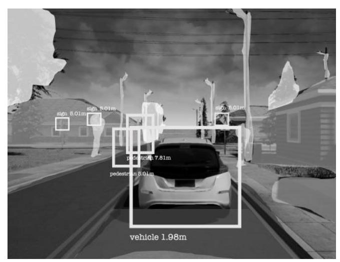
Spring 2022 Principles of Safe Autonomy
Lecture 4:
Basic Perception -> Edge Detection

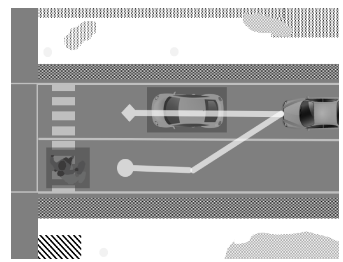
Sayan Mitra

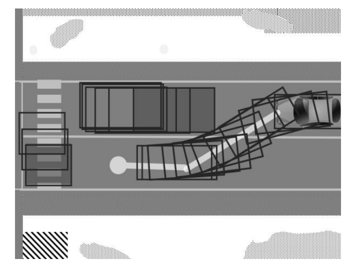
slides adapted from Svetlana Lazebnik

GEM platform

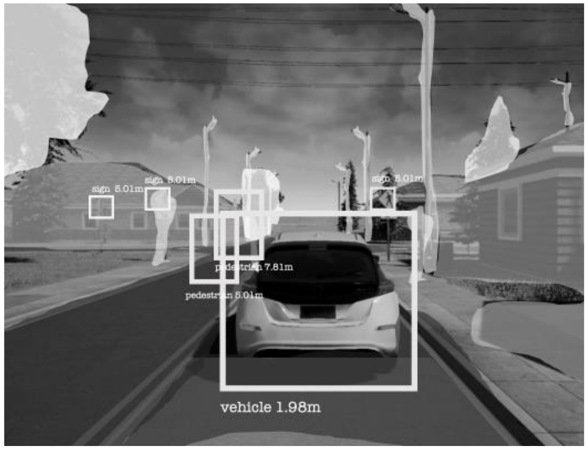
Autonomy pipeline





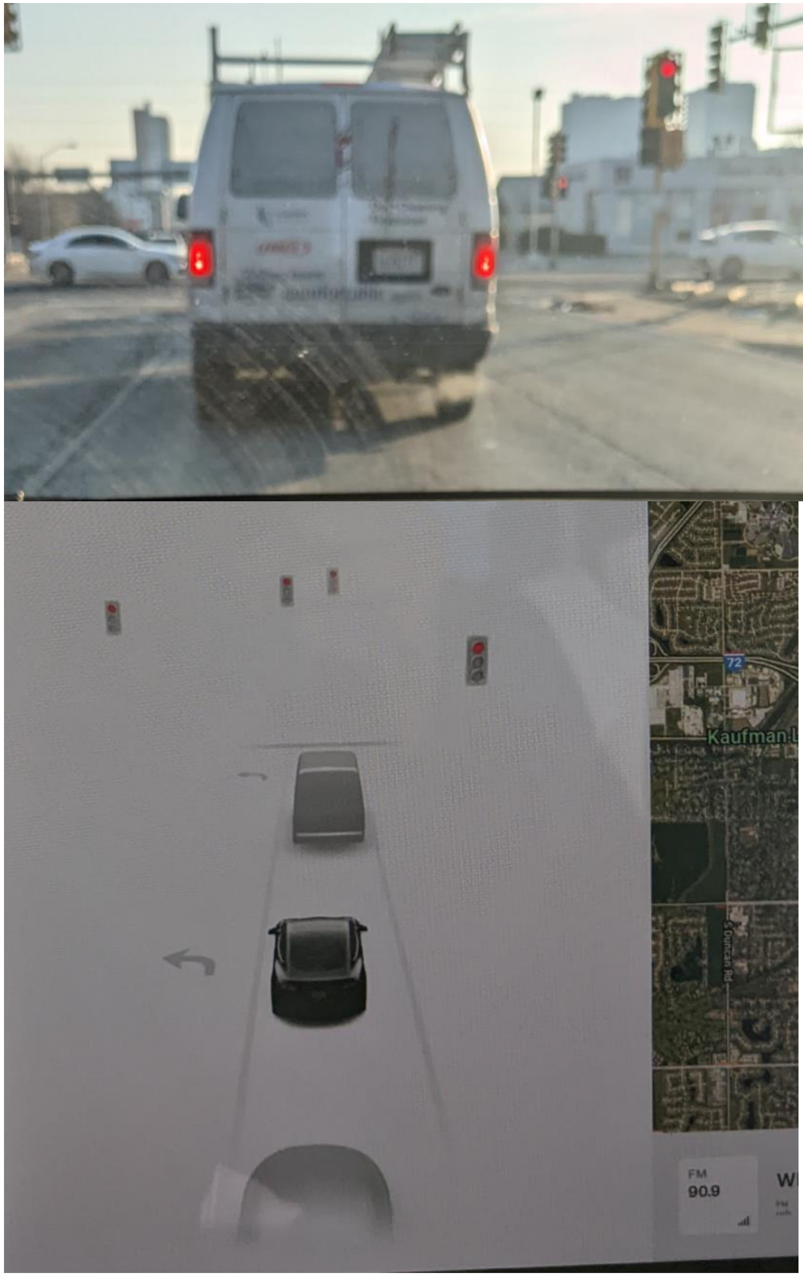


Sensing Physics-based models of camera, LIDAR, RADAR, GPS, etc.	**Perception** Programs for object detection, lane tracking, scene understanding, etc.	**Decisions and planning** Programs and multi-agent models of pedestrians, cars, etc.	**Control** Dynamical models of engine, powertrain, steering, tires, etc.



Perception

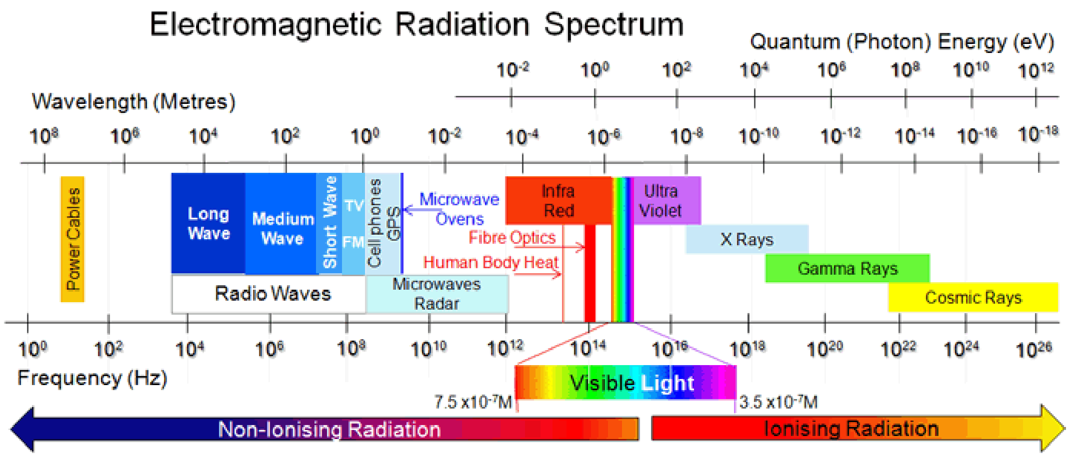
Programs for object detection, lane tracking, scene understanding, etc.



Perception subsystem converts signals from the environment into meaningful bits

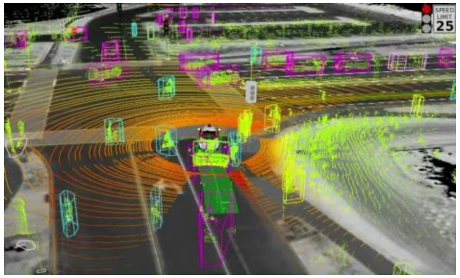
Perception: EM to objects

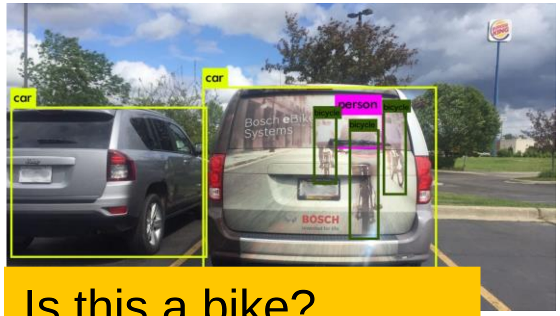
Problem: Process electromagnetic radiation from the environment to construct a *model* of the world, so that the constructed model is close to the real world



Challenging for computers: millions of years of evolution

Ill-defined problem: impossibility of defining meaning “car”, “bicycle”, etc.

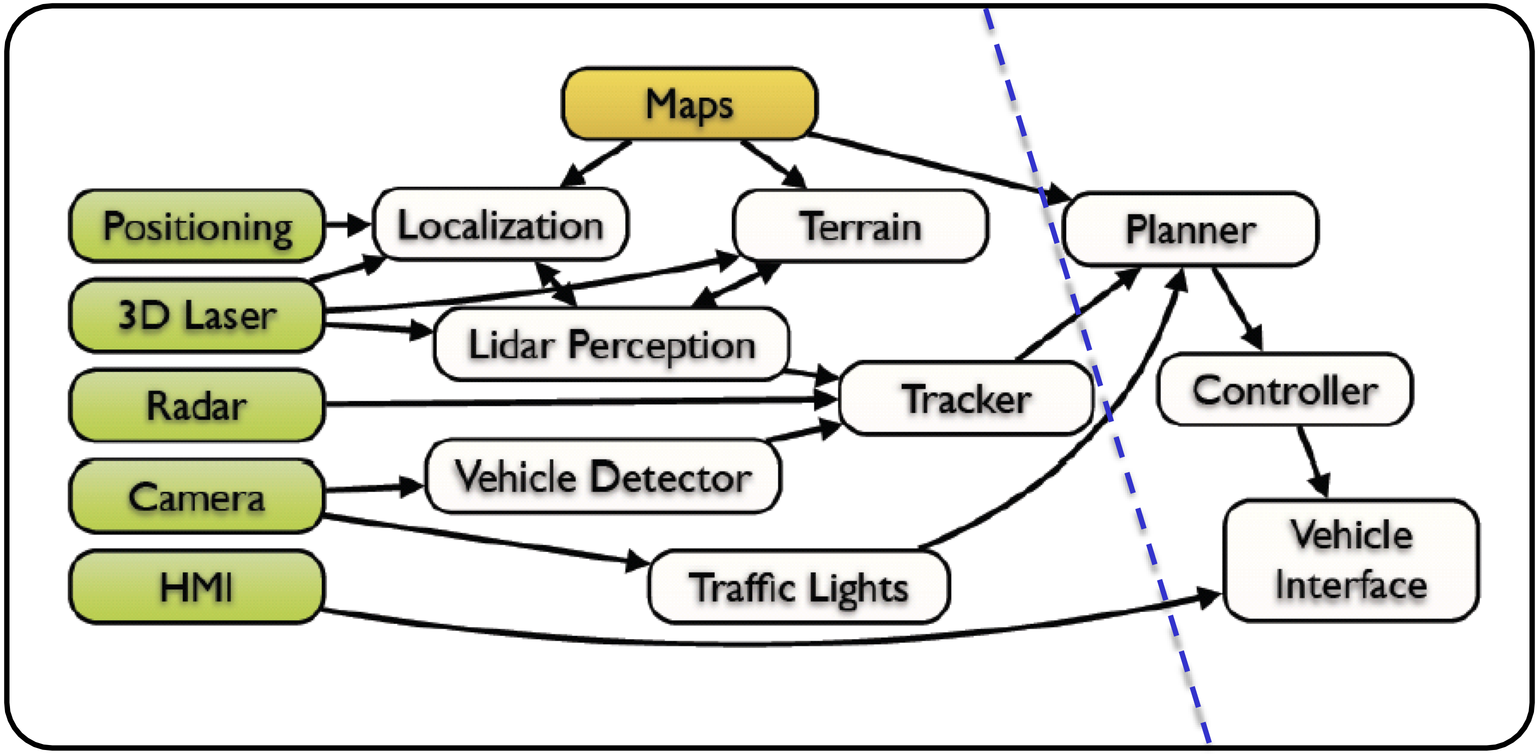




Is this a bike?

Is this a car?

