# Lecture 17: Decision-Making cont. & Formal Safety

Professor Katie Driggs-Campbell April 1, 2021

ECE484: Principles of Safe Autonomy



### Administrivia - Schedule

- MP3 due next week 4/9
  - Demo on Thursday / Report due Friday
- MP4 due Friday 4/30
- Next week: Final Safety Lecture and MP4 Walkthrough
- No class on Tuesday 4/13
- No class on Tuesday 5/4 extra office hours
- Guest Lectures on 4/15, 4/20, and 4/22
  - Attendance worth double
  - Google form to collect questions will be posted on discord
- Project presentations on 4/27 and 4/29
  - Information will be provided on 4/8 during lecture
- Report due on Friday 5/14 (last day of finals)
  - Rubric and guidelines to be posted soon



#### Administrivia - Oral Exam Protocol

- **Dates:** Monday 4/12 to Tuesday 5/4
- **Times:** 15-minute time slots on Mondays 5-6pm, Tuesdays 5-6pm, Wednesdays 11am-12pm, or by appointment (if needed)
  - Signup will be posted on discord first come, first serve
- Content: Questions and rubric (with bonus allocation) released this weekend
  - You will be asked one of six questions, chosen by rolling a die
- Protocol: We will follow CBTF protocol
  - Log in with both computer and phone
  - Position phone so we can see screen and workspace
- A word of caution: If we suspect cheating, we will schedule a final exam



#### Today's Plan

- MDP Policies and Value Iteration
- Simple Example
- Introduction to Safety
- Responsibility Sensitive Safety



#### **MDP** Policies

Policies map states to actions

 $\pi$ :  $x \rightarrow u$ 

- We want to find a policy that maximizes future pay off
  - Suppose T = 1:  $\pi_1(x) = \operatorname{argmax}_u r(x, u)$
- We write the Value Function for given  $\pi$ :

$$V_1(x) = \gamma \max_u r(x, u)$$

 Generally, we want to find the sequence of actions that optimize the expected cumulative discounted future payoff

$$R_T = \mathbb{E}\left[\sum_{\tau=0}^T \gamma^\tau r_{t+\tau}\right]$$



# Recall: $V_1(x) = \gamma \max r(x, u)$ Value Functions For longer time horizons, we define V(x) recursively: T=2: pick an action that max sum Vi+1-step pay $\pi_{2}(x) = \operatorname{argmax}[r(x,u) + \int V_{1}(x')p(x'(x,u)dx')$ $X V_2(x) = \chi max [$ For Finite T: $\pi_{\tau}(x) = \operatorname{argmax} \left[ r(x, u) + \int \mathcal{V}_{\tau_{\tau}}(x) \rho(x'|x, u) dx \right]$ $V_T(x) = \chi max f$



#### Value Functions

• In the infinite time horizon, we tend to reach equilibrium:

$$V_{\infty}(x) = \gamma \max_{u} \left[ r(x, u) + \int V_{\infty}(x') p(x'|x, u) \, dx' \right]$$

- This is the *Bellman Equation* 
  - Satisfying this is necessary and sufficient for an optimal policy



## Computing the (Approximate) Value Function

- Initial guess for  $\widehat{V}$ 
  - $\hat{V}(x) \leftarrow r_{min}, \forall x$
- Successively update for increasing horizons
  - $\widehat{V}(x) \leftarrow \gamma \max_{u} \left[ r(x,u) + \int \widehat{V}(x') p(x'|x,u) dx' \right]$
- Value iteration converges if  $\gamma < 1$
- Given estimate  $\widehat{V}(x)$ , policy is found:
  - $\pi(x) = \operatorname{argmax}_u \left[ r(x, u) + \int \widehat{V}(x') p(x'|x, u) dx' \right]$
- Often, we use the discrete version:

•  $\pi(x) = \operatorname{argmax}_{u} \left[ r(x, u) + \sum_{x'}^{r} \widehat{V}(x') p(x'|x, u) \right]$ 



Example: Create an MDP (S, A, T, Y, R)Stop X = Z X, X Z, X ZA= 27,03 XZ D(x)(x,u) is deterministic s if x, + u=0 0 will move up P=1 0 will stop P=1 X3 R = 7 N 



