Principles of Safe Autonomy: Lecture 15: Filtering applications and SLAM

Sayan Mitra

Reference: Probabilistic Robotics by Sebastian Thrun, Wolfram Burgard, and Dieter Fox
Slides: From the book's website

Outline of filtering and state estimation module

- Applications of Particle filter
    - Monte Carlo localization (MCL)
- Kahoot
- Overview of SLAM

Particle Filters

- Represent belief by finite number of parameters (just like histogram filter)
- But, they differ in how the parameters (particles) are generated and populate the state space
- Key idea: represent belief $bel(x_t)$ by a random set of state samples
- Advantages
  - The representation is approximate and nonparametric and therefore can represent a broader set of distributions than e.g., Gaussian
  - Can handle nonlinear tranformations
- Related ideas: Monte Carlo filter, Survival of the fittest, Condensation, Bootstrap filter, Filtering: [Rubin, 88], [Gordon et al., 93], [Kitagawa 96], Dynamic Bayesian Networks: [Kanazawa et al., 95]d

Particle filtering algorithm

$X_t = x_t^{[1]}, x_t^{[2]}, \ldots x_t^{[M]}$ particles

Algorithm Particle_filter(X_{t-1}, u_t, z_t):
$\bar{X}_{t-1} = X_t = \emptyset$

for all m in [M] do:

sample $x_t^{[m]} \sim p(x_t | u_t, x_{t-1}^{[m]})$

$w_t^{[m]} = p\left(z_t \middle| x_t^{[m]}\right)$

$\bar{X}_t = \bar{X}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$

end for

for all m in [M] do:

draw i *with probability* $\propto w_t^{[i]}$

add $x_t^{[i]}$ *to* X_t

end for

return X_t

ideally, $x_t^{[m]}$ is selected with probability prop. to $p(x_t \mid z_{1:t}, u_{1:t})$

$\bar{X}_{t-1}$ is the temporary particle set

// sampling from state transition dist.

// calculates *importance factor* w_t or weight

// resampling or importance sampling; these are distributed according to $\eta\, p\left(z_t \middle| x_t^{[m]}\right) \overline{bel}(x_t)$

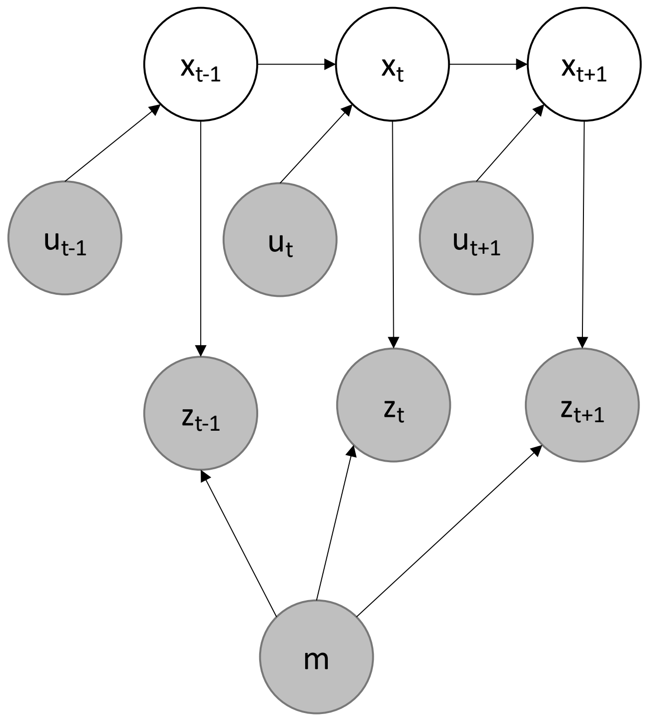
// survival of fittest: moves/adds particles to parts of the state space with higher probability

Localization as coordinate transformation

Shaded known:
map (m), control inputs (u), measurements(z). White nodes to be determined (x)

maps (m) are described in global coordinates. Localization = establish ***coord transf.*** between m and robot's local coordinates

Transformation used for objects of interest (obstacles, pedestrians) for decision, planning and control

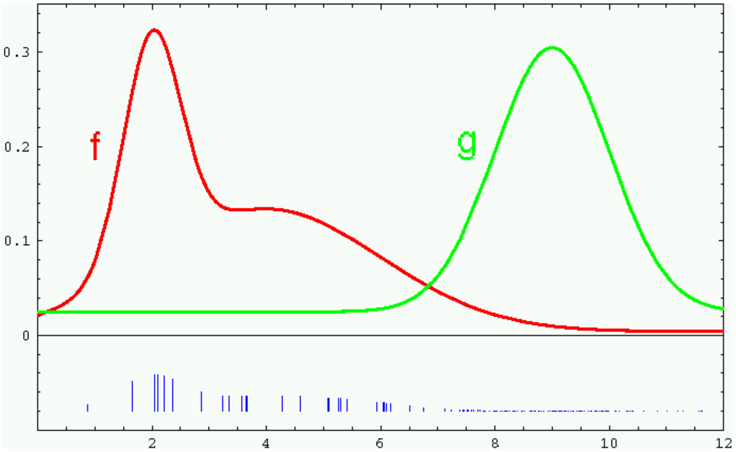


Monte Carlo Localization

- Represents beliefs by particles

Importance Sampling

suppose we want to compute $E_f[I(x \in A)]$ but we can only sample from density g



$E_f[I(x \in A)]$

$= \int f(x) I(x \in A) dx$

$= \int \frac{f(x)}{g(x)} g(x) I(x \in A) dx$, provided $g(x) > 0$

$= \int w(x) g(x) I(x \in A) dx$

$= E_g[w(x) I(x \in A)]$

We need $f(x) > 0 \Rightarrow g(x) > 0$

Weight samples: $w = f / g$

Monte Carlo Localization (MCL)

$X_t = x_t^{[1]}, x_t^{[2]}, \ldots x_t^{[M]}$ particles

Algorithm MCL(X_{t-1}, u_t, z_t,m):

$\bar{X}_{t-1} = X_t = \emptyset$

for all m in [M] do:

$$x_t^{[m]} = \boldsymbol{sample\_motion\_model}(u_t\ x_{t-1}^{[m]})$$

$$w_t^{[m]} = \boldsymbol{measurement\_model}(z_t, x_t^{[m],m})$$

$$\bar{X}_t = \bar{X}_t + \langle\, x_t^{[m]}, w_t^{[m]} \rangle$$

end for

for all m in [M] do:

draw i $with\ probability\ \propto w_t^{[i]}$

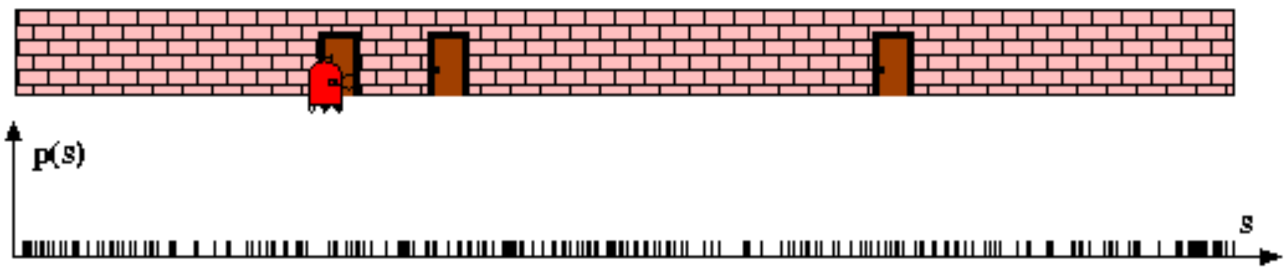
add $x_t^{[i]}$ $to\ X_t$

end for

return X_t

Plug in motion and measurement models in the particle filter

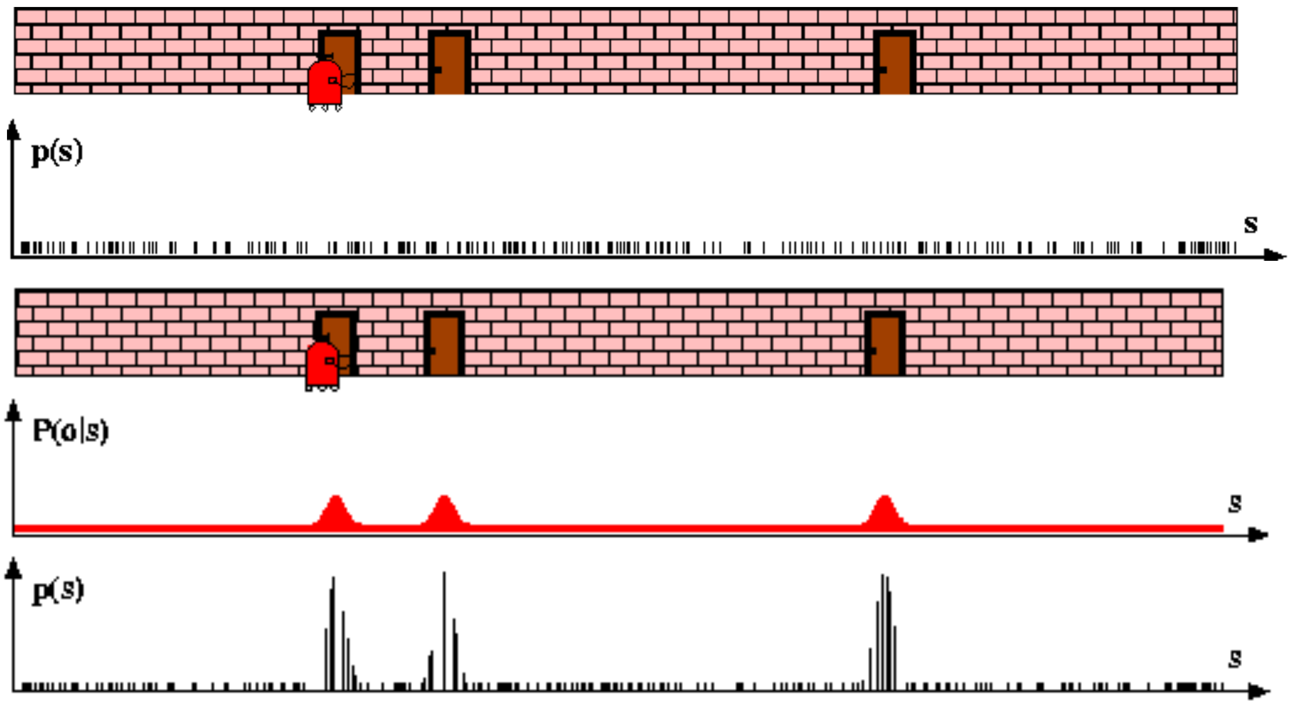
Particle Filters



Sensor Information: Importance Sampling

$$Bel(x) \leftarrow \alpha\ p(z\,|\,x)\ Bel^{-}(x)$$

$$w \leftarrow \frac{\alpha\ p(z\,|\,x)\ Bel^{-}(x)}{Bel^{-}(x)} = \alpha\ p(z\,|\,x)$$



Robot Motion

$$Bel^{-}(x) \leftarrow \int p(x \mid u, x')\, Bel(x')\ \mathrm{d}x'$$

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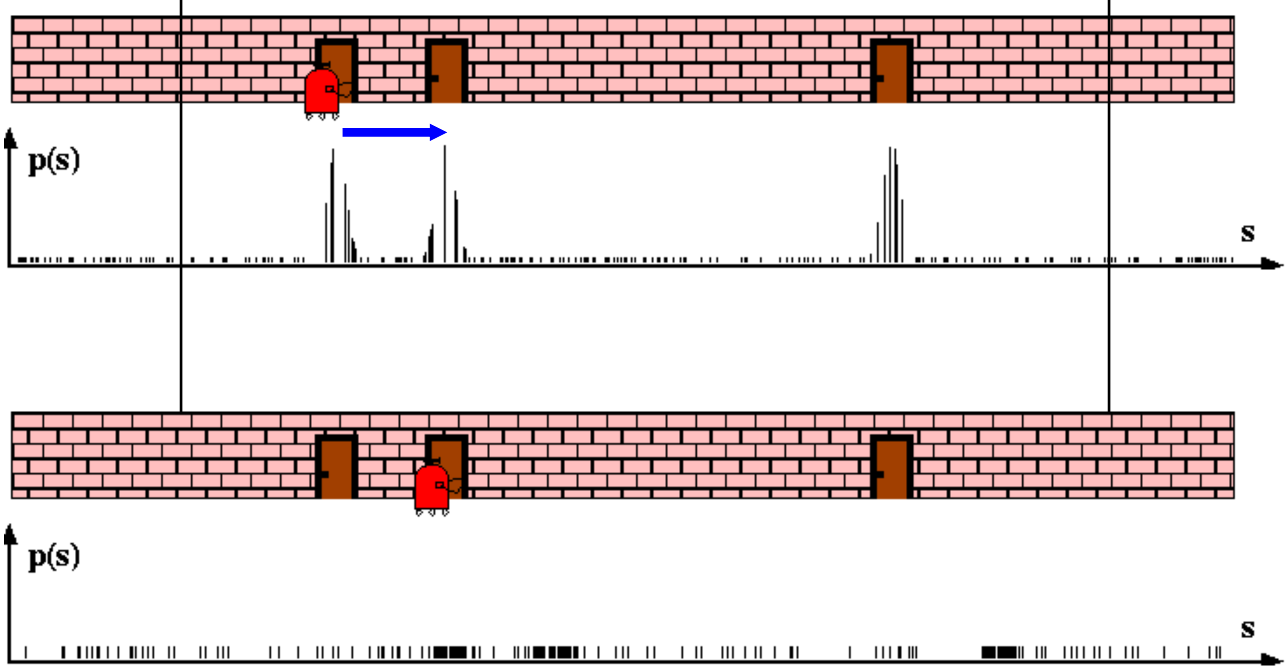

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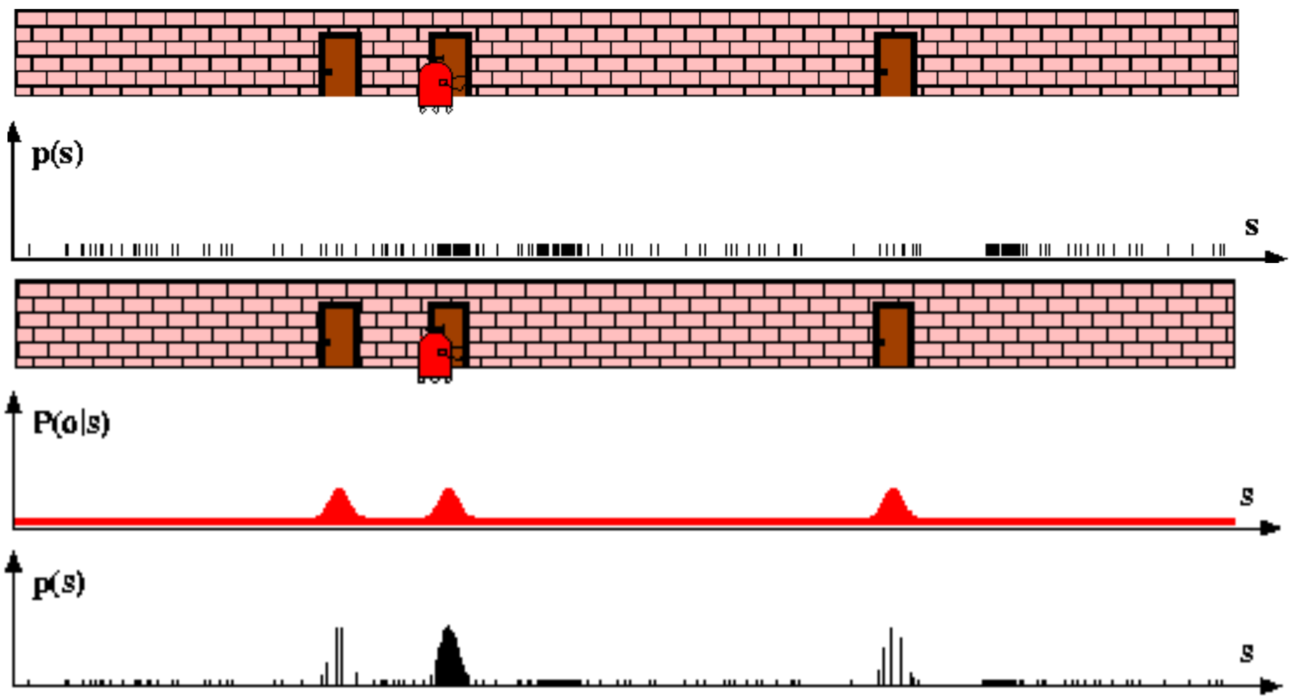