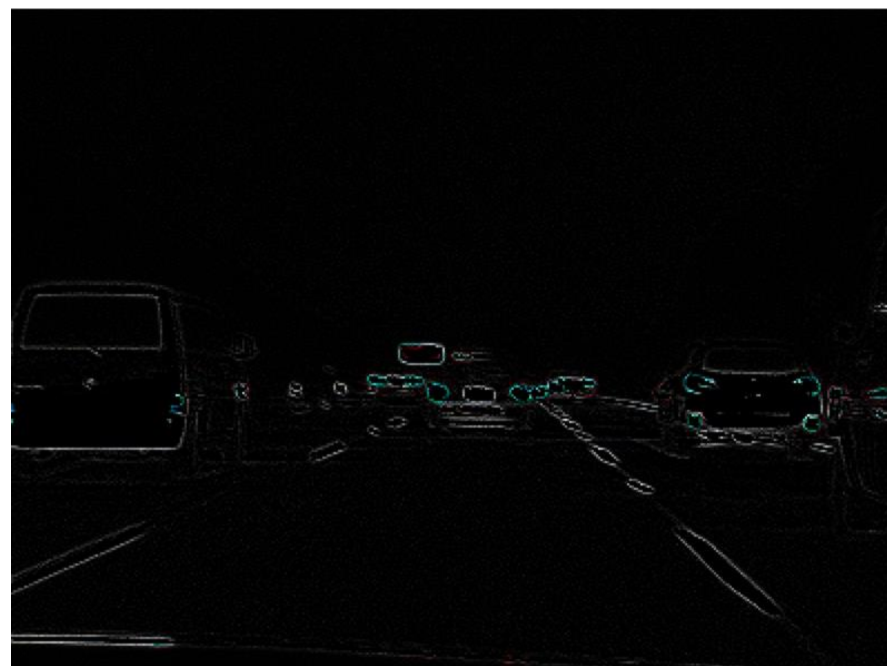
A practical perception pipeline in an AV has many pieces



This architecture from a slide from M. James of Toyota Research Institute, North America

Outline

- Linear filtering
- Edge detection



Motivation: Image denoising

- How can we reduce noise in a photograph?

Image representation

Images are represented as 2D arrays of pixels. Each pixel is represented by (array of) value(s) representing its color.

```
# read an image
img = cv2.imread('images/noguchi02.jpg')

# show image format (basically a 3-d array of pixel color info, in BGR format)
print(img)
```

```
[
   [[72 99 143]  [76 103 147] [78 106 147]  ...,
   [[74 101 145] [77 104 148] [77 105 146]  ...,
   [[76 103 147] [77 104 148] [76 104 145]  ...,
   ...,
   [[39 78 130] [39 78 130] [40 79 131]  ...,   [
   [[32 71 123] [32 71 123] [32 71 123]  ...,   [
   [[39 78 130] [39 78 130] [39 78 130]  ...,   [
]
```

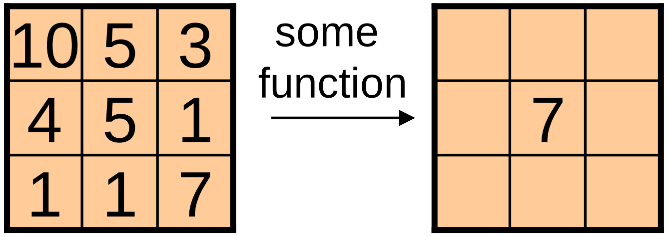
Where [72 99 143] is the blue, green, and red values of that pixel.

We will work with grayscale images

Denote by img[i,j] (or f[i,j]) the value of the i,j-th pixel

What is filtering?

Modify the pixels in an image based on some function of a local neighborhood of the pixels.



Bright(img,k): for all i,j

- img’[i][j] = k*img[i][j]

Shifting right by s Shift(img,s):

- img’[k] = img[k-s]; img’[0]…img’[s-1] is undefined

Simplest: Linear filtering

- replace each pixel by a linear combination of neighbors

Moving average

- Let's replace each pixel with a *weighted* average of its neighborhood
- The weights are called the *filter kernel*
- What are the weights for the average of a 3x3 neighborhood?

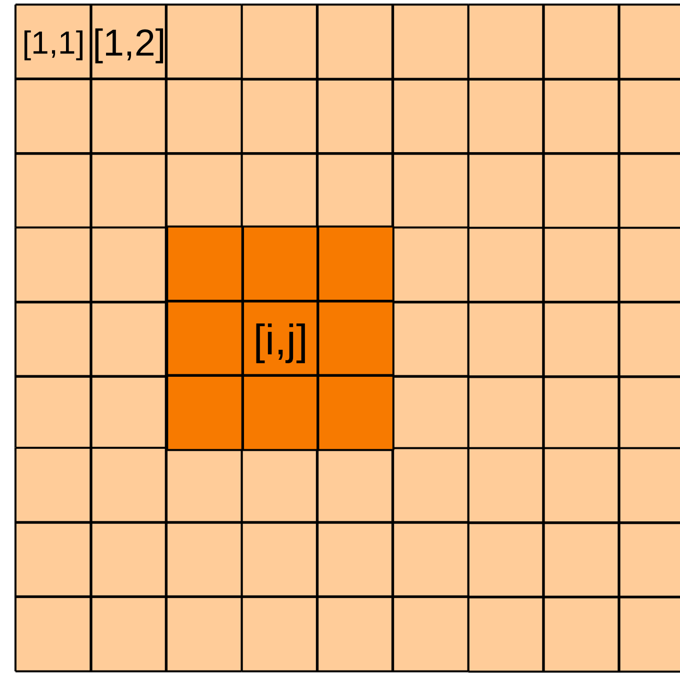
$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

"box filter"

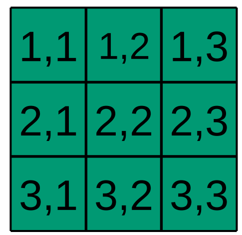
Source: D. Lowe

Convolution

image[i,j]



convolution
mask g[,]



Output or convolved image
f = g * img

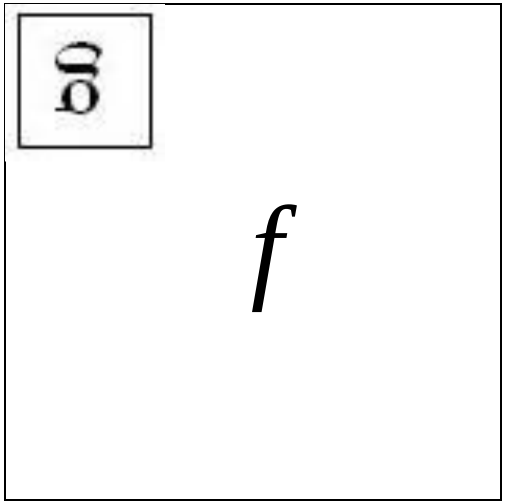
f[i,j] = g[1,1] img[i-1,j-1] + g[1,2] img[i-1,j] + g[1,3] img[i-1,j+1]
+ g[2,1] img[i,j-1] + g[2,2] img[i,j] + g[2,3] img[i,j+1]
+ g[3,1] img[i+1,j-1] + g[3,2] img[i+1,j] + g[3,3] img[i+1,j+1]

Defining convolution

- Let *f* be the image and *g* be the kernel. The output of convolving *f* with *g* is denoted *f* * *g*.

$$(f*g)[m,n]=\sum_{k,l} f[m-k,n-l]\,g[k,l]$$

Convention: kernel is “flipped”



Source: F. Durand

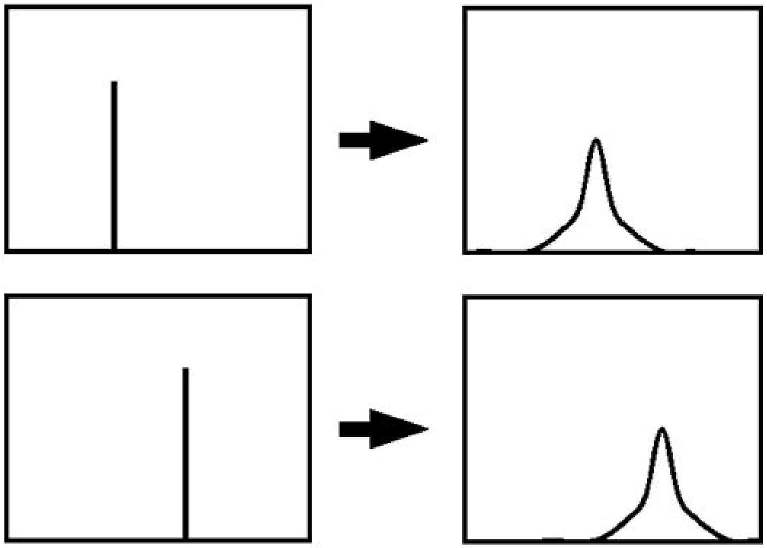
For analysis we will work with 1D images

- Let *f* be the image and *g* be the kernel. The output of convolving *f* with *g* is denoted *f* * *g*.

$$(f * g)[m] = \Sigma_k f[m-k]g[k]$$

Key properties: Prove the first two

- **Shift invariance:** same behavior regardless of pixel location: filter(shift(f)) = shift(filter(f))



- **Linearity:** filter(f_1 + f_2) = filter(f_1) + filter(f_2)

- **Theoretical result:** any linear shift-invariant operator can be represented as a convolution

Properties in more detail

- Commutative: $a * b = b * a$
  - Conceptually no difference between filter and signal
- Associative: $a * (b * c) = (a * b) * c$
  - Often apply several filters one after another: $(((a * b_1) * b_2) * b_3)$
  - This is equivalent to applying one filter: $a * (b_1 * b_2 * b_3)$
- Distributes over addition: $a * (b + c) = (a * b) + (a * c)$
- Scalars factor out: $ka * b = a * kb = k (a * b)$
- Identity: unit impulse $e = [\ldots, 0, 0, 1, 0, 0, \ldots]$,
  $a * e = a$

openCV: filter2D

Output image same size as input

Multi-channel: each channel is processed independently

Extrapolation of border

Examples

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

?

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

Filtered
(no change)

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

Shifted *left*
By 1 pixel

Practice with linear filters

Original

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

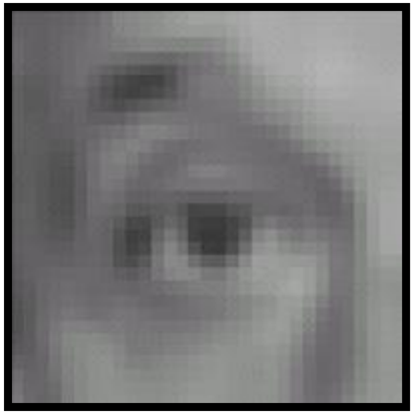
?

Source: D. Lowe

Practice with linear filters

Original

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$



Blur (with a box filter)

Source: D. Lowe

Practice with linear filters

Original

$$\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 2 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} - \frac{1}{9}\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

?

