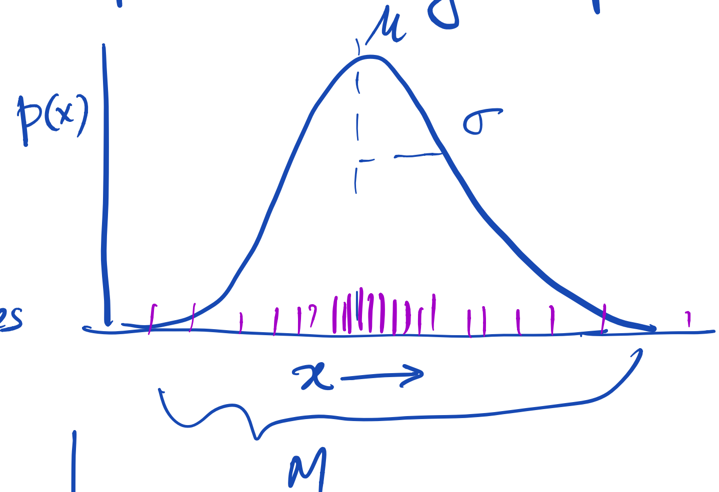
Lecture 14: Filtering
November 2, 2020
Particle filtering

- non-parametric method (Expressive)
- Particles for representing $bel(x_t)$
  ↳ nonlinear operations can be implemented with ease

## Overview

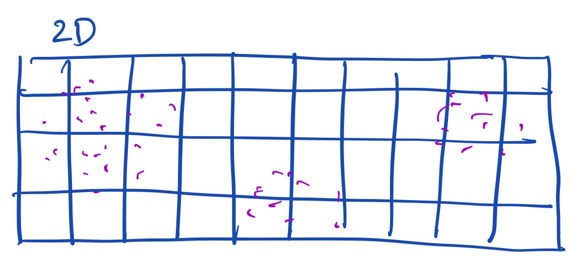
$bel(x_t)$ will be represented by samples

$N(\mu, \sigma)$
mean and s.d




Same number of samples can represent many diff distributions

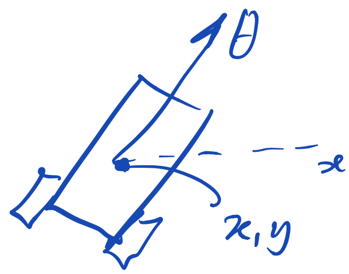
* Expressive
* Nonlinear ops will be easy
M



Particles : Samples of the dist $bel(x_t)$
are called particles

$M$ : Number of particles
Denote the set of particles $\{x_t^{[1]}, x_t^{[2]}, \ldots x_t^{[M]}\}$
$X_t$ $\quad |X_t| = M \approx 1000$

each $x_t^{[m]}$ $\quad 1 \leq m \leq M$
is a state vector for this system
e.g. 3D vehicle model
$x_t^{[m]} \in \mathbb{R}^3 \quad \langle x_t^{[m]}, y_t^{[m]}, \theta_t^{[m]} \rangle$




(informal meaning)
$x_t^{[m]}$ is included in $X_t$
with probability $bel(x_t)$

$= p(x_t \mid z_{t-1}, u_{t-1})$

Only holds asymptotically as $M \to \infty$

PF Algorithm $(X_{t-1}, u_t, z_t)$

(1) Prediction step $\bar{X}_t \leftarrow X_{t-1}, u_t$ Control input

(2) Correction / Measurement $X_t \leftarrow \bar{X}_t, z_t$

$X_t = x_t^{[1]}, x_t^{[2]}, \dots x_t^{[M]}$ particles

**Algorithm Particle_filter($X_{t-1}, u_t, z_t$):**

$\bar{X}_{t-1} = X_t = \emptyset$

for all $m$ in [M] do:

  sample $x_t^{[m]} \sim p(x_t | u_t, \boxed{x_{t-1}^{[m]}})$ *

  $w_t^{[m]} = p(z_t | x_t^{[m]})$

  $\bar{X}_t = \bar{X}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$

end for

for all $m$ in [M] do:

  draw $i$ *with probability* $\propto w_t^{[i]}$

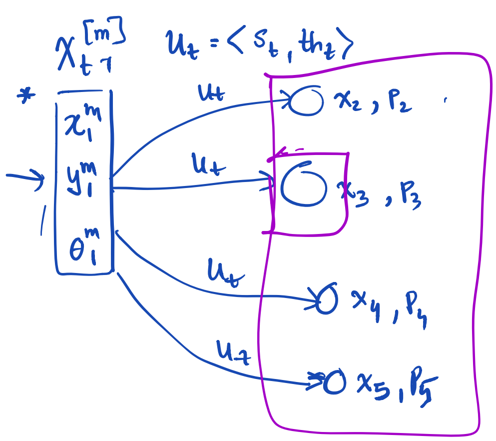
  add $x_t^{[i]}$ *to* $X_t$

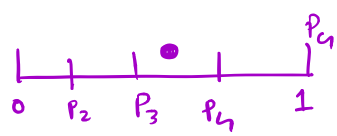
end for

return $X_t$

P — Sequential importance sampling

C — importance sampling



# Importance Sampling

"Survival of the fittest"

fitness ~ measurement

particle is more likely to be sampled from $\bar{X}$ if it has higher prob. of generating the measurement $z$

