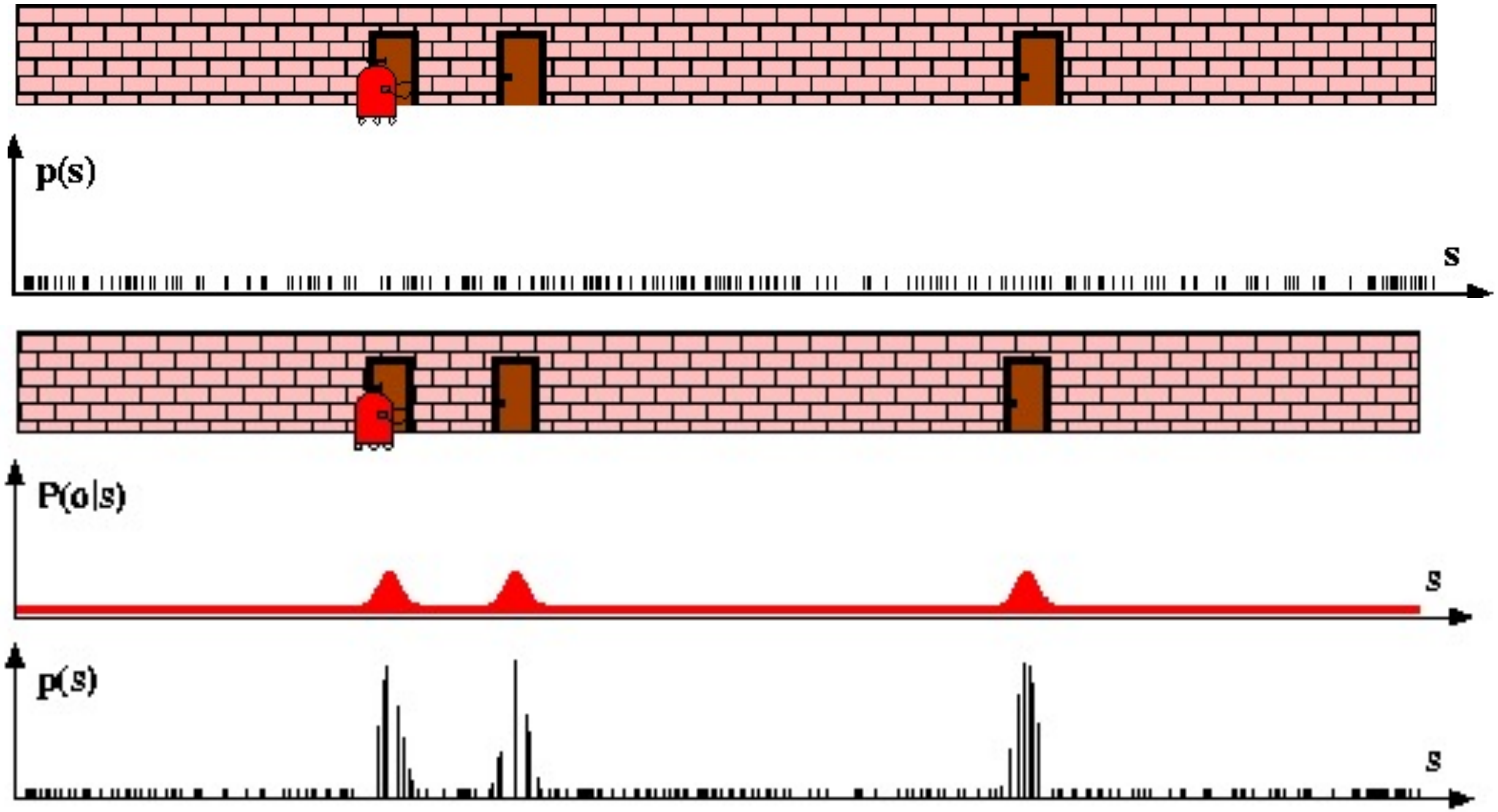
# Particle Filters



# Sensor Information: Importance Sampling

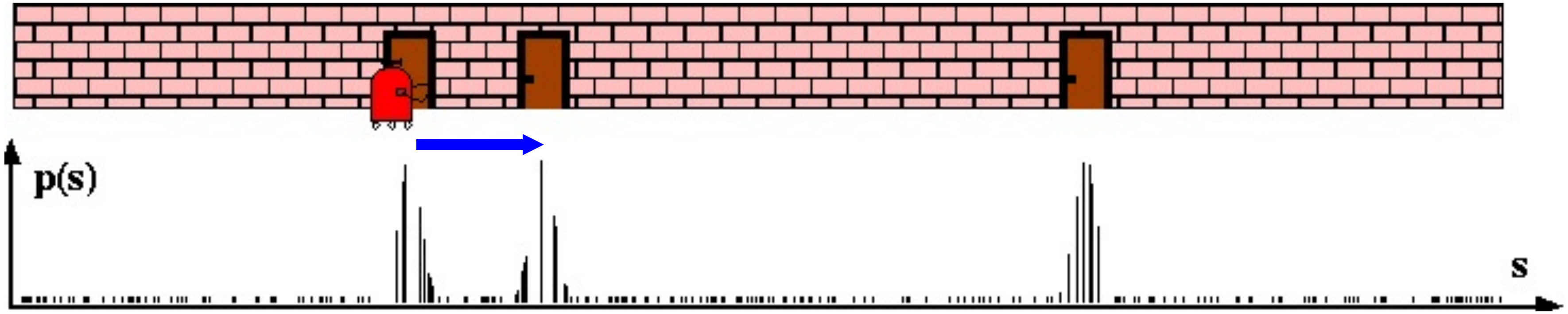
$$Bel(x) \leftarrow \alpha \, p(z \mid x) \, Bel^{-}(x)$$

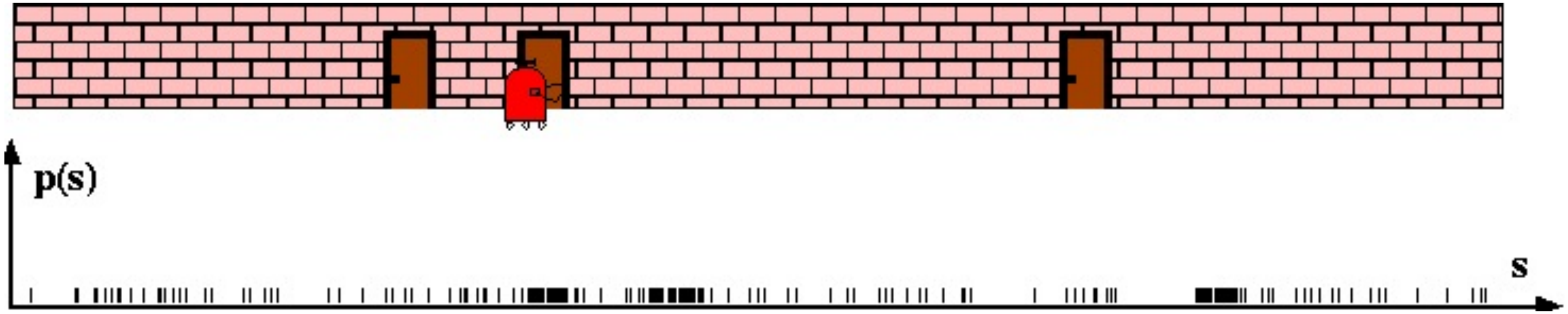
$$w \leftarrow \frac{\alpha \, p(z \mid x) \, Bel^{-}(x)}{Bel^{-}(x)} = \alpha \, p(z \mid x)$$



# Robot Motion

$$Bel^{-}(x) \quad \leftarrow \quad \int p(x \mid u, x')\, Bel(x') \;\mathrm{d}\, x'$$



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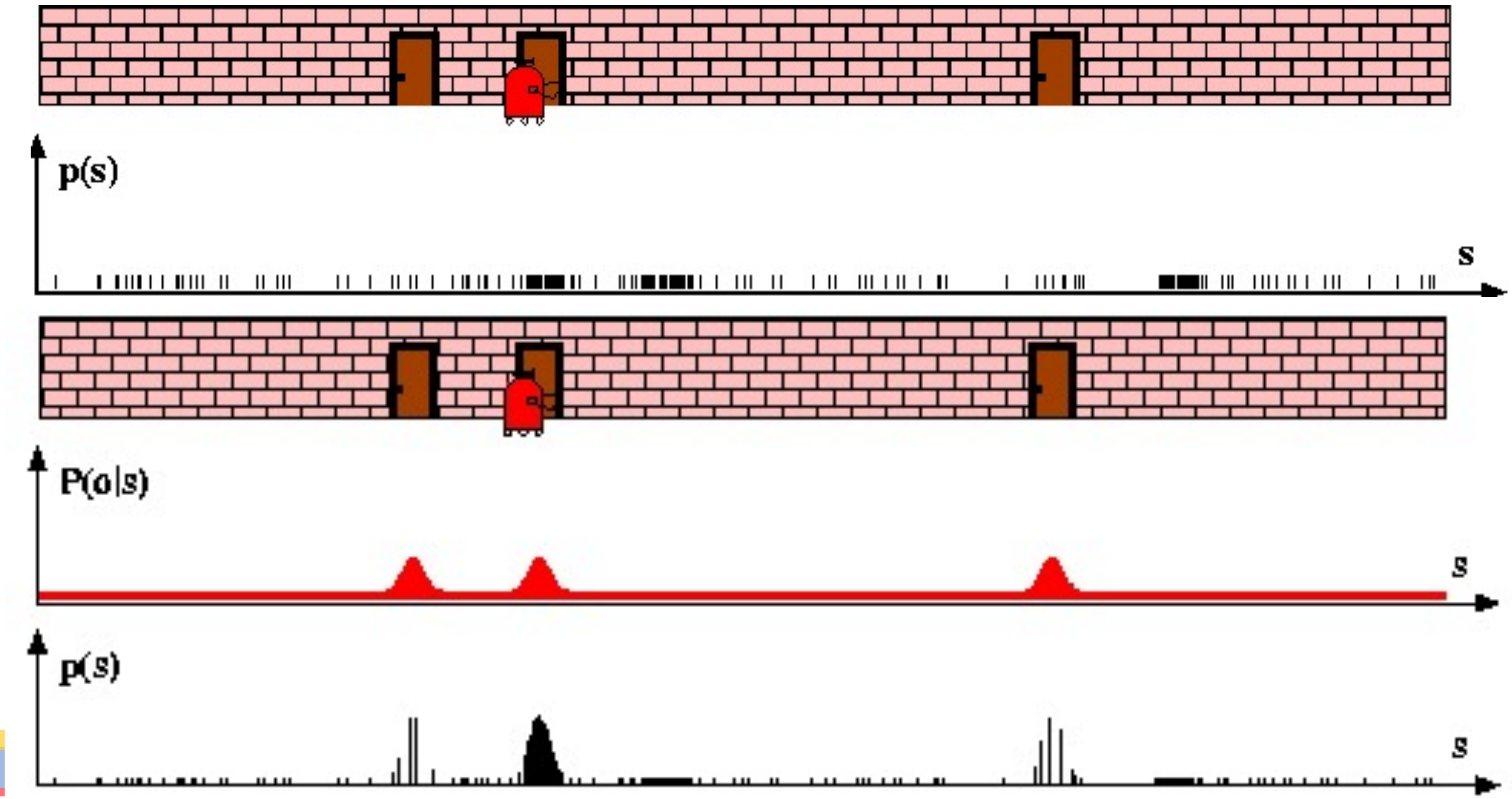