Example: Value Iteration vector rep:  $\hat{V} = [\hat{V}(x_1) \hat{V}(x_2) \hat{V}(x_3)]$  $\rightarrow init: \hat{v} = [0 \ 0 \ 0] \quad u=0$  $V_{s}$  )  $\hat{v} = [\chi V_{s} 0 0]$  $\hat{V}_{1}(x_{1}) = \chi \max(0+0, r_{3}+0) = \chi$  $\hat{U}(x_2) = \chi max(0+0) = 0$ Ĵz= Lgrs Z  $\hat{\mathcal{J}}_{1}(\chi_{2}) = \bigotimes = \bigcirc$  $V_{1}(x_{1}) = \gamma r_{5}$  $\hat{V}_2(x_2) = \chi m_{\mu}^{0,\mu} (0 + \delta V_3, 0 + 0) = \chi^2 V_3 / 1$ 013 013  $\sqrt[n]{\sqrt{2}(\chi_2)} = 0$ 

## Grid world example

- States: cells in 10 × 10 grid
- Actions: up, down, left, right
- Transition model: 0.7 chance of moving in intended direction, uniform in other directions
- Reward:
  - two states with cost
  - two terminal states with rewards
  - -1 for wall crash
- Discount is 0.9



- <mark>0</mark> .2	- <b>0</b> .1	- <mark>0</mark> .1	- <b>0</b> .1	- <mark>0</mark> .2					
- <mark>0</mark> .1	0	0	0	0	0	0	0	0	- <mark>0</mark> .1
- <mark>0</mark> .1	0	0	0	0	0	0	3	0	- <mark>0</mark> .1
- <mark>0</mark> .1	0	0	0	0	0	0	0	0	- <mark>0</mark> .1
- <mark>0</mark> .1	0	0	-5	0	0	0	0	0	- <mark>0</mark> .1
- <mark>0</mark> .1	0	0	0	0	0	0	0	0	- <mark>0</mark> .1
<mark>0</mark> .1	0	0	0	0	0	0	0	0	- <mark>0</mark> .1
<mark>0</mark> .1	0	0	<b>_1</b> 0	0	0	0	0	10	- <mark>0</mark> .1
<b>-0.1</b>	0	0	0	0	0	0	0	0	- <b>0</b> .1
-0.2	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.2



— <mark>0</mark> .35	— <mark>0</mark> .16	- <mark>0</mark> .14	— <mark>0</mark> .14	— <mark>0</mark> .14	— <mark>0</mark> .14	— <mark>0</mark> .14	1. <mark>0</mark> 5	— <mark>0</mark> .16	-0.35
	_	_	_	_	_	_	_		
-0.16	- <mark>0</mark> .03	- <mark>0</mark> .01	- <mark>0</mark> .01	- <mark>0</mark> .01	- <mark>0</mark> .01	1. <mark>3</mark> 5	1.88	1.33	-0.16
-0.14	-0.01	— <mark>0</mark>	-0.04	— <mark>0</mark>	1.19	1.89	3	1.88	1.05
<b>-0.14</b>	-0.01	-0.08	-0.45	-0.08	0	1.36	1.89	1.35	-0.14
-0.14	— <mark>0</mark> .06	— <mark>0</mark> .45	— <mark>5</mark> .4	— <mark>0</mark> .45	- <mark>0</mark> .04	0	1.19	-0.01	-0.14
-0.14	-0.01	— <mark>0</mark> .08	-0.53	- <mark>0</mark> .08	0	0	-0	3. <mark>9</mark> 5	-0.14
-0.14	-0.01	- <b>0.16</b>	-0.94	-0.16	0	0	4. <mark>5</mark> 4	6. <mark>2</mark> 9	4.4
-0.14	-0.1	-0.9	-10.8	L — 0.9	-0.08	3.97	6.3	10	6.73
-0.16	-0.03	-0.19	-0.92	-0.18	-0.01	-0.01	4.52	6.27	4.37
-0.35	-0.16	-0.14	-0.28	-0.14	-0.14	<b>-0.14</b>	-0.14	3.81	-0.35

