Problem 1

Show that the convolution operation is *distributes over addition*. That is \( a \ast (b+c) = a \ast b + a \ast c \). To simplify things, you may consider convolution over infinite, 1-dimensional vectors. Then, the \( k^{th} \) pixel of the convolved image \( f \ast g \) is given by:

\[
(f \ast g)[k] = \sum_{m=-\infty}^{\infty} f[m]g[k-m].
\]

Problem 2

Given a finite state automaton \( \mathcal{A} = (Q, Q_0, A, D) \) and a set of states \( S \subseteq Q \)

(A) (5 points) Define \( \text{Post}_{\mathcal{A}}(S) \)

(B) (5 points) Define the set of states of \( \mathcal{A} \) that are reachable from the initial set \( Q_0 \) in exactly \( k \) steps (or transitions)

\[ \text{Reach}_{\mathcal{A}}(Q_0, k) = \]

(C) (10 points) Suppose you have a subroutine for \( \text{Post}_{\mathcal{A}}(S) \). How will use this subroutine to check whether the automaton hits any unsafe state in the set \( U \subseteq Q \) within \( k \) steps? You can assume that the initial set \( Q_0 \) is safe. Fill out the following pseudocode to give an algorithm.

```
Input: \( \mathcal{A} = (Q, Q_0, A, D) \), \( k, U \)
Output: safe/unsafe
S = Q_0
S1 = __________
i = 0
While (i \__________)
    S1 = Post(S)
    If __________
```
(D) (5 points) What is the relationship between \( \text{Reach}_A(Q_0,i) \) and the set \( S \) computed in the above algorithm at the end of the \( i \)th iteration of the while loop.

Problem 3

Consider the linear system
\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
4 & 2
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
1 \\
2
\end{bmatrix} u = Ax + Bu
\]

Defining the full state feedback control law:
\[
u = -[k_1, k_2] \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} = -Kx
\]

(a) Write down the characteristic equation of the closed loop system \( A_{cl} = A - BK \) (the equation that needs to be solved to find the eigenvalues of \( A_{cl} \)).

(b) Choose some negative values of \( k_1 \) and \( k_2 \) that will make \( A_{cl} \) Hurwitz. Write the corresponding real parts of the eigenvalues of \( A_{cl} \).

Short questions

Define the clustering problem. Why is it called unsupervised learning?

What is the clustering objective function \( J_{\text{clust}} \)?
In classification using the “classical” pipeline (bag of visual words without pyramids), explain why spatial information about the visual words is lost.

There are 6 different datasets noted as A,B,C,D,E,F. Each dataset is clustered using two different methods, and one of them is K-means. All results are shown in Figure 2. Circle out which result is more likely to be generated by K-means method. (Hint: check the state when K-means converges; Centers for each cluster have been noted as X; Since x and y axis are scaled proportionally, you can determine the distance to centers geometrically). The distance measure used here is the Euclidean distance. (From CMU10-601 FA15 HW3)

Figure 2: Clustered results for 6 datasets

A: A₂, B₂, C₂, D₁, E₂, F₂

State whether the following are true or false

i. ROS is a centralized network where a master ROS node manages communication among other nodes.

ii. A ROS topic can have only one publisher and one subscriber at a time.

iii. Sum of values in any smoothing kernel should always equal to 1.

iv. Sum of values in any derivative kernel should always equal to 1.
Write down kernels for (a) left shift by 5 pixels, (b) box-filter of size 5, (c) a $5 \times 5$ Gaussian derivative kernel in the x-direction. Given before-after images, you should be able to tell which filter has been applied.

Consider the 2-dimensional ODE system described by: $\dot{x} = x^2 + \sin(y); \dot{y} = xy + a$, where $a$ is a parameter of the model. Find all the equilibria of this system.