(Note that filter sums to 1)

Practice with linear filters

Original

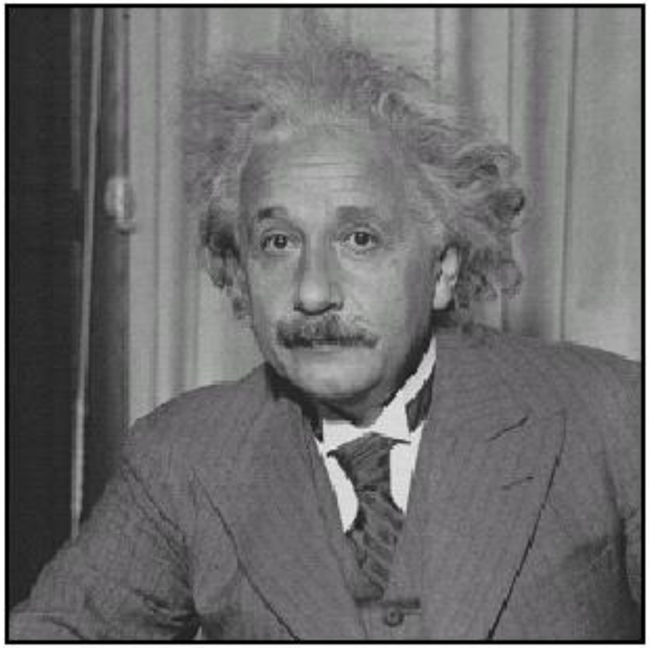
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Sharpening filter

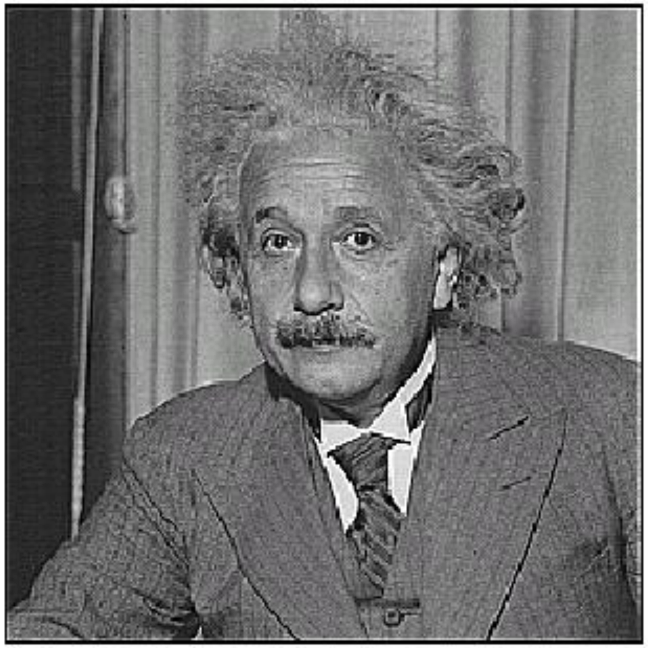
- Accentuates differences with local average

Source: D. Lowe

Sharpening



before



after

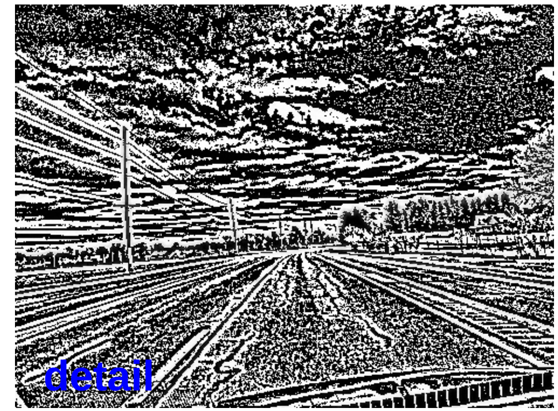
Source: D. Lowe

Sharpening

What does blurring take away?

−

=



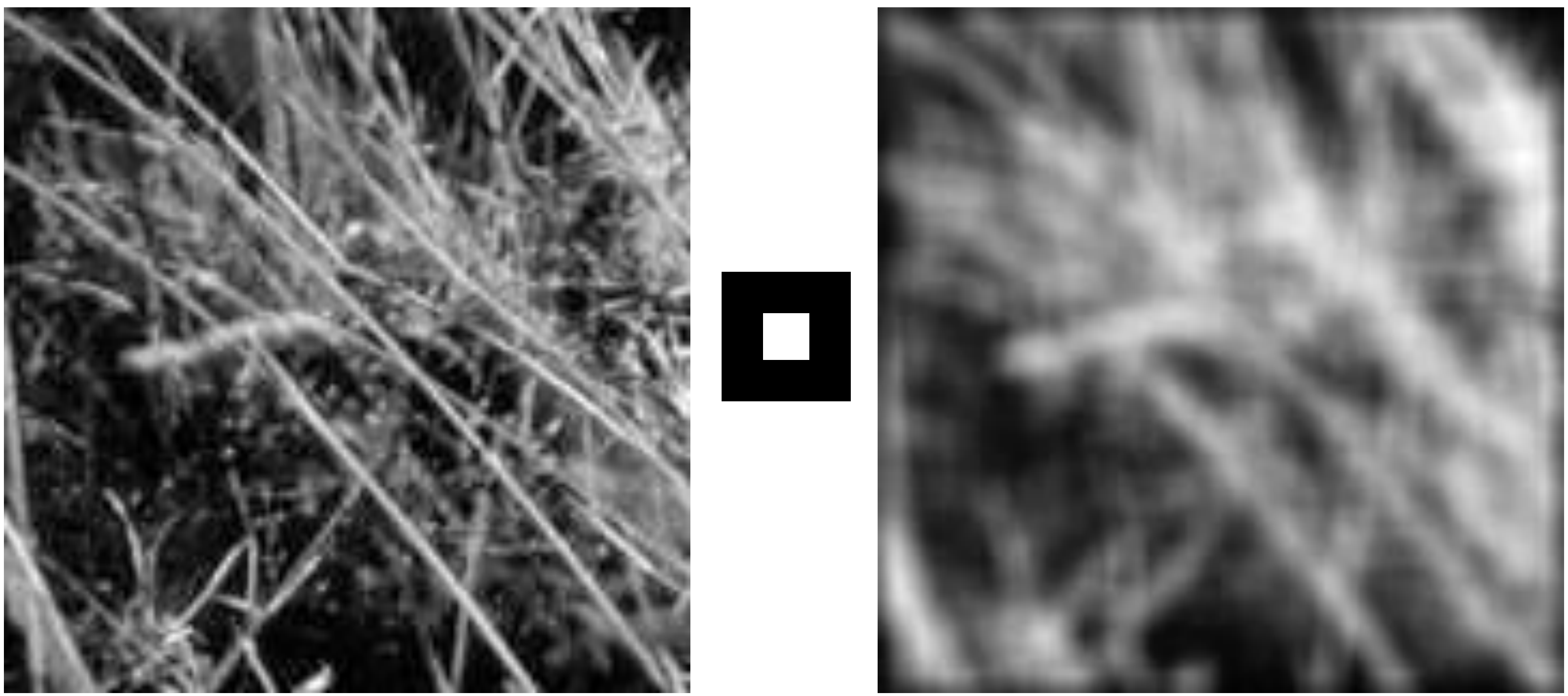
Let’s add it back:

+

=

Smoothing with box filter revisited

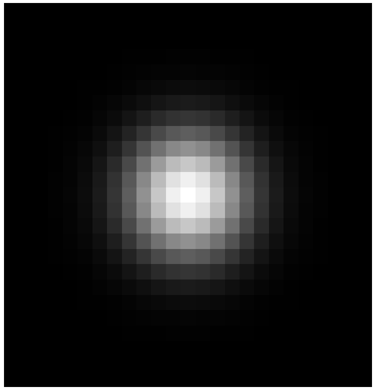
- What's wrong with this picture?
- What's the solution?



Source: D. Forsyth

Smoothing with box filter revisited

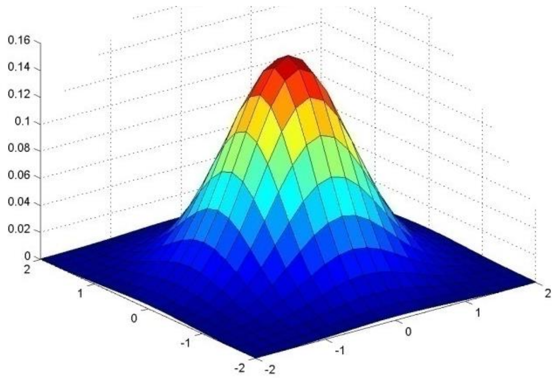
- What's wrong with this picture?
- What's the solution?
  - To eliminate edge effects, weight contribution of neighborhood pixels according to their closeness to the center

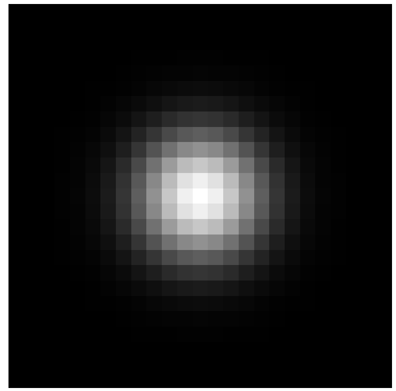


"fuzzy blob"

Gaussian Kernel

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$





0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

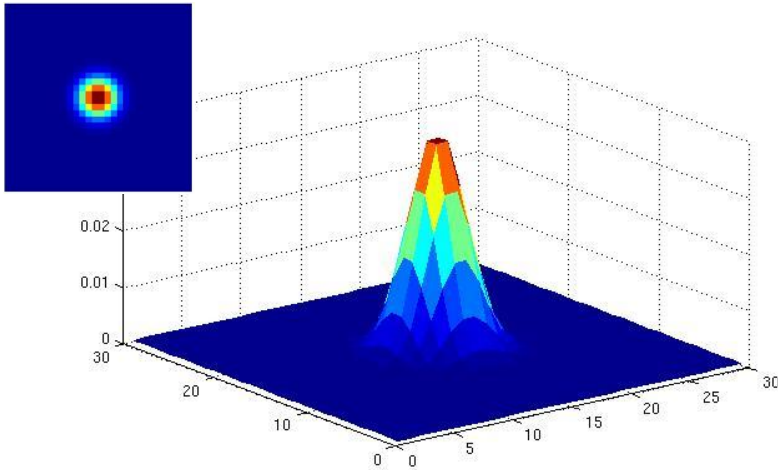
5 x 5, σ = 1

Constant factor at front makes volume sum to 1 (can be ignored when computing the filter values, as we should renormalize weights to sum to 1 in any case)

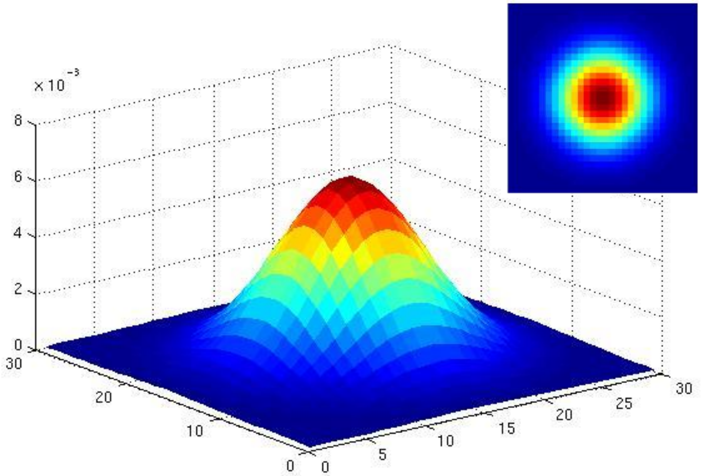
Source: C. Rasmussen

Gaussian Kernel

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$



σ = 2 with 30 x 30 kernel

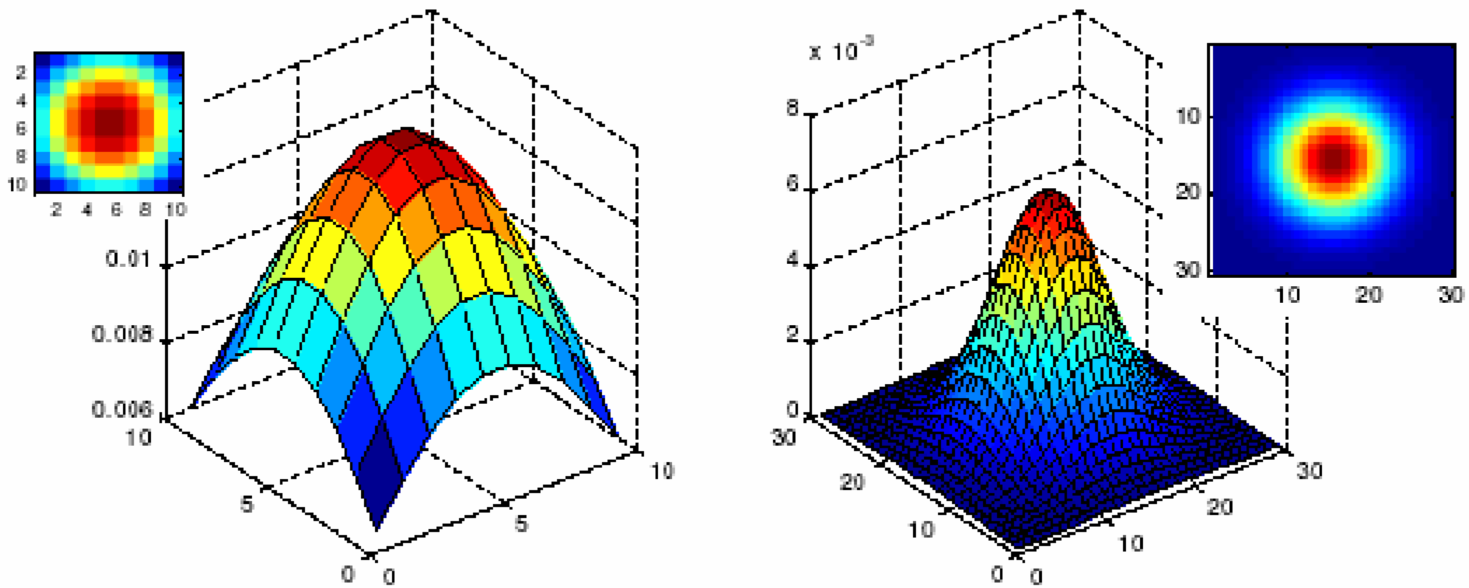


σ = 5 with 30 x 30 kernel

Standard deviation σ: determines extent of smoothing

Choosing kernel width

The Gaussian function has infinite support, but discrete filters use finite kernels