# Robot Motion

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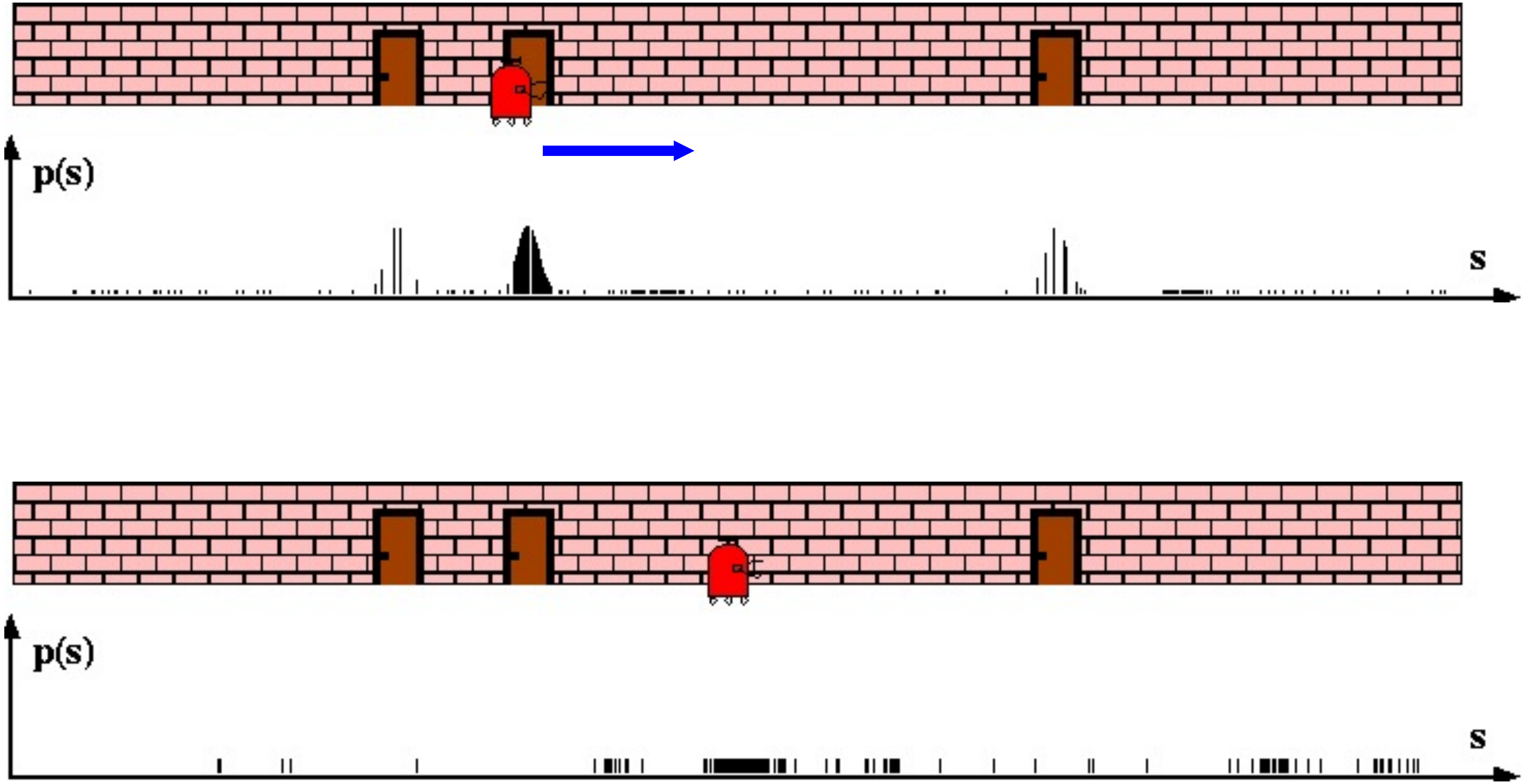
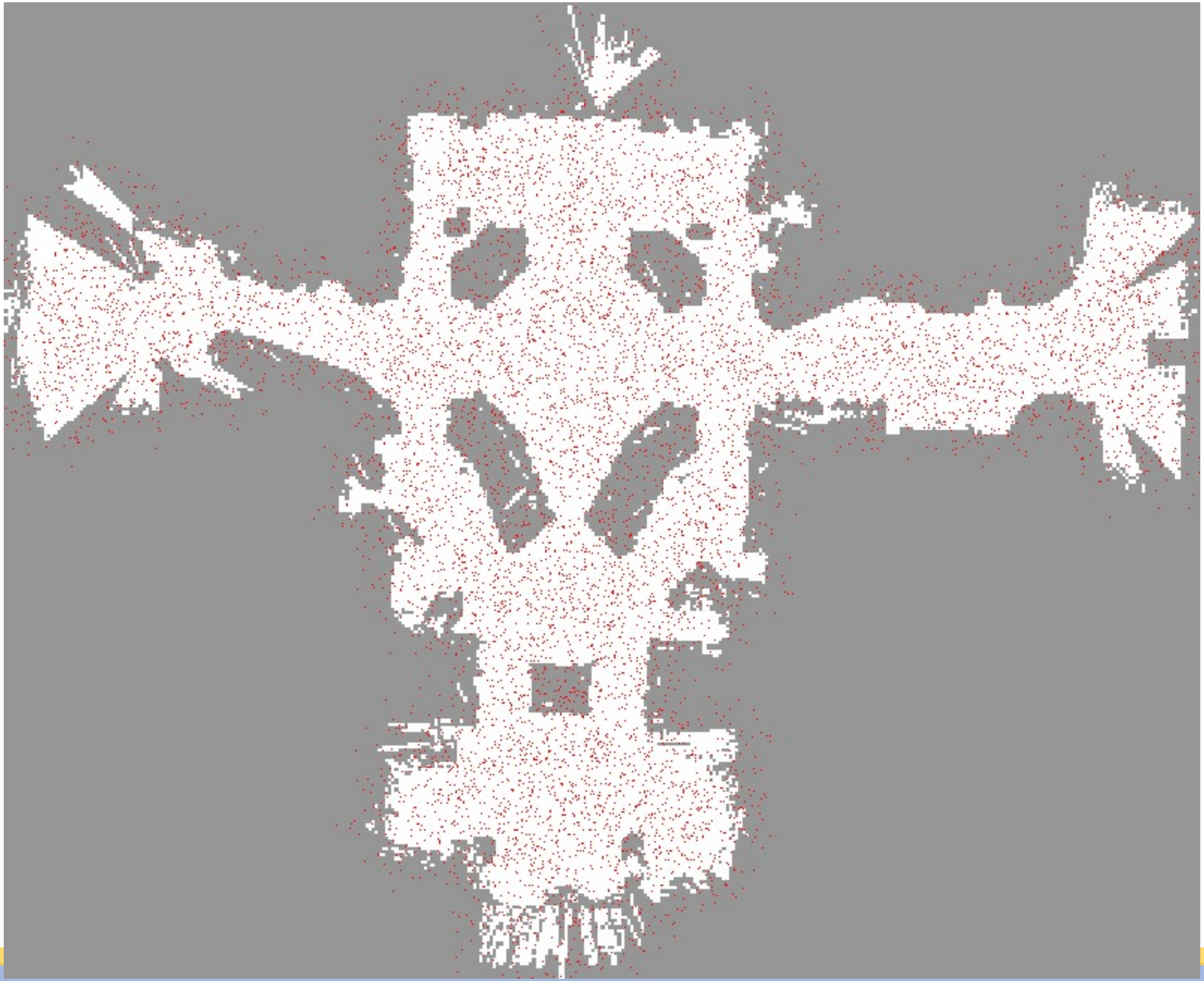
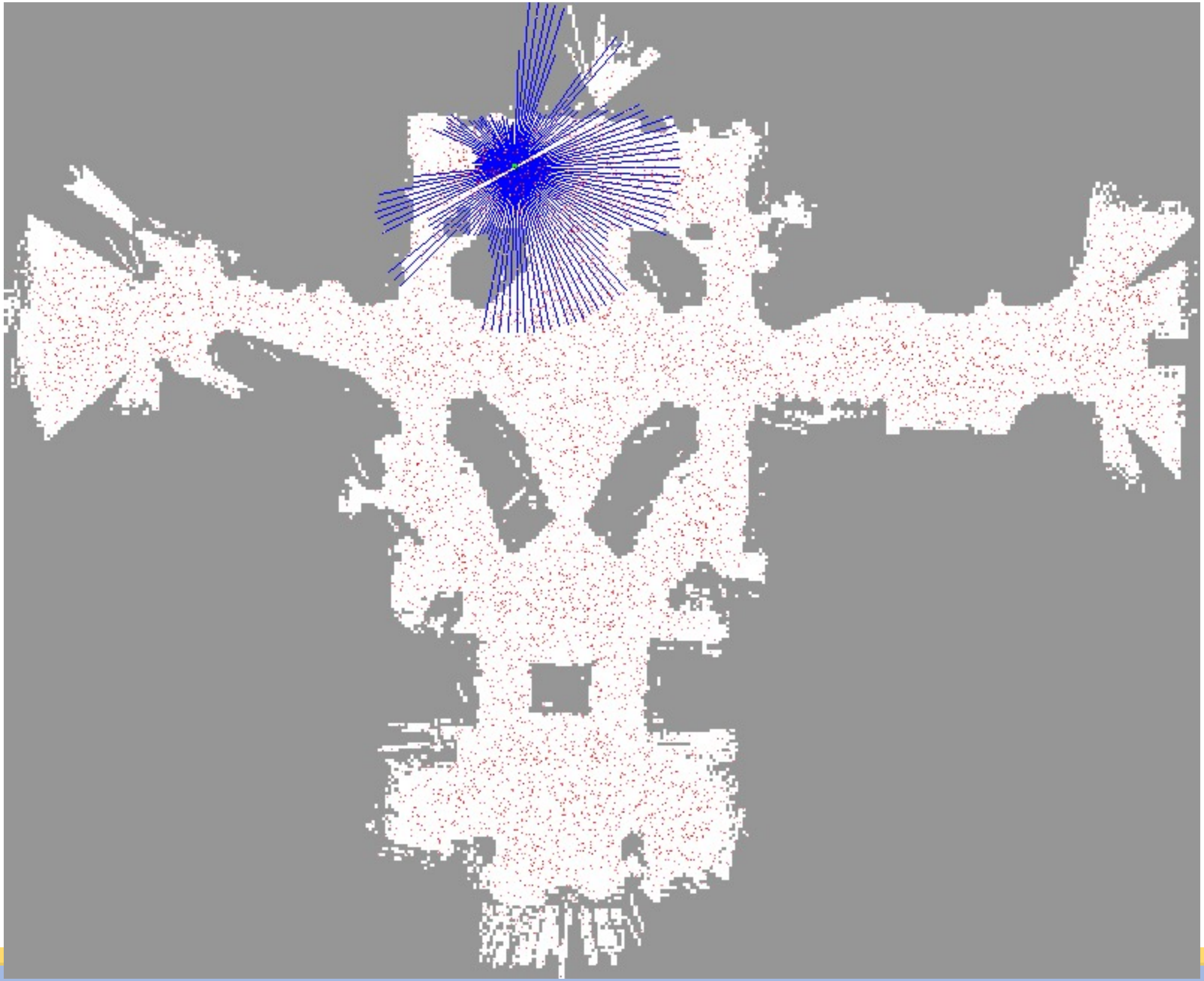
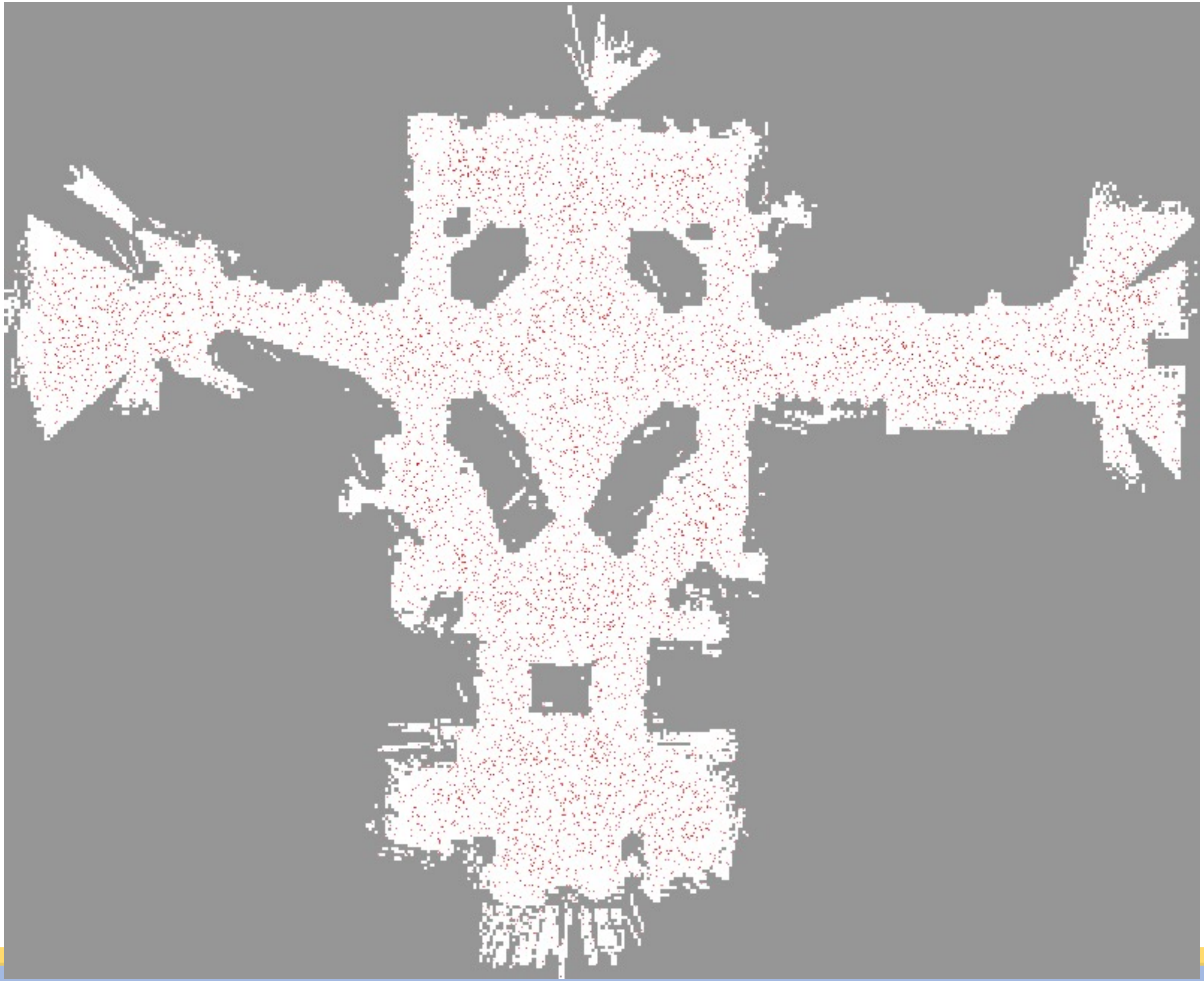
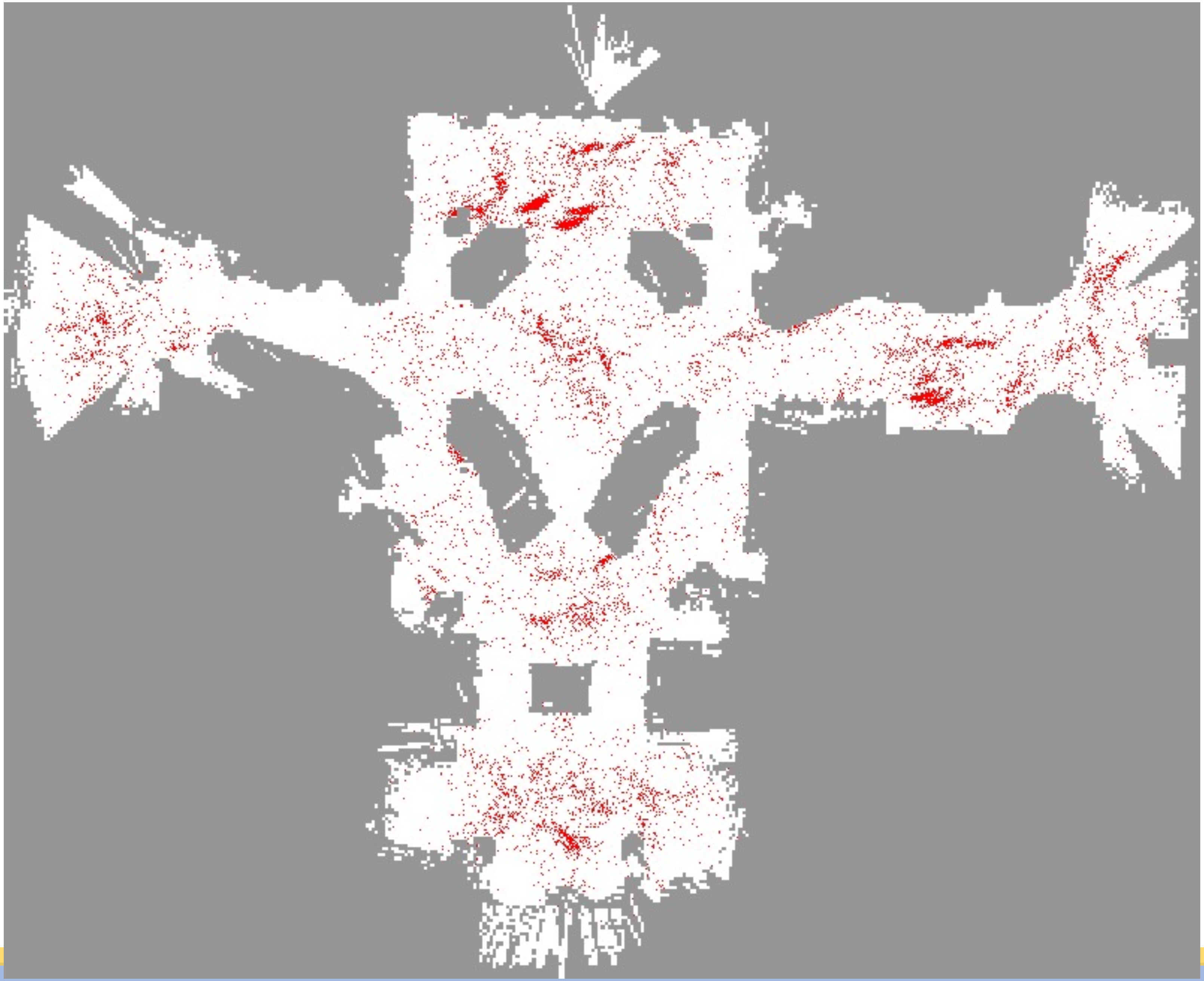
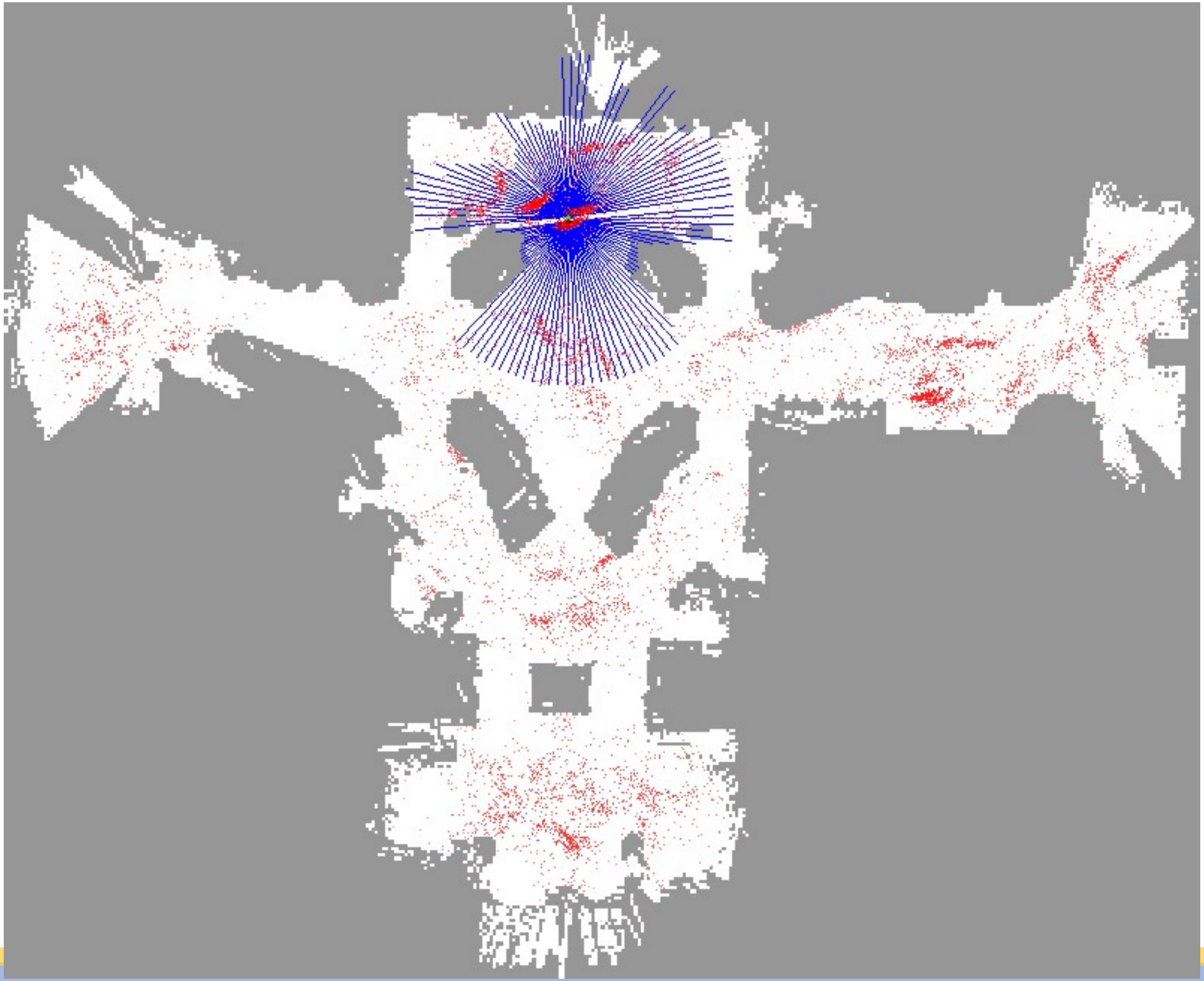