Robot Motion

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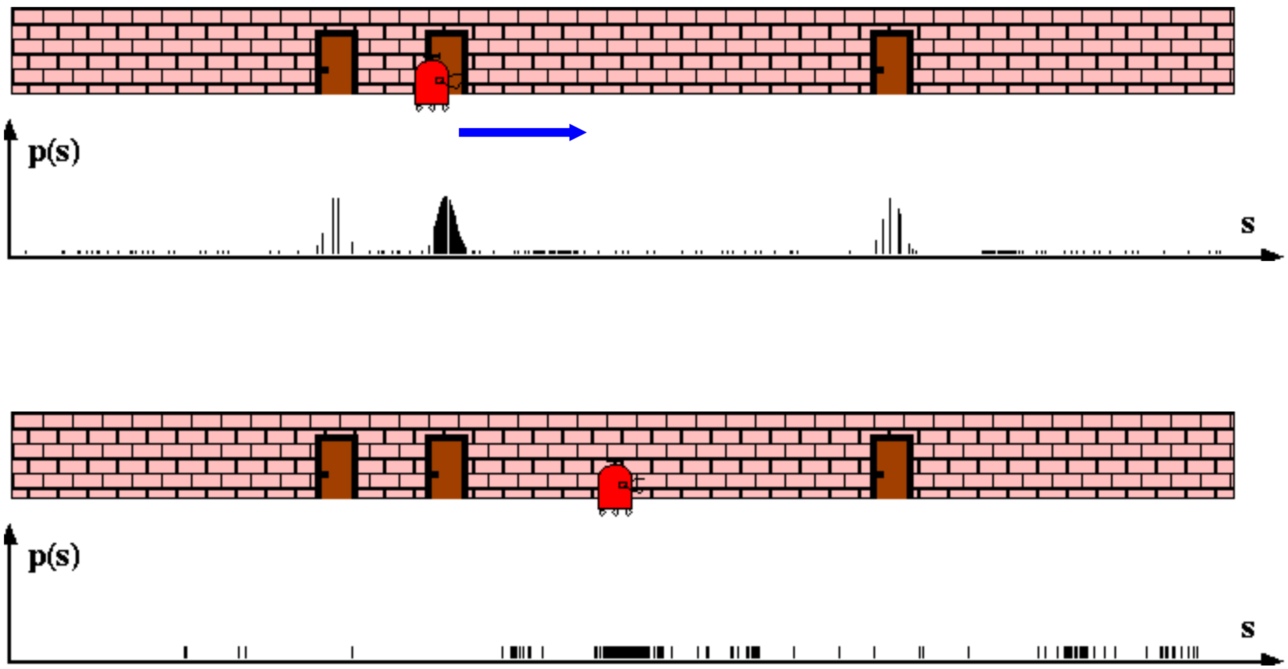
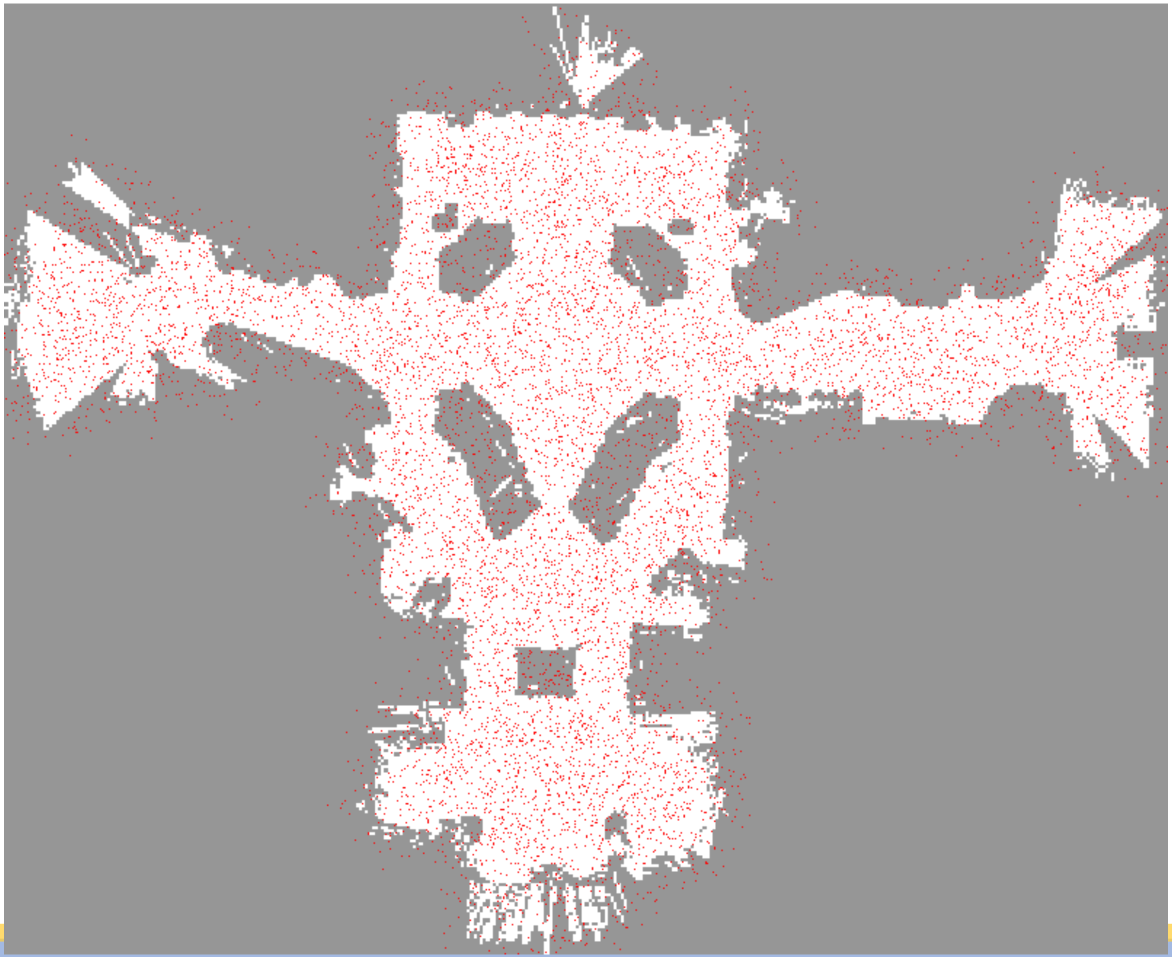
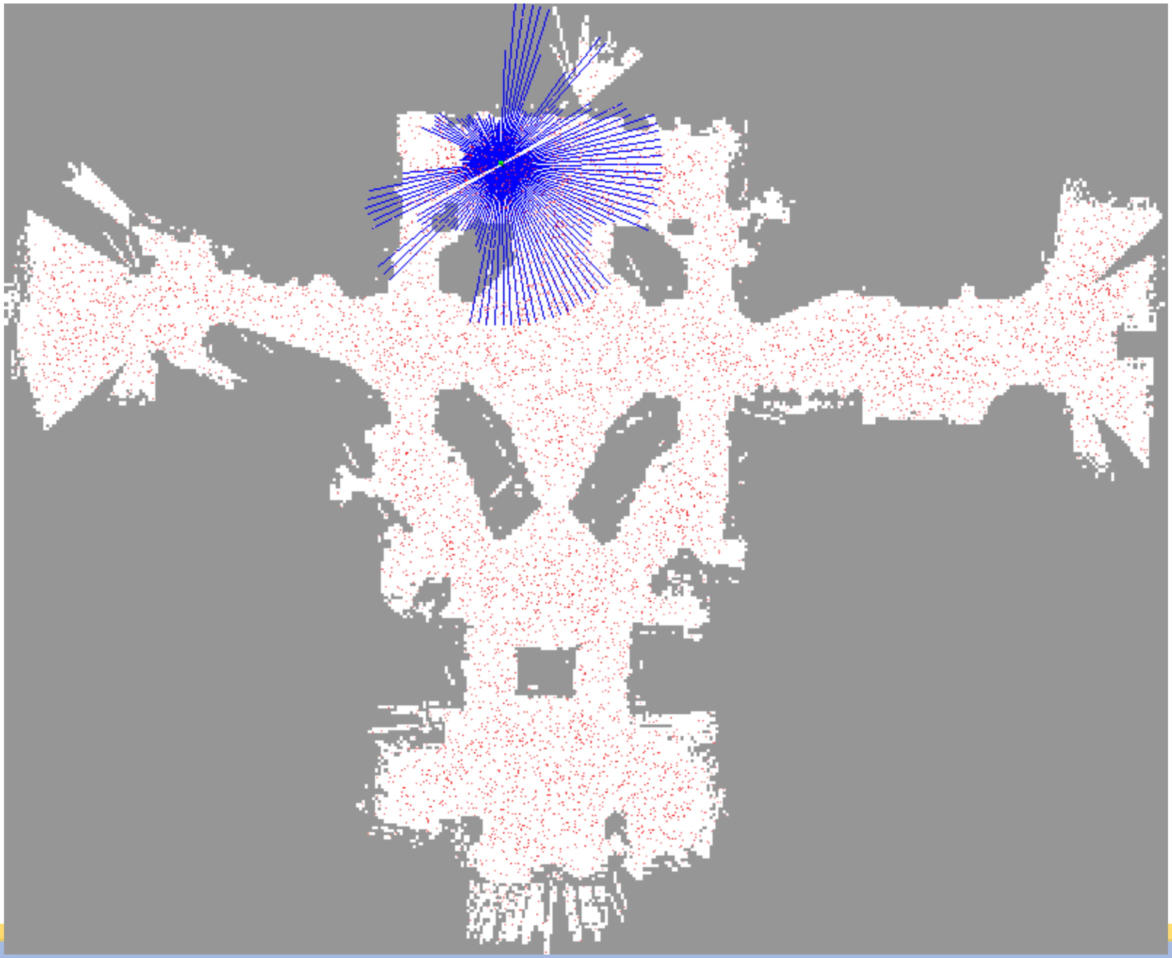
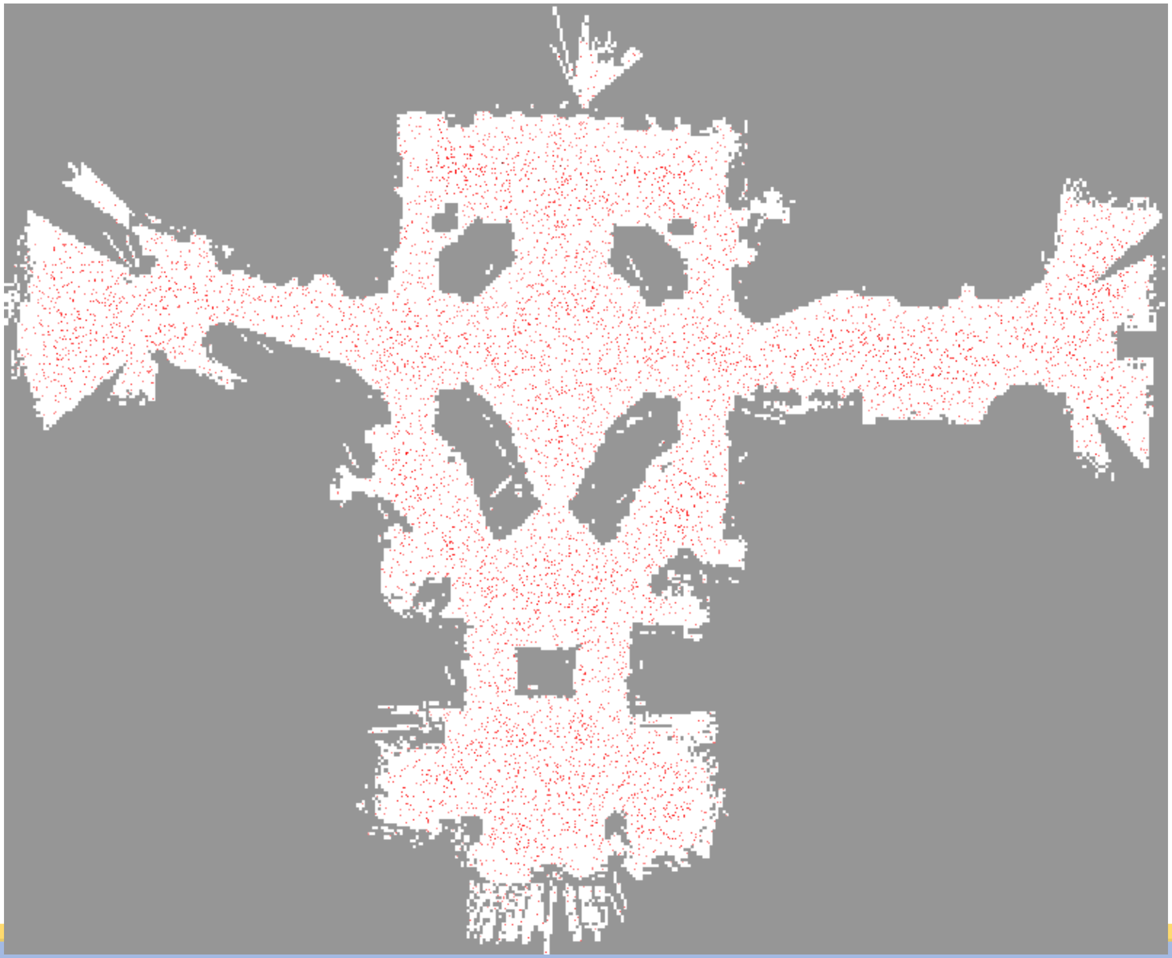
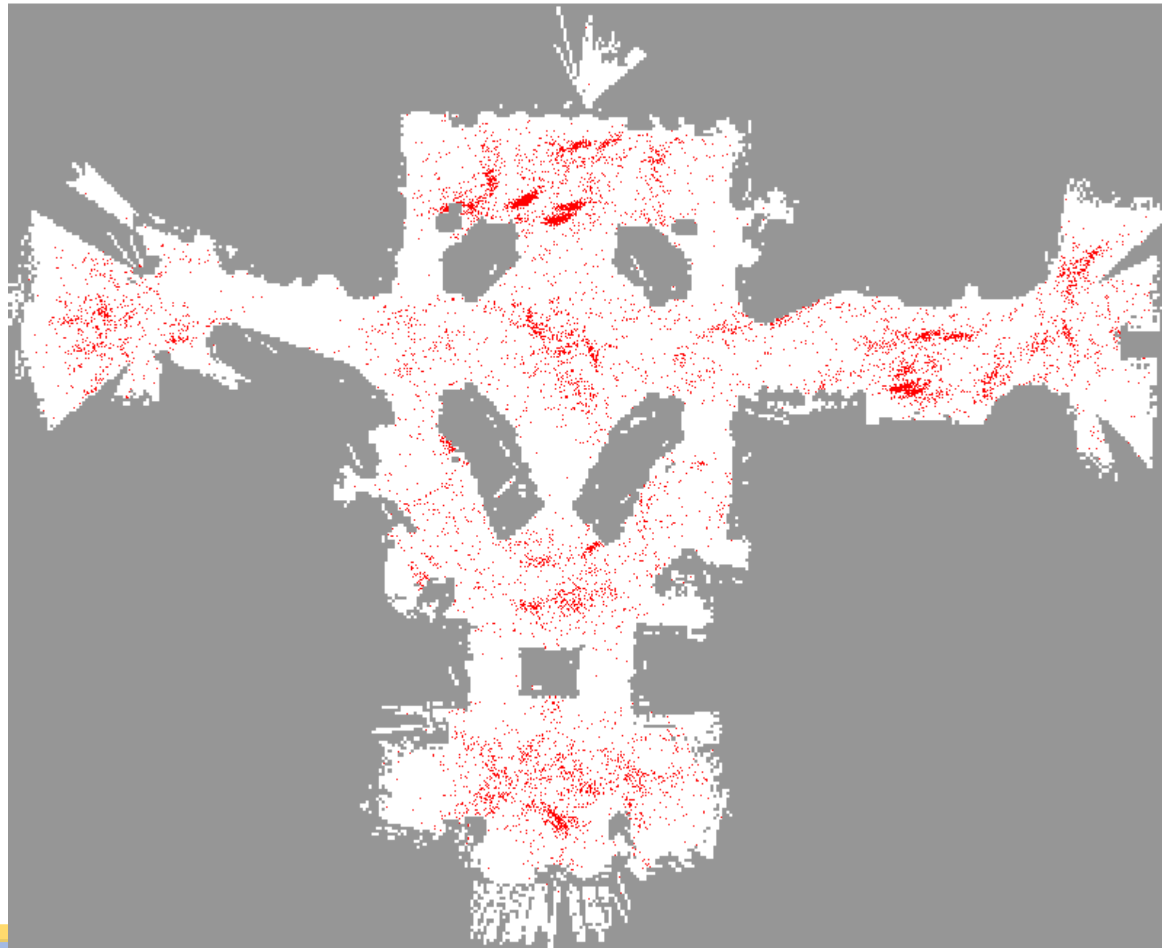
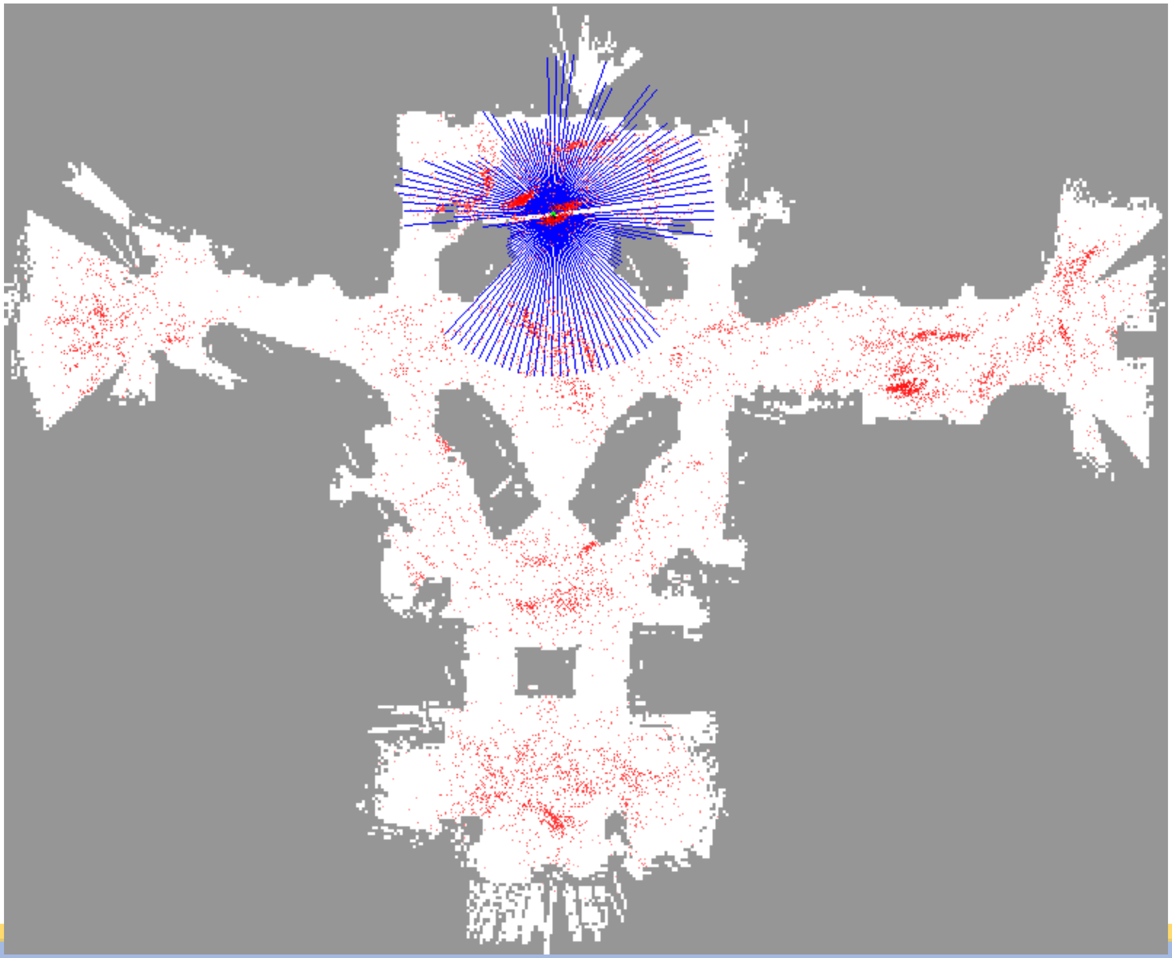