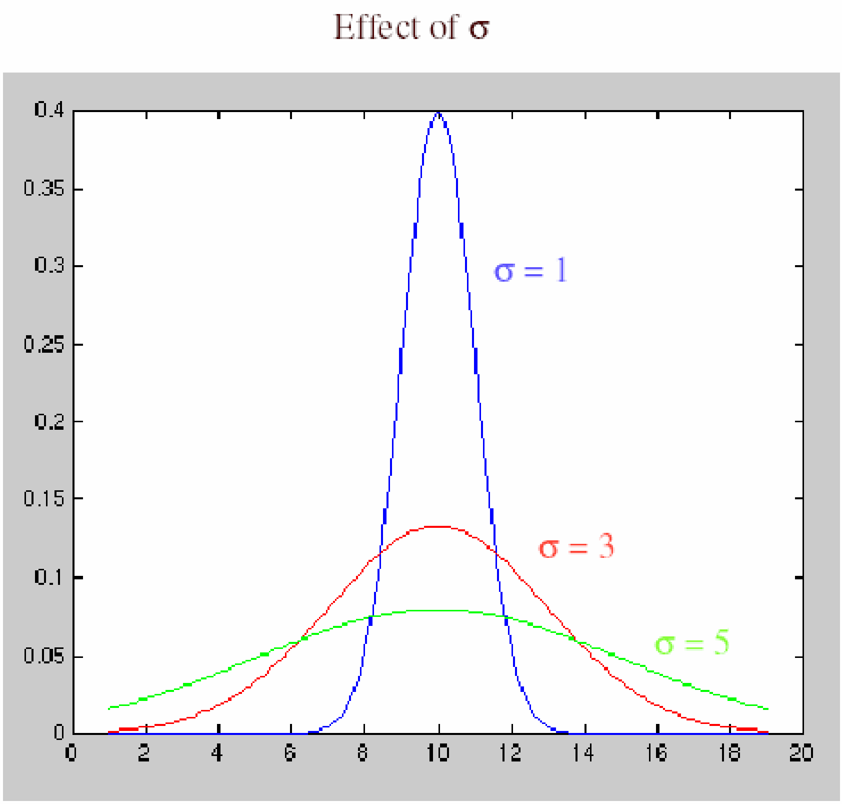


σ = 5 with 10x10 kernel

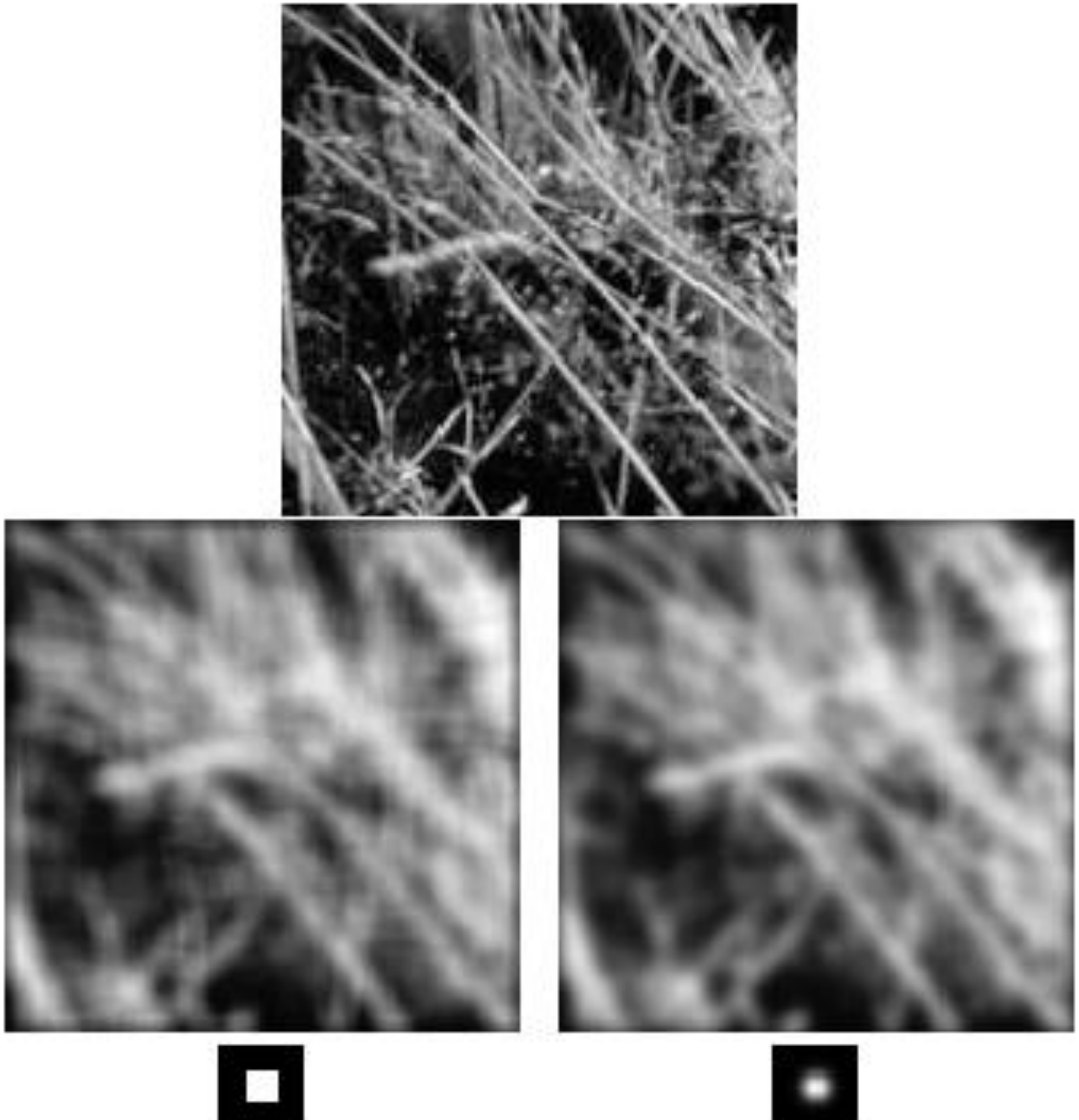
σ = 5 with 30x30 kernel

Choosing kernel width

Rule of thumb: set filter half-width to about 3σ



Gaussian vs. box filtering



Gaussian filters

- Remove high-frequency components from the image (*low-pass filter*)
- Convolution with self is another Gaussian
  - So can smooth with small-σ kernel, repeat, and get same result as larger-σ kernel would have
  - Convolving two times with Gaussian kernel with std. dev. σ is same as convolving once with kernel with std. dev. $\sigma\sqrt{2}$
- *Separable* kernel
  - Factors into product of two 1D Gaussians
  - Discrete example:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Source: K. Grauman

Separability of the Gaussian filter

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2 + y^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{x^2}{2\sigma^2}} \right) \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{y^2}{2\sigma^2}} \right)$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

Source: D. Lowe

Why is separability useful?

- Separability means that a 2D convolution can be reduced to two 1D convolutions (one along rows and one along columns)
- What is the complexity of filtering an $n \times n$ image with an $m \times m$ kernel?
  - $O(n^2 m^2)$
- What if the kernel is separable?
  - $O(n^2 m)$

Noise

Original

Salt and pepper noise

Impulse noise

Gaussian noise

- **Salt and pepper noise**: contains random occurrences of black and white pixels
- **Impulse noise:** contains random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution

Source: S. Seitz

Reducing salt-and-pepper noise

3x3

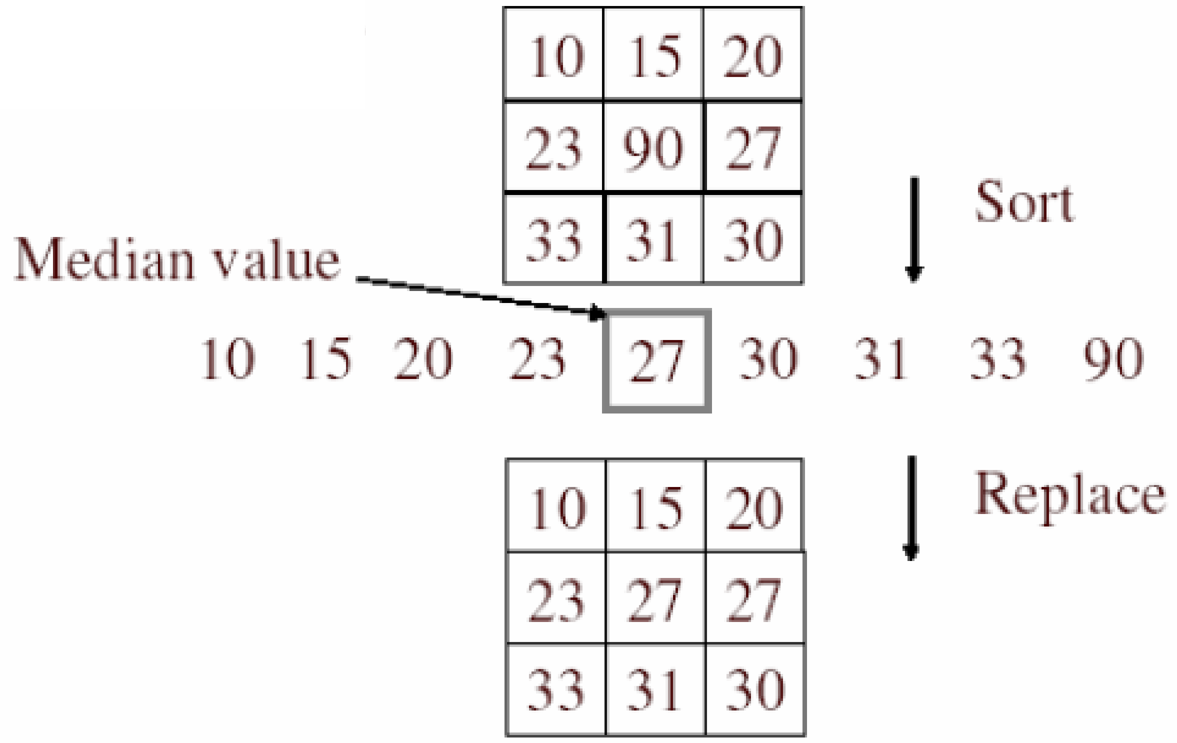
5x5

7x7

What's wrong with the results?