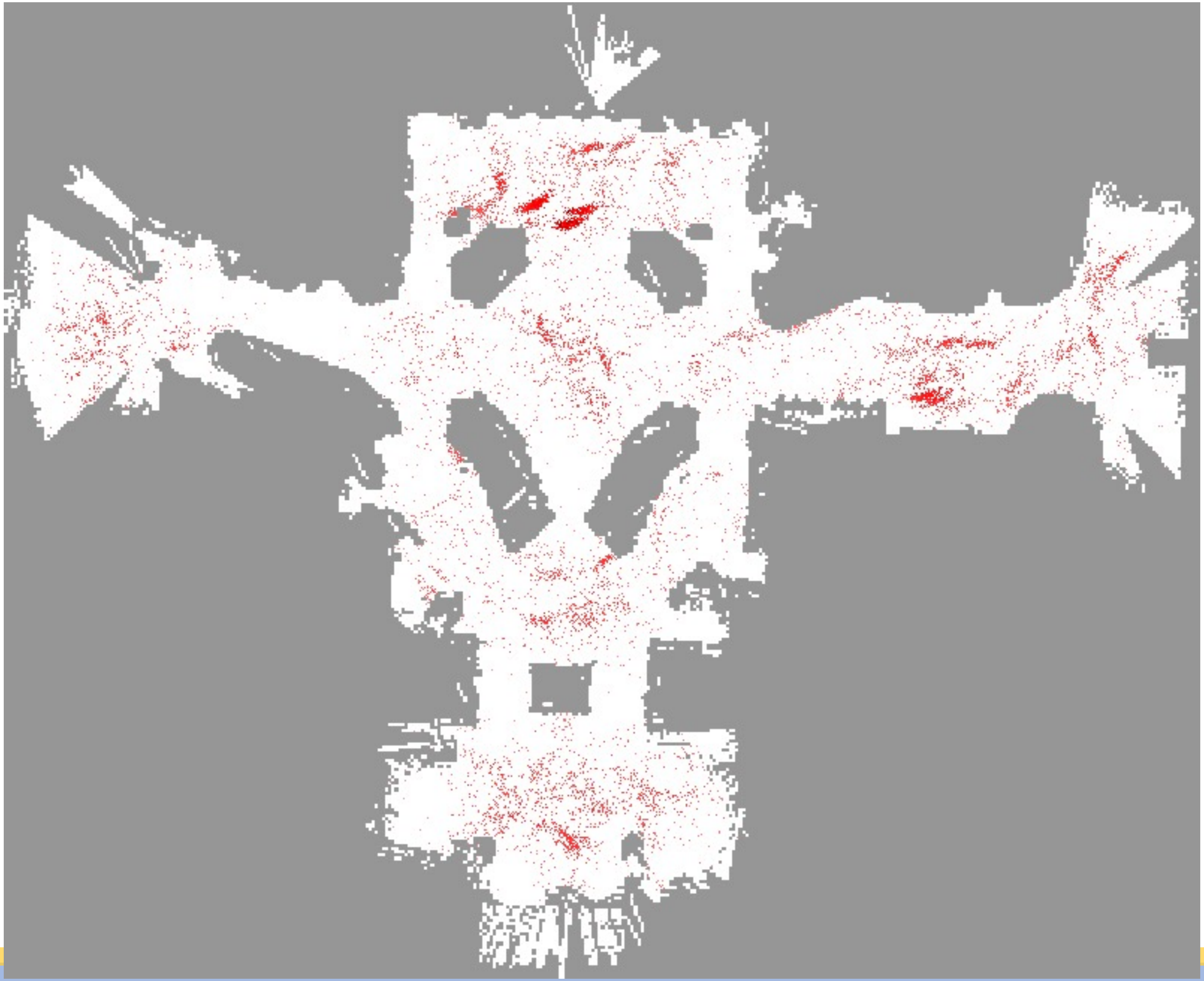


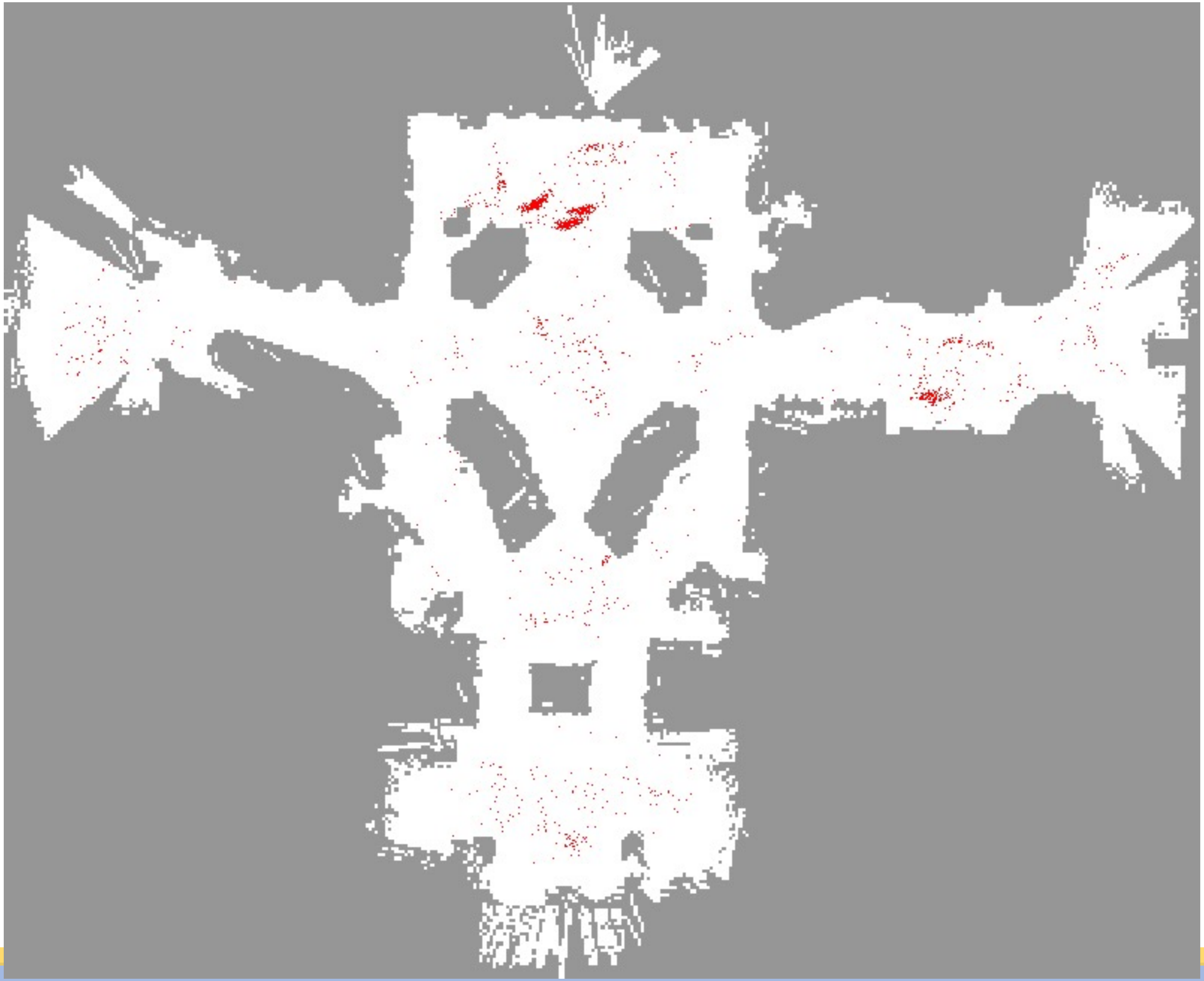


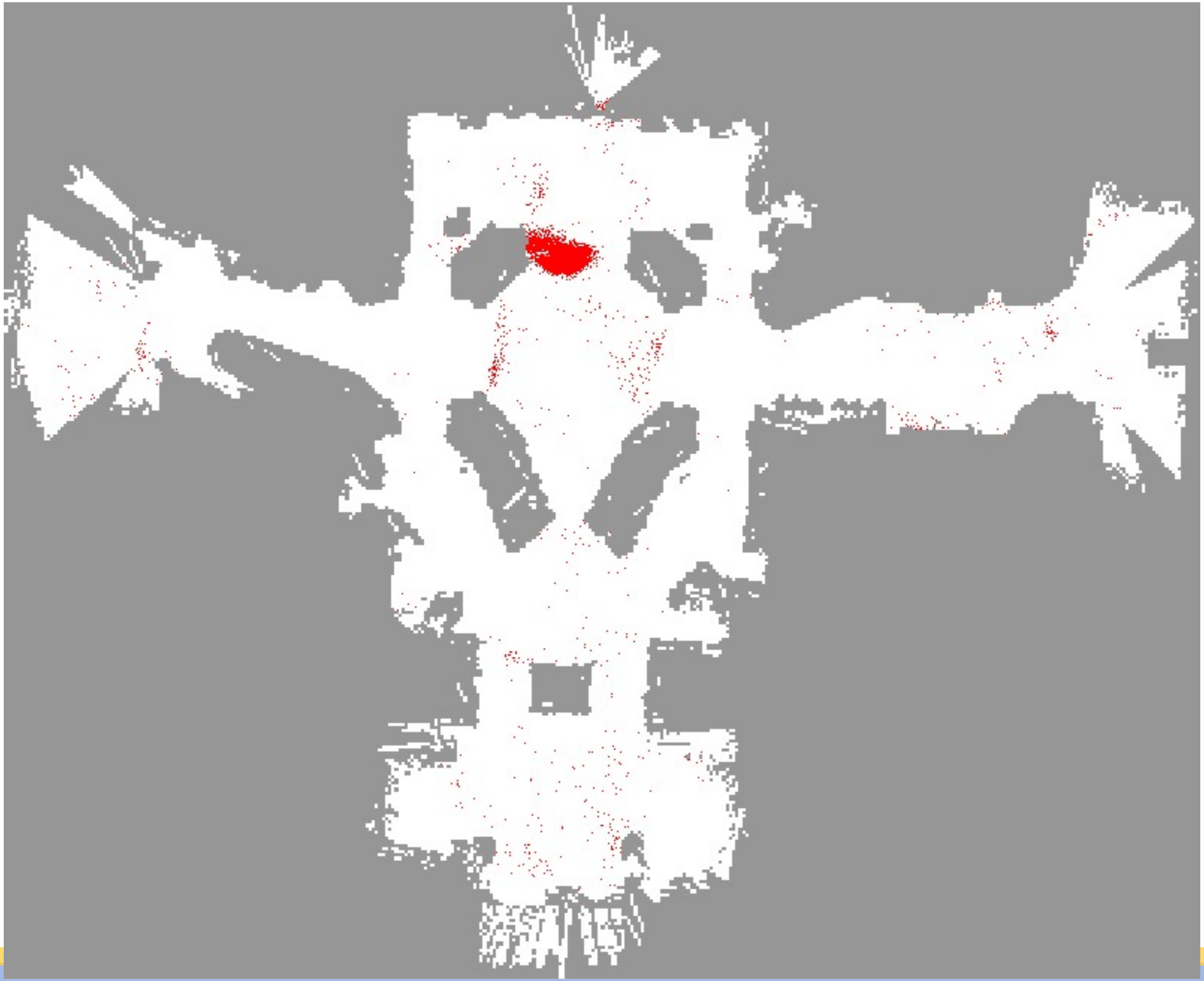


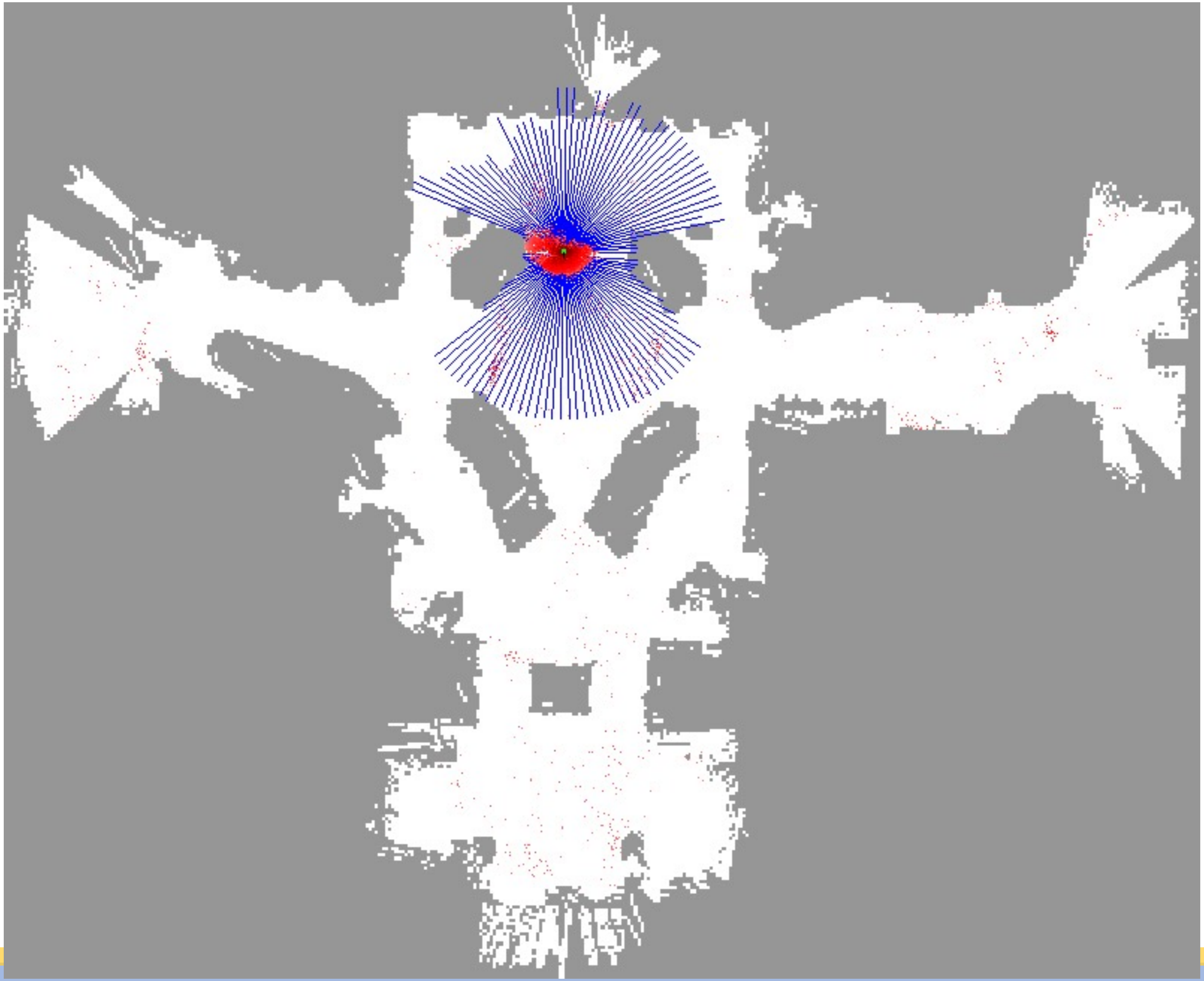