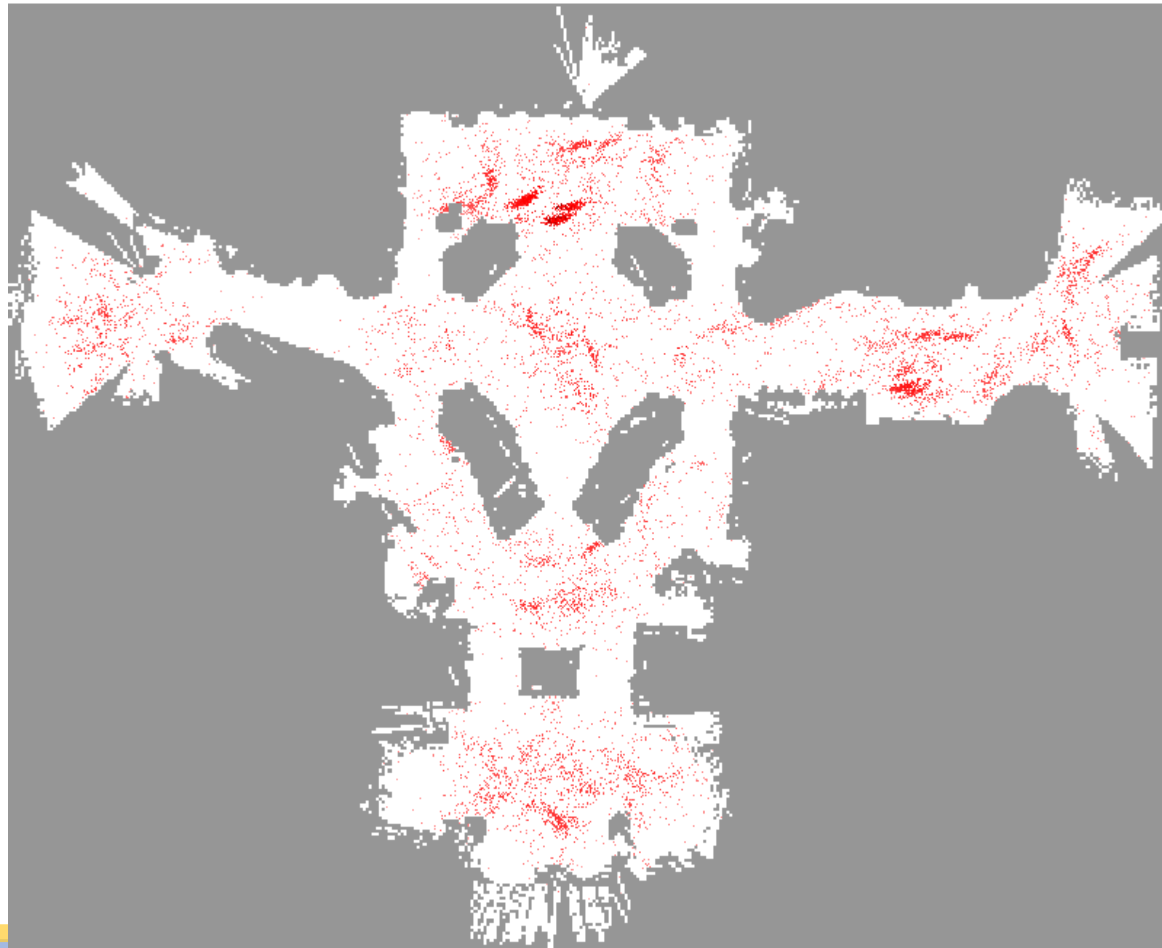


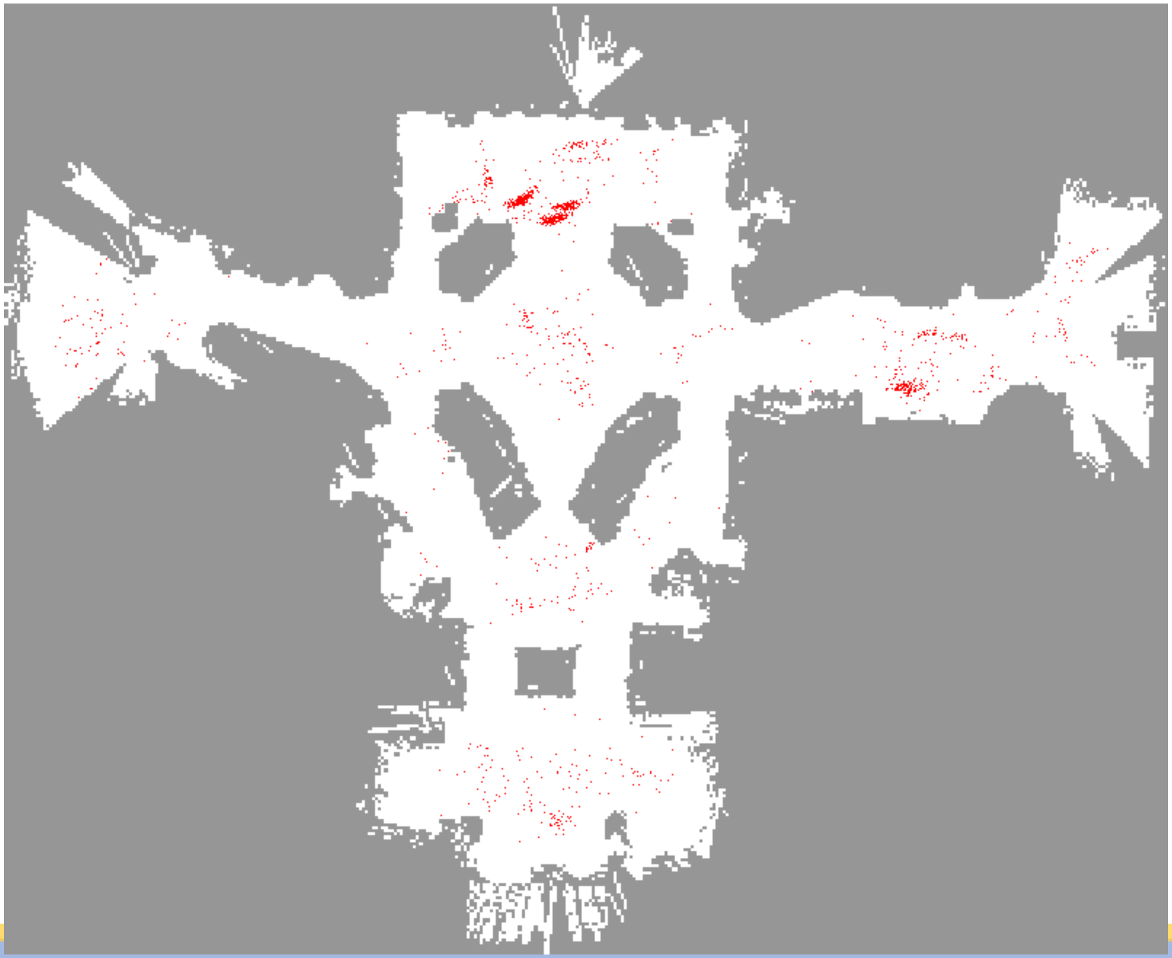


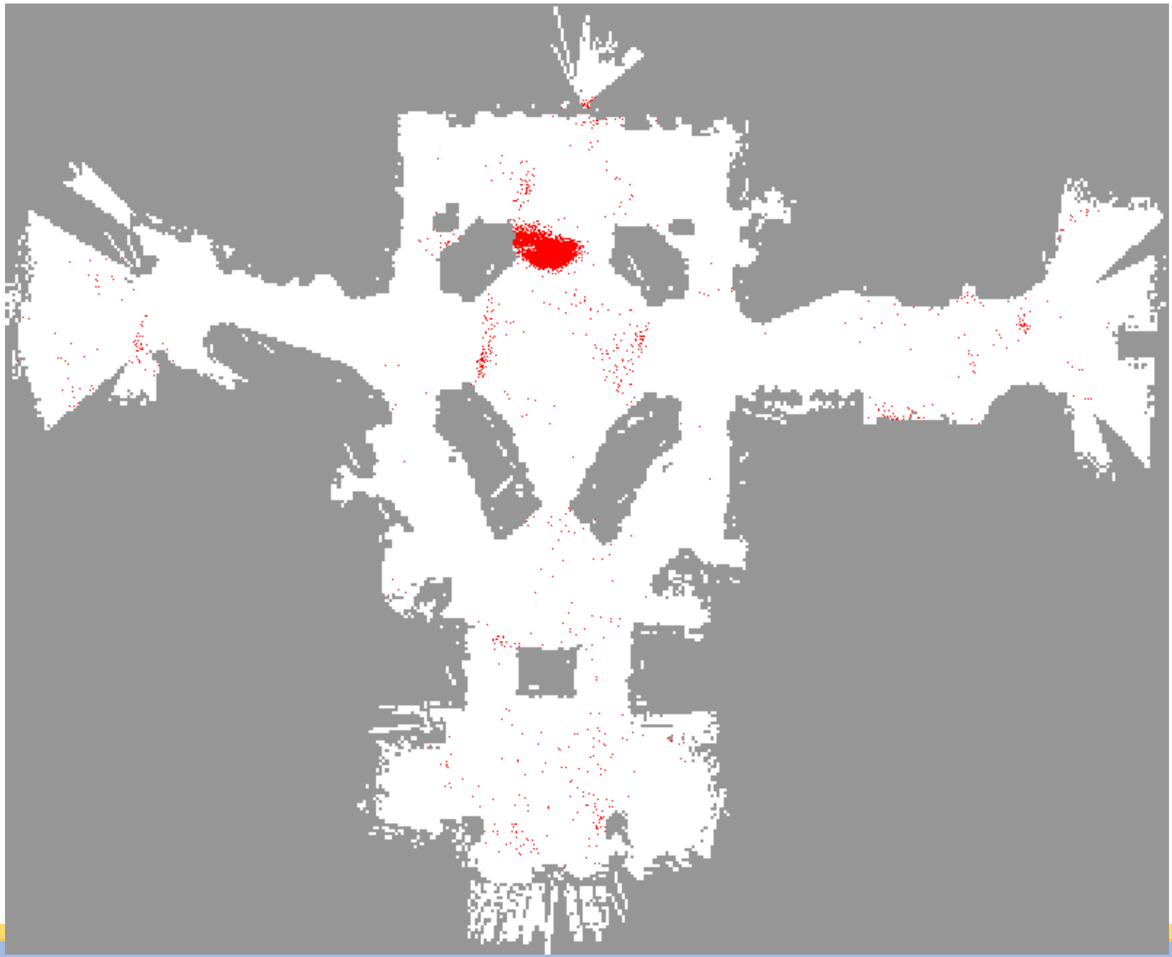


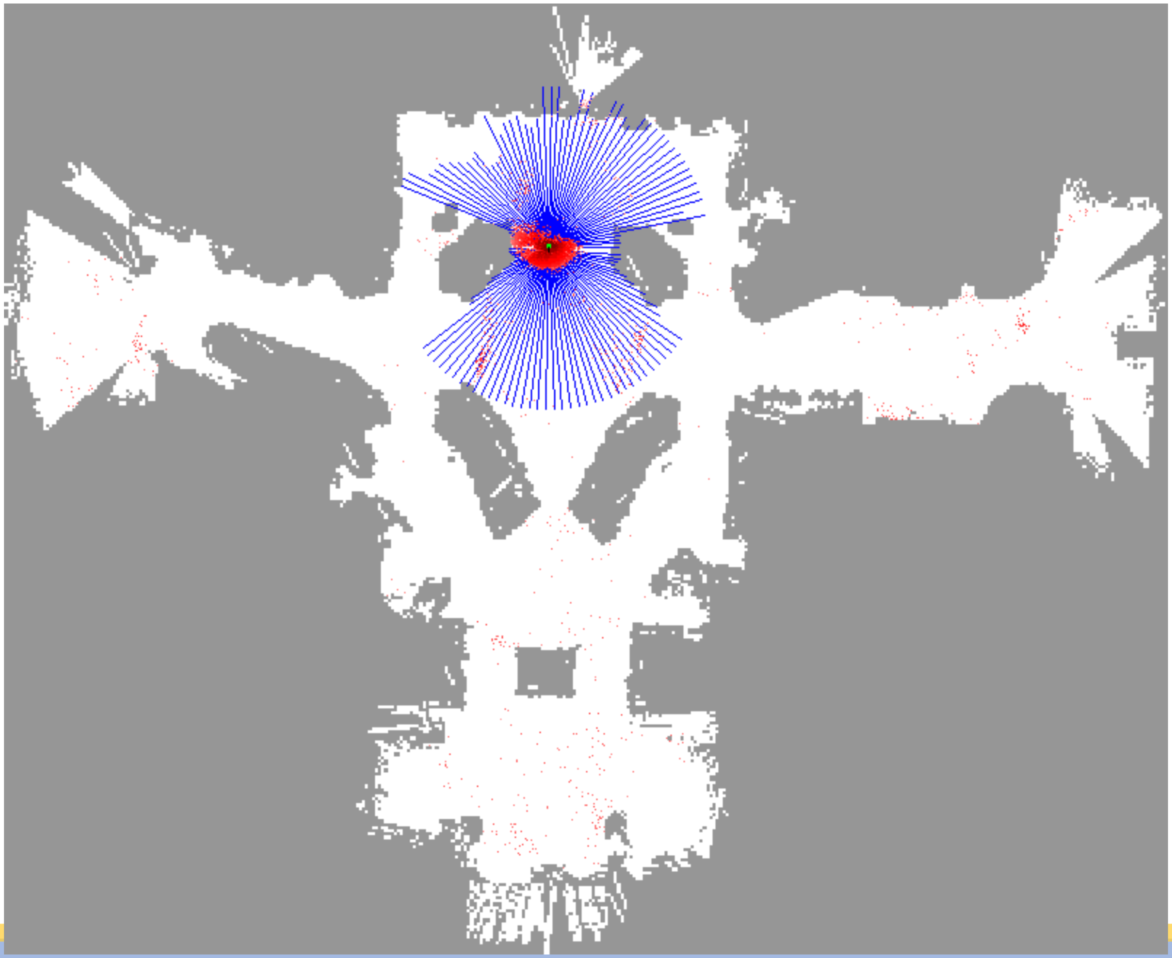


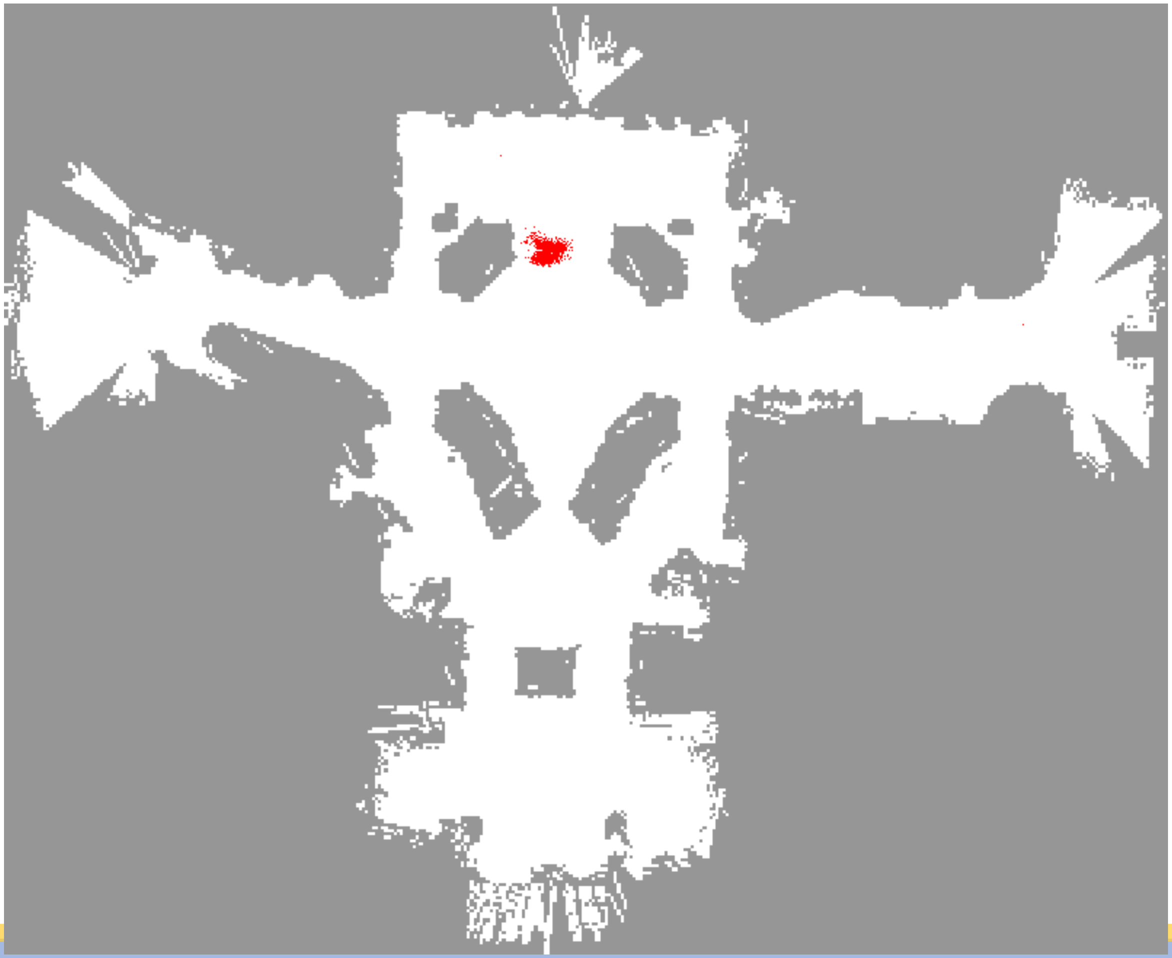


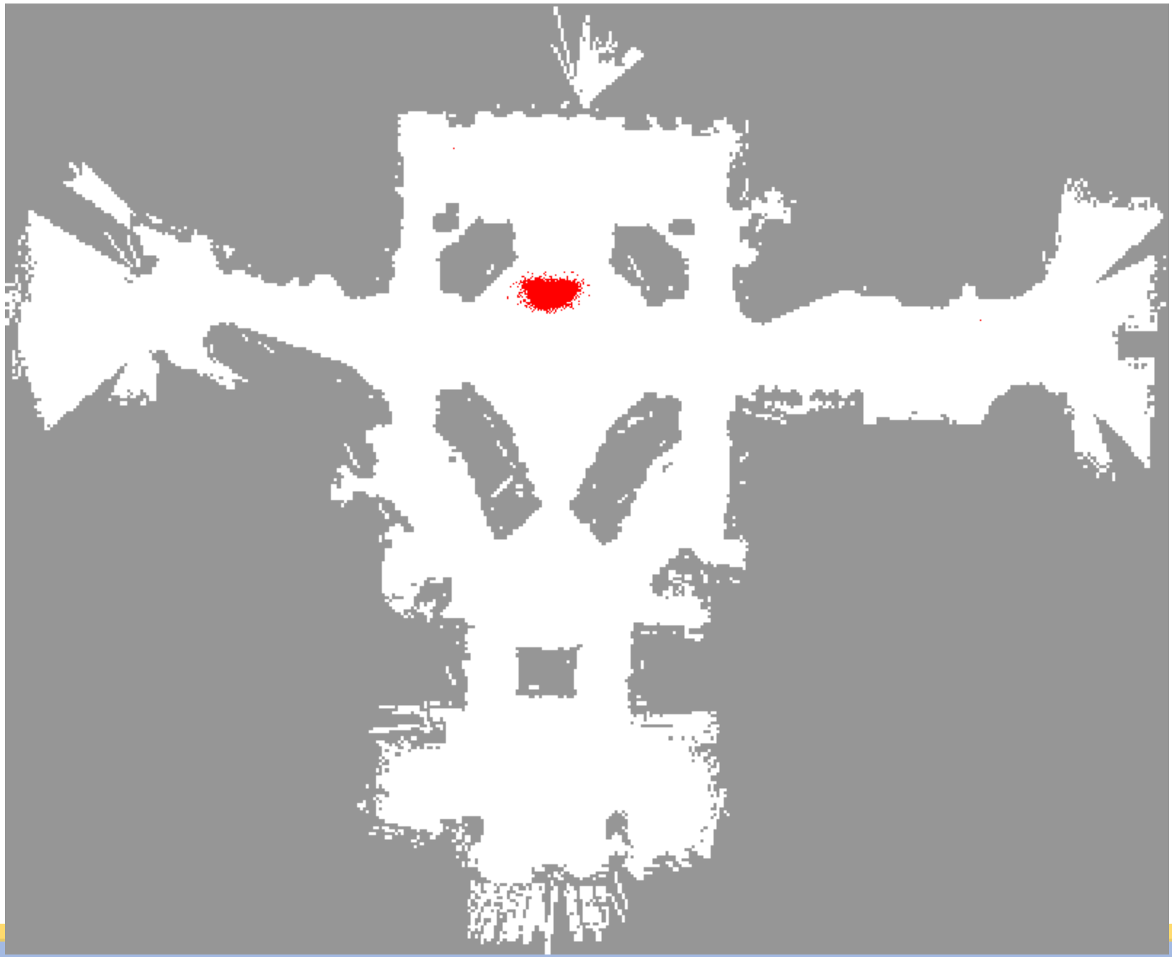


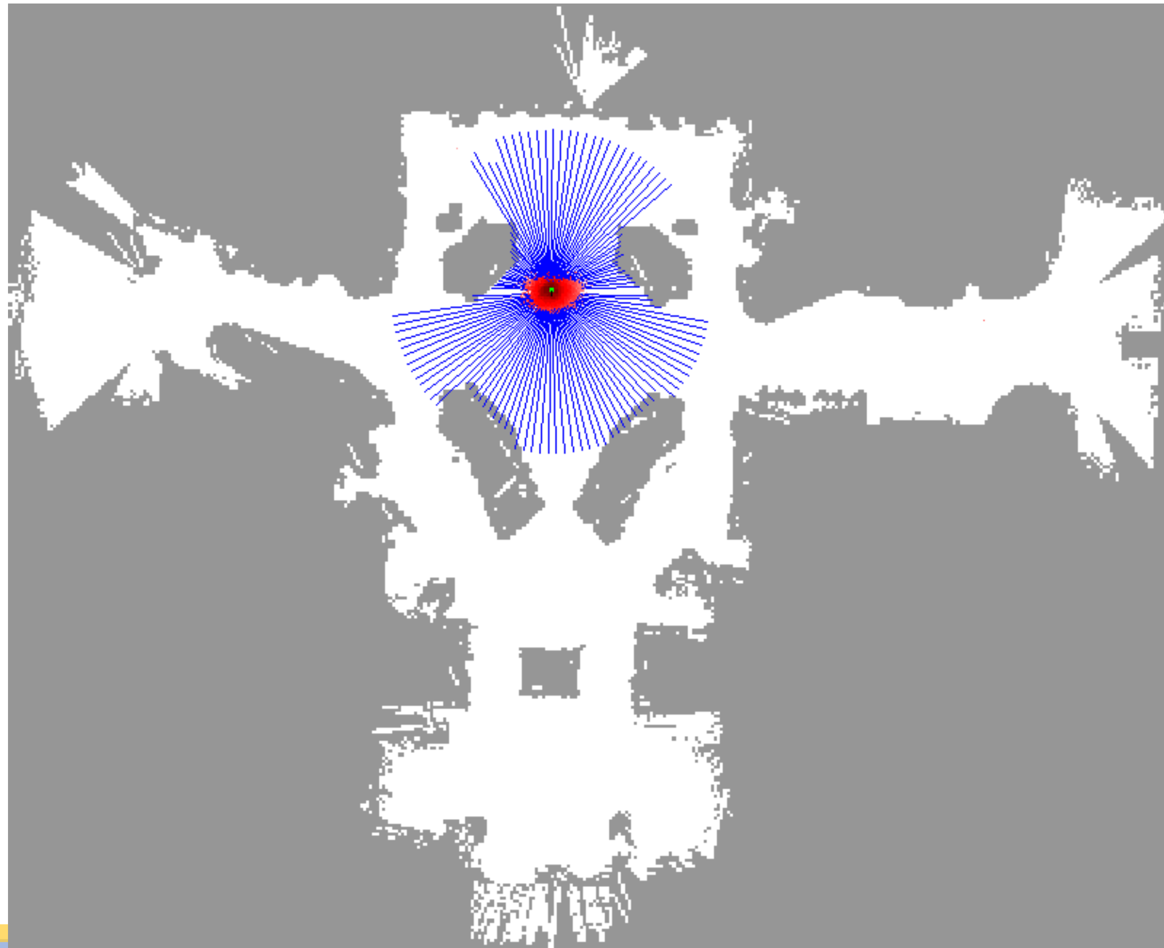