Alternative idea: Median filtering

- A **median filter** operates over a window by selecting the median intensity in the window

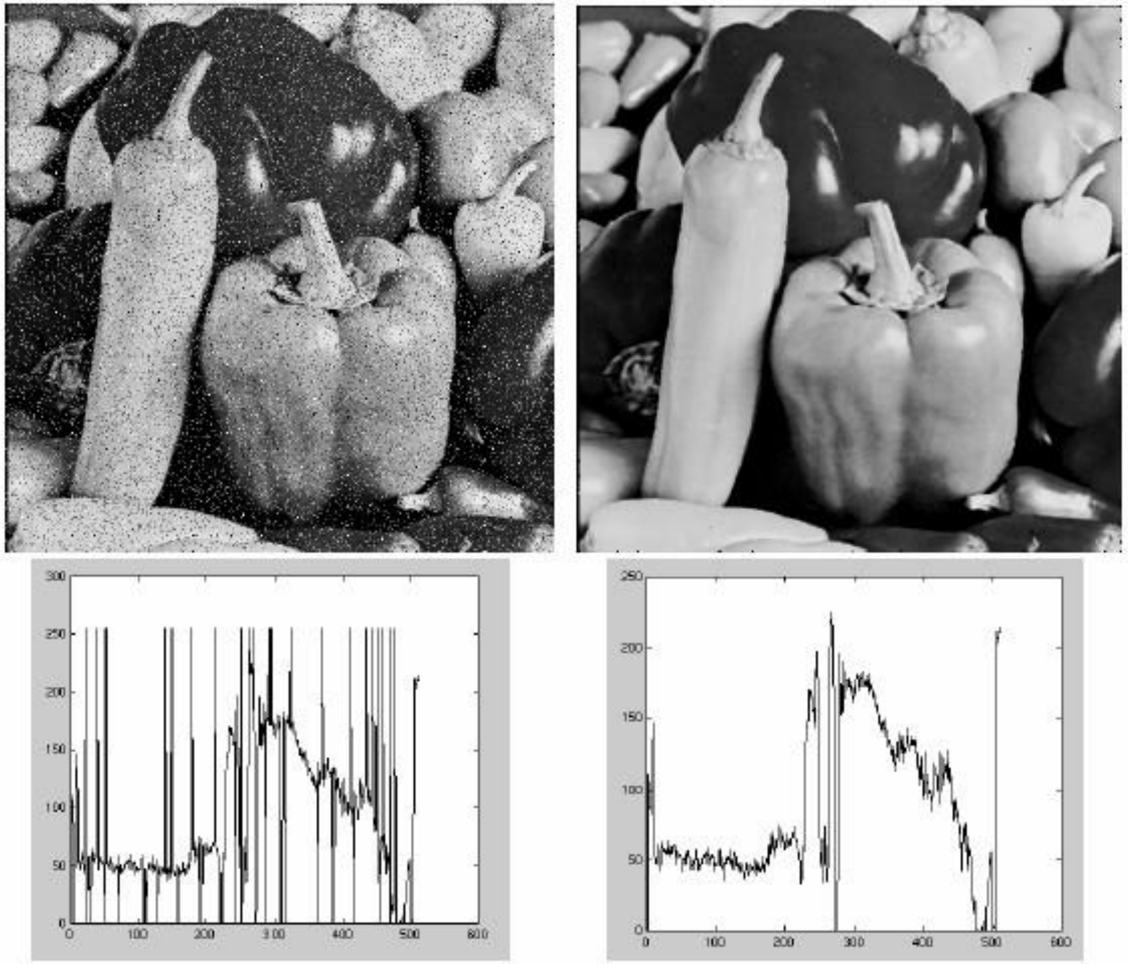


- Is median filtering linear?

Median filter

Salt-and-pepper noise

Median filtered



open cv: cv2.medianBlur (input, output,ksize)

Source: M. Hebert

Median filter

- Is median filtering linear?
- Let’s try filtering

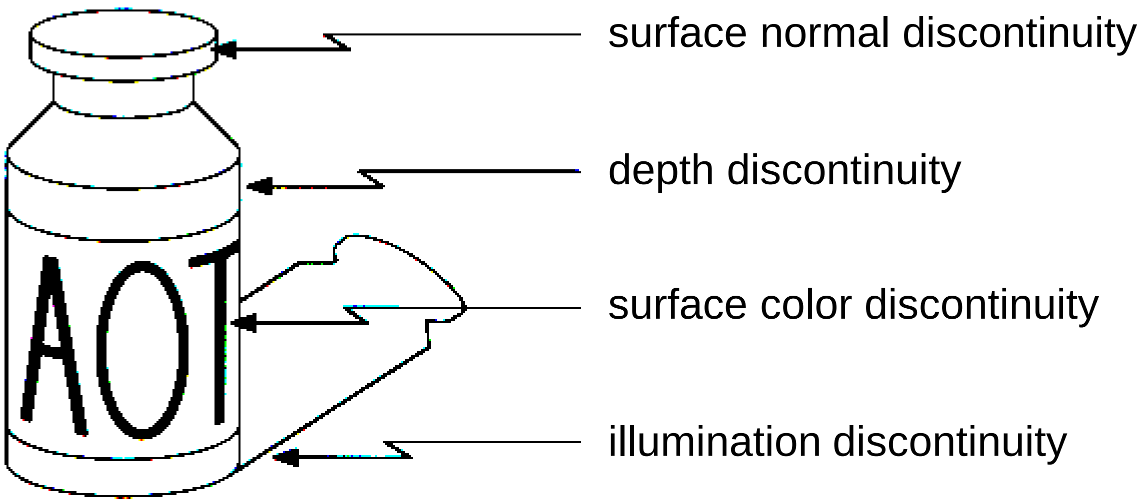
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Edge detection

Winter in Kraków photographed by Marcin Ryczek

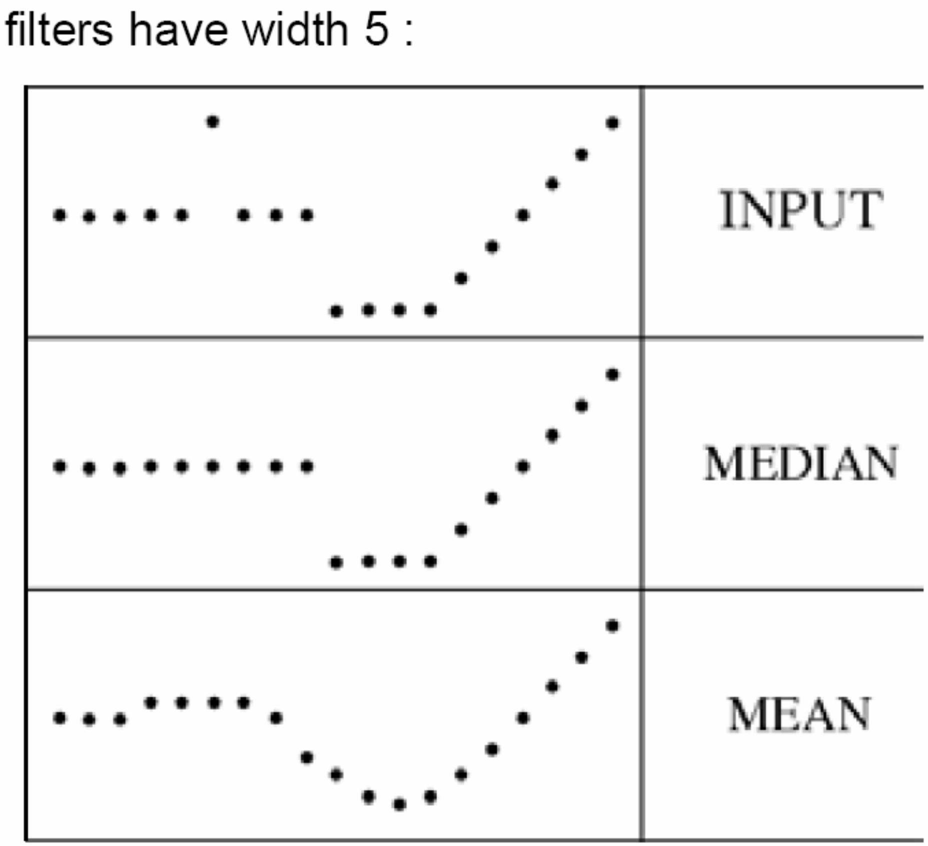
Edge detection

- **Goal:** Identify sudden changes (discontinuities) in an image
- Intuitively, edges carry most of the semantic and shape information from the image
  - E.g., Lanes, traffic signs, cars



Median filter

- What advantage does median filtering have over Gaussian filtering?
  - Robustness to outliers



Gaussian vs. median filtering

3x3 5x5 7x7

Gaussian

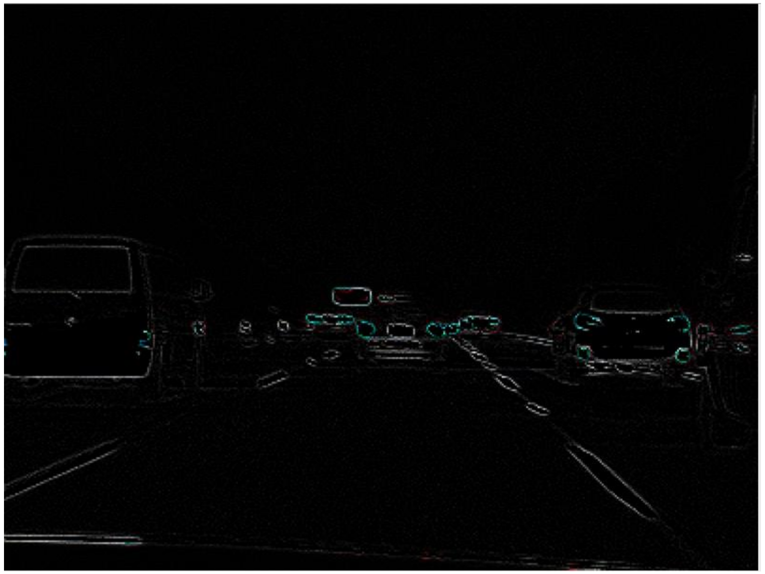
Median

Review: Image filtering

- Convolution
- Box vs. Gaussian filter
- Separability
- Median filter

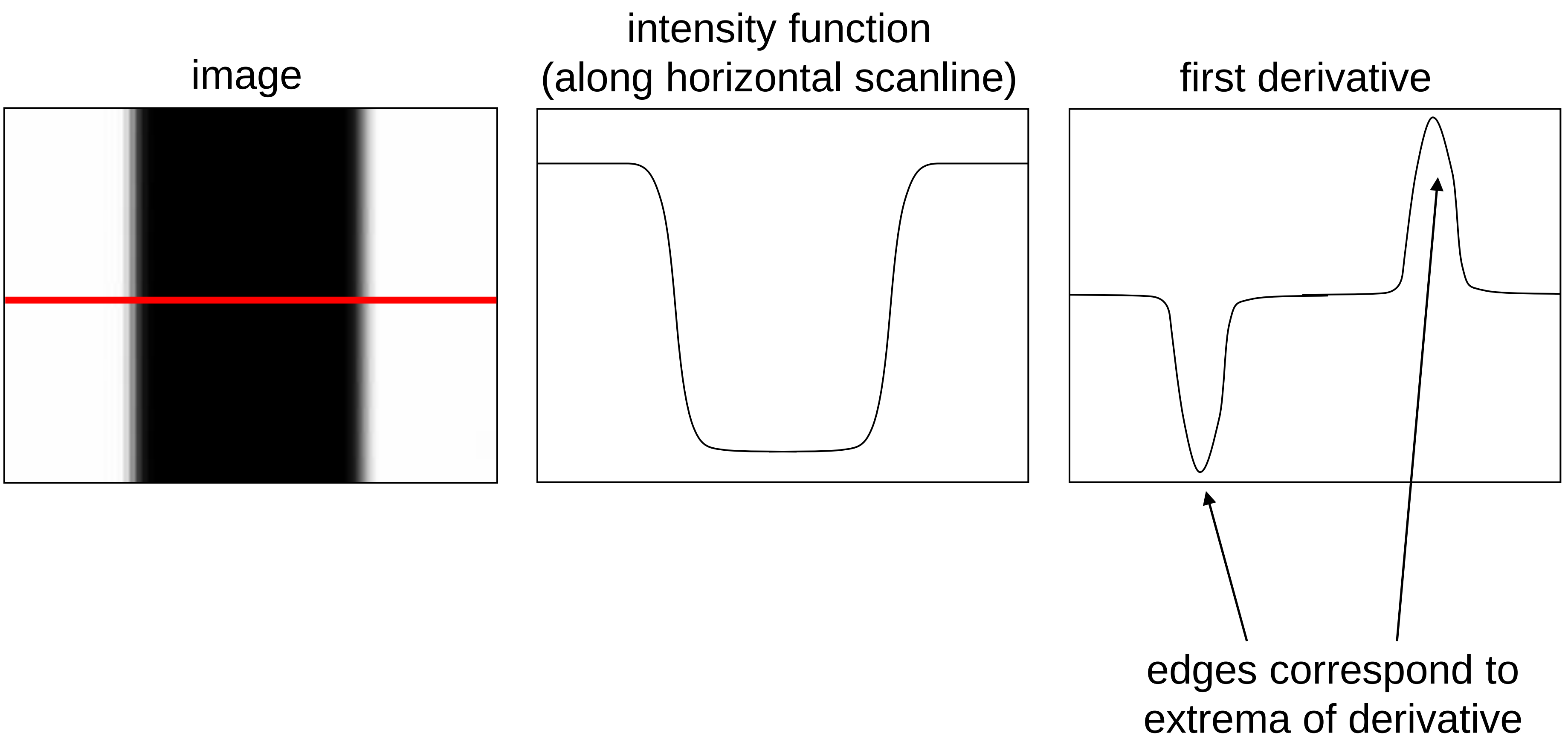
Outline

- Filtering
  - Convolution: Linearity, shift invariance, associativity, commutativity, …
  - Kernels: Gaussian, box, …
  - Separability of Gaussian
  - Median filter
- Today: Edge detection
- Object recognition: Classification