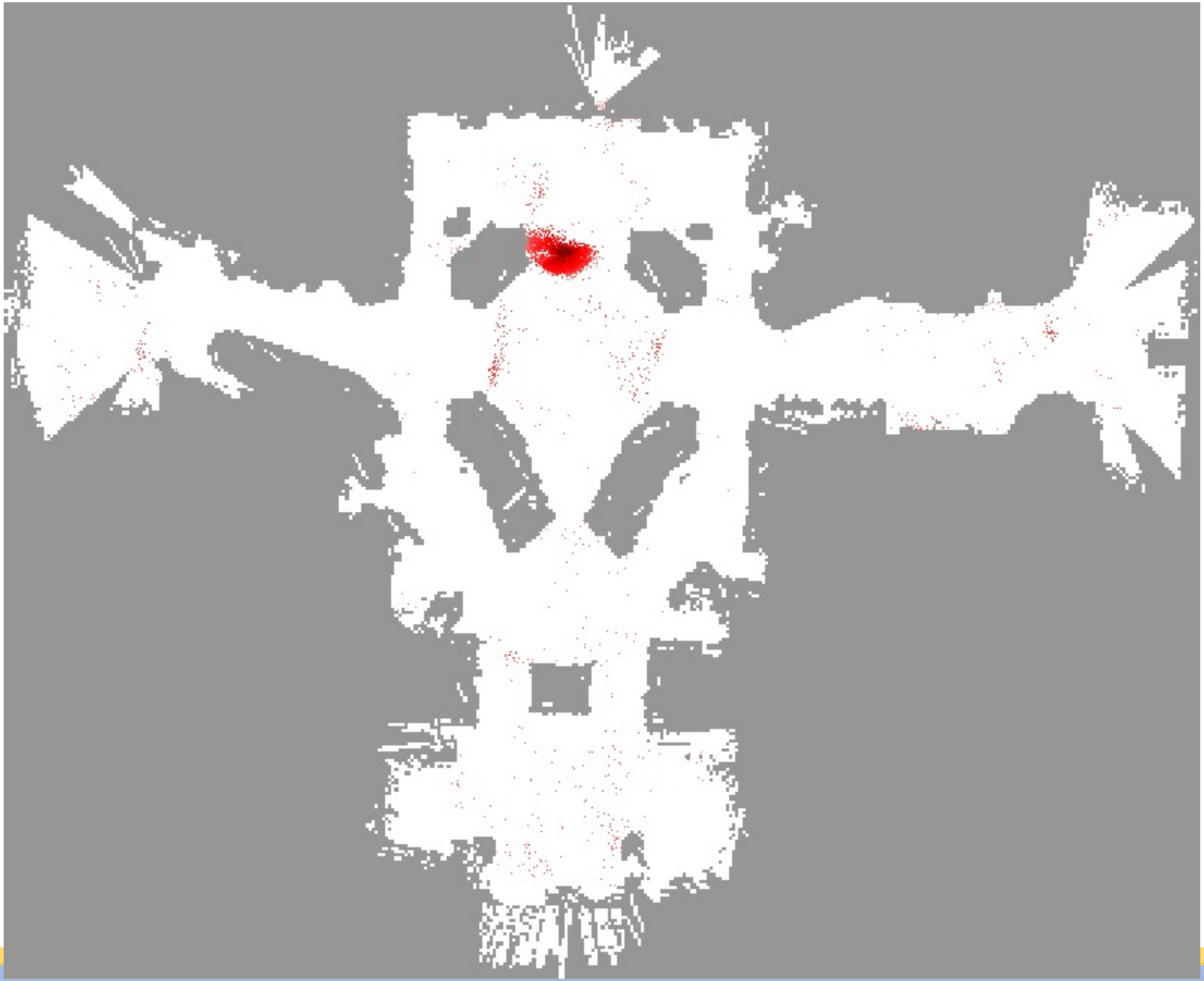


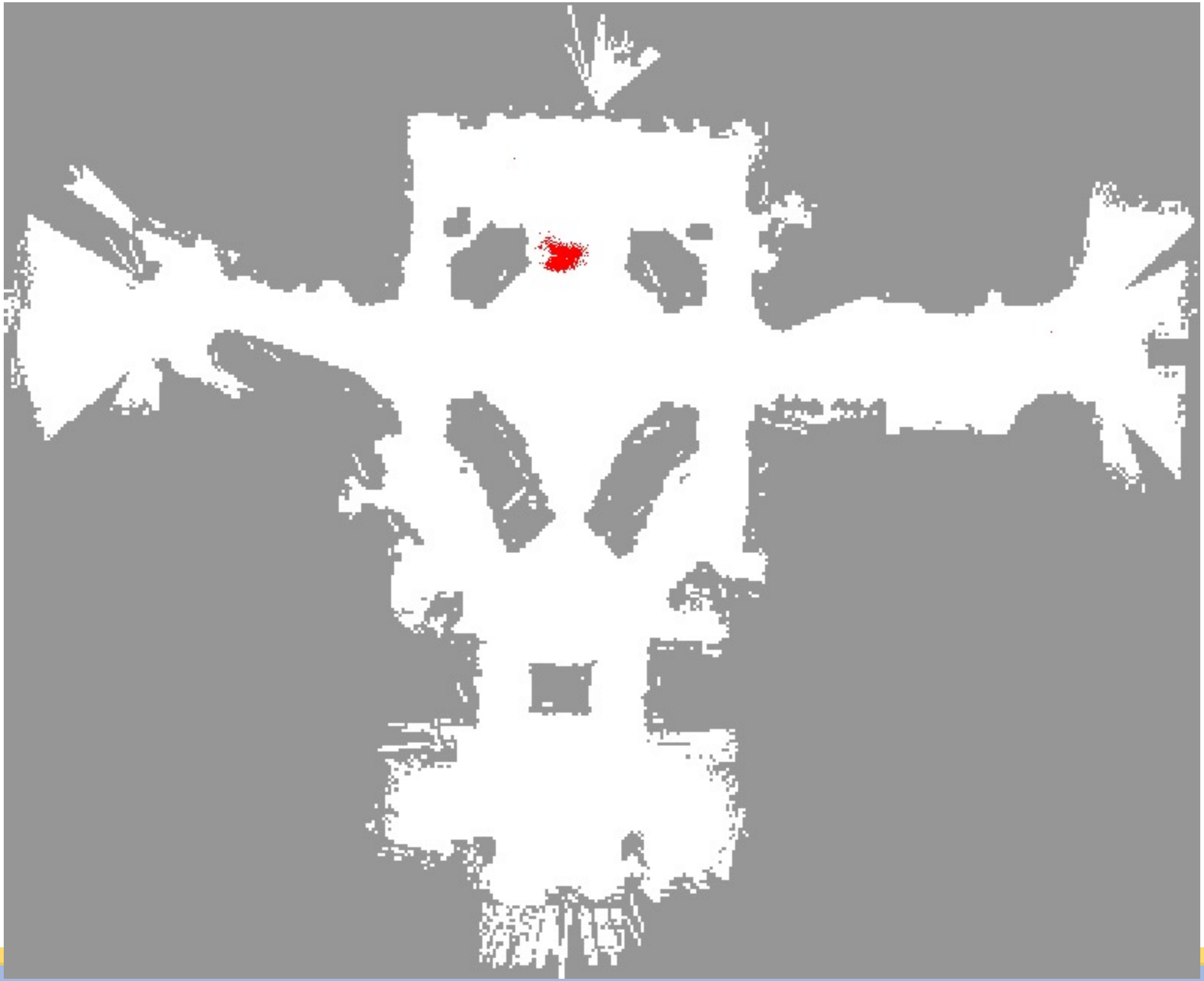


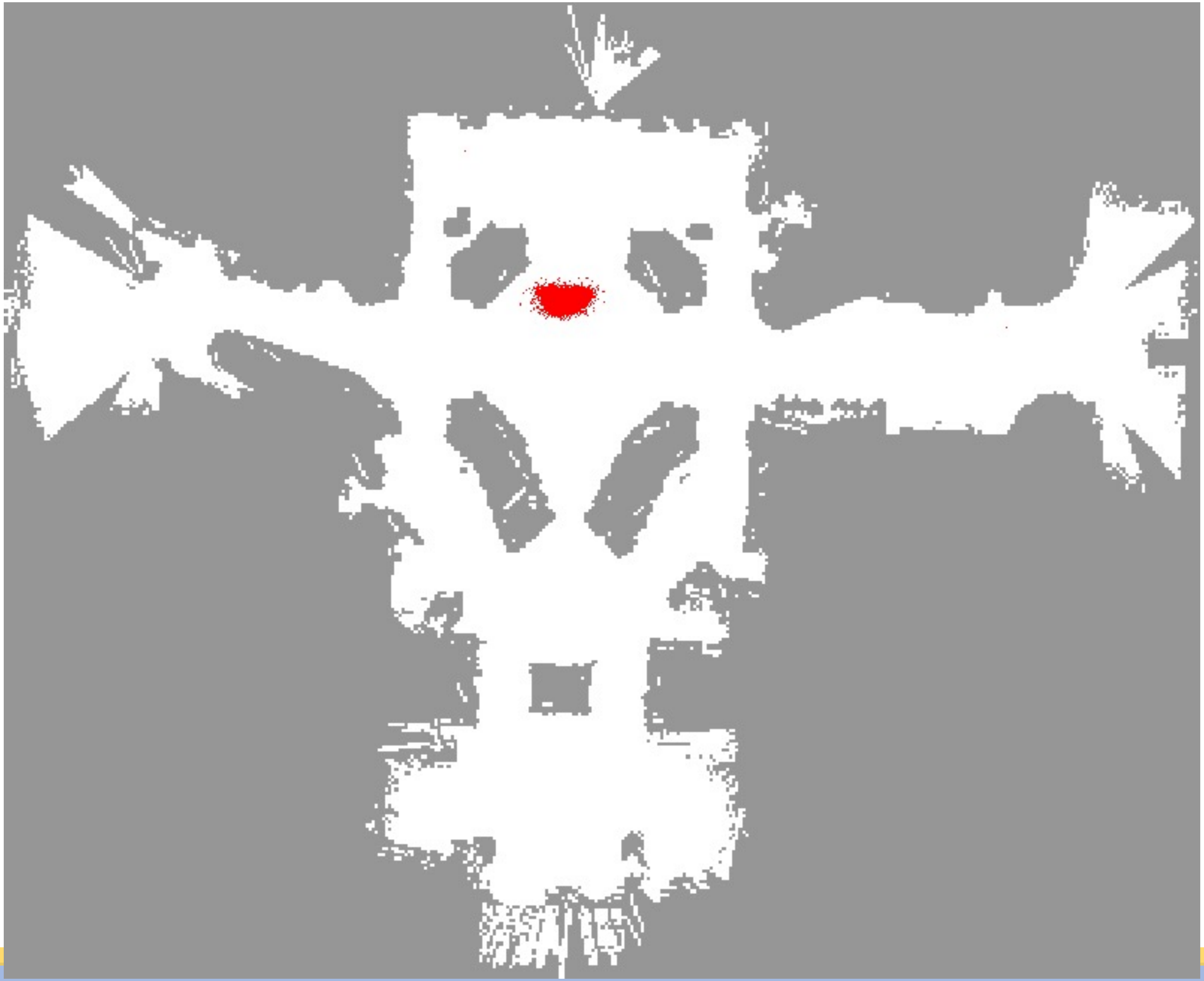


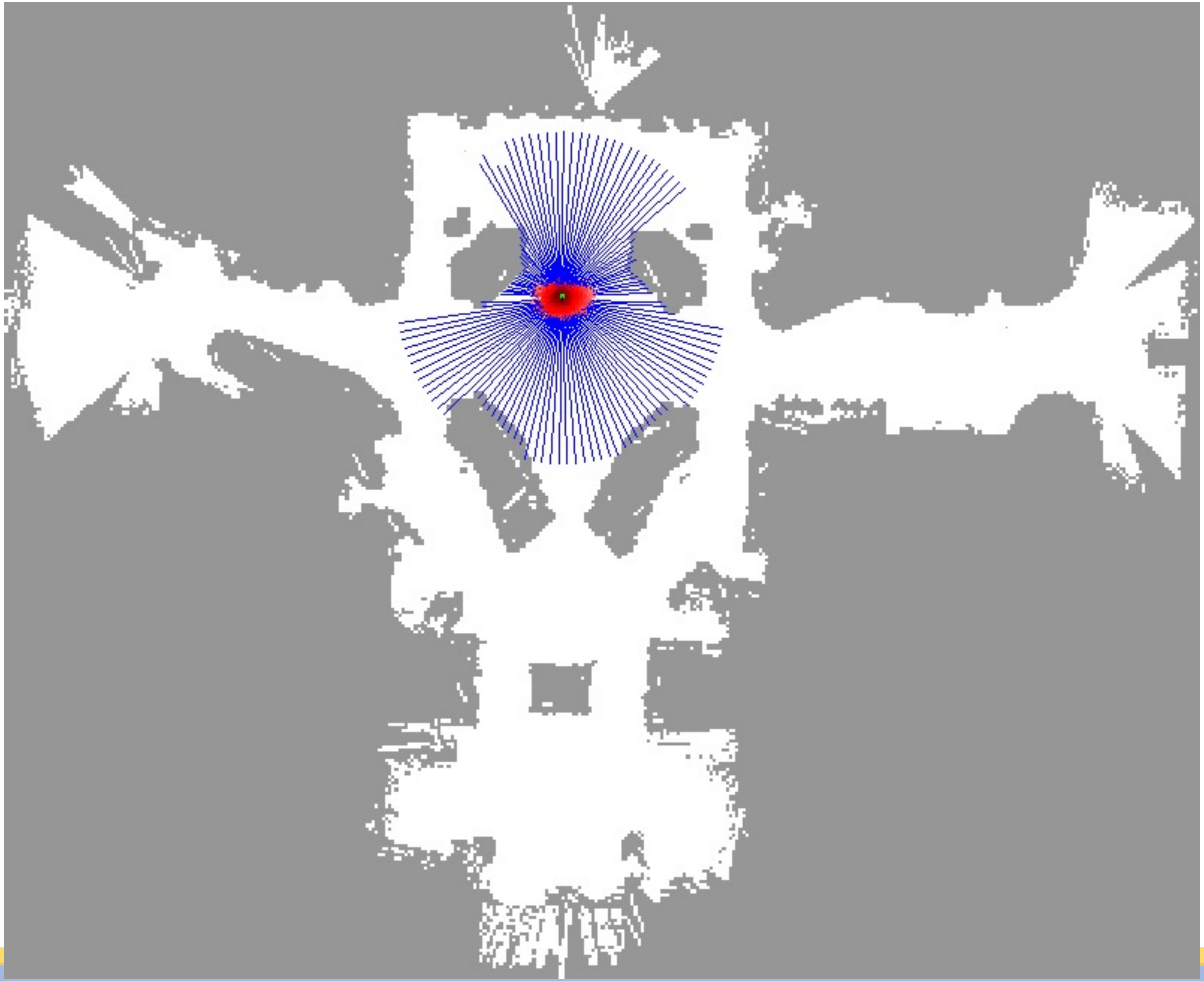