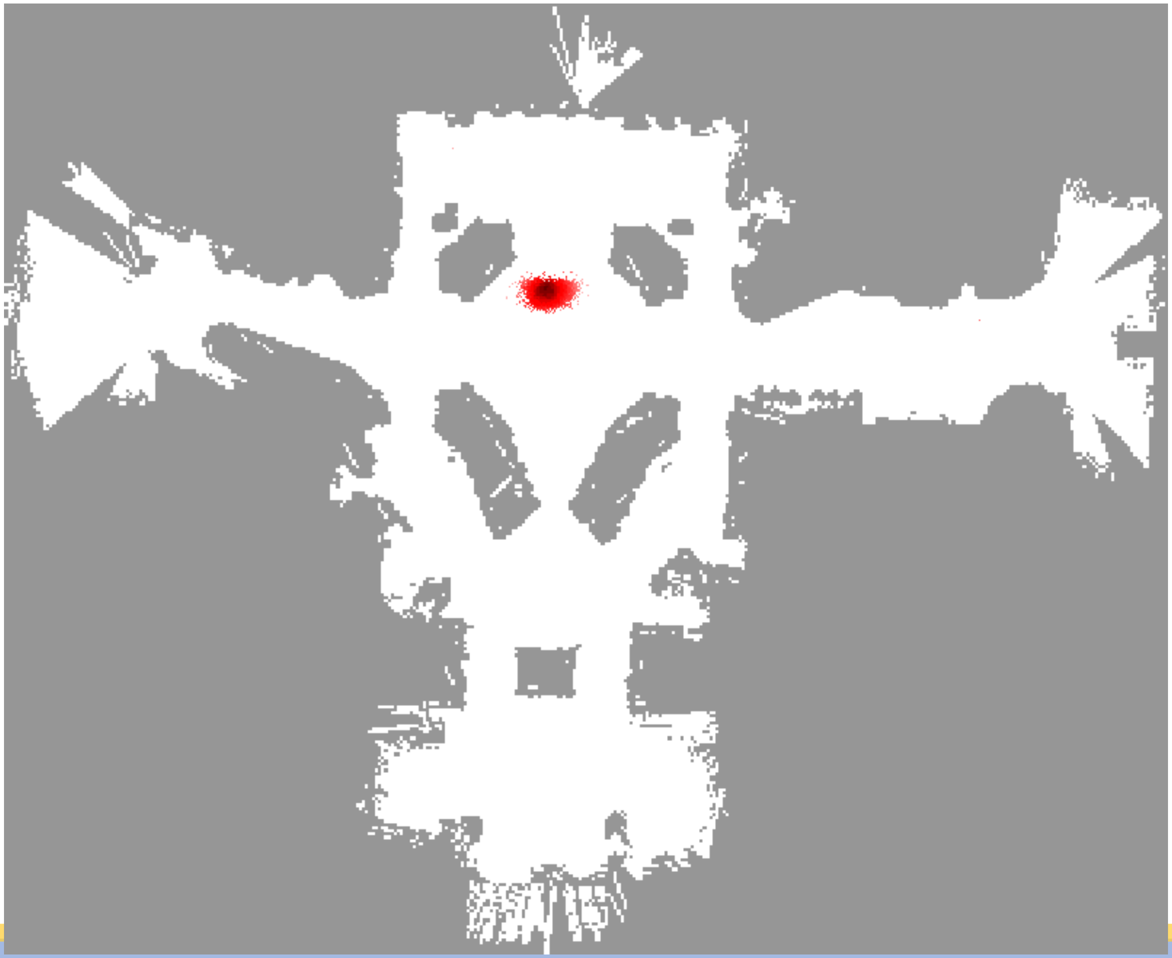


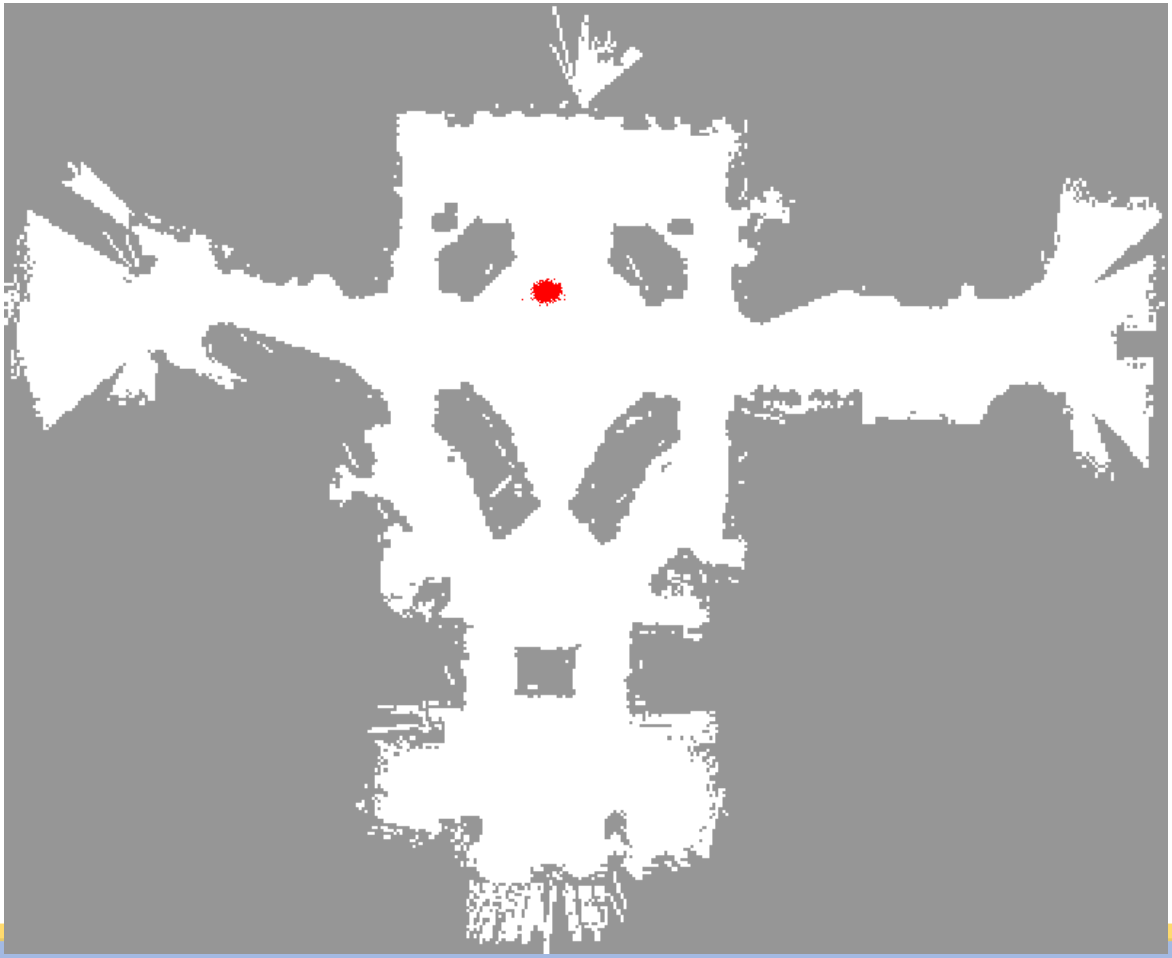


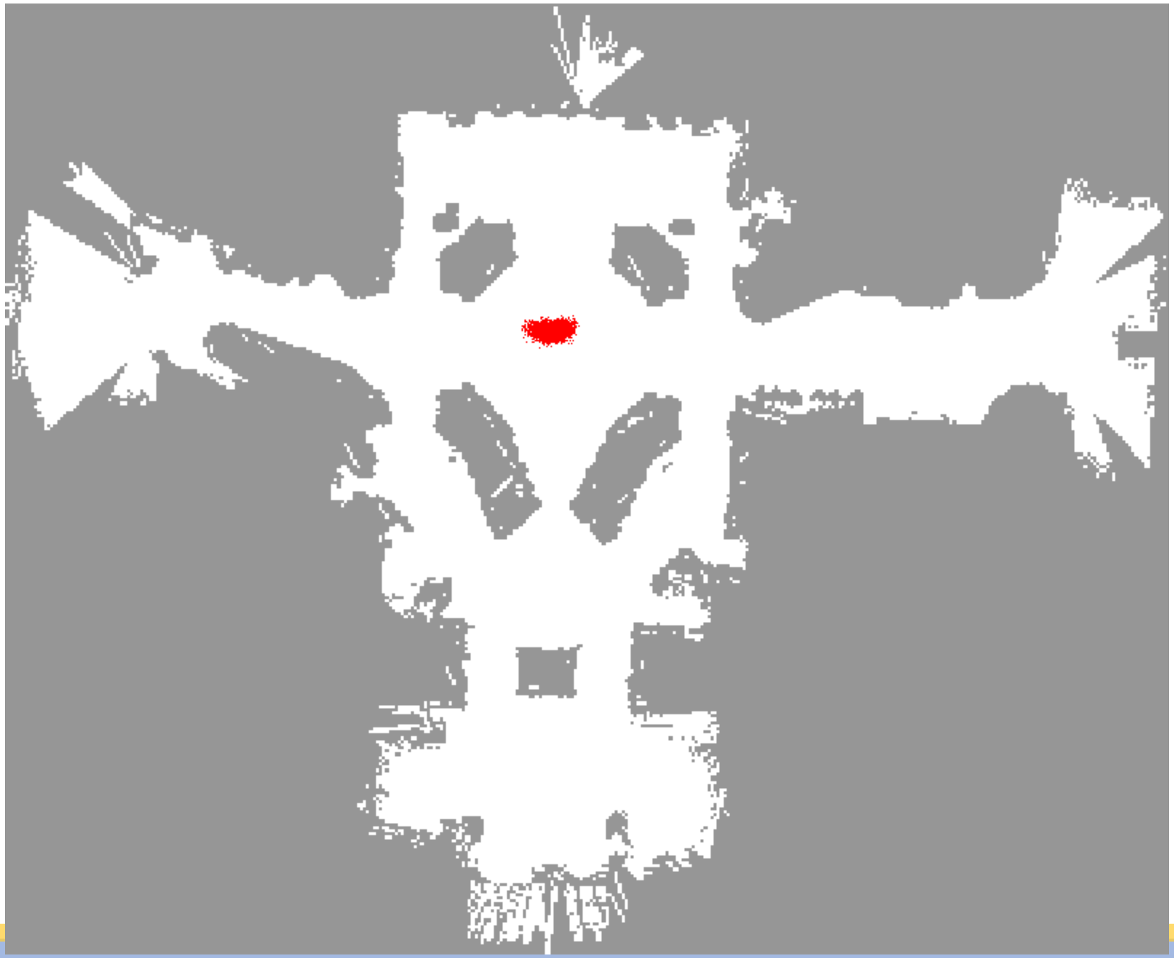


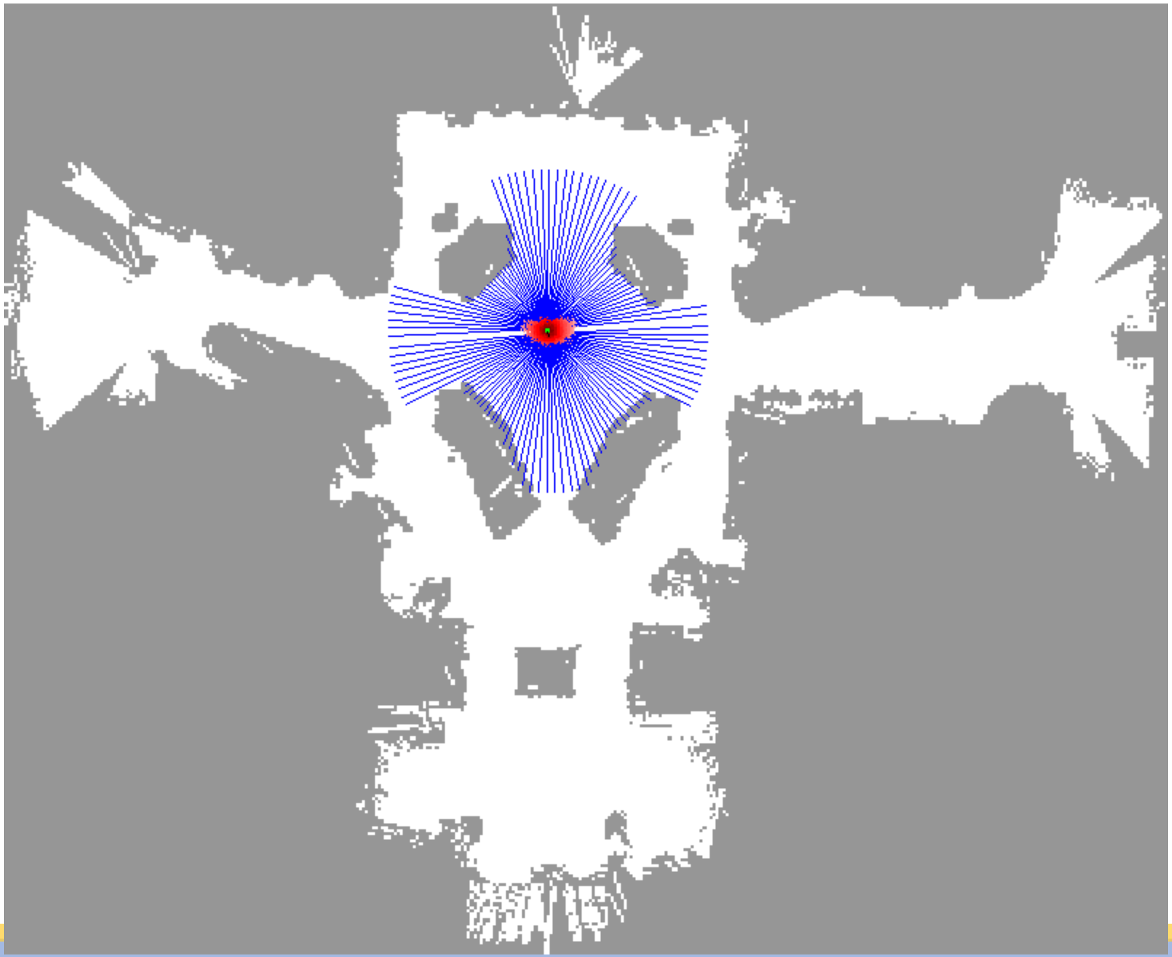


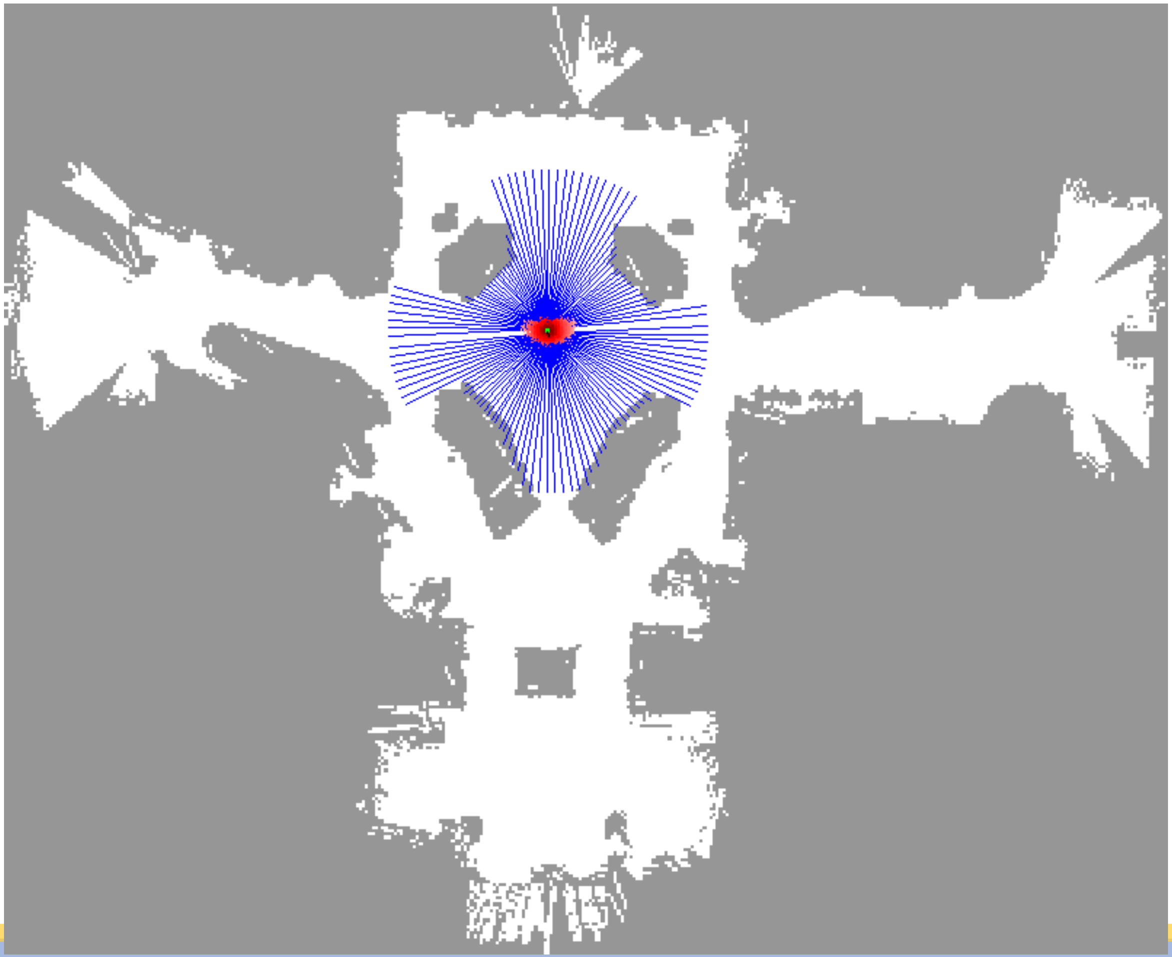




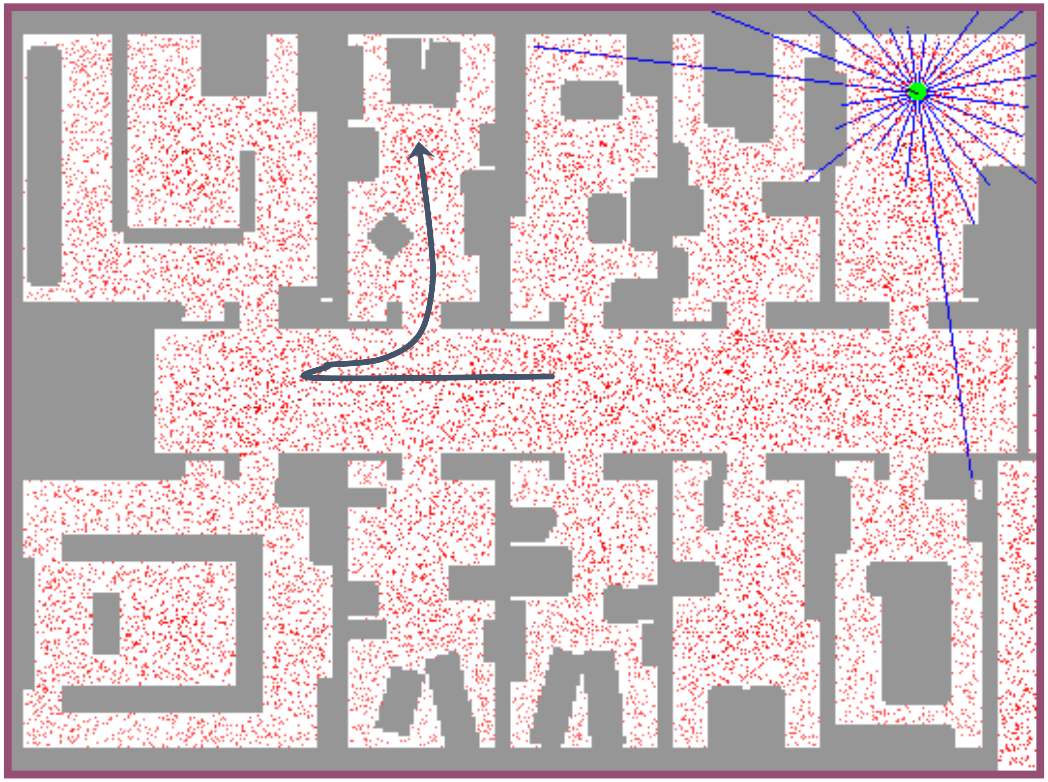