— <mark>0</mark> .38	— <mark>0</mark> .18	— <mark>0</mark> .15	— <mark>0</mark> .15	— <mark>0</mark> .15	— <mark>0</mark> .15	0. <mark>8</mark> 2	1. <mark>1</mark> 5	0. <mark>7</mark> 9	- <mark>0</mark> .38
-0.18	— <mark>0</mark> .04	— <mark>0</mark> .02	-0.03	- <mark>0</mark> .02	0.94	1. <mark>3</mark> 5	2. <mark>2</mark> 3	1. <mark>3</mark> 2	0.79
-0.15	-0.02	-0.02	-0.04	0.74	1.19	2.24	3	2.23	1.15
-0.15	- <b>0.0</b> 3	- <b>0.0</b> 8	- <mark>0.5</mark> 3	-0.08	0.95	1.36	2.24	1.35	0.82
-0.17	— <mark>0</mark> .06	-0.54	-5.41	-0.53	— <mark>0</mark> .04	0.96	1.19	2.7	-0.15
-0.15	— <mark>0</mark> .03	-0.11	-0.63	-0.1	- <mark>0</mark> .01	<b>_0</b>	3. <mark>3</mark> 2	3. <mark>9</mark> 5	3
-0.15	<b>-0.04</b>	-0.18	-1.14	-0.17	-0.02	3.21	4. <mark>5</mark> 3	7. <mark>4</mark> 6	5. <mark>0</mark> 9
-0.2	<b>-0.1</b>	-1.07	-10.82	2 —1.06	2.42	3.96	7.47	10	7.6
-0.18	-0.11	-0.2	-1.13	-0.18	-0.04	3.19	4.52	7.44	5.07
-0.38	-0.18	-0.26	-0.31	-0.24	-0.15	-0.15	3.07	4.15	2.83

<b>-0.4</b>	— <mark>0</mark> .19	- <mark>0</mark> .15	— <mark>0</mark> .16	— <mark>0</mark> .15	0. <mark>5</mark> 4	0. <mark>9</mark> 1	1. <mark>5</mark> 5	0. <mark>8</mark> 7	0.3
		_				_	_	_	
-0.19	— <mark>0</mark> .04	- <mark>0</mark> .03	-0.03	0.64	0.94	1. <mark>7</mark> 7	2. <mark>2</mark> 3	1. <mark>7</mark> 4	0.87
-0.15	-0.03	-0.02	0.41	0.74	1.65	2.24	3	2.23	1.55
-0.16	-0.04	-0.11	-0.53	0.57	0.95	1.79	2.24	2. <mark>1</mark> 8	0.91
<b>-0.18</b>	— <mark>0</mark> .09	-0.54	-5.48	-0.53	0.64	0.96	2. <mark>6</mark> 2	2. <mark>7</mark>	2. <mark>0</mark> 9
-0.16	— <mark>0</mark> .06	-0.14	-0.66	-0.13	-0.01	2.47	3. <mark>3</mark> 2	5.51	3. <mark>7</mark> 2
<b>-0.17</b>	-0.06	-0.24	<b>-1.16</b>	-0.22	2.23	3.21	5.97	7.52	6. <mark>0</mark> 8
<b>-0.21</b>	-0.15	<b>-1.07</b>	-10.9	ō 0.52	2.39	5.5	7.47	10	7.8
-0.24	-0.12	-0.29	-1.14	-0.25	2.2	3.19	5.94	7.54	6.07
-0.24	-0.12	-0.29	-1.14	-0.25	2.2	3.19	5.94	7.54	6.07

<b>-0.4</b>	— <mark>0</mark> .19	- <mark>0</mark> .15	— <mark>0</mark> .16	— <mark>0</mark> .15	0. <mark>5</mark> 4	0. <mark>9</mark> 1	1. <mark>5</mark> 5	0. <mark>8</mark> 7	0.3
		_				_	_	_	
-0.19	— <mark>0</mark> .04	- <mark>0</mark> .03	-0.03	0.64	0.94	1. <mark>7</mark> 7	2. <mark>2</mark> 3	1. <mark>7</mark> 4	0.87
-0.15	-0.03	-0.02	0.41	0.74	1.65	2.24	3	2.23	1.55
-0.16	-0.04	-0.11	-0.53	0.57	0.95	1.79	2.24	2. <mark>1</mark> 8	0.91
<b>-0.18</b>	— <mark>0</mark> .09	-0.54	-5.48	-0.53	0.64	0.96	2. <mark>6</mark> 2	2. <mark>7</mark>	2. <mark>0</mark> 9
-0.16	— <mark>0</mark> .06	-0.14	-0.66	-0.13	-0.01	2.47	3. <mark>3</mark> 2	5.51	3. <mark>7</mark> 2
<b>-0.17</b>	-0.06	-0.24	<b>-1.16</b>	-0.22	2.23	3.21	5.97	7.52	6. <mark>0</mark> 8
<b>-0.21</b>	-0.15	<b>-1.07</b>	-10.9	ō 0.52	2.39	5.5	7.47	10	7.8
-0.24	-0.12	-0.29	-1.14	-0.25	2.2	3.19	5.94	7.54	6.07
-0.24	-0.12	-0.29	-1.14	-0.25	2.2	3.19	5.94	7.54	6.07