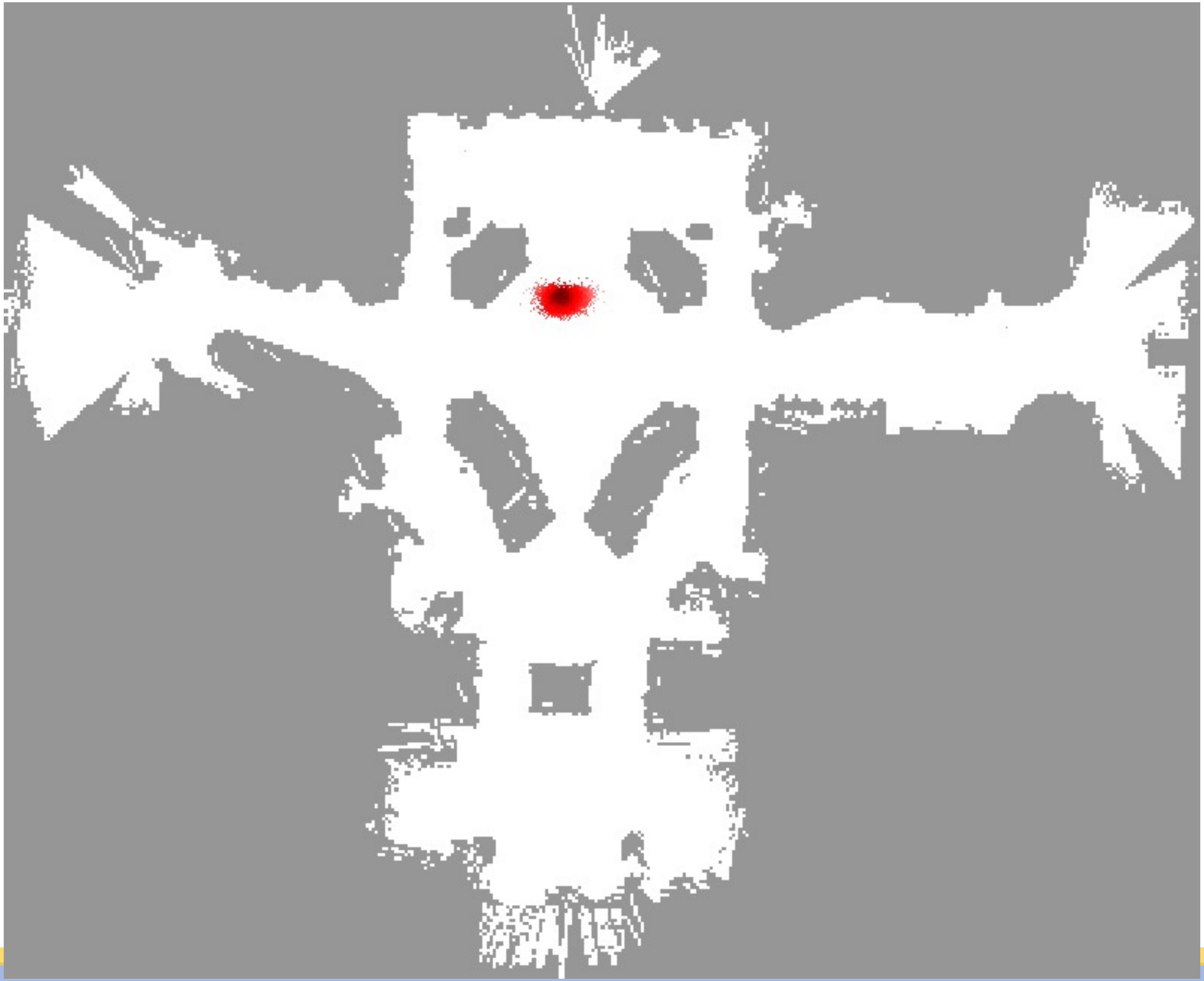


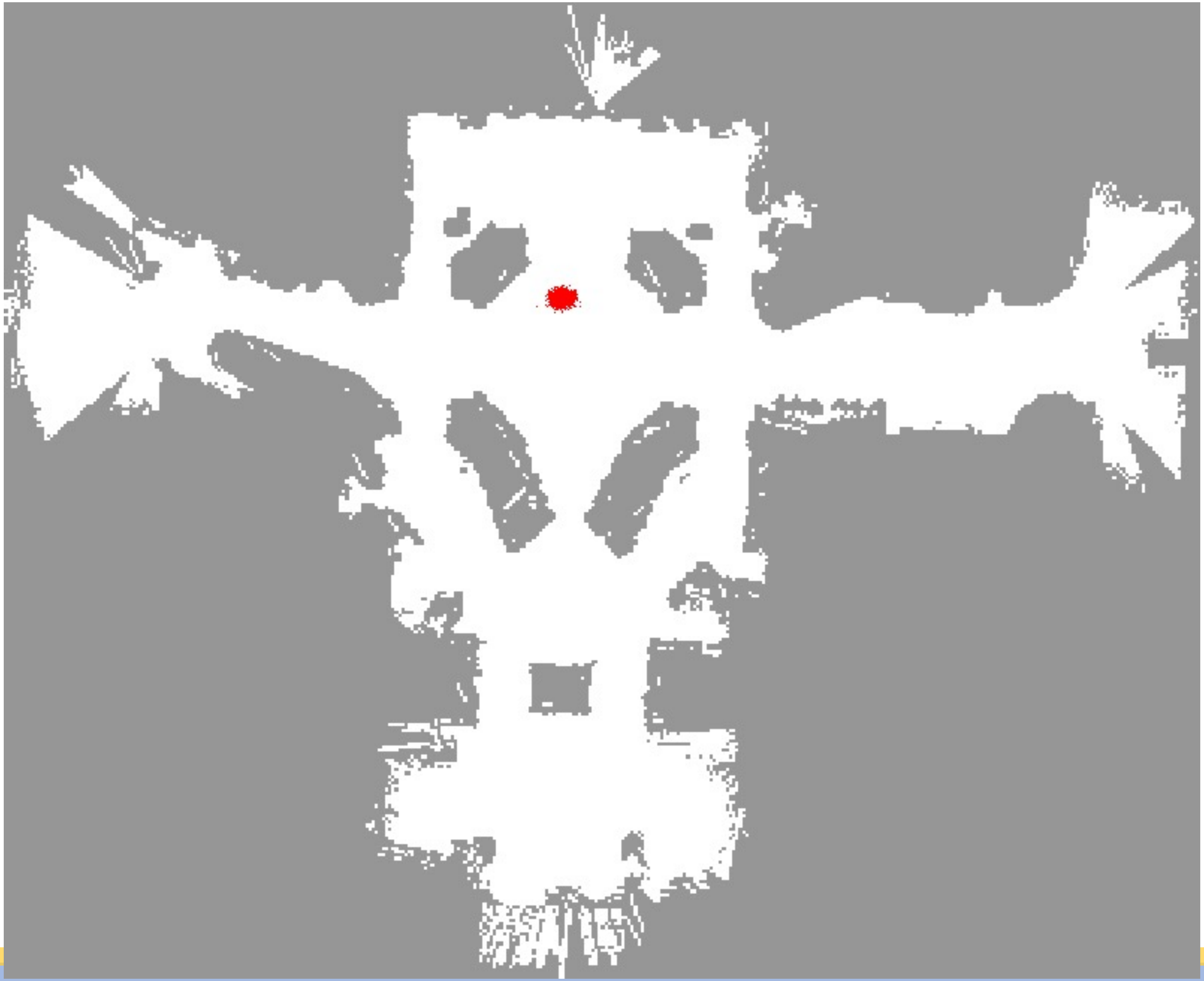


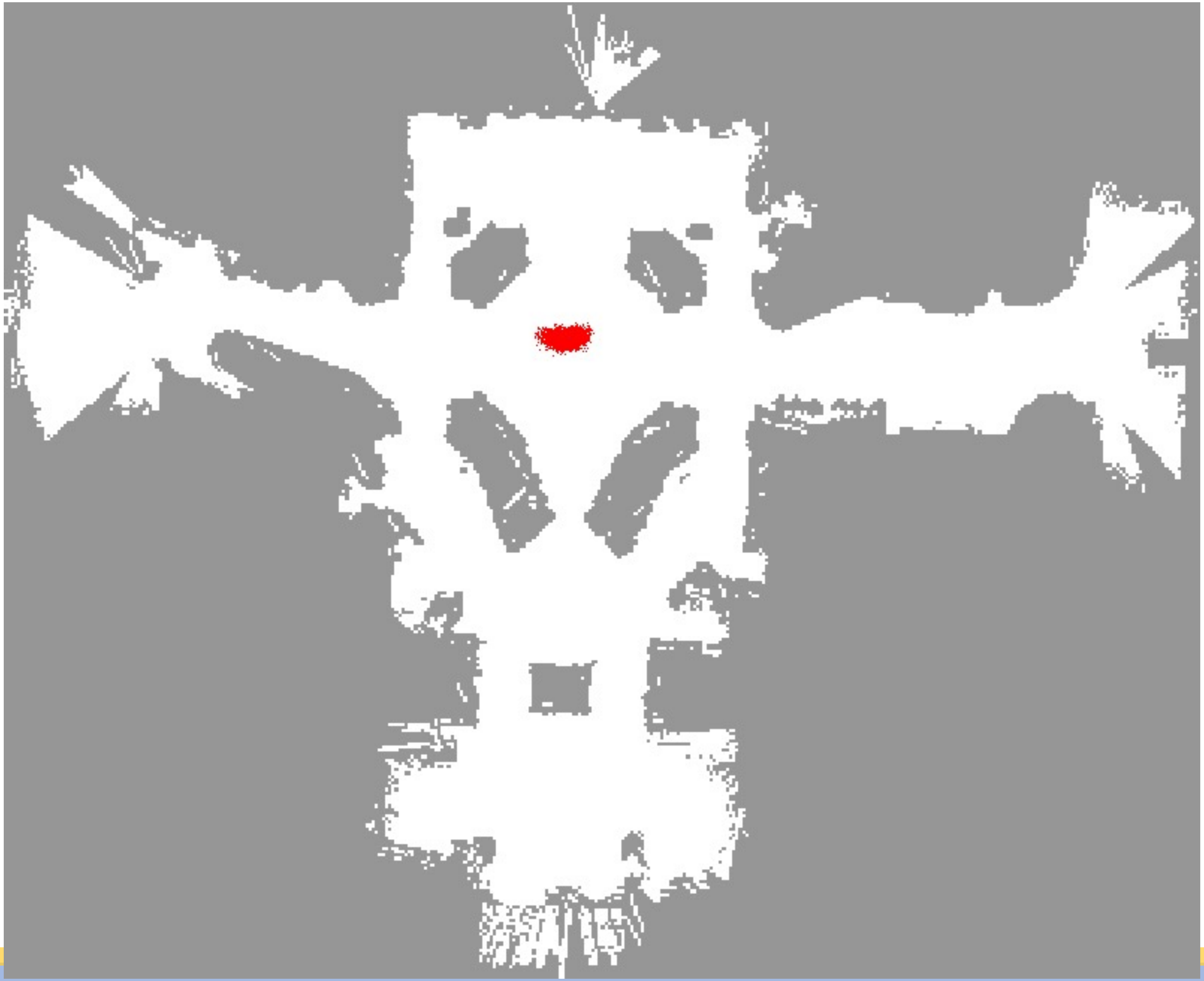


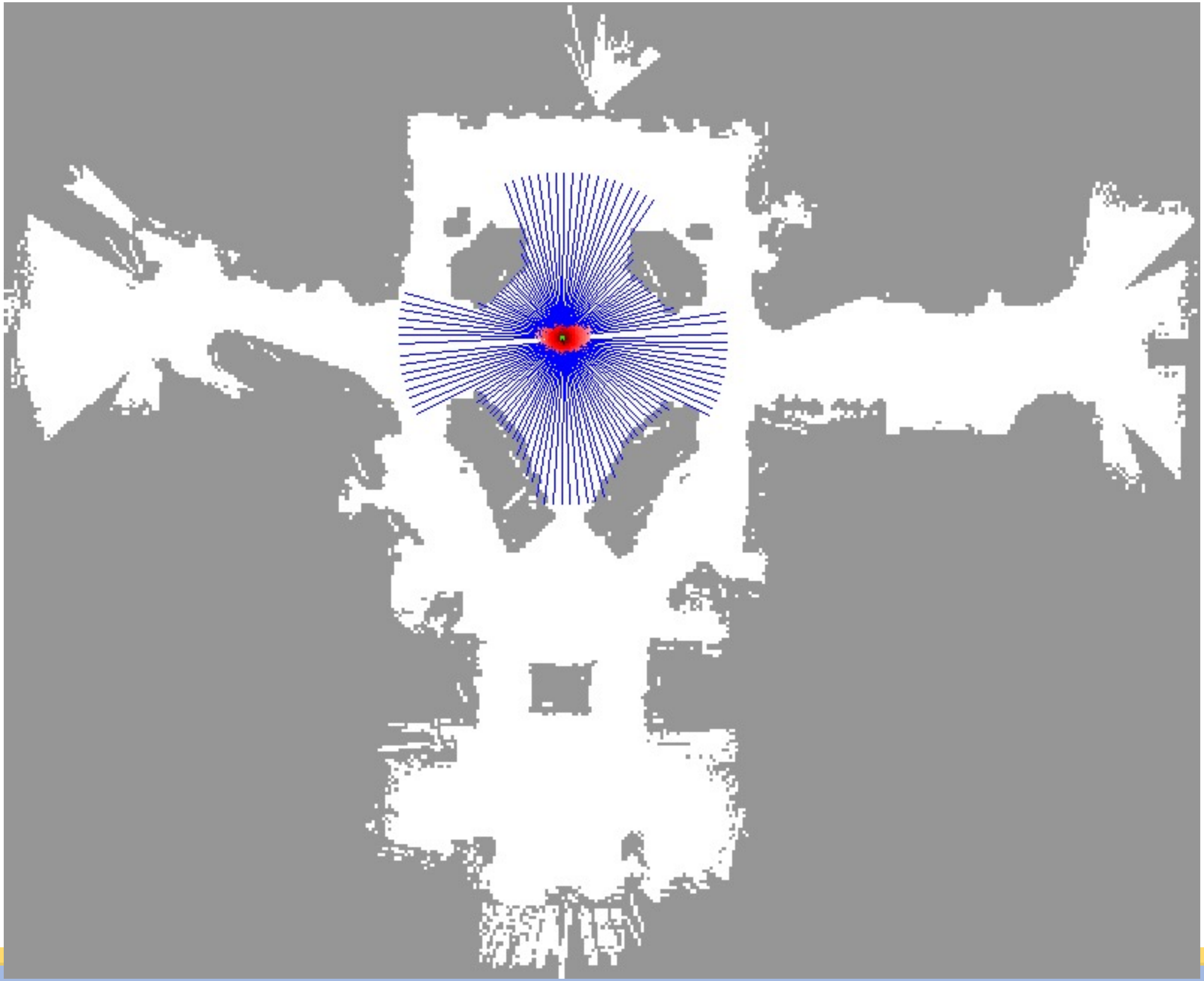


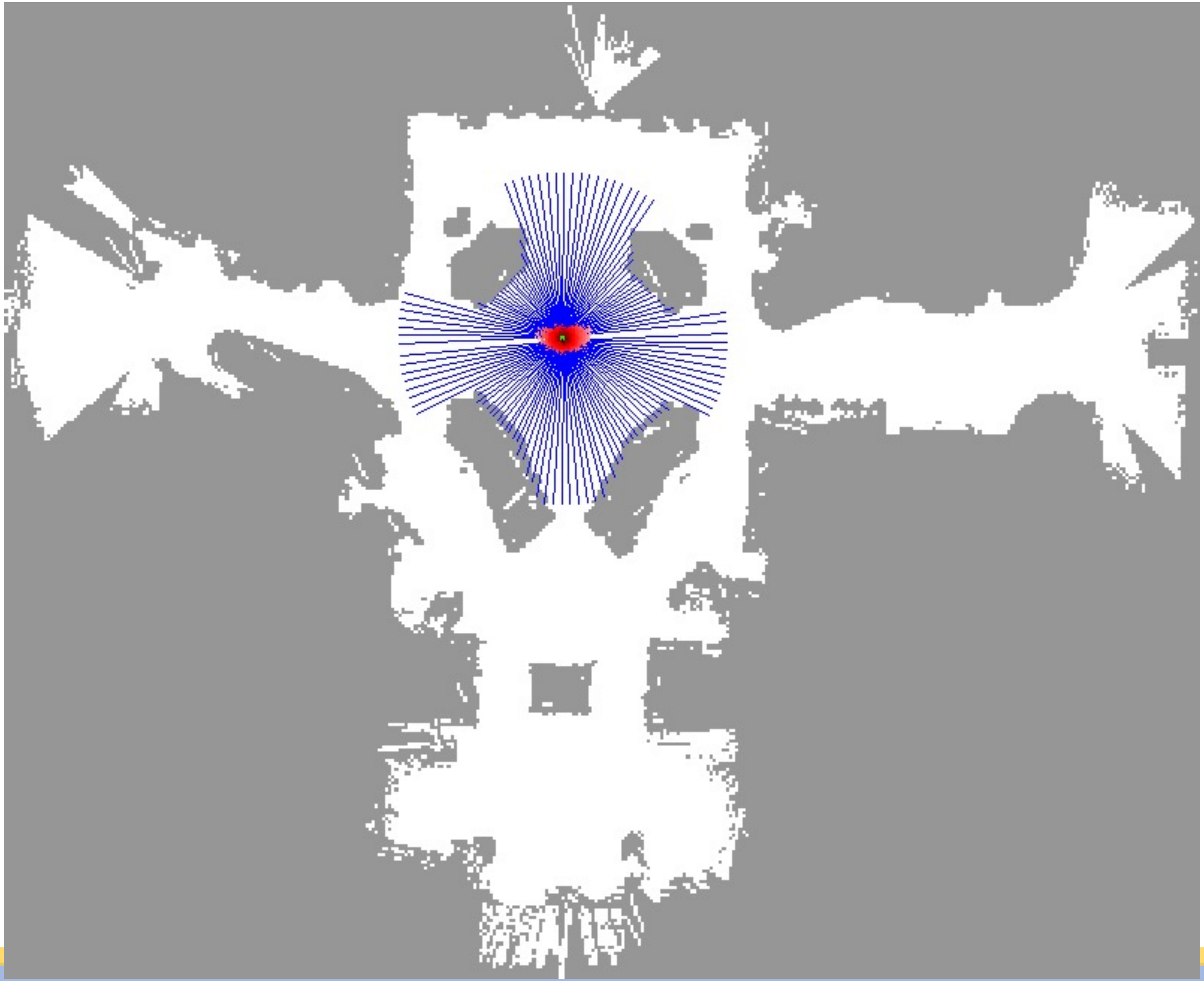