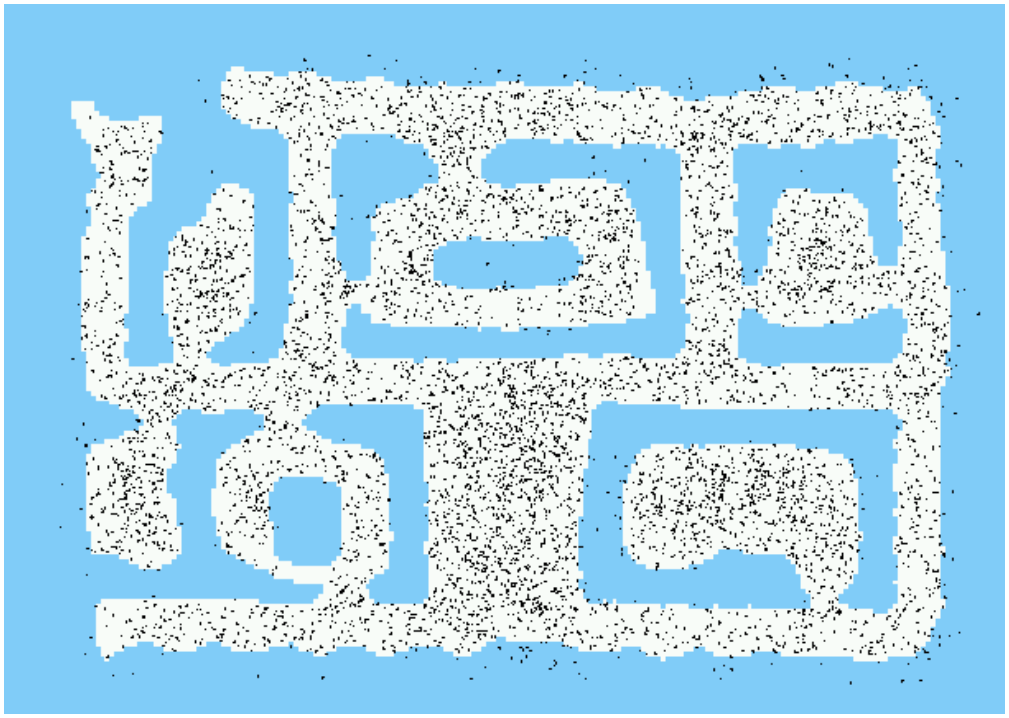




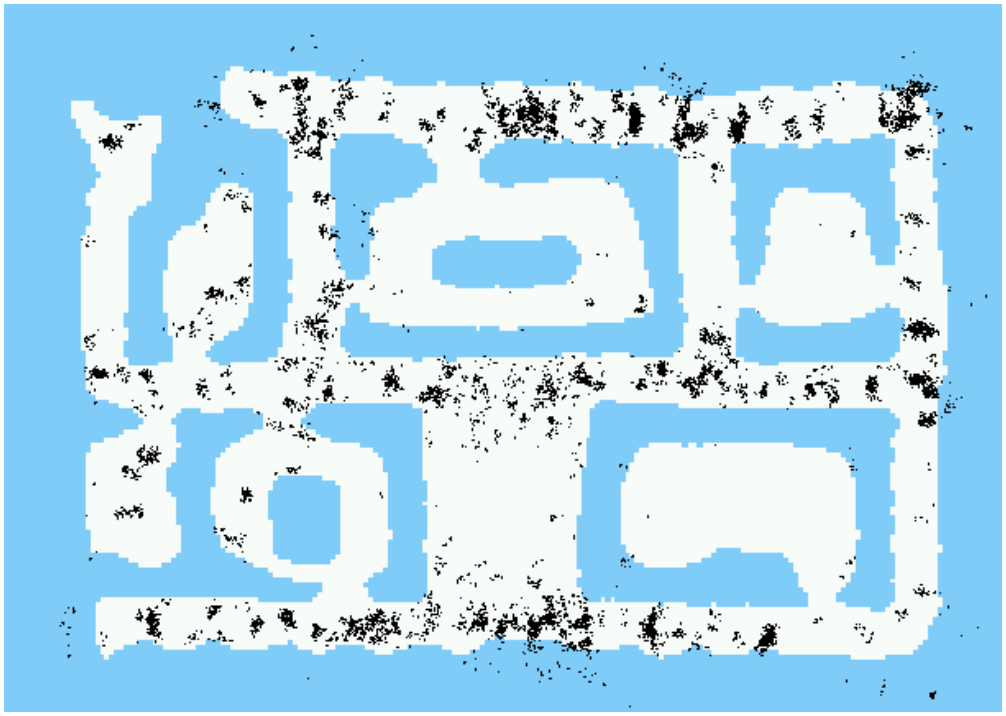
Sample-based Localization (sonar)



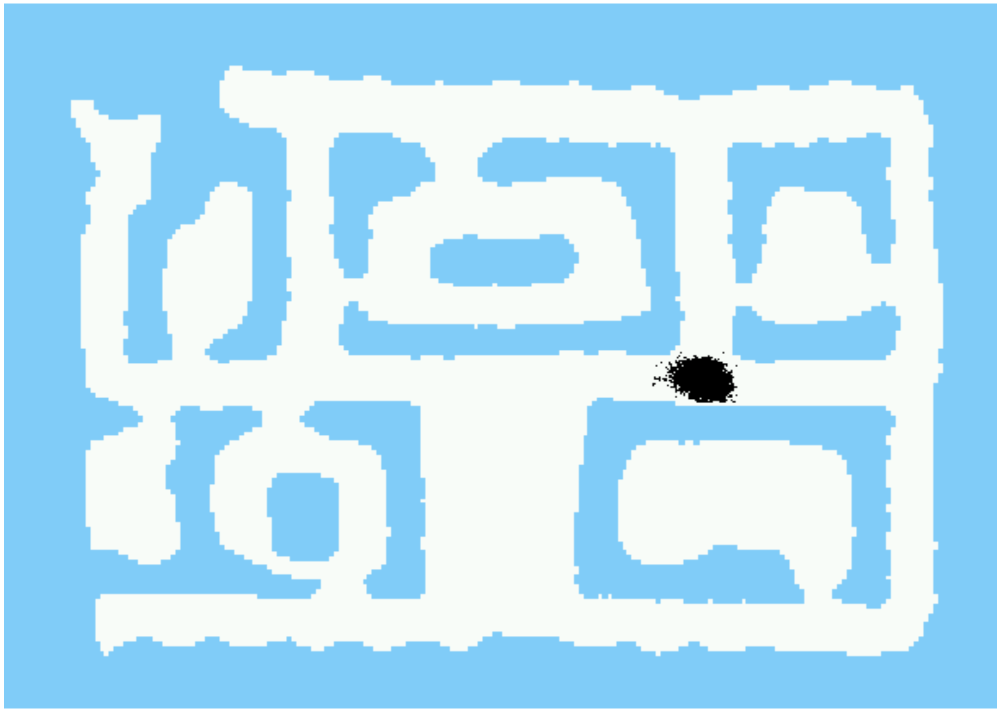
Initial Distribution



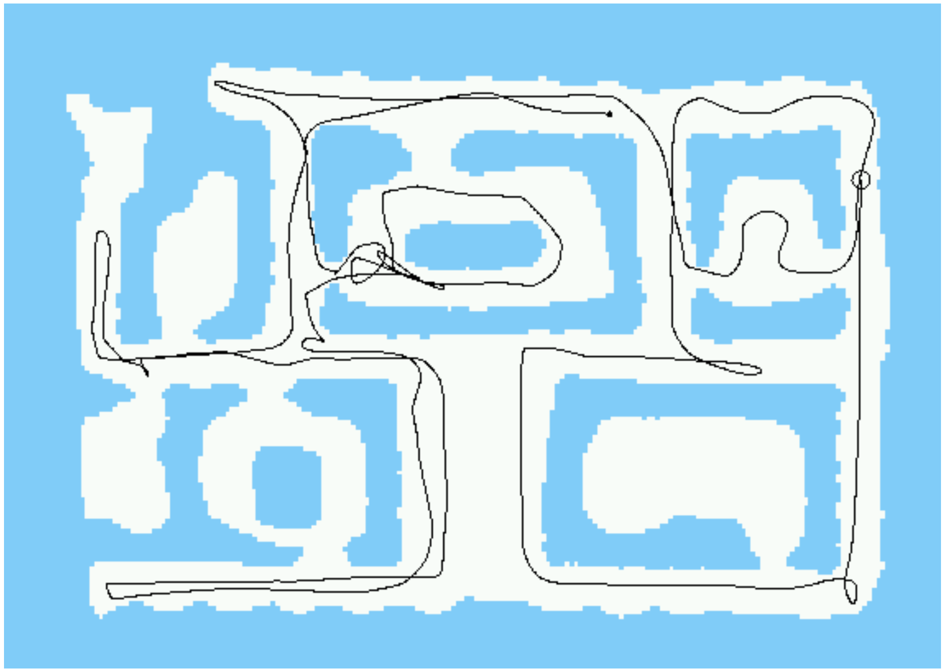
After Incorporating Ten Ultrasound Scans



After Incorporating 65 Ultrasound Scans