- <mark>0</mark> .41	- <mark>0</mark> .19	- <mark>0</mark> .17	- <mark>0</mark> .16	0. <mark>3</mark> 3	0. <mark>6</mark> 1	1. <mark>2</mark> 9	1. <mark>6</mark> 1	1. <mark>2</mark> 4	0.48
-0.19	-0.06	— <mark>0</mark> .04	0.43	0.64	1.37	1. <mark>7</mark> 8	2. <mark>3</mark> 5	1.77	1.24
-0.17	-0.04	0.24	0.41	1.19	1.65	2.36	3	2.38	1.61
-0.17	-0.05	-0.12	-0.11	0.57	1.38	1.79	2.48	2. <mark>1</mark> 9	1.68
-0.2	-0.09	-0.57	-5.49	-0.05	0.64	2.09	2. <mark>6</mark> 2	4. <mark>0</mark> 9	2. <mark>7</mark> 6
-0.18	— <mark>0</mark> .07	-0.17	-0.69	-0.14	1.8	2.46	4. <mark>7</mark> 1	5. <mark>6</mark> 2	4. <mark>7</mark> 5
-0.19	-0.09	-0.25	-1.21	1.33	2.22	4.68	6	7.88	6. <mark>3</mark> 7
-0.25	-0.16	-1.13	-9.98	0.48	3.91	5.5	7.87	10	8
-0.25	-0.16	-0.3	-1.2	1.31	2.18	4.63	6.01	7.88	6.39
-0.44	-0.26	-0.34	-0.43	-0.29	1.44	2.5	4.64	5.8	4.81

- <mark>0</mark> .41	- <mark>0</mark> .21	- <mark>0</mark> .17	0. <mark>1</mark> 7	0. <mark>3</mark> 7	0. <mark>9</mark> 6	1. <mark>3</mark> 4	1. <mark>7</mark> 5	1. <mark>3</mark> 1	0.78
-0.21	-0.06	0.27	0.42	1.04	1.38	1. <mark>9</mark> 4	2. <mark>3</mark> 5	1. <mark>9</mark> 4	1.31
-0.17	0.13	0.24	0.8	1.18	1.84	2.36	3	2.39	1.81
-0.18	-0.06	0.09	-0.11	0.96	1.38	2.09	2.48	3. <mark>1</mark> 7	2. <mark>1</mark> 3
-0.21	-0.11	-0.58	-5.15	-0.05	1.6	2.09	3. <mark>7</mark> 5	4. <mark>2</mark> 2	3. <mark>6</mark> 6
-0.2	— <mark>0</mark> .09	-0.18	-0.7	1.19	1.8	3.74	4. <mark>7</mark> 4	6. <mark>1</mark> 9	5. <mark>1</mark>
-0.21	-0.1	-0.28	-0.14	1.32	3.58	4.7	6.52	7. <mark>9</mark> 2	6. <mark>6</mark> 5
-0.27	-0.19	-1.05	-10.02	<sup>2</sup> 1.8	3.9	6. <b>1</b> 5	7.88	10	8.07
-0.29	-0.18	-0.34	-0.14	1.28	3.52	4.7	6.5	7.94	6.65
-0.46	-0.3	-0.36	-0.46	0.86	1.77	3.59	4.85	6.23	5.21

