The solution of Quiz 1

1. **Problem 1**
    a. Yes, he should use a small sigma.
    Advantages: more detail with sharpening edges, less blur so it's good for edge detector
    Disadvantages: Also include more noises
    b. Reason: Sigma value is too small; threshold on the image after the Sobel filter is too small
    Fix: Use a higher sigma value; Use the median filter; set up a higher threshold
    c. F(ROS is not centralized) F T F
    d. More clusters -> more computational time, accuracy might increase but not always
    Higher k in kNN -> more computational time, accuracy might increase but not always

2. **Problem 2**
    a. Since Lipschitz continuous
    $|f(x_1) - f(x_2)| \leq K_f|x_1 - x_2|$
    $|g(x_1) - g(x_2)| \leq K_g|x_1 - x_2|$
    To show f+g is Lipschitz continuous, need to show
    $|(f+g)(x_1) - (f+g)(x_2)| \leq K(x_1 - x_2)$
    Therefore
    $$\begin{aligned}|(f+g)(x_1) - (f+g)(x_2)| &= |f(x_1) + g(x_1) - f(x_2) - g(x_2)| \\ &= |f(x_1) - f(x_2) + g(x_1) - g(x_2)| \\ &\leq |f(x_1) - f(x_2)| + |g(x_1) - g(x_2)| \\ &\leq K_f|x_1 - x_2| + K_g|x_1 - x_2| \\ &= (K_f + K_g)|x_1 - x_2|\end{aligned}$$

    b. (1) b
    (2) a
    (3) d or c
    (4) e
    (5) c or d

3. **Problem 3**
    a. The statement is not an invariant of the SimpleCar model.
    Consider at time t, $x_{goal} - v_{init} < x(t) < x_{goal}$.
    Then for time t+1, $v(t+1) = v(t) = v_{init}$
    $x(t+1) = x(t) + v(t+1) > x_{goal} - v_{init} + v_{init} > x_{goal}$

The statement is violated. Therefore, the statement is not an invariant of the SimpleCar model

b. The statement is an invariant of the SimpleCar model.
   Base case:
   $x(0) = 0 \le x_{goal}$, $v(0) = v_{init} \le v_{init}$
   Inductive Hypothesis: For time t, $x(t) < x_{goal} + v_{init}$, $v(t) \le v_{init}$
   Inductive Step:
   Suppose $x(t) \le x_{goal}$, $v(t+1) = v(t) \le v_{init}$
   $x(t+1) = x(t) + v(t+1) \le x(t) + v_{init} \le x_{goal} + v_{init}$
   Suppose $x(t) > x_{goal}$, $v(t+1) = 0$
   $x(t+1) = x(t) + v(t+1) = x(t) < x_{goal} + v_{init}$
   Therefore, the statement is an invariant of the SimpleCar model

4. **Problem 4**
   a. The eigen values of the A matrix
      A = [1, 1;2, -2] is given by
      eig(A) = 1.5616, -2.5616
      Therefore, the open loop system is not stable.
   b. $A_{cl} = A - BK = [1 - k_1, 1 - k_2; 2, -2]$
   c. The characteristic equation is given by
      $det(\lambda I - A_{cl}) = det([\lambda - 1 + k_1, k_2 - 1; -2, \lambda + 2])$
      $= \lambda^2 + (k_1 + 1)\lambda + 2(k_1 + k_2 - 2)$

   d. Since $\lambda = -2, -3$
      We get
      $2k_2 - 2 = 0$
      $2k_2 - k_1 + 2 = 0$
      Therefore, we get $k_1 = 4$, $k_2 = 1$