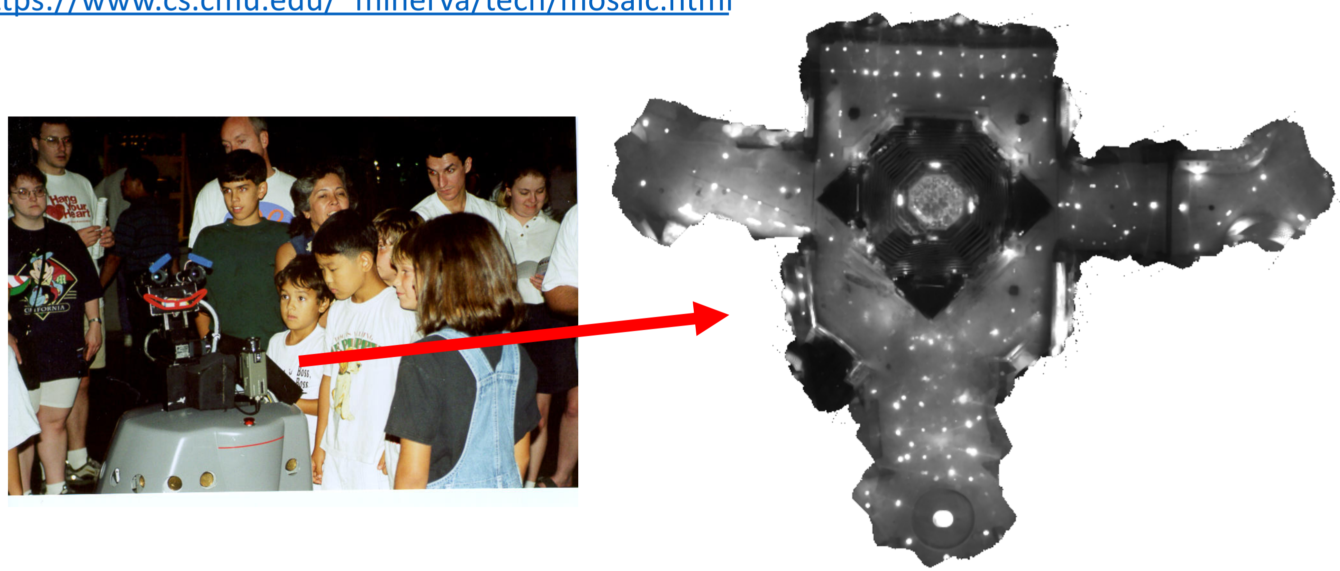


Estimated Path

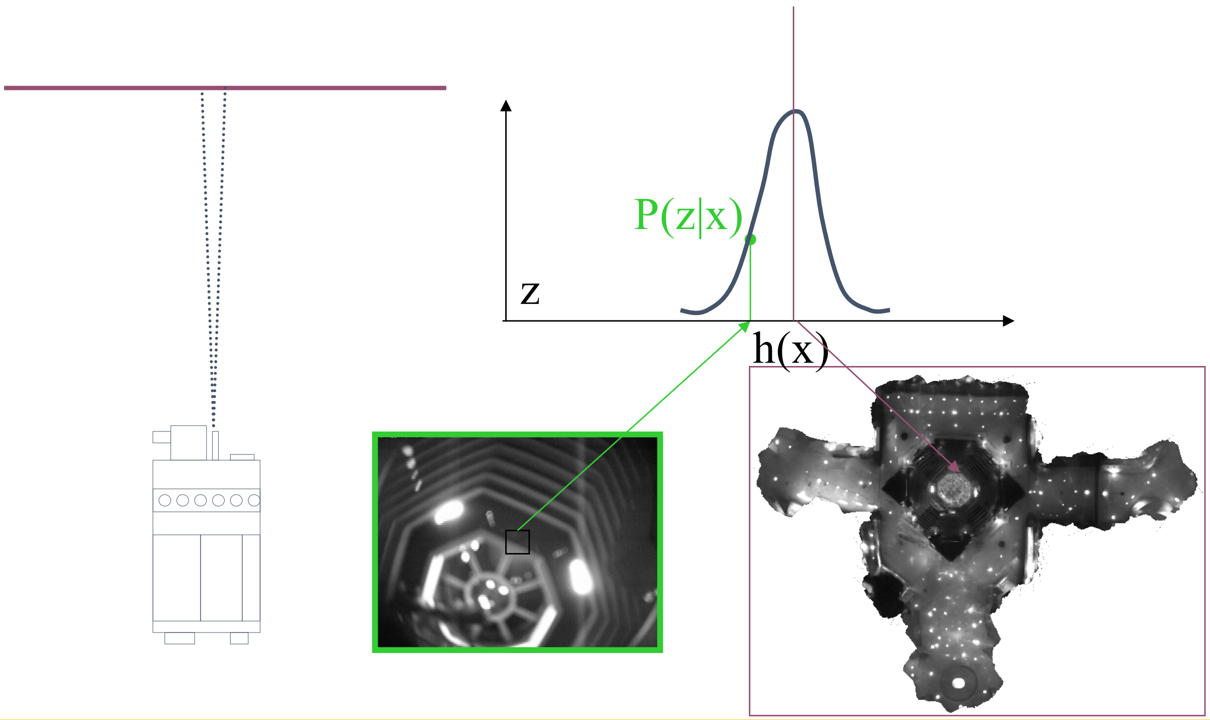


Using Ceiling Maps for Localization

Sensor: Upward looking camera
Map / model of the world: Ceiling Mosaic (construction is nontrivial)
https://www.cs.cmu.edu/~minerva/tech/mosaic.html