-0.42	- <mark>0</mark> .21	0. <mark>0</mark> 5	0. <mark>2</mark>	0. <mark>6</mark> 9	1. <mark>0</mark> 1	1. <mark>4</mark> 8	1. <mark>7</mark> 8	1. <mark>4</mark> 7	0.89
-0.21	0.15	0.27	0.77	1.05	1.57	1. <mark>9</mark> 4	2.4	1. <mark>9</mark> 5	1.47
-0.07	0.13	0.55	0.8	1.41	1.84	2.42	3	2. <mark>6</mark>	1.88
-0.19	0.06	0.09	0.22	0.96	1.71	2.09	3. <mark>1</mark> 1	3. <mark>2</mark> 9	2. <mark>8</mark> 5
-0.22	-0.12	-0.43	-5.16	0.74	1.6	3.03	3. <mark>7</mark> 8	4. <mark>8</mark> 5	4. <mark>0</mark> 1
-0.21	- <mark>0</mark> .11	-0.2	0.26	1.18	2.93	3.76	5. <mark>3</mark> 4	6. <mark>2</mark> 5	5. <mark>4</mark> 3
-0.22	-0.12	-0.19	-0.16	2.51	3.59	5.32	6.55	8. <mark>0</mark> 4	6. <mark>7</mark> 5
-0.29	-0.2	-1.08	-8.98	1.79	4.67	6.1 <b>6</b>	8.03	10	8.12
-0.3	-0.21	-0.23	-0.16	2.44	3.59	5.29	6.57	8.05	6.76
-0.49	-0.31	-0.39	0.36	1.17	2.72	3.86	5.32	6.37	5.49

-0.43	— <mark>0</mark> .06	0. <mark>0</mark> 7	0. <mark>4</mark> 7	0. <mark>7</mark> 3	1. <mark>1</mark> 7	1. <mark>5</mark> 1	1. <mark>8</mark> 4	1. <mark>5</mark>	1. <mark>0</mark> 2
-0.07	0.14	0.55	0.77	1.24	1.58	2. <mark>0</mark> 1	2.4	2. <mark>1</mark> 2	1.51
-0.06	0.36	0.55	1.03	1.41	1.95	2.42	3	2. <mark>6</mark> 9	2. <mark>2</mark> 3
-0.1	0.06	0.33	0.22	1.29	1.71	2.6	3. <mark>1</mark> 3	3. <mark>8</mark> 2	3. <mark>1</mark> 5
-0.23	-0.03	-0.44	-4.53	0.74	2.39	3.05	4. <mark>3</mark> 5	4. <mark>9</mark> 4	4. <mark>3</mark> 8
-0.23	-0.12	0.1	0.25	2.16	2.94	4.38	5. <mark>3</mark> 7	6. <mark>4</mark> 7	5. <mark>5</mark> 7
-0.24	-0.12	-0.21	0.78	2.52	4.26	5.34	6.75	8. <mark>0</mark> 6	6. <mark>8</mark> 4
-0.3	-0.22	-0.96	-9	2.58	4.69	6.43	8.03	10	8.15
-0.33	-0.2	-0.25	0.74	2.51	4.22	5.36	6.75	8.07	6.84
-0.51	-0.34	0.04	0.62	1.97	3.01	4.32	5.44	6.51	5.62

0.41	0. <mark>7</mark> 4	0. <mark>9</mark> 6	1. <mark>1</mark> 8	1. <mark>4</mark> 3	1. <mark>7</mark> 1	1. <mark>9</mark> 8	2. <mark>1</mark> 1	2. <mark>3</mark> 9	2. <mark>0</mark> 9
0.74	1.04	1.27	1.52	1. <mark>8</mark> 1	2. <mark>1</mark> 5	2. <mark>4</mark> 7	2.58	3. <mark>0</mark> 2	2. <mark>6</mark> 9
0.86	1.18	1.45	1.76	2.15	2. <mark>5</mark> 5	2. <mark>9</mark> 7	3	3. <mark>6</mark> 9	3. <mark>3</mark> 2
0.84	1.11	1.31	1.55	2.45	3.01	3. <mark>5</mark> 6	4. <mark>1</mark>	4. <mark>5</mark> 3	4. <mark>0</mark> 4
0.91	1. <mark>2</mark>	1. <mark>0</mark> 9	-3	2.48	3.53	4. <mark>2</mark> 1	4. <mark>9</mark> 3	5. <mark>5</mark>	<b>4.</b> 88
_	_	_							
1.1	1.46	1.79	2.24	3.42	4.2	4.97	5. <mark>8</mark> 5	6. <mark>6</mark> 8	5. <mark>8</mark> 4
1.06	1.41	1.7	2.14	3.89	4.9	5.85	6.92	8. <mark>1</mark> 5	6.94
0.92	1. <mark>1</mark> 8	0. <mark>7</mark>	-7.39	3.43	5.39	6.67	8.15	10	8.19
1.09	1.45	1.75	2.18	3.89	4.88	5.84	6.92	8.15	6.94
1.07	1.56	2.05	2.65	3.38	4.11	4.92	5.83	6.68	5.82

Converged!