Edge detection

An edge is a place of rapid change in the image intensity function



Derivatives with convolution

For 2D function f(x,y), the partial derivative w.r.t *x* is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

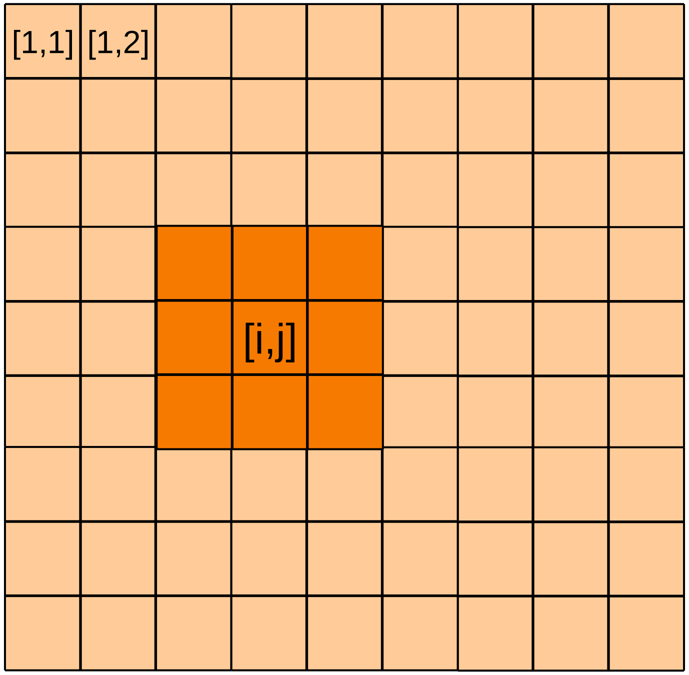
For discrete data, we can approximate using finite differences:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

To implement the above as convolution, what would be the associated filter?

Convolution

image[i,j]



convolution mask g[,]

-1	1

Output or convolved image

f = g * img

f[i,j] = -1.img[i,j-1] + 1. img[i,j]

Partial derivatives of an image

$$\frac{\partial f(x,y)}{\partial x}$$

$$\begin{bmatrix} -1 & 1 \end{bmatrix}$$

$$\frac{\partial f(x,y)}{\partial y}$$

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Which shows changes with respect to x?

Finite difference filters

Other approximations of derivative filters exist:

Prewitt: $M_x = \begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -1 & 0 & 1 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$; $M_y = \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -1 & -1 \\ \hline \end{array}$

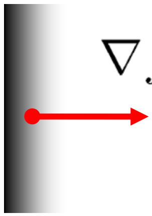
Sobel: $M_x = \begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$; $M_y = \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$

Roberts: $M_x = \begin{array}{|c|c|}\hline 0 & 1 \\ \hline -1 & 0 \\ \hline \end{array}$; $M_y = \begin{array}{|c|c|}\hline 1 & 0 \\ \hline 0 & -1 \\ \hline \end{array}$

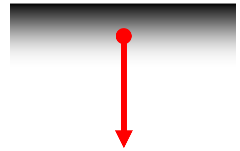
Source: K. Grauman

Image gradient

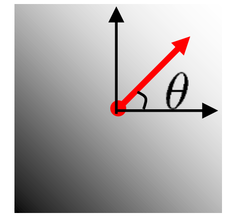
The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$



$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

The gradient points in the direction of most rapid increase in intensity

- How does this direction relate to the direction of the edge?

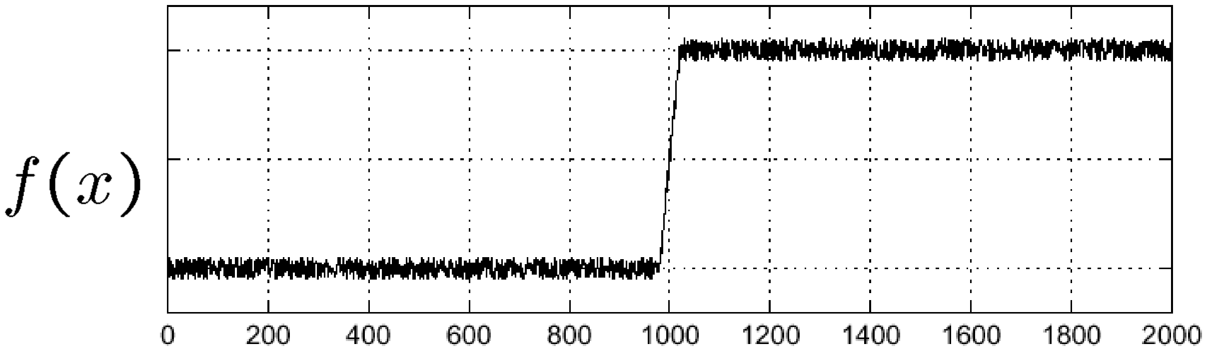
The gradient direction is given by $\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$

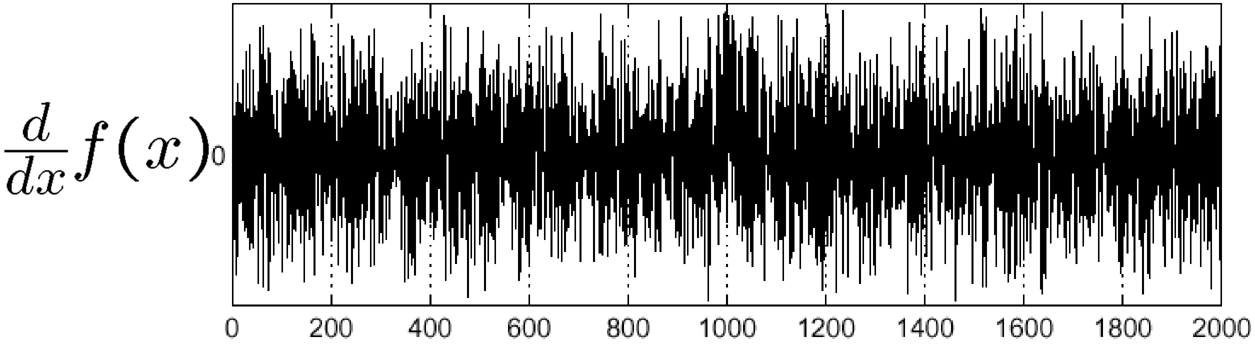
The edge strength is given by the gradient magnitude (norm)

$$||\nabla f|| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Effects of noise

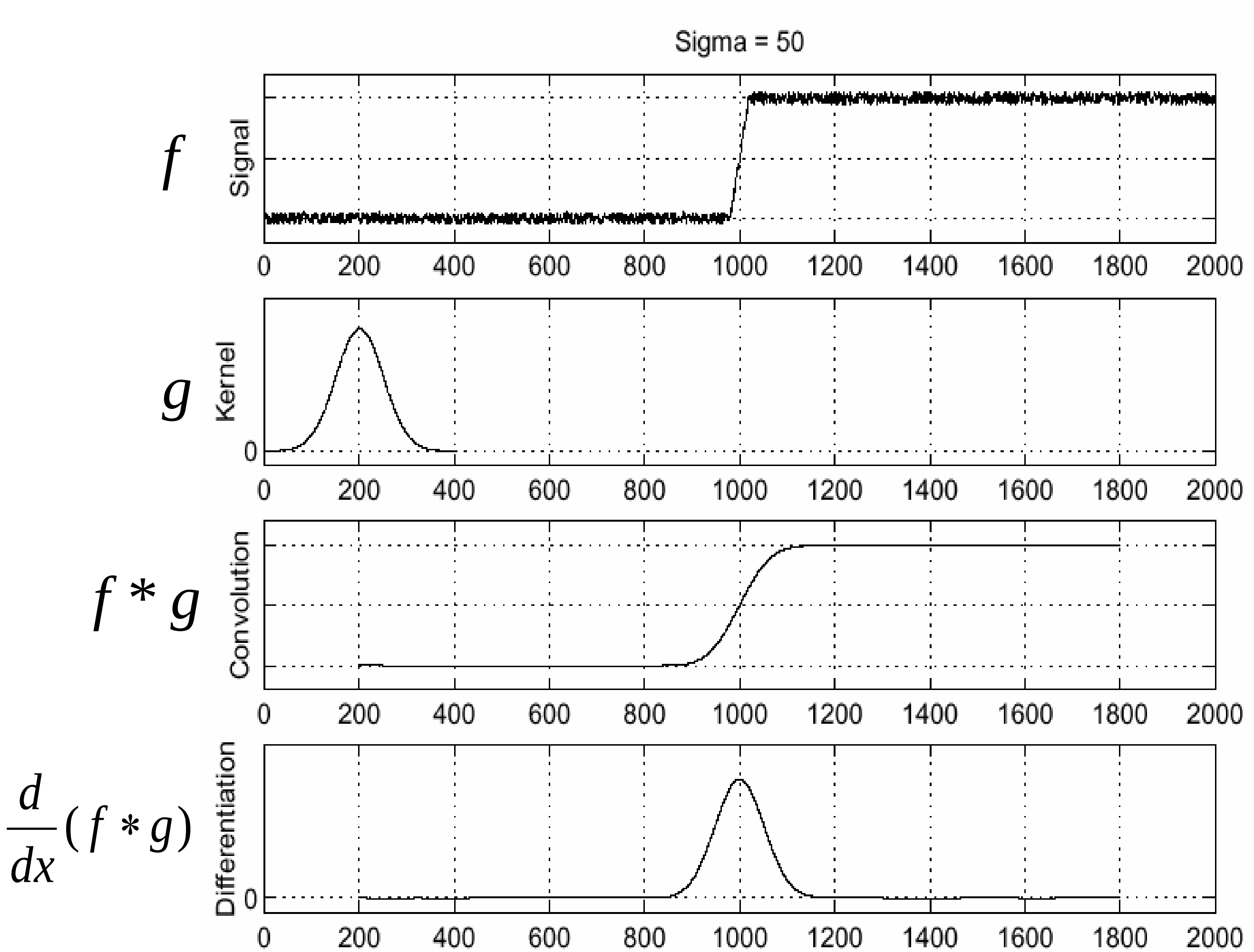
Consider a single row or column of the image





Where is the edge?