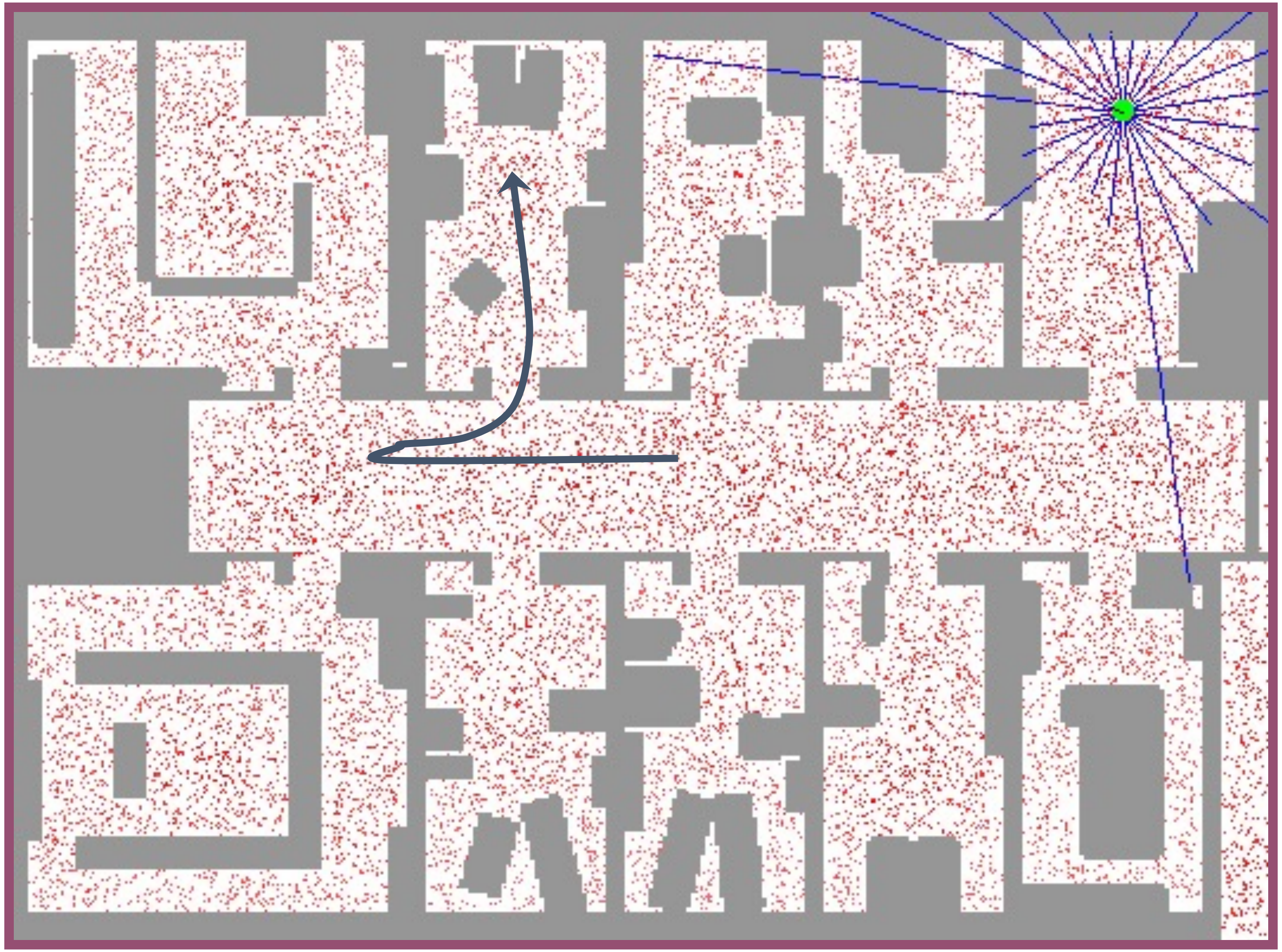




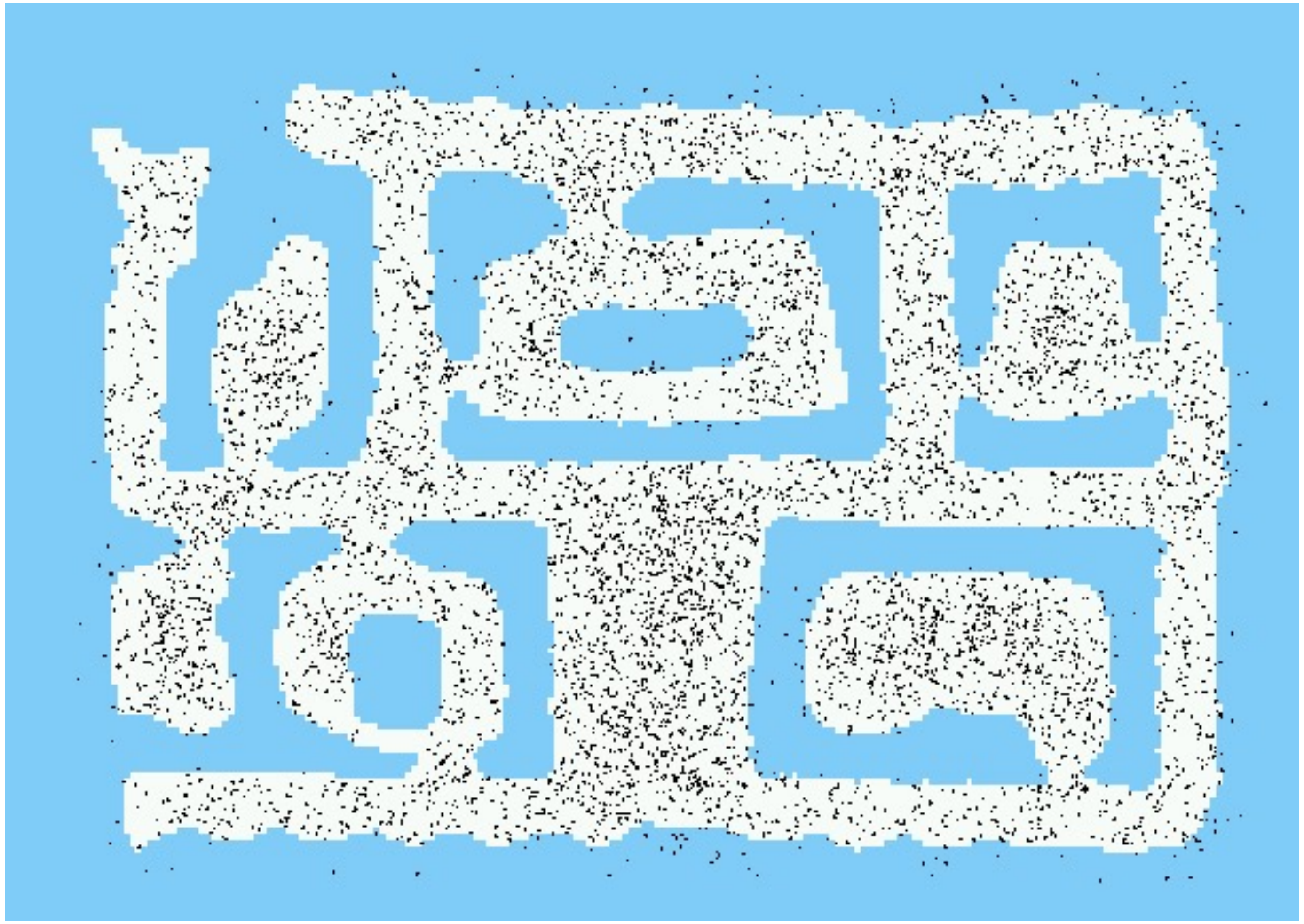




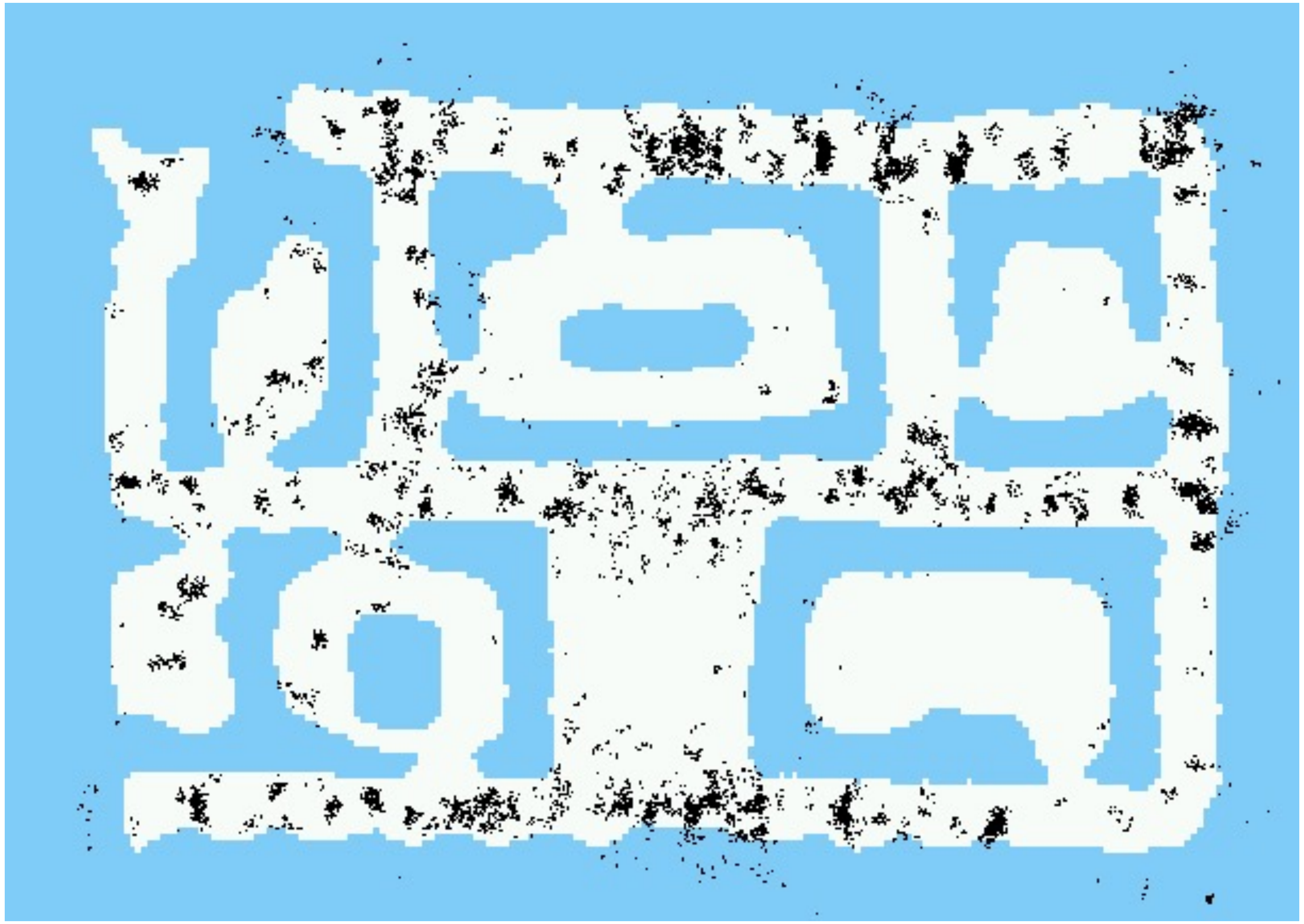
# Sample-based Localization (sonar)