We want to sample from $\boxed{bel(x_t)}$

→ We do not have access to this

→ We do have $\boxed{\overline{bel}(x_t)} = \bar{X}_t$

How to sample from dist $f$ when we only have access to samples from $g$

$P_f(x \in A)$ but we can only sample from $g$.

$$P_f(x \in A) = E_f[\mathbb{I}(x \in A)] \qquad \mathbb{I}_A(x) = 1 \quad x \in A$$

$$= 0 \quad x \notin A$$

$$= \int_{x \in X} f(x)\, \mathbb{I}(x \in A)\, dx \quad \text{[Def of Expect]}$$

$$= \int_{x \in X} \underbrace{\boxed{\frac{f(x)}{g(x)}}}_{= W(x)} g(x)\, \mathbb{I}(x \in A)\, dx$$

[$g(x) > 0$ when $f(x) > 0$]

$$= \int_{x \in X} \underline{w(x)}\, g(x)\, \underline{\mathbb{I}(x \in A)}\, dx$$

$$= \mathbb{E}_g\left[\underbrace{\underline{w(x)}\, \mathbb{I}(x \in A)}_{\text{adjusted weights}}\right]$$

Next.

→ Limitations of PF

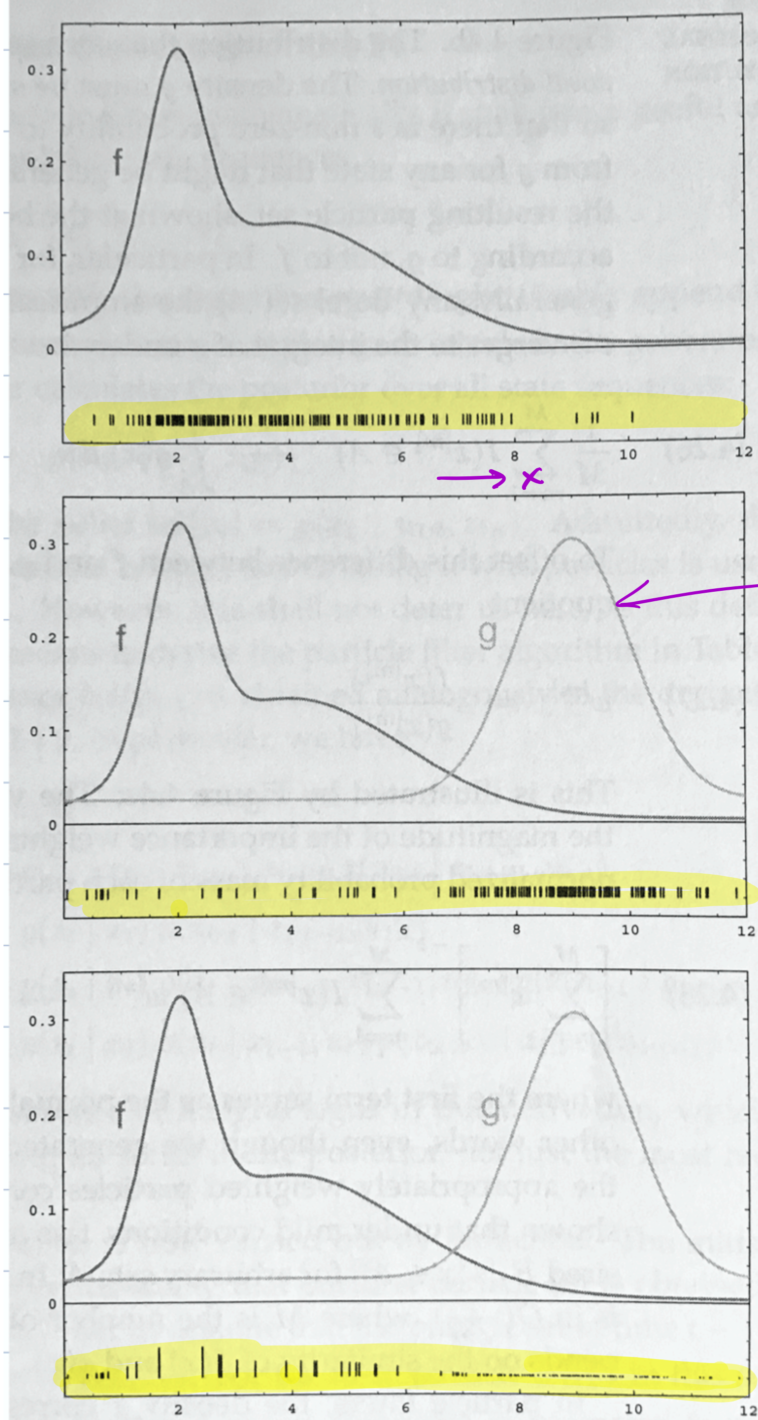
→ Why IS

→ Applications.

Review: Notes, Notes from Sp 20

Videos of Sp 20

Book ch 9.




we have samples from g

$$\boxed{w(x) = \frac{f(x)}{g(x)}}$$