Robot estimating state of a door w. camera

initially robot knows nothing

prior distribution

X_0 RV state of the door at $t=0$

belief $bel(X_0 = open) = 1/2$

$bel(X_0 = closed) = 1/2$

Measurement model / Sensor

$P(z_t = Sense\_open \mid x_t = open) = 0.6$

$P(z_t = Sense\_closed \mid x_t = open) = 0.4$

$P(z_t = Sense\_open \mid x_t = closed) = 0.2$

$P(z_t = Sense\_closed \mid x_t = closed) = 0.8$

Motion model / manipulator

$P(x_t = open \mid x_{t-1} = open \; u_t = push) = 1$

$P(closed \mid open \; push) = 0$

$P(open \mid closed \; push) = 0.8$

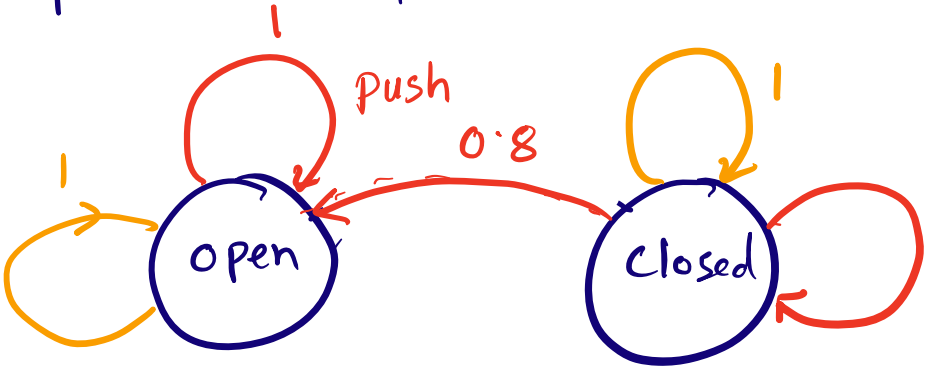
$P(closed \mid closed \; push) = 0.2$

$P(x_t = \text{open} \mid x_{t-1} = \text{open}, u_t = \text{do\_nothing}) = 1$

$P(x_t = \text{closed} \mid x_{t-1} = \text{closed}, u_t = \text{do\_no}) = 1$

$P(x_t = \text{closed} \mid x_{t-1} = \text{open}, \ldots) = 0$

$P(x_t = \text{open} \mid x_{t-1} = \text{closed}, \ldots) = 0$



MDP

Suppose at $t = 1$ $u_t = \text{do\_nothing}$ $z_t = \text{sense\_open}$

How to calculate the posterior distribution?

$bel(x_0 = \text{open}) = 1/2 = bel(x_0 = \text{closed})$

Prediction (Motion) Correction (Measurement)

$$\overline{bel}(x_1) = \sum_{x_0} P(x_1 \mid u_1, x_0)\, bel(x_0)$$

$$= P(x_1 \mid u_1 = \text{do\_no}\ x_0 = \text{is\_open})\, bel(x_0 = \text{is\_open}) + P(x_1 \mid u_1 = \text{do\_no}\ x_0 = \text{is\_closed})\, bel(x_0 = \text{is\_closed})$$

$$\overline{bel}(x_1 = \text{is\_open}) = 1 \cdot 1/2 + 0 \cdot 1/2 = 1/2$$

$\overline{bel}(x_1 = is\_closed) = 0 + 1 \cdot 1/2 = 1/2$

$bel(x_1) = \eta \; P(z_1 = sense\_open \mid x_1) \; \overline{bel}(x_1)$

$bel(x_1 = is\_open) = \eta \; 0.6 \cdot 1/2 = \eta \; 0.3 = 3/4$

$bel(x_1 = is\_closed) = \eta \; P(z_1 = sense\_open \mid x_1 = is\_closed) \cdot \overline{bel}(x_1 = closed)$

$= \eta \; 0.2 \cdot 1/2 = \eta \; 0.1 = 1/4$

$bel(x_1 = is\_open) = 3/4 \quad bel(x_1 = is\_closed) = 1/4$

$u_2 = push \quad z_2 = sense\_open$

$bel(x_2 = is\_open) = 0.983$

$bel(x_2 = is\_closed) = 0.017$