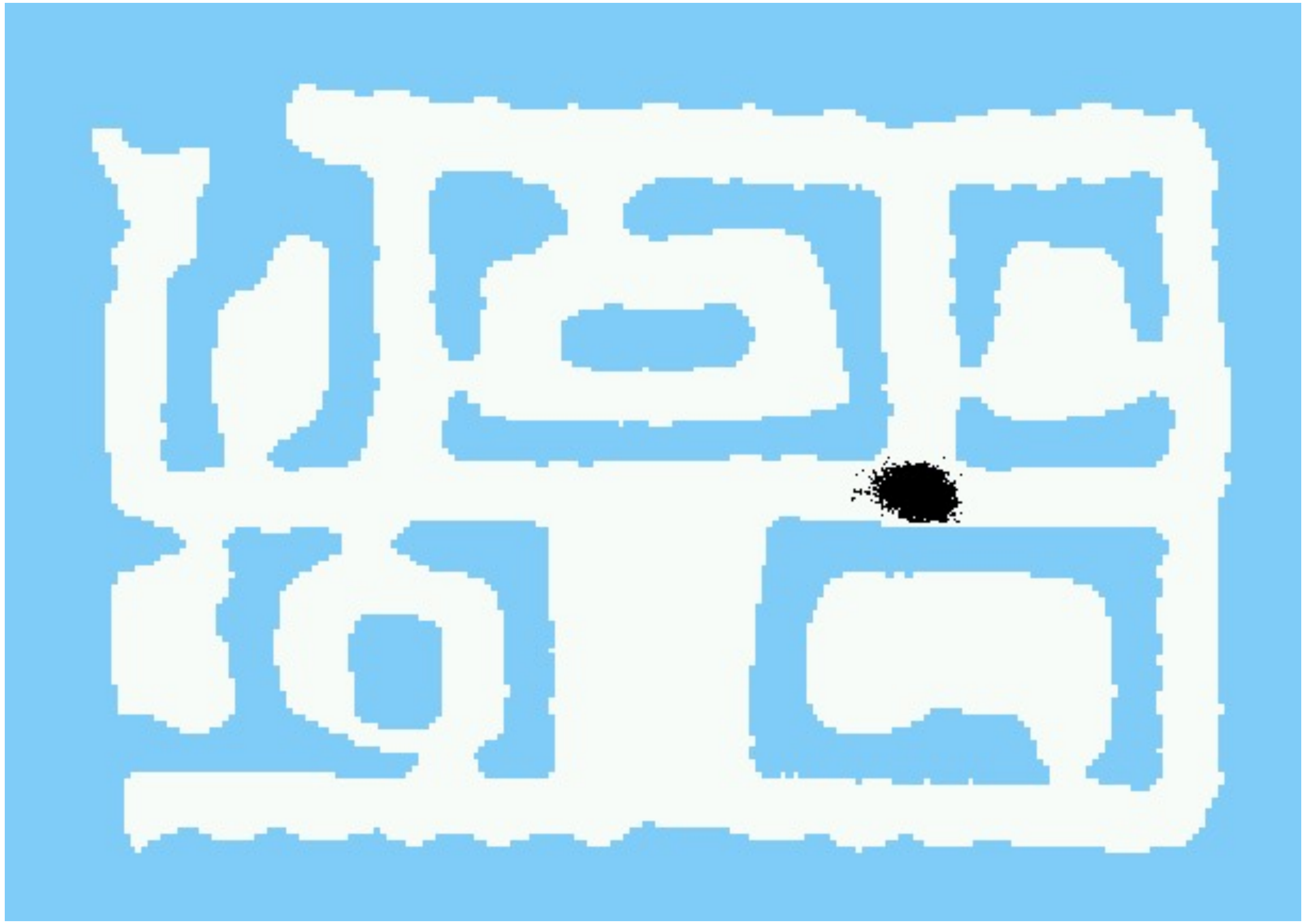
# Initial Distribution



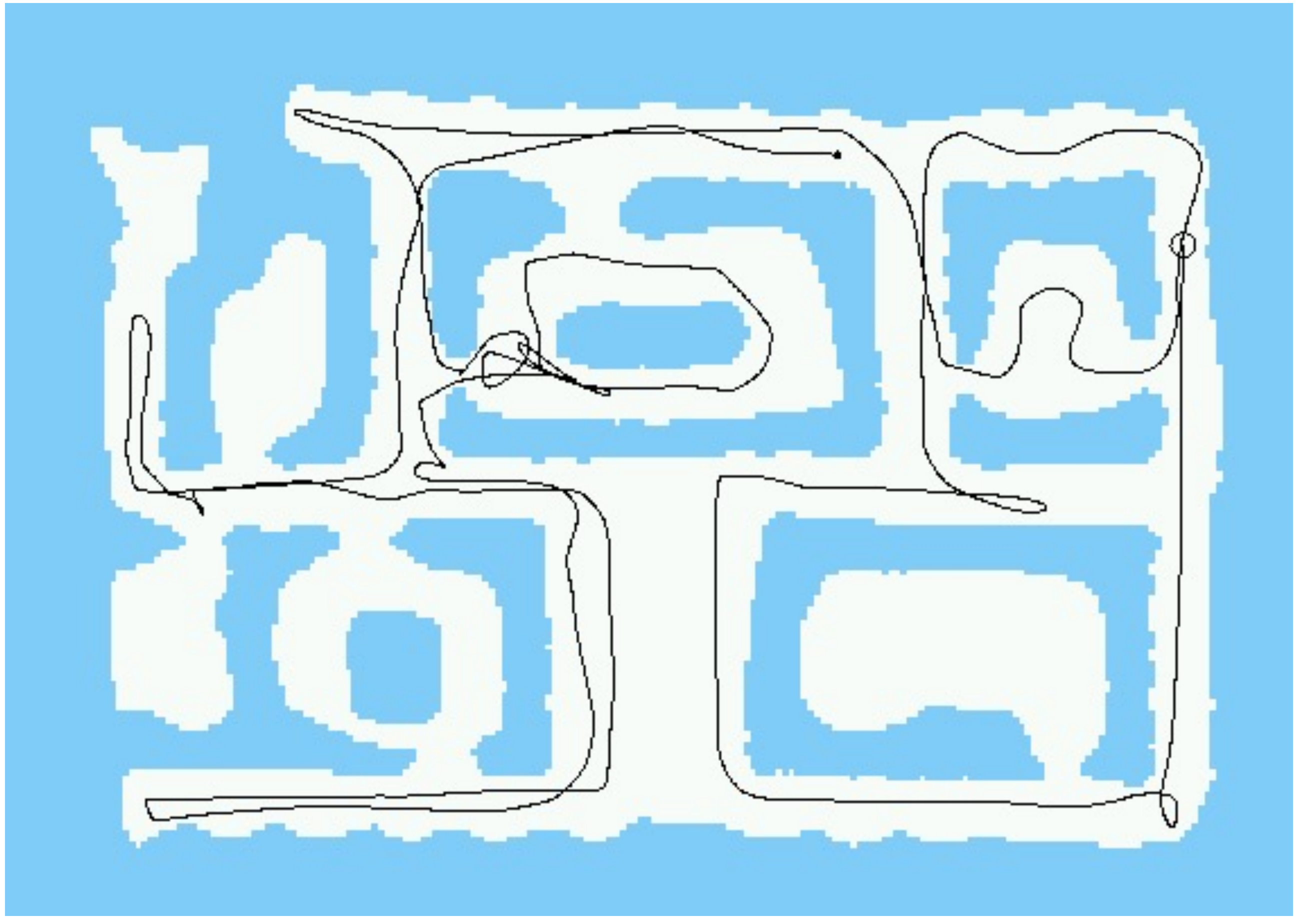
# After Incorporating Ten Ultrasound Scans



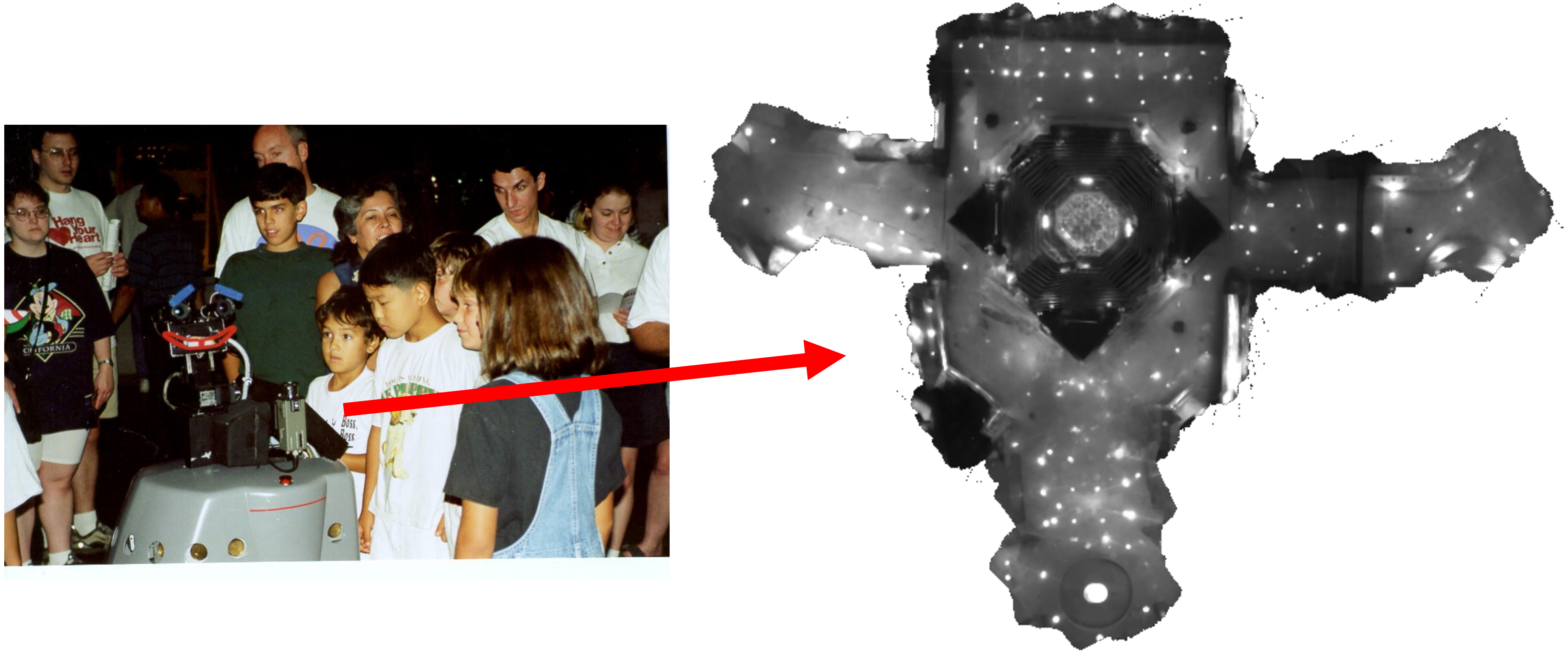
# After Incorporating 65 Ultrasound Scans