 $\gamma = 0.9$ 

 $\gamma = 0.5$ 

0.41	0. <mark>7</mark> 4	0. <mark>9</mark> 6	1. <mark>1</mark> 8	1. <mark>4</mark> 3	1. <mark>7</mark> 1	1. <mark>9</mark> 8	2. <mark>1</mark> 1	2. <mark>3</mark> 9	2. <mark>0</mark> 9	-0.28	- <mark>0</mark> .13	— <mark>0</mark> .12	- <mark>0</mark> .11	— <mark>0</mark> .09	— <mark>0</mark> .04	0. <mark>0</mark> 8	0. <mark>3</mark> 1	0. <mark>0</mark> 7	— <mark>0</mark> .19
0.74	1.04	1.27	1.52	1.8 <mark>1</mark>	2. <mark>1</mark> 5	2. <mark>4</mark> 7	2.58	3. <mark>0</mark> 2	2. <mark>6</mark> 9	-0.13	-0.01	0	0.02	0.07	0.18	0. <mark>4</mark> 6	1. <mark>1</mark> 1	0. <mark>4</mark> 5	0.07
0.86	1.18	1.45	1.76	2.15	2. <mark>5</mark> 5	2. <mark>9</mark> 7	3	3. <mark>6</mark> 9	3. <mark>3</mark> 2	-0.12	-0	0.01	0.04	0.15	0.42	1.12	3	1.11	0.31
0.84	1.11	1.31	1.55	2.45	3.01	3. <mark>5</mark> 6	4. <mark>1</mark>	4. <mark>5</mark> 3	4. <mark>0</mark> 4	-0.12	-0.01	<b>-0.02</b>	-0.24	0.05	0.19	0.47	1.12	0.48	0.09
0.91	1. <mark>2</mark>	1. <mark>0</mark> 9	-3	2.48	3.53	4. <mark>2</mark> 1	4. <mark>9</mark> 3	5. <mark>5</mark>	4.8 <mark>8</mark>	-0.13	-0.02	<b>-0.27</b>	-5.12	-0.23	0.08	0.2	0.46	0. <mark>5</mark> 4	0.13
1.1	1.46	1.79	2.24	3.42	4.2	4.97	5.85	6. <mark>6</mark> 8	5.84	-0.12	_ <mark>0</mark> .01	-0.04	-0.28	0.02	0.11	0.28	0. <mark>6</mark> 5	1. <mark>3</mark> 9	0. <mark>5</mark> 3
1.06	1.41	1.7	2.14	3.89	4.9	5.85	6.92	8.15	6.94	-0.12	-0.02	-0.06	-0.51	0.05	0.26	0.64	1.5 <mark>5</mark>	3. <mark>7</mark> 2	1. <mark>4</mark> 9
0.92	1. <mark>1</mark> 8	0. <mark>7</mark>	-7.39	3.43	5.39	6.67	8.15	10	8.19	-0.13	-0.04	-0.53	-10.19	0 -0.33	0.5	1.39	3.72	10	3.74
1.09	1.45	1.75	2.18	3.89	4.88	5.84	6.92	8.15	6.94	-0.14	-0.03	-0.07	-0.51	0.04	0.25	0.63	1.55	3.72	1.49
1.07	1.56	2.05	2.65	3.38	4.11	4.92	5.83	6.68	5.82	-0.28	-0.14	-0.15	-0.18	-0.1	-0.01	0.16	0.54	1.32	0.43

### Decision-Making Summary

- Given an MDP, we defined Expected Cumulative Payoff, which plays a key role in optimizing actions over planning horizons
- Used value iteration to determine the "value" of a particular state, which helps us determine the best action to take considering future payoff
- We generally assumed the transition and reward function are known exactly – but what if we don't have access to this information?
  - Will post notes on basic Q-learning for RL!



### Today's Plan

- MDP Policies and Value Iteration
- Simple Example
- Introduction to Safety
- Responsibility Sensitive Safety



## (Safety) Challenges for AVs

- Each module is challenging to develop
- AVs are safety critical systems, but:
  - It is impossible to guarantee zero accidents
  - Statistical approaches are problematic
  - Any software or hardware update requires new certification
- Scalability remains a challenge
  - How to mass produce expensive computation and sensors?
  - How to ensure you can drive everywhere?