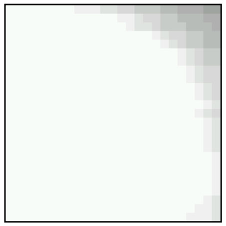


Vision-based Localization



Under a Light

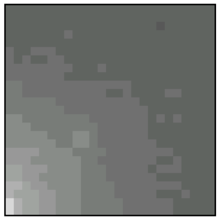
Measurement z:



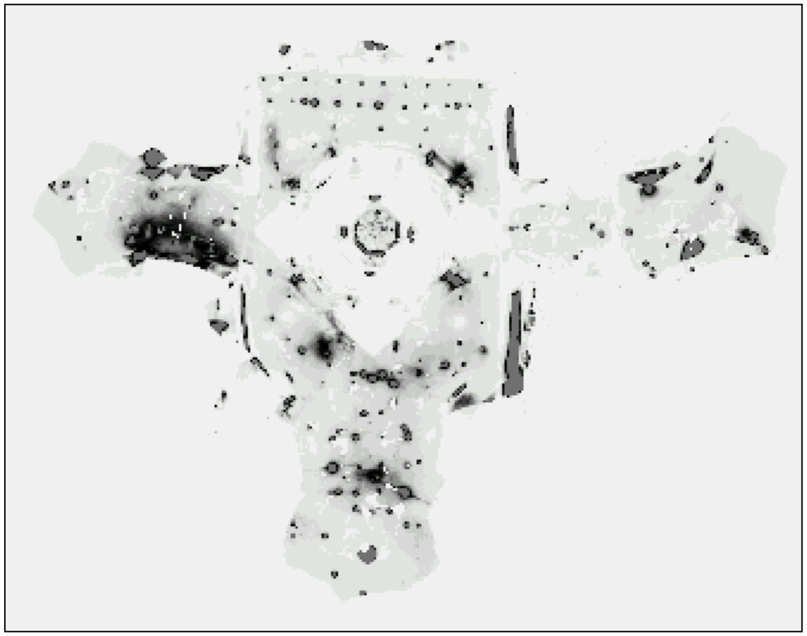
***P(z|x)*:**

Next to a Light

Measurement z:



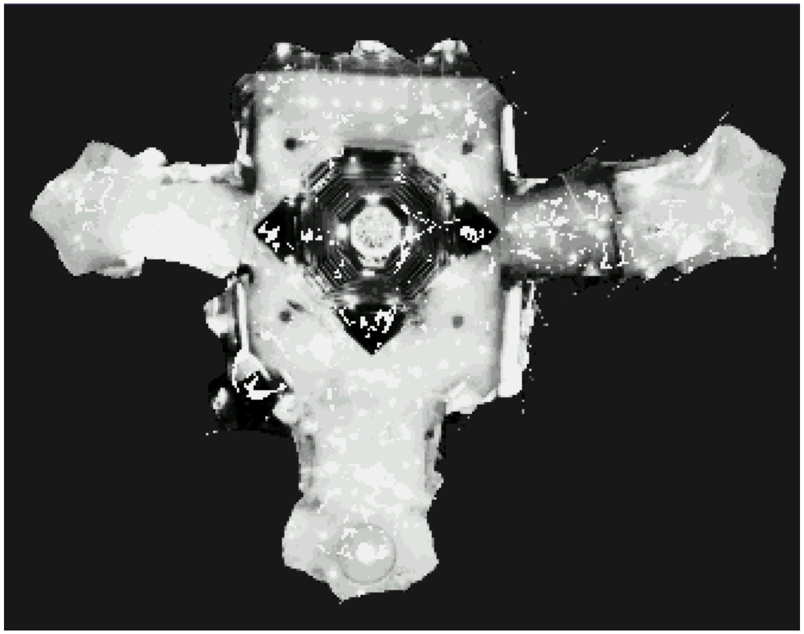
***P(z|x)*:**



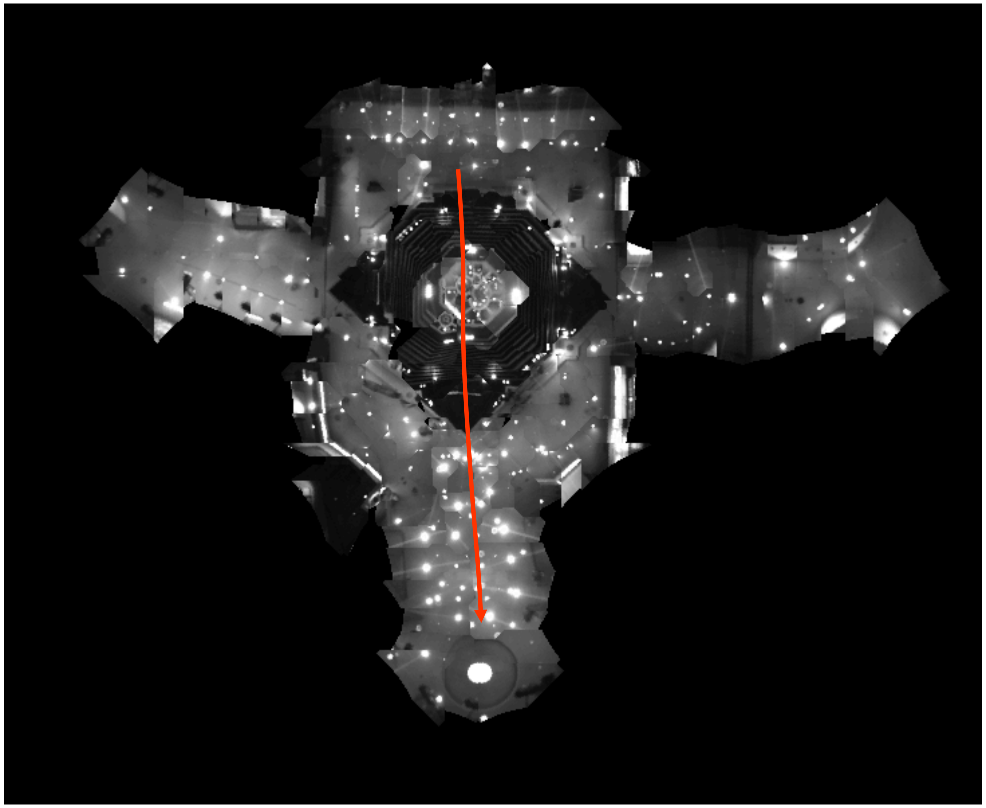
Elsewhere

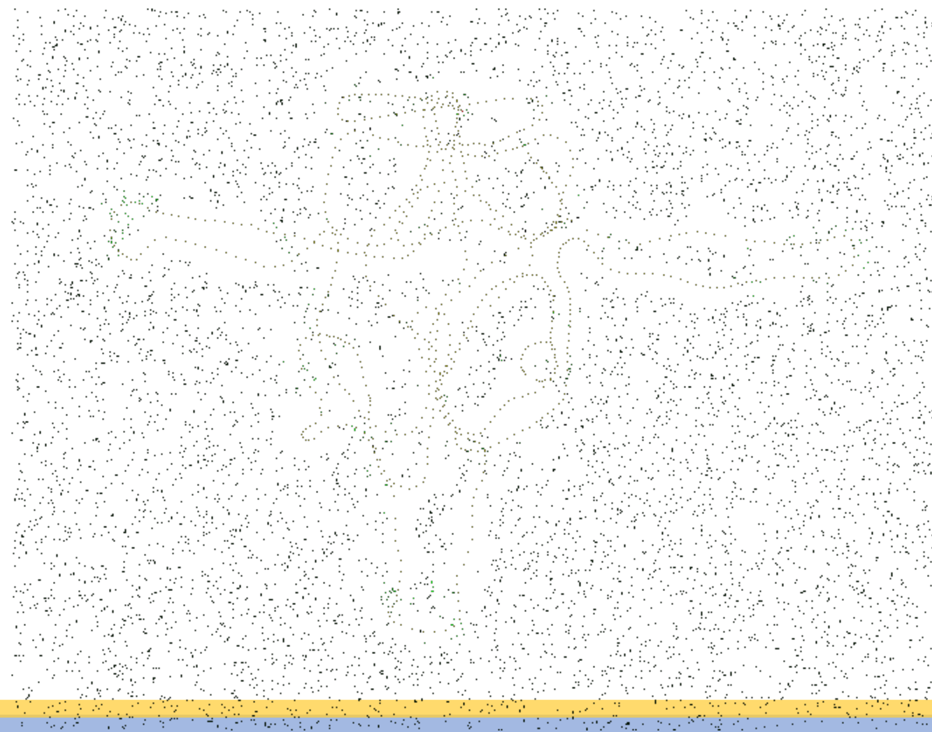
Measurement z:

***P(z|x)*:**



Global Localization Using Vision





Kahoot

- https://play.kahoot.it/v2/?quizId=3f040019-06e6-4fbe-9c98-780be526f271

Summary: Advantages and Limitations of MCL

Advantages of particle filtering-based localization (MCL)

- Solves global localization
- Can approximate any distributions (non-parametric)
- Increasing M improves accuracy of approximation (clear trade-off)
  - Possible to have adaptive implementations
  - track the pose of a mobile robot and to

Disadvantages

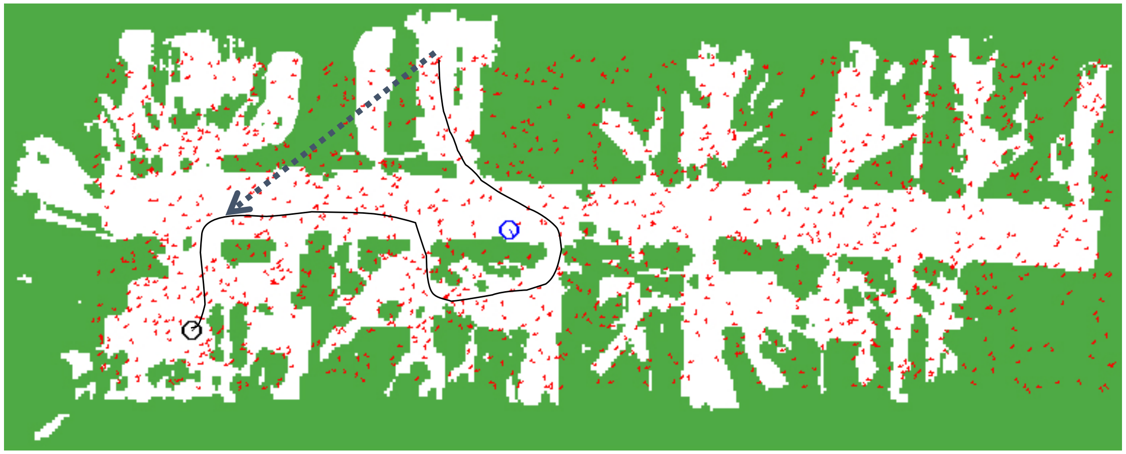
- Cannot solve global localization failures or kidnapped robot problem
  - Disappearance of diversity: particles other than the most likely positions disappear; only near a single pose “survive”; cannot recover if the pose is wrong
    - Can be resolved by injecting some random particles; how many? from what distribution?
    - Add particles based on some estimate of localization performance $p(z_t \,|z_{t-1}) = \frac{1}{M} \sum w_t^{[m]}$
- Particle deprivation: if $p(x_t \,|x_{t-1}, u_t)$ is very different from $p(x_t \,|z_t)$ then many more particles are needed; if the measurement model has no uncertainty---no noise---MCL fails
  - Simple solution trick: use noisy sensors;

Random Samples
Vision-Based Localization

936 Images, 4MB, .6secs/image

Trajectory of the robot:

Kidnapping the Robot



The SLAM Problem

- SLAM stands for simultaneous localization and mapping
- The task of building a map while estimating the pose of the robot relative to this map

- Why is SLAM hard?
  Chicken and egg problem:
  a map is needed to localize the robot and
  a pose estimate is needed to build a map

The SLAM Problem

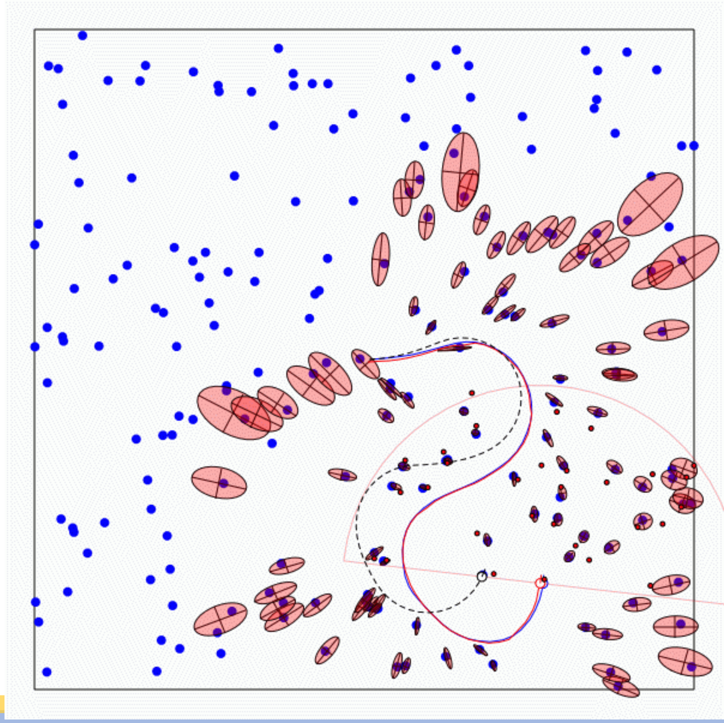
A robot moving though an unknown, static environment

Given:

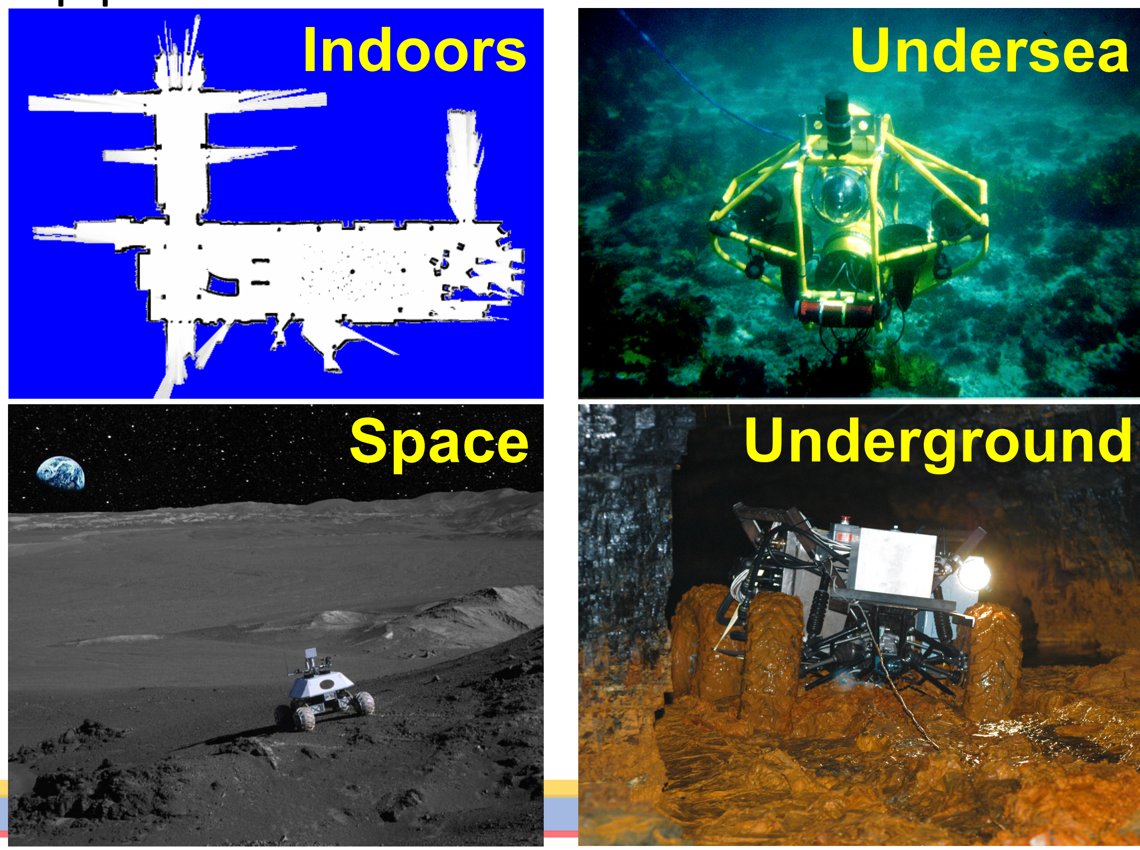
- The robot's controls
- Observations of nearby features

