ECE 498SM: Principles of Safe Autonomy

Fall 2020 – Quiz 2

1. This is a closed book exam
2. You may use extra sheets of blank paper for answers as needed
3. You may use one sheet of pre-written notes
4. You may use a calculator, but this is not necessary
5. Absolutely no interaction between students is allowed
6. Illegible handwriting will be graded as incorrect

Name:________________________________

NetID:________________________________

Problems

1. Particle Filter: 25 Points
2. Clustering: 25 Points
3. Informed Search: 25 Points
4. Feedback control: 25 Points

1. Particle filter

Consider a world with only three possible robot locations: $X = x_1, x_2, x_3$. Consider a Monte Carlo Localization (Particle filter) algorithm which uses $N \geq 1$ *particles*. Initially, the particles are uniformly distributed over the three locations. Let Z be a boolean sensor characterized by the following *measurement model*:

$$
\begin{aligned}
p(Z = 1|X = x_1) &= 0.8, \qquad & p(Z = 0|X = x_1) &= 0.2 \\
p(Z = 1|X = x_2) &= 0.4, \qquad & p(Z = 0|X = x_2) &= 0.6 \\
p(Z = 1|X = x_3) &= 0.1, \qquad & p(Z = 0|X = x_3) &= 0.9
\end{aligned}
$$

Roughly, we have a high probability of observing $Z = 1$ at location x_1, and a high probability of observing $Z = 0$ at location x_3.

Based on the prior uniform distribution, calculate the true posterior $p(X = x_i|Z = 1)$ for each of the location $X = \{x_1, x_2, x_3\}$. **Show the intermediate expressions and the final result.**

2. Clustering

(a) Write the pseudocode for K-means clustering.

(b) Suppose K-means clustering on a given data set has converged and after that all the data points are scaled by a constant factor of α. Show that this change in data will not cause any changes in the clustering.

(c) Given a dataset {0,2,4,6,24,26}, initialize the k-means clustering algorithm with 2 cluster centers $c_1 = 3$ and $c_2 = 4$. What are the values of c_1 and c_2 after one iteration of k-means? What are the values of c_1 and c_2 after the second iterations of k-means?

(d) What are some advantages and disadvantages of K-means clustering? (give 2 for each)

3. Informed Search

Best First Search

(a) Explain why best-first search is not optimal.

(b) Give an example to illustrate why it is not optimal.

(c) Suggest a fix to best-first search to overcome the non-optimality.

Hybrid A*

The discrete coordinates are integers for each node *n* in the tree. The discrete coordinates are obtained by rounding the continuous coordinates to the nearest integers (Assume x.5 is rounded to x+1). Consider the heuristic function:

$$h_1(n) = |(x_n - x_{goal})| + |(y_n - y_{goal})|.$$

Consider the goal position $g\ at\ (0,0)$. The *cost-to-come* function is defined as $g(n) = g(previous(n)) + 1$. The steering angle $\delta \in \{0, -30°, 30°\}$; the speed $v = 1.0$. For heading angle θ, x-axis points to $0°$. The vehicle dynamic model is:

$$\theta_{t+1} = \theta_t + \delta$$
$$x_{t+1} = x_t + v * \cos(\theta_{t+1})$$
$$y_{t+1} = y_t + v * \sin(\theta_{t+1})$$

(i) The *open set* in the algorithm pops out a node n_0 with continuous coordinates $(x, y, \theta) = (3.0, 5.0, 90°)$ and g(n_0) = 13. Calculate and write the **continuous coordinates** of all the new nodes that will be expanded from n_0 and the **corresponding g values**.

(ii) Suppose, after n_0 is popped out, the open set has one node n_1 with continuous coordinates $(x, y, \theta) = (2.8\,, 5.7, 90°)$ and g(n_1) = 14. Write down the **continuous coordinates** (x, y, θ) of all the nodes in open set after this iteration is finished and the **corresponding h values**. *[Recall, in hybrid A*, if two nodes have the same discrete coordinates then only the one with the lower cost (f = g + h) is added to the tree.]*

Node name	**Cont. coord**	**Disc. coord**	**g**	**h**	**f**

4. Sampling Based Planning

RRT

1) If a path to the **goal** area exists, is the RRT algorithm guaranteed to find it eventually if given enough time? Why or why not?

2) Is RRT guaranteed to find the shortest path eventually? Why or why not?

3) In the RRT algorithm, when generating a random sample Xrand, suppose with some probability δ > 0 we pick the goal Xgoal. Will the algorithm preserve probabilistic completeness? How will the behavior of the algorithm change? Explain your answer

Conditional independence.

(a) When can we say that two events A and B are conditionally independent, given another event C with $P(C) > 0$.

(b) If A and B are conditionally independent, then show that $P(A|B, C) = P(A|C)$.