Solution: smooth first



- To find edges, look for peaks in $\frac{d}{dx}(f * g)$

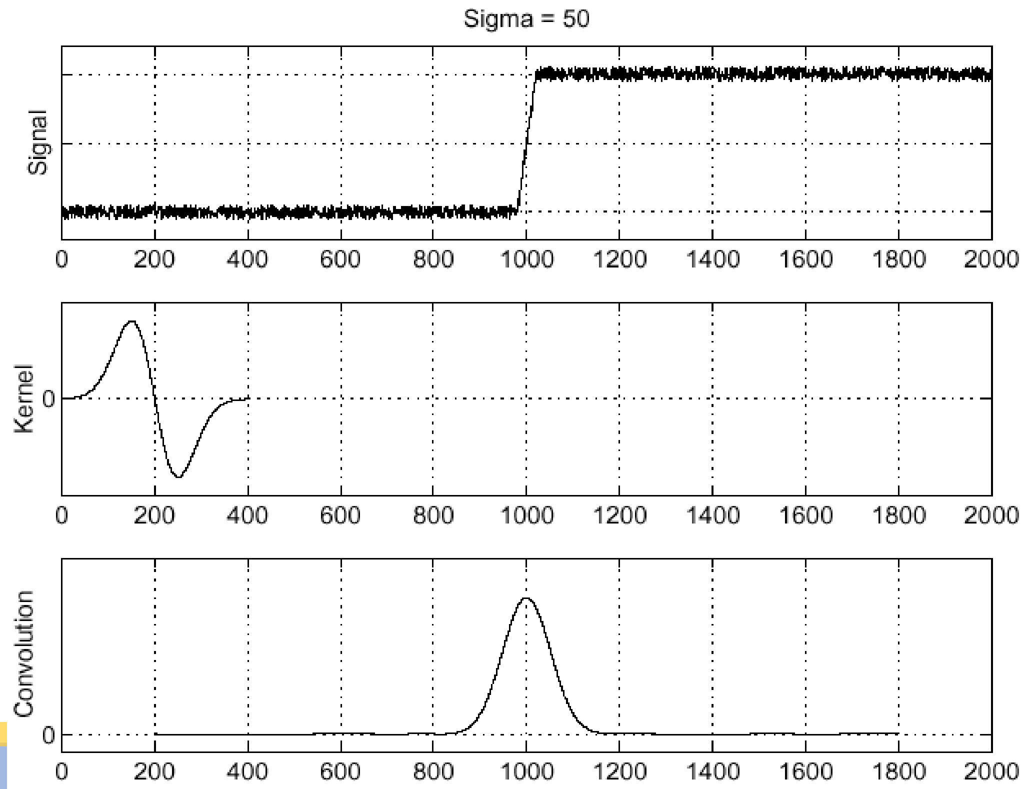
Derivative theorem of convolution

- Differentiation is convolution, and convolution is associative: $$\frac{d}{dx}(f*g) = f*\frac{d}{dx}g$$
- This saves us one operation:

f

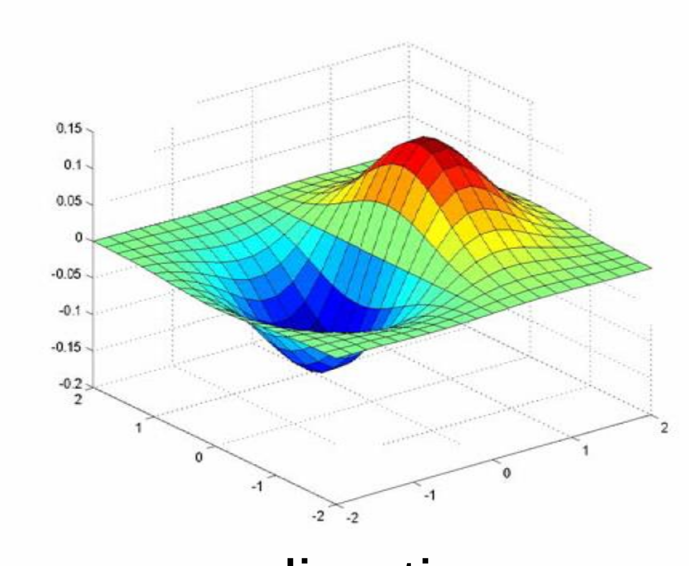
$\frac{d}{dx}g$

$f*\frac{d}{dx}g$

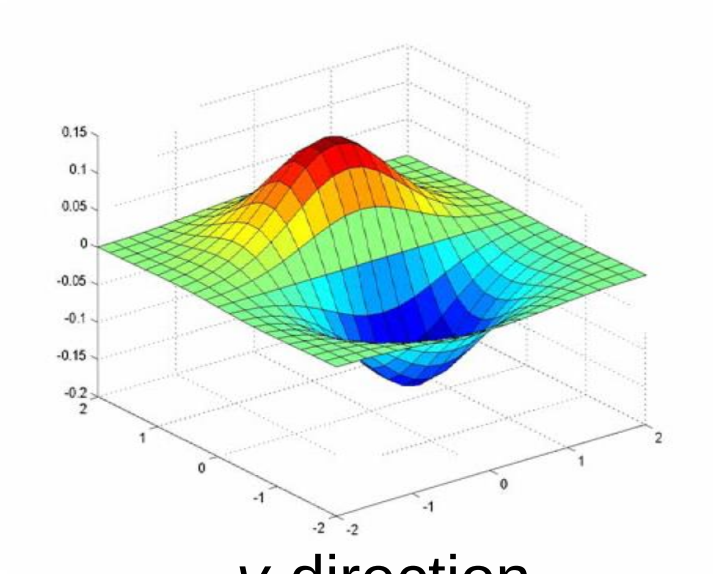


Source: S. Seitz

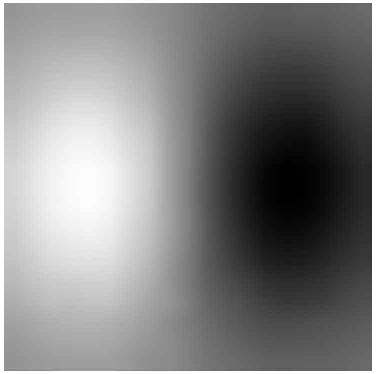
Derivative of Gaussian filters

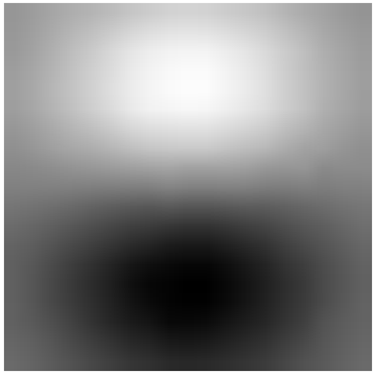


x-direction



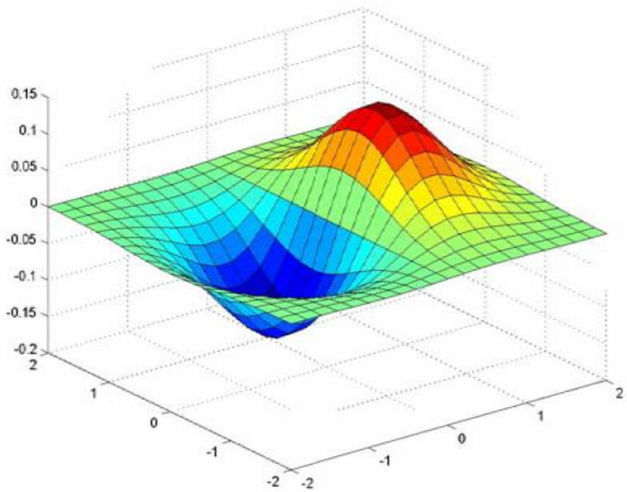
y-direction



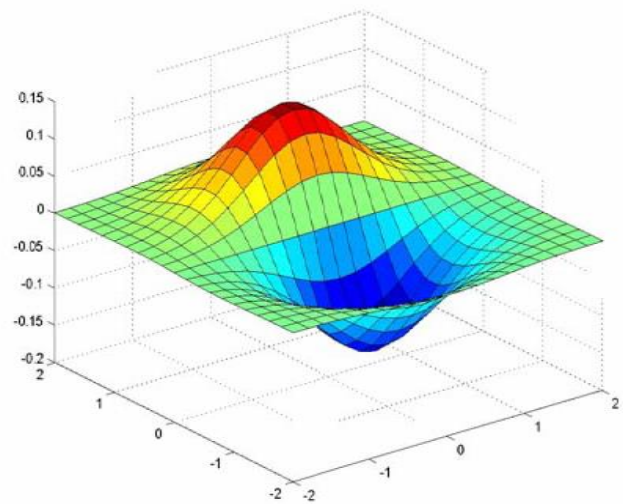


Which one finds horizontal/vertical edges?

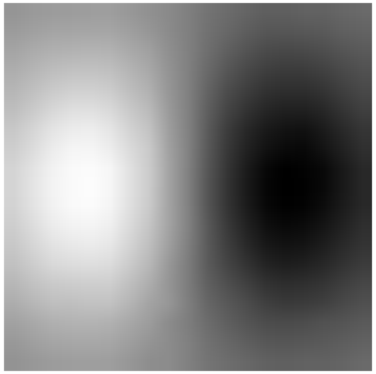
Derivative of Gaussian filters

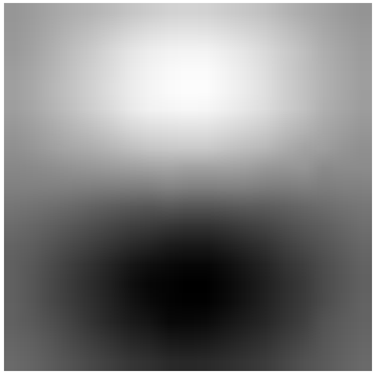


x-direction



y-direction





Are these filters separable?

Recall: Separability of the Gaussian filter

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

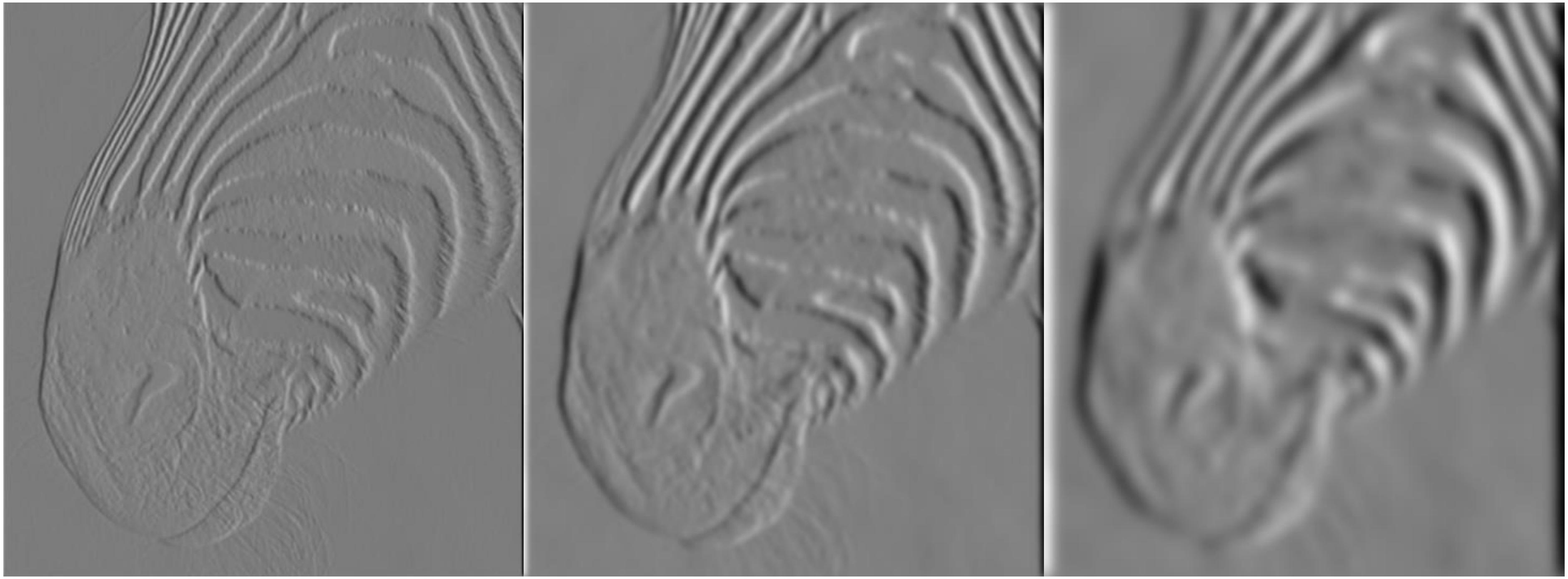
$$= \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{x^2}{2\sigma^2}}\right) \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{y^2}{2\sigma^2}}\right)$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

Source: D. Lowe

Scale of Gaussian derivative filter

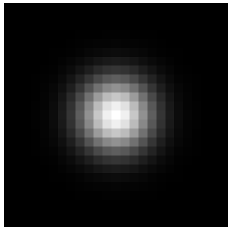


1 pixel 3 pixels 7 pixels

Smoothed derivative removes noise, but blurs edge

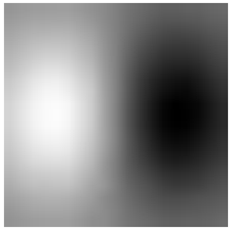
Also finds edges at different “scales”

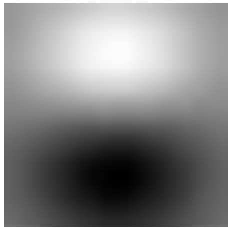
Review: Smoothing vs. derivative filters



Smoothing filters

- Gaussian: remove "high-frequency" components; "low-pass" filter
- Can the values of a smoothing filter be negative?
- What should the values sum to?
  - **One:** constant regions are not affected by the filter