#### How to ensure multi-agent safety?

#### Absolute safety is impossible 😕



#### Statistical guarantees are infeasible $\ensuremath{\mathfrak{S}}$ Typically, to show $p_1$ likelihood of failure, we aim to gather at least $\frac{1}{p_1}$ samples

- Are all miles equal?
- 10 disengagements per 300 million miles is not enough
- Any change to the software will require a new data collection



### **On-Road Testing**

- Data required to guarantee 10<sup>-9</sup> probability of failure: 10<sup>9</sup> hours of driving or 30 billion miles
- However, this is insufficient to show the difference between an AV with error rate of  $p_1$  and a perfect system
  - We want system with 10<sup>-9</sup> accident rate per hour, but we have an AV system with 10<sup>-8</sup> accident rate per hour.
  - If we drive for 10<sup>8</sup> hours, there is still a constant probability that the testing process is not telling us the true accident rate
- $\rightarrow$  Purely data-based approach for safety is naïve at best



#### Safety Guarantees Automata

- An **automata** is a mathematical model for describing computations or processes evolving in discrete steps
- *States* can be discrete or continuous valued
- State *transitions* define how the states can change
  - Transitions can be *non-deterministic:* multiple next states from a single state
- No inherent notion of *time*, but each transition can be thought of as passage of a fixed amount of time

An automaton is a tuple  $\mathcal{A}=\langle Q,\Theta, A, \mathcal{D} \rangle$  where

- Q is a set of states
- $\Theta \subseteq Q$  is the set of initial or start states
- *A* is a set of actions or labels
- D⊆Q×A×Q is the set of transitions
   → A transition can be thought of as a triple (u,a,u')





#### Safety Guarantees: Automata

• 
$$a_f[t] = a_{min}$$
  
•  $v_f[t+1] = v_f[t] + a_f[t]\Delta$   
•  $c_f[t+1] = c_f[t] + v_f[t]\Delta + \frac{1}{2}a[t]\Delta^2$ 



•  $a_r[t] \in [a_{min}, a_{max}]$ 

- $v_r[t+1] = v_r[t] + a_r[t]\Delta$
- $c_r[t+1] = c_r[t] + v_r[t]\Delta + \frac{1}{2}a_r[t]\Delta^2$

• 
$$X = \{a_f, v_f, c_f, a_r, v_r, c_r\}$$
  
•  $Q = \mathbb{R}^6$ 

•  $A = (a_1, a_2)$  the acceleration choices  $\mathcal{D} \subseteq \mathbb{R}^6 \times A \times \mathbb{R}^6$ 



#### Executions, Reachability, & Invariants

An execution of A is an alternating (possibly infinite) sequence of states s<sub>t</sub> and actions u<sub>t</sub>: α = s<sub>0</sub>u<sub>1</sub>s<sub>1</sub>u<sub>2</sub>s<sub>2</sub> ... such that:
s<sub>0</sub> ∈ Θ

•  $\forall i$  in the sequence the transition function is applied  $(s_i \xrightarrow{u_{i+1}} s_{i+1})$ 

- A state *s* is **reachable** if there exists an execution that ends at *s*
- $\rightarrow$  The set of reachable states is denoted by  $Reach_A$



#### Formal Invariants

• What does it mean for I to hold "always" for A?

- I holds at all states along any execution  $s_0u_1s_1u_2s_3$
- I holds in all reachable states of A
- $Reach_{\mathcal{A}} \subseteq [[I]]$

Invariants capture most properties that you will encounter in practice

- safety: "aircraft always maintain separation"
- bounded reaction time: "within 15 seconds of press, light must turn to walk"
- How to verify if *I* is an invariant?
  - Does there exist reachable state s such that  $s \neq I$ ?



#### **Reachability Problem**

#### **Automata**

Given a directed graph G = (V, E), and two sets of vertices  $S, T \subseteq V, T$  is reachable from S if there is a path from S to T.