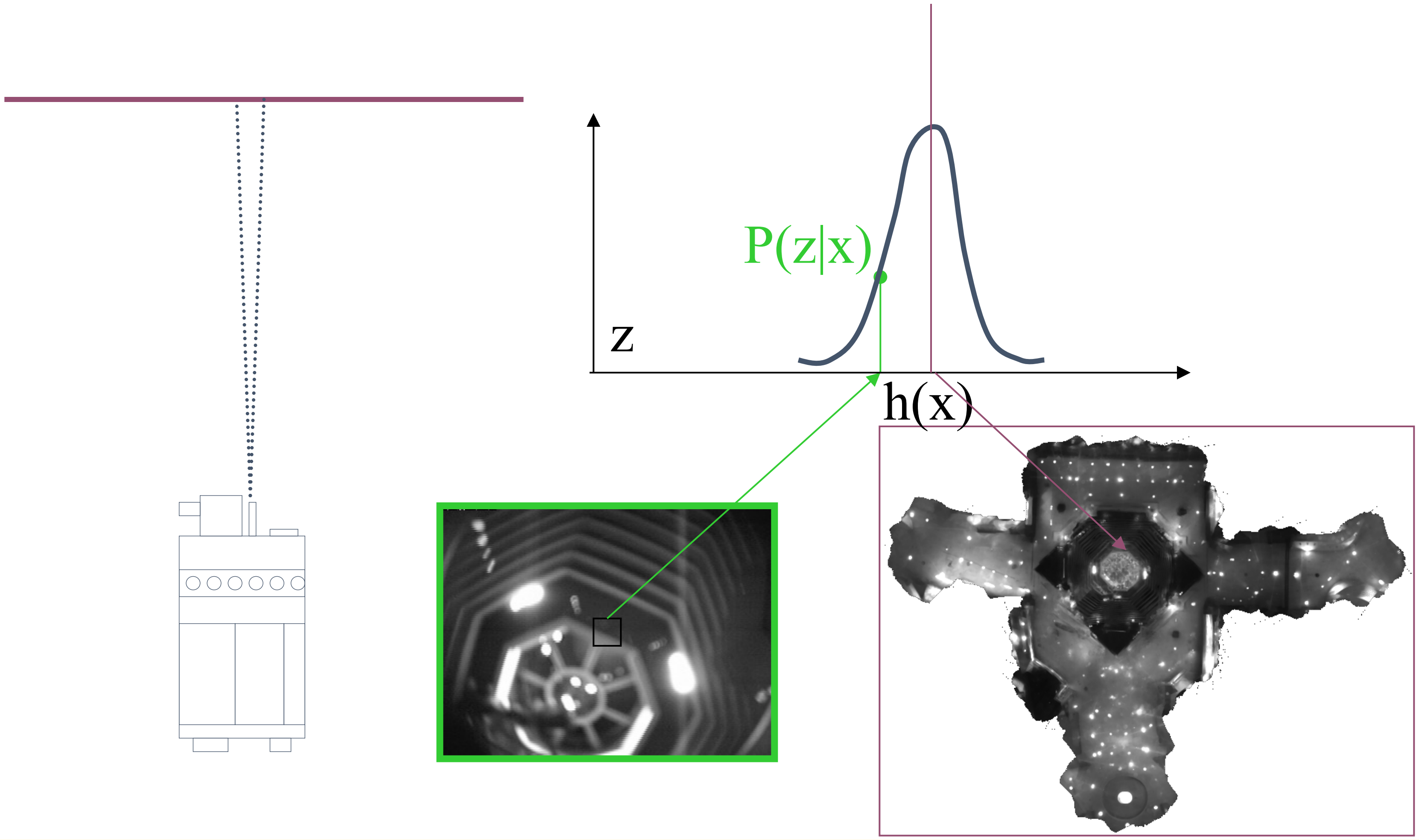
# Estimated Path



# Using Ceiling Maps for Localization

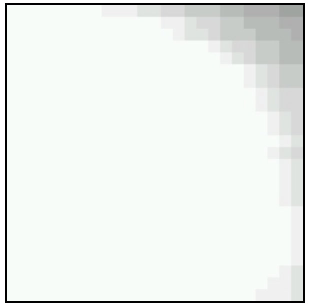


# Vision-based Localization

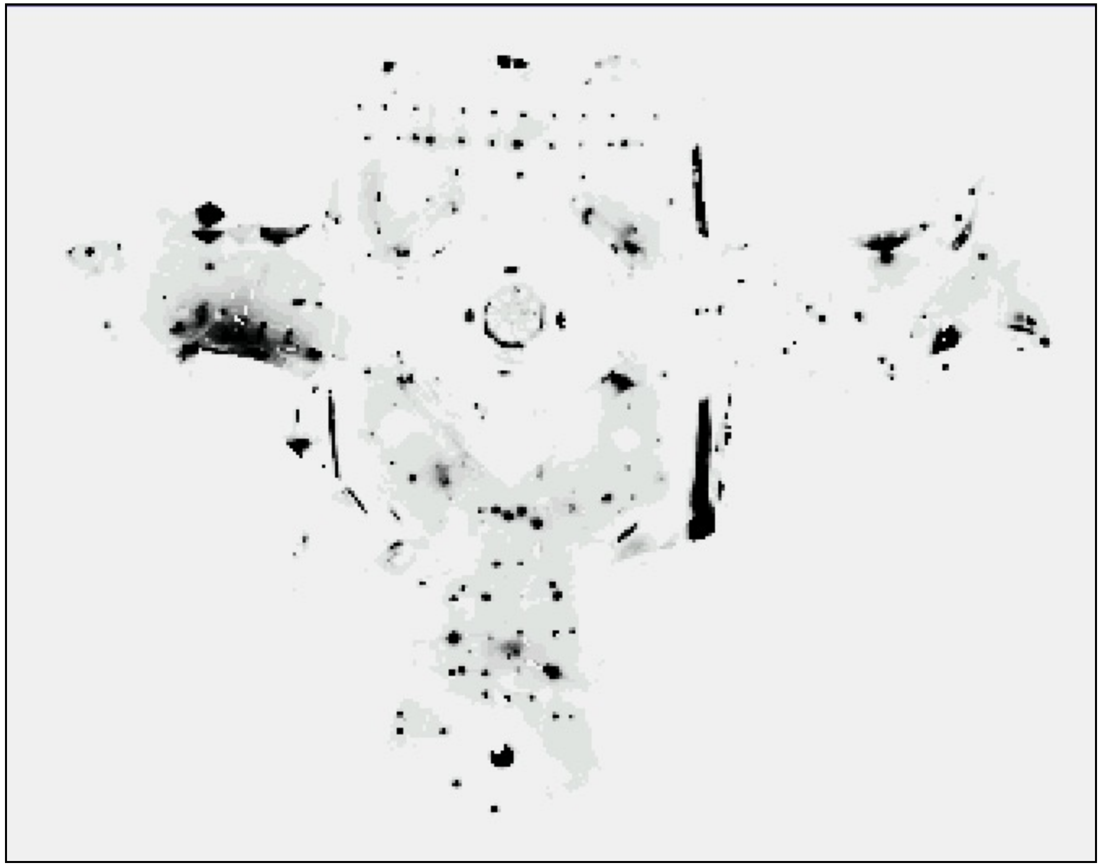


# Under a Light

**Measurement z:**

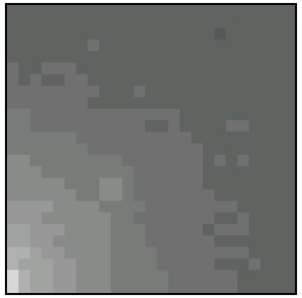


***P(z|x)*:**

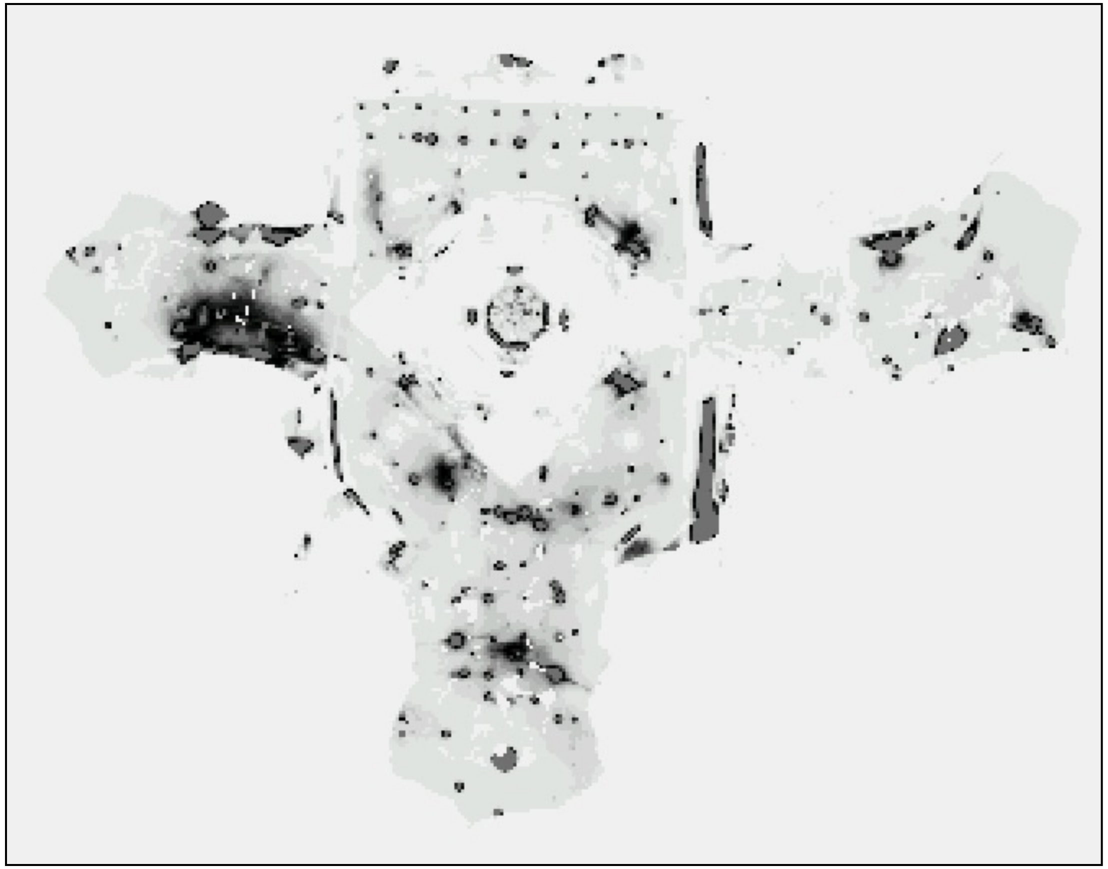


# Next to a Light

**Measurement z:**



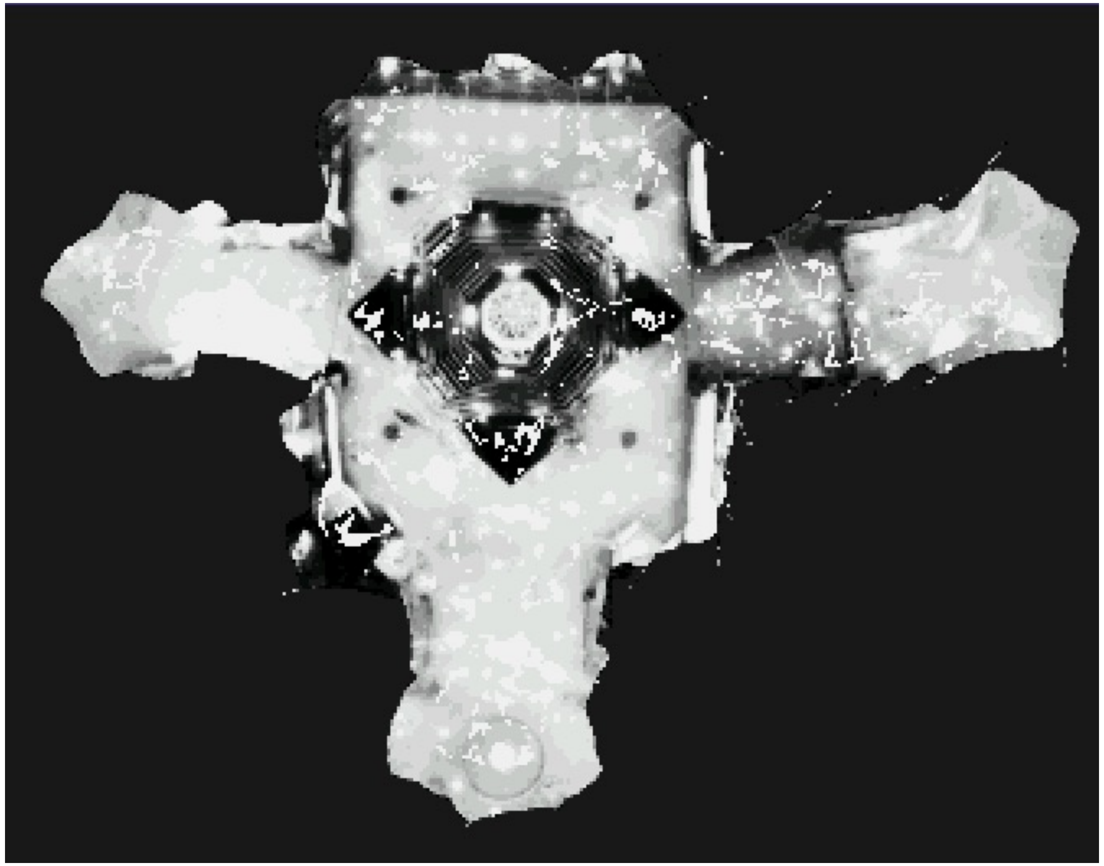
***P(z|x)*:**



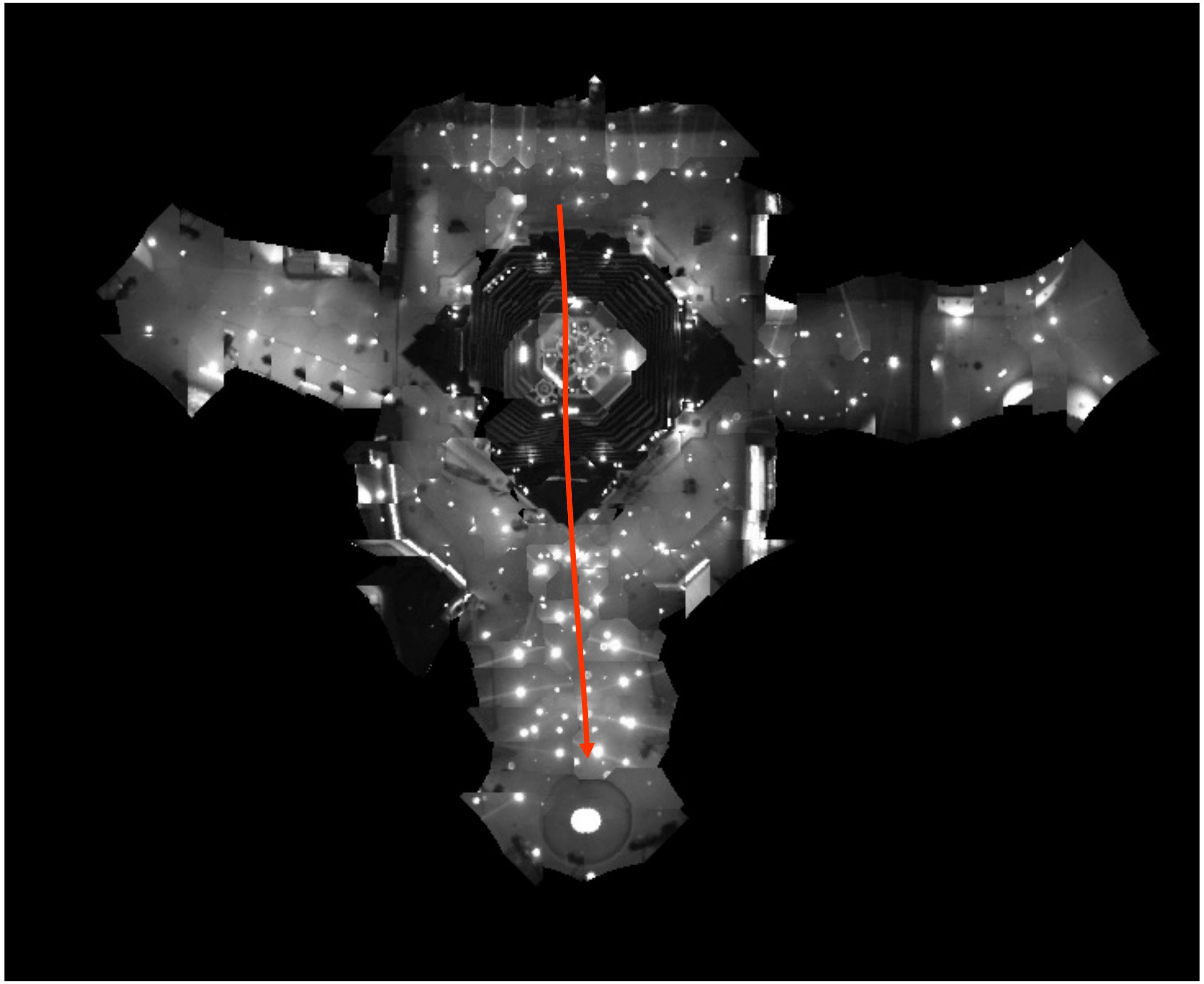
# Elsewhere

**Measurement z:**

***P(z|x)*:**



# Global Localization Using Vision



# Limitations

- The approach described so far is able to
  - track the pose of a mobile robot and to
  - globally localize the robot.

- Can we deal with localization errors (i.e., the kidnapped robot problem)?
- How to handle localization errors/failures?
  - Particularly serious when the number of particles is small

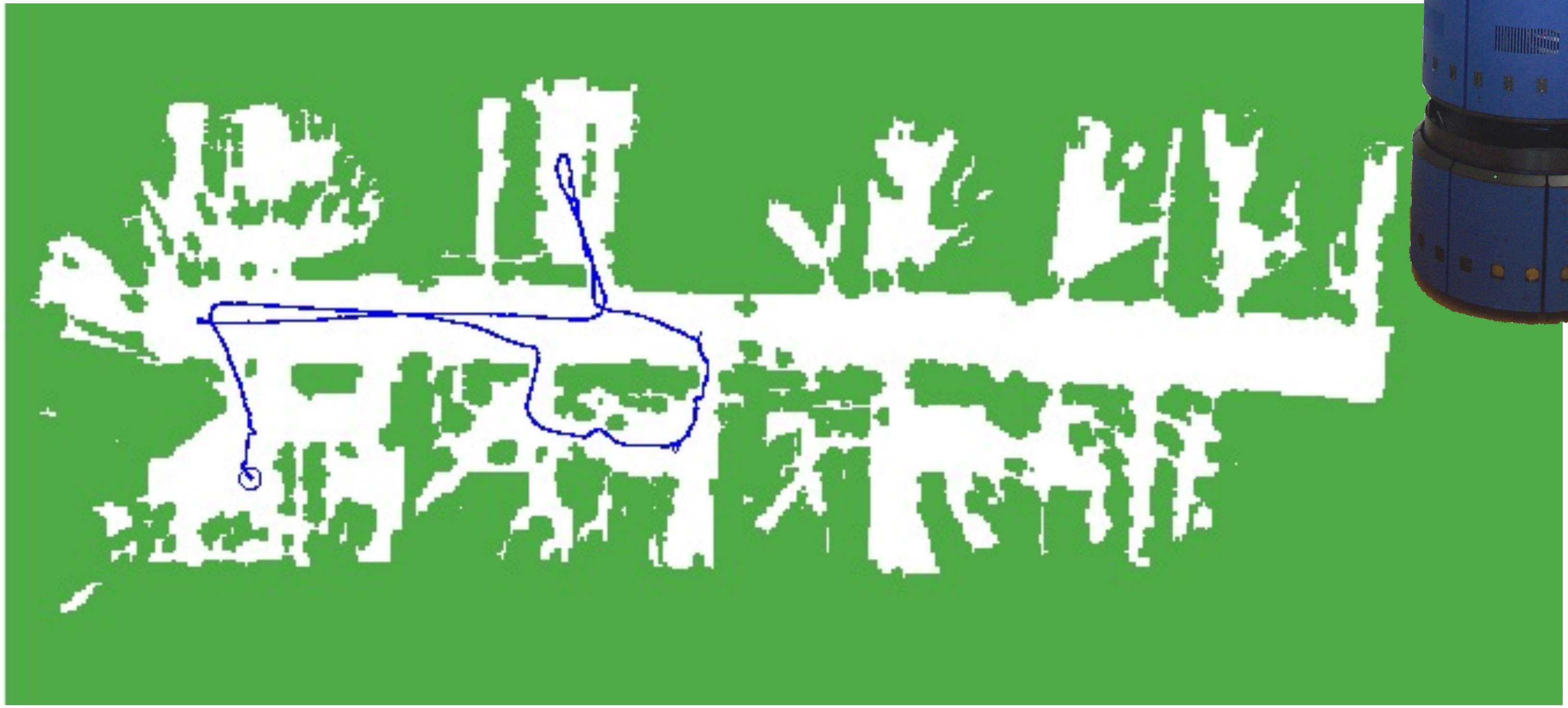
# Approaches

- Randomly insert samples
  - Why?
  - The robot can be teleported at any point in time
- How many particles to add? With what distribution?
  - Add particles according to localization performance
  - Monitor the probability of sensor measurements $p(z_t | z_{1:t-1}, u_{1:t}, m)$
  - For particle filters: $p(z_t | z_{1:t-1}, u_{1:t}, m) \approx \frac{1}{M} \sum w_t^{[m]}$
- Insert random samples proportional to the average likelihood of the particles (the robot has been teleported with higher probability when the likelihood of its observations drops).

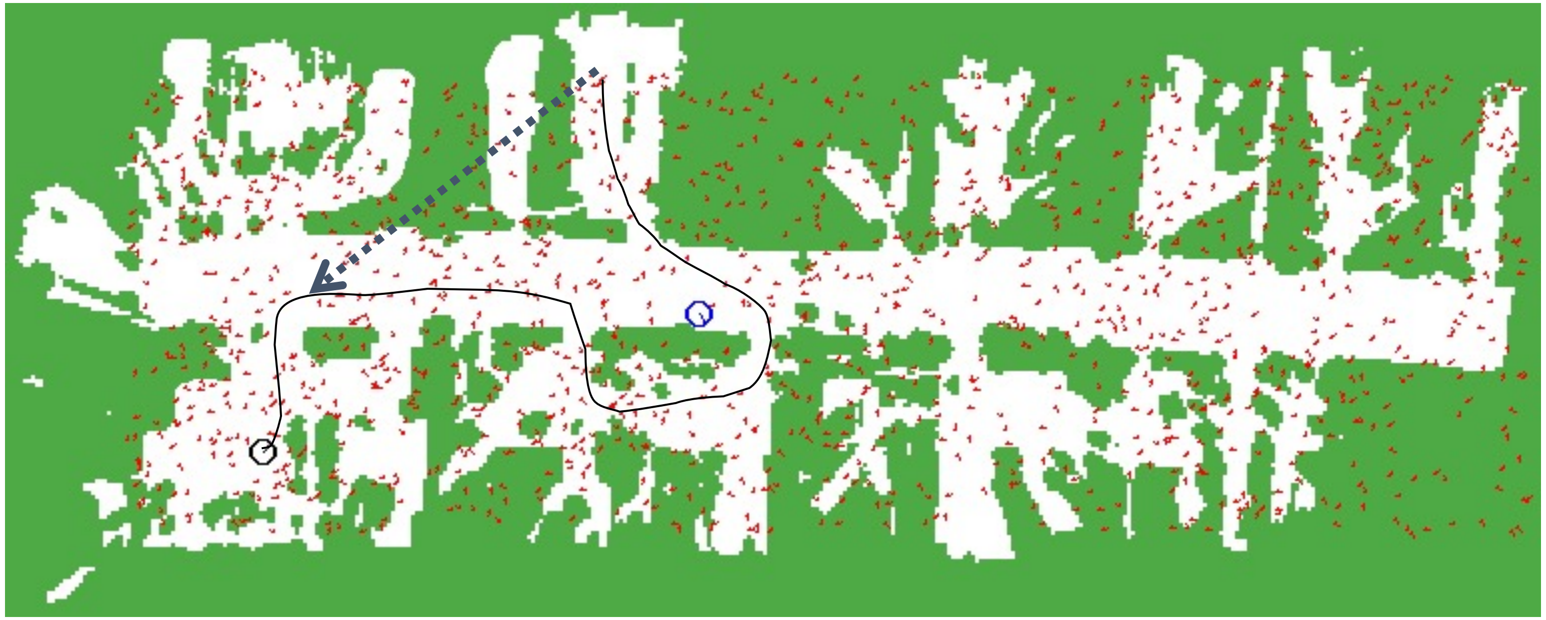
# Random Samples
# Vision-Based Localization

936 Images, 4MB, .6secs/image

Trajectory of the robot:



# Kidnapping the Robot



# Summary

- Particle filters are an implementation of recursive Bayesian filtering
- They represent the posterior by a set of weighted samples.
- In the context of localization, the particles are propagated according to the motion model.
- They are then weighted according to the likelihood of the observations.
- In a re-sampling step, new particles are drawn with a probability proportional to the likelihood of the observation.