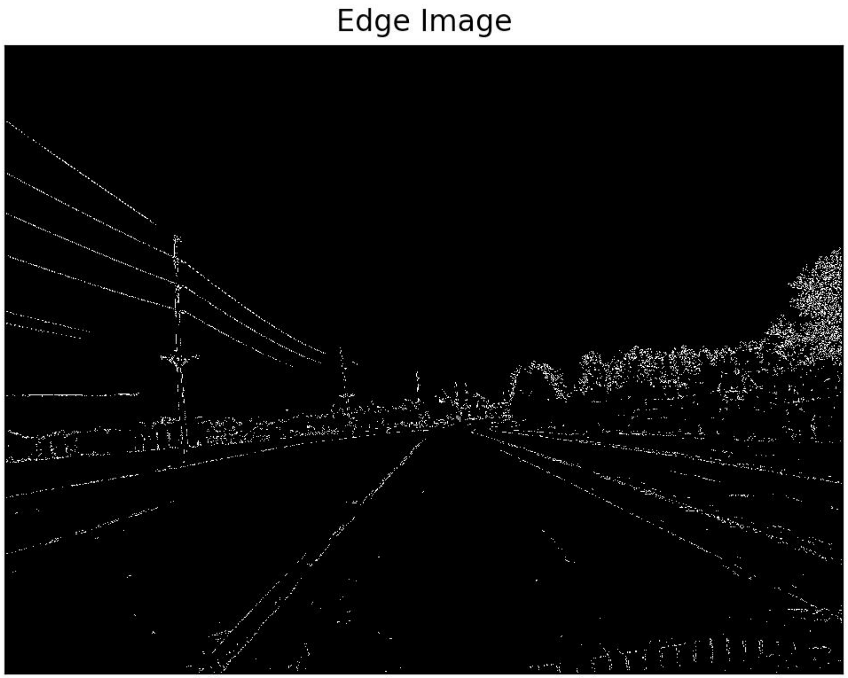


Derivative filters

- Derivatives of Gaussian
- Can the values of a derivative filter be negative?
- What should the values sum to?
  - **Zero:** no response in constant regions

Building an edge detector

original image



Grad output

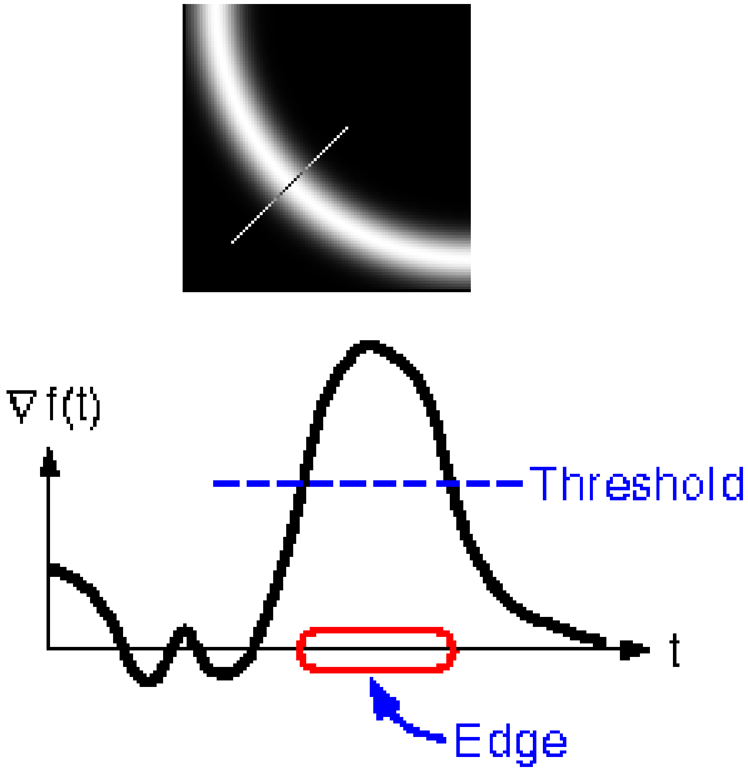
norm of the gradient $\|\nabla f\| = \sqrt{(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2}$

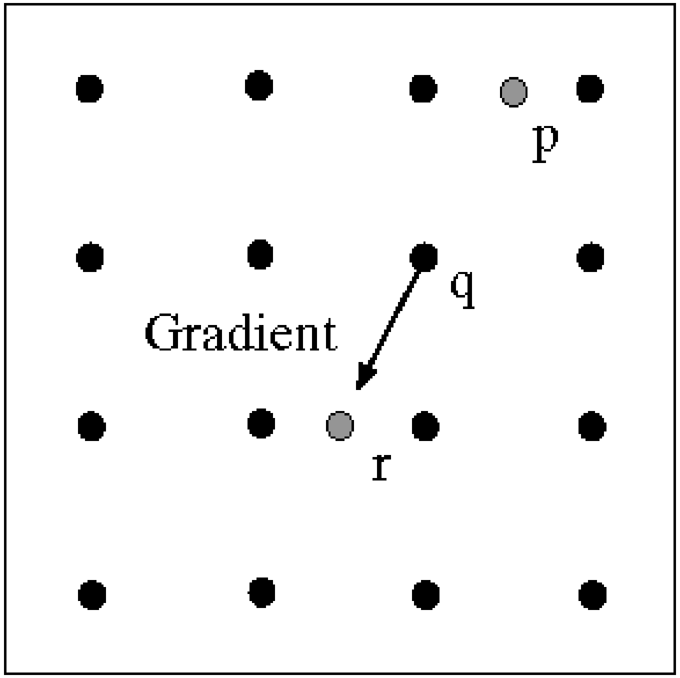
Building an edge detector

How to turn these thick regions of the gradient into curves?

Thresholded norm of the gradient

Non-maximum suppression





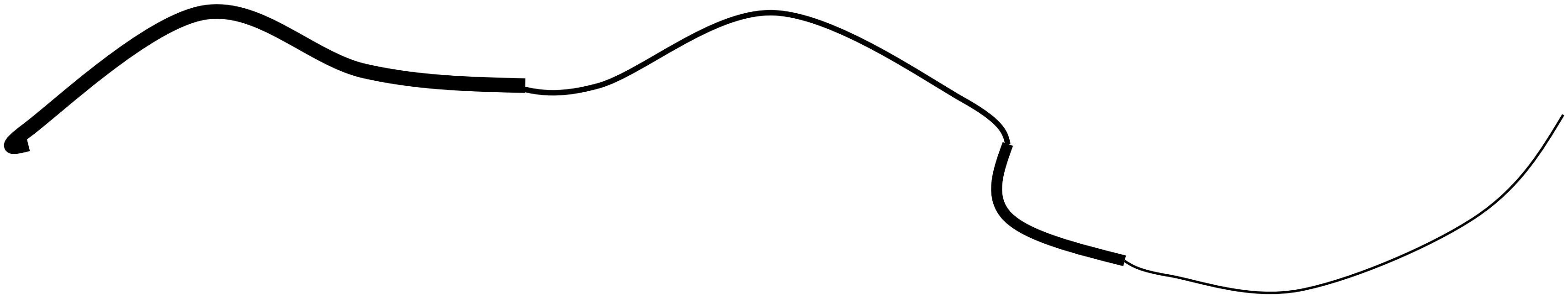
- For each location q above threshold, check that the gradient magnitude is higher than at neighbors p and r along the direction of the gradient
  - May need to interpolate to get the magnitudes at p and r

Non-maximum suppression

Another problem: pixels along this edge didn’t survive thresholding

Hysteresis thresholding

Use a high threshold to start edge curves, and a low threshold to continue them.



Source: Steve Seitz

Hysteresis thresholding

original image

high threshold (strong edges)

low threshold (weak edges)

hysteresis threshold

Source: L. Fei-Fei

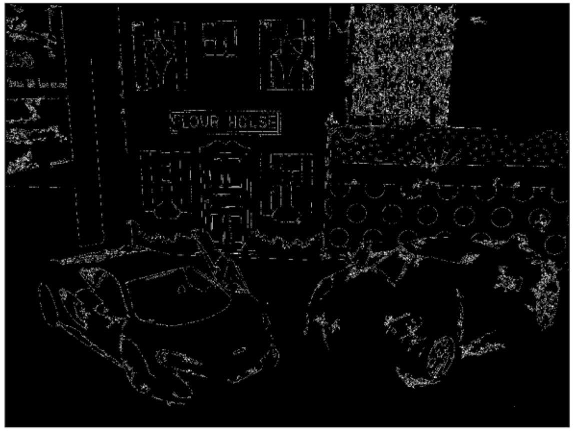
Recap: Canny edge detector

1. Compute x and y gradient images
2. Find magnitude and orientation of gradient
3. **Non-maximum suppression**:
   - Thin wide “ridges” down to single pixel width
4. **Linking and thresholding** (**hysteresis**):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

opencv: `canny(image,th1,th2)`

Original Image

Edge Image



J. Canny, [A Computational Approach To Edge Detection](), IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

Summary

- Convolution as translation invariant linear operations on signals and images
- Definition of convolution and its properties (associativity, commutativity, etc.)
- Artifacts of of hard-edge kernels
- Gaussian kernel, its definition and properties (separability)
- Median filter, sharpening
- Derivatives as convolution (Sobel, etc.)

Sharpening

What does blurring take away?

–

=

Let’s add it back:

+

=

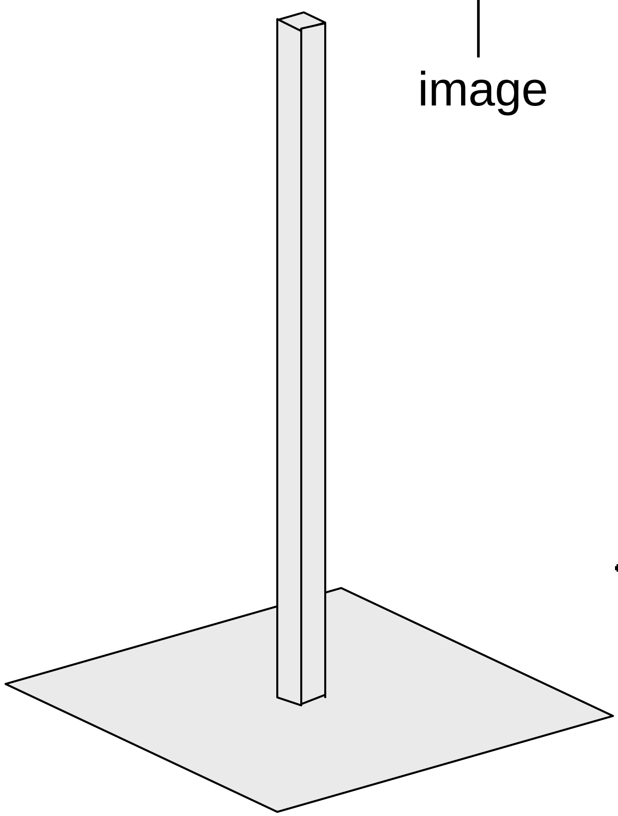
Unsharp mask filter

$$f + \alpha(f - f * g) = (1+\alpha) f - \alpha\, f * g = f * ((1+\alpha)e - g)$$

image

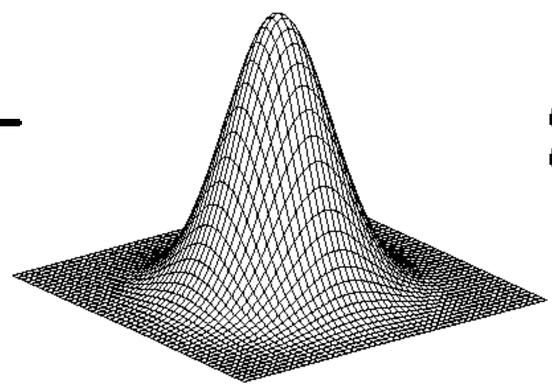
blurred image

unit impulse (identity)



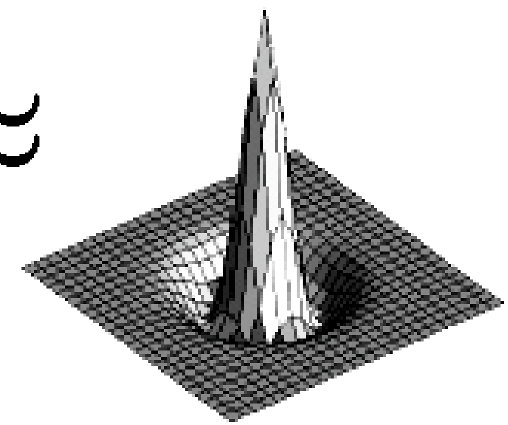
unit impulse

$-$



Gaussian

$\approx$



Laplacian of Gaussian