#### Reachability Problem (G, S, T): is *T* is reachable from *S* in *G*?





#### **Reachable Sets**





#### City Safety: Emergency Braking

Image Credit: Volvo





# Responsibility Sensitive Safety

developed by Intel / MobilEye

Instead of looking at absolute safety, introduce a safety notion that depends on *responsibility* 

 $\bigcirc$ 

- $\rightarrow$  AV should never be responsible for an accident <u>RSS Rules:</u>
- 1. Keep a safe longitudinal distance from the car ahead.
- 2. Keep a safe lateral distance from the cars on your sides.
- 3. Respect right-of-way rules (multiple geometries, traffic lights, pedestrians, unstructured roads).
- 4. Be cautious of occluded areas. ~



#### RSS Example: Safe Following Distance

The longitudinal distance between a car  $(c_r)$  that drives behind another car  $(c_f)$  is safe w.r.t. a response time  $\rho$  if:

- $c_f$  applies at most  $a_{max}^{brake}$  .
- $c_r$  will apply at most  $a_{max}^{accel}$  during response time
- After ho,  $c_r$  will brake by at least  $a_{min}^{
  m brake}$  until full stop
- $\rightarrow c_{\gamma}$  will not collide with  $c_f$

Remarks:

- 1. This is basic reachability!
- 2. The safe distance depends on a set of parameters that can be determined by regulation.
- 3. The parameters can be different for a robotic car and a human driver.
- 4. The parameters can be different for different road conditions.



#### RSS Example: Safe Following Distance

- Let  $v_r$ ,  $v_f$  be the longitudinal velocities of the cars
- The minimal safe distance is:

$$d_{min} = \left[ v_r \cdot \rho + \frac{1}{2} a_{max}^{\text{accel}} \cdot \rho^2 + \frac{\left( v_r + \rho \cdot a_{max}^{\text{accel}} \right)^2}{2a_{min}^{\text{brake}}} - \frac{v_f^2}{2a_{max}^{\text{brake}}} \right]_+$$



#### Summary

- Discussed the challenges with defining safety and introduced the ideas behind verification and formal guarantees for safety
  - Reachability is as key tool for guaranteeing specified properties
- While useful, often require well-defined specifications, require highfidelity models (and assumptions), consider worst case scenarios, and/or only assess one piece of the puzzle
  - Making formal guarantees on the full autonomous stack remains a challenge
- Introduced RSS, a framework that aims to define safety in an intuitive manner



# Extra Slides



# Certifying airworthiness of aviation software

What fraction of the cost of developing a new aircraft is in SW?

#### DO178C

Primary document by which FAA & EASA approves software-based aerospace systems.

DAL establishes the rigor necessary to demonstrate compliance

Dev.Assuranc e Level (DAL)	Hazard Classification	Objectives
A	Catastrophic	71
В	Hazardous	69
С	Major	62
D	Minor	26
E	No Effect	0

**Statement Coverage:** Every statement of the source code must be covered by a test case

FLIGH

**Condition Coverage:** Every condition within a branch statement must be covered by a test case

**"Special credits":** For using formal methods based tools recently introduced



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#### Safety Certification (cont)

- A component's DAL level is determined from a safety assessment process and hazard analysis through examination of the effects of a failure condition in the system. The failure conditions are categorized by their effects on the aircraft, crew, and passengers.
  - For example, Level A is assigned for "Catastrophic Outcome," and Level E is for "No Safety Effect."
- DAL level then establishes the rigor necessary to demonstrate compliance with DO-178C
- E.g., components that command, control, or monitor safety critical functions are Level A. The standard requires any Level A software to be tested to cover every statement, branch, and function call, and also to pass the Modified Condition Decision Coverage (MC/DC) tests
  - Requires that (i) each entry and exit point in the code be invoked, (ii) each decision take every possible value, and (iii) each condition in a decision take every possible value
  - For certain levels, DO-178C requires that the testing, verification, and validation be performed by a team that is independent of the software development team
- Again, certification is process-based and does not eliminate possibility of bad logic, interference in the the control code
- Dozens of commercial tools (e.g., MATLAB, Esterel, Cantata, VectorCAST, Rapita Systems, and CodeSonar) can support DO-178C certification by applying **formal verification**.



#### The formal verification problem

Example requirements:

Safety: "For all nominal behaviors of the car, the separation between the cars must be always > 1 m"

Efficiency: "For all nominal driver inputs, the air-fuel ratio must be in the range [1,4]"

Privacy: "Using GPS does not compromise user's location"

Fairness: "Similar people's loan approval are decided similarly"





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