The solution of Quiz 1

1. Problem 1
   a. Yes, he should use a small sigma.
      Advantages: more detail with sharpening edges, less blur so it’s good for edge detector
      Disadvantages: Also include more noises
   b. Reason: Sigma value is too small; threshold on the image after the Sobel filter is too small
      Fix: Use a higher sigma value; Use the median filter; set up a higher threshold
   c. F(ROS is not centralized) F T F
   d. More clusters -> more computational time, accuracy might increase but not always
      Higher k in kNN -> more computational time, accuracy might increase but not always

2. Problem 2
   a. Since Lipschitz continuous
      \[ |f(x_1) - f(x_2)| \leq K_f |x_1 - x_2| \]
      \[ |g(x_1) - g(x_2)| \leq K_g |x_1 - x_2| \]
      To show \( f+g \) is Lipschitz continuous, need to show
      \[ |(f+g)(x_1) - (f+g)(x_2)| \leq K (x_1 - x_2) \]
      Therefore
      \[ |(f+g)(x_1) - (f+g)(x_2)| = |f(x_1) + g(x_1) - (f(x_2) + g(x_2))| \]
      \[ = |f(x_1) - (f(x_2) + g(x_1) - g(x_2))| \]
      \[ \leq |f(x_1) - f(x_2)| + |g(x_1) - g(x_2)| \]
      \[ \leq K_f |x_1 - x_2| + K_g |x_1 - x_2| \]
      \[ = (K_f + K_g) |x_1 - x_2| \]
   b. (1) b
      (2) a
      (3) d or c
      (4) e
      (5) c or d

3. Problem 3
   a. The statement is not an invariant of the SimpleCar model.
      Consider at time \( t \), \( x_{goal} - v_{init} < x(t) < x_{goal} \).
      Then for time \( t+1 \), \( v(t+1) = v(t) = v_{init} \)
      \( x(t+1) = x(t) + v(t+1) > x_{goal} - v_{init} + v_{init} > x_{goal} \)
The statement is violated. Therefore, the statement is not an invariant of the SimpleCar model

b. The statement is an invariant of the SimpleCar model.

Base case: 
\[ x(0) = 0 \leq x_{\text{goal}}, \quad v(0) = v_{\text{init}} \leq v_{\text{init}} \]

Inductive Hypothesis: For time t, \( x(t) < x_{\text{goal}} + v_{\text{init}} \), \( v(t) \leq v_{\text{init}} \)

Inductive Step: 
Suppose \( x(t) \leq x_{\text{goal}} \), \( v(t + 1) = v(t) \leq v_{\text{init}} \)
\[ x(t + 1) = x(t) + v(t + 1) \leq x(t) + v_{\text{init}} \leq x_{\text{goal}} + v_{\text{init}} \]
Suppose \( x(t) > x_{\text{goal}} \), \( v(t + 1) = 0 \)
\[ x(t + 1) = x(t) + v(t + 1) = x(t) < x_{\text{goal}} + v_{\text{init}} \]

Therefore, the statement is an invariant of the SimpleCar model

4. Problem 4
a. The eigen values of the A matrix
\[ A = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix} \]
\[ \text{eig}(A) = 1.5616, -2.5616 \]
Therefore, the open loop system is not stable.

b. \( A_{cl} = A - BK = \begin{bmatrix} 1 - k_1 & 1 - k_2 \\ 2 & -2 \end{bmatrix} \]

c. The characteristic equation is given by
\[ \text{det}(\lambda I - A_{cl}) = \text{det}([\lambda - 1 + k_1, k_2 - 1; -2, \lambda + 2]) \]
\[ = \lambda^2 + (k_1 + 1)\lambda + 2(k_1 + k_2 - 2) \]

d. Since \( \lambda = -2, -3 \)
We get
\[ 2k_2 - 2 = 0 \]
\[ 2k_2 - k_1 + 2 = 0 \]
Therefore, we get \( k_1 = 4, \quad k_2 = 1 \)