Estimate:

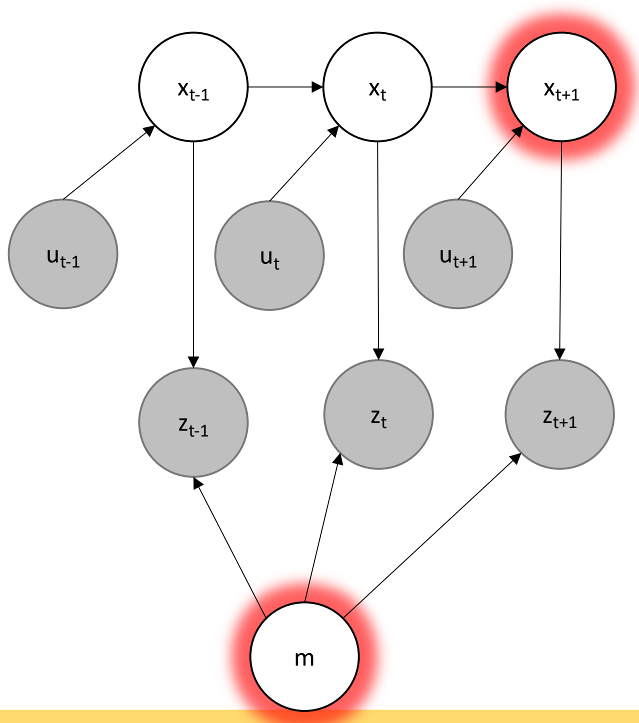
- Map of features
- Path of the robot



SLAM Applications



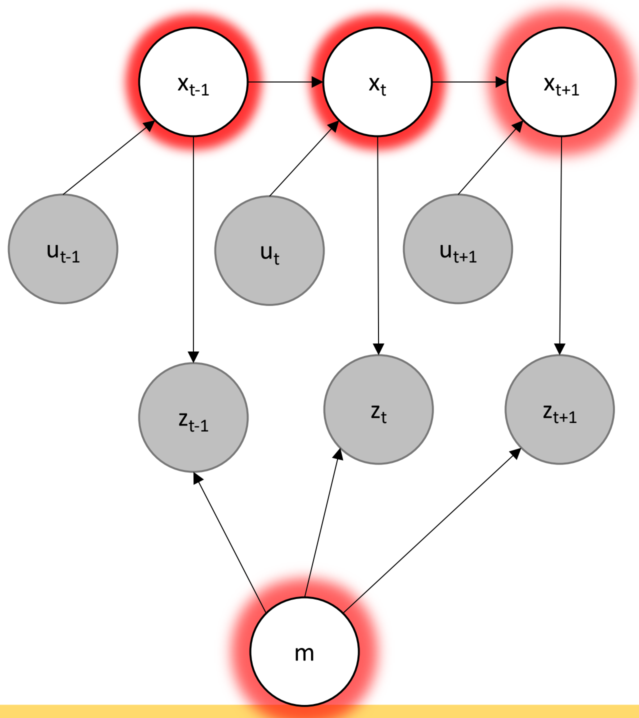
Online SLAM



Shaded known:
control inputs (u),
measurements(z).
White nodes to be
determined (x,m)

want to calculate
$p(x_t, m | z_{1:t}, u_{1:t})$

Full SLAM



Shaded known: control inputs (u), measurements(z). White nodes to be determined (x,m)

want to calculate
$p(x_{1:t}, m | z_{1:t}, u_{1:t})$

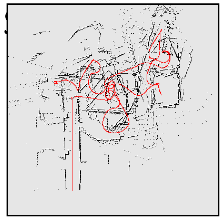
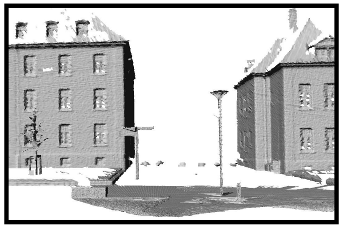
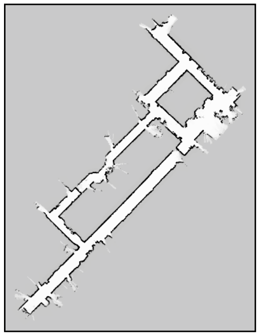
Continuous unknowns: $x_{1:t}, m$
Discrete unknowns: Relationship of detected objects to new objects

$$p(x_{1:t}, c_t, m | z_{1:t}, u_{1:t})$$

c_t: corrsnpondence variable

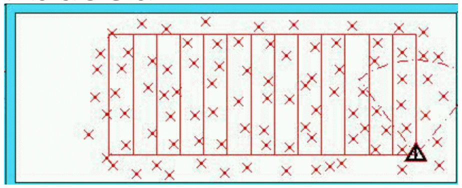
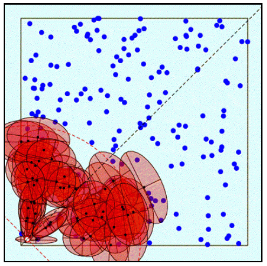
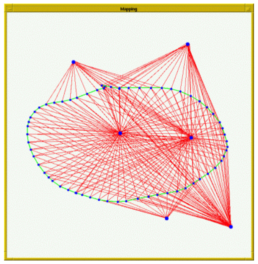
Representations

- Grid maps

[Lu & Milios, 97; Gutmann, 98: Thrun 98; Burgard, 99; Konolige & Gutmann, 00; Thrun, 00; Arras, 99; Haehnel, 01;…]

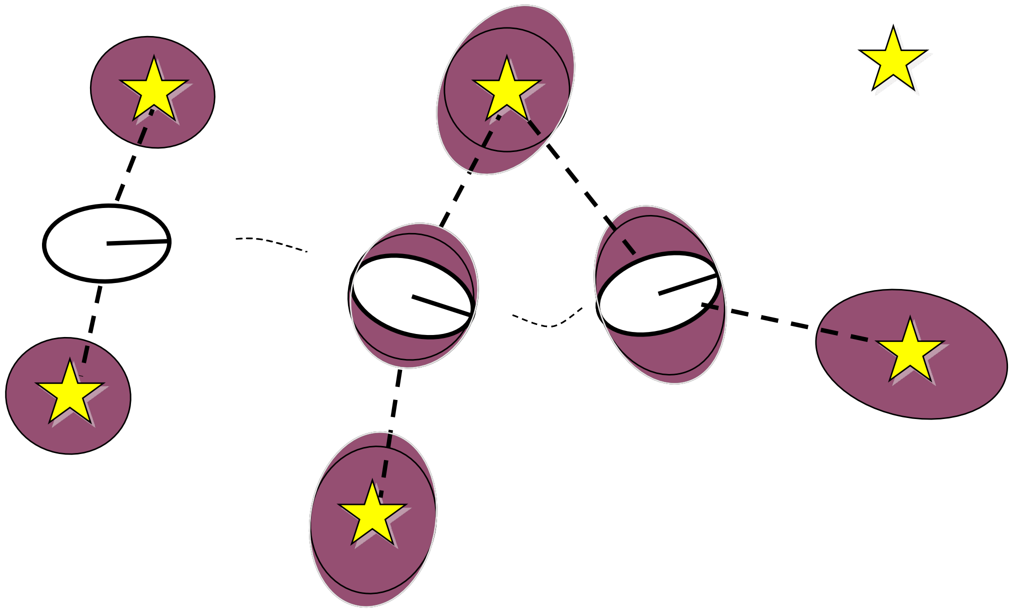
- Landmark-based

[Leonard et al., 98; Castelanos et al., 99: Dissanayake et al., 2001; Montemerlo et al., 2002;…

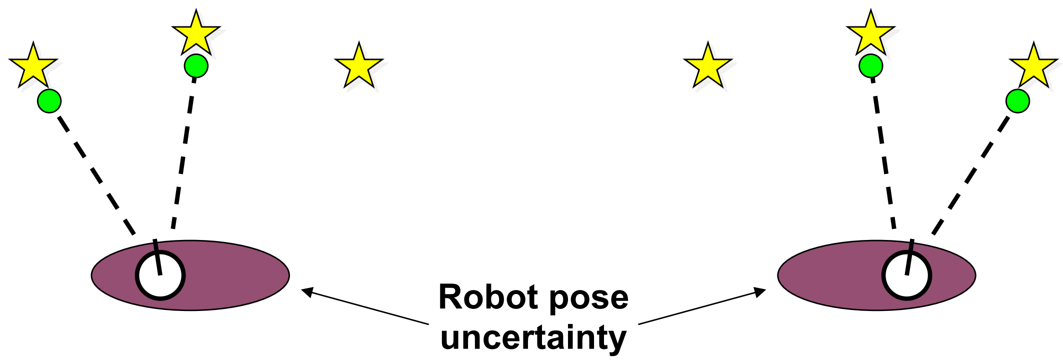
Why is SLAM a hard problem?

SLAM: robot path and map are both **unknown**



Robot path error correlates errors in the map

Why is SLAM a hard problem?



- In the real world, the mapping between observations and landmarks is unknown
- Picking wrong data associations can have catastrophic consequences
- Pose error correlates data associations

SLAM:
Simultaneous Localization and Mapping

- Full SLAM:

Estimates entire path and map!

$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t})$$

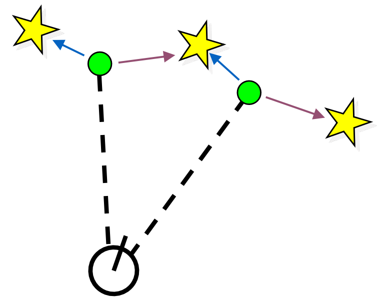
- Online SLAM:

$$p(x_t, m \mid z_{1:t}, u_{1:t}) = \int\int \ldots \int p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) \, dx_1 dx_2 \ldots dx_{t-1}$$

Integrations typically done one at a time

Estimates most recent pose and map!

Data Association Problem



- A data association is an assignment of observations to landmarks
- In general there are more than $\binom{n}{m}$ (n observations, m landmarks) possible associations
- Also called "assignment problem"

Particle Filters

- Represent belief by random samples
- Estimation of non-Gaussian, nonlinear processes

- Sampling Importance Resampling (SIR) principle
  - Draw the new generation of particles
  - Assign an importance weight to each particle
  - Resampling

- Typical application scenarios are tracking, localization, …

Localization vs. SLAM

- A particle filter can be used to solve both problems
- Localization: state space $<x, y, \theta>$
- SLAM: state space $<x, y, \theta, map>$
  - for landmark maps = $<l_1, l_2, \ldots, l_m>$
  - for grid maps = $<c_{11}, c_{12}, \ldots, c_{1n}, c_{21}, \ldots, c_{nm}>$
- **Problem:** The number of particles needed to represent a posterior grows exponentially with the dimension of the state space!

- Naïve implementation of particle filters to SLAM will be crushed by the curse of dimensionality

Dependencies

- Is there a dependency between the dimensions of the state space?
- If so, can we use the dependency to solve the problem more efficiently?

Dependencies

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- If so, can we use the dependency to solve the problem more efficiently?

- In the SLAM context
  - The map depends on the poses of the robot.
  - We know how to build a map given the position of the sensor is known.

Conditional Independence

- A and B are conditionally independent given C if P(A, B | C) = P(A|C) P(B|C)

- Height and vocabulary are not independent
- But they are conditionally independent given age

Factored Posterior (Landmarks)

poses map observations & movements

$$p(x_{1:t}, l_{1:m} \mid z_{1:t}, u_{0:t-1}) =$$

$$p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot p(l_{1:m} \mid x_{1:t}, z_{1:t})$$

SLAM posterior

Robot path posterior

landmark positions

Does this help to solve the problem?

Factorization first introduced by Murphy in 1999

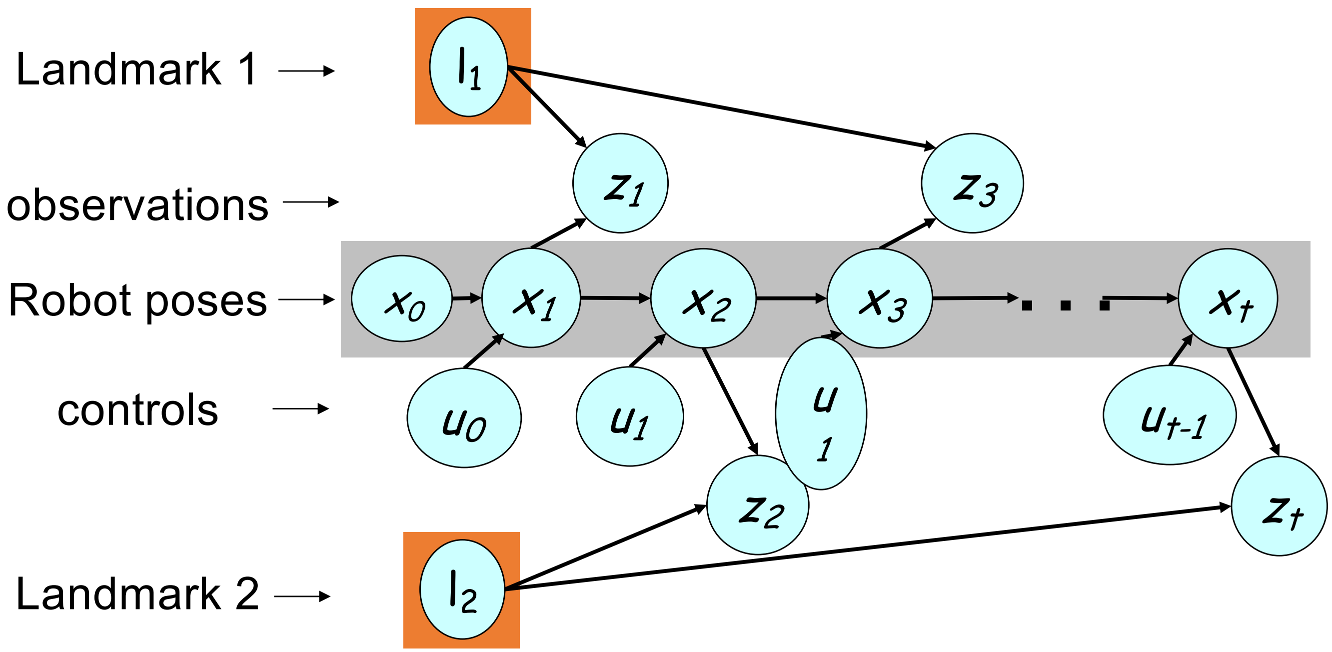
Factored Posterior (Landmarks)

poses map observations & movements

$$p(x_{1:t}, l_{1:m} \mid z_{1:t}, u_{0:t-1}) =$$
$$p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot p(l_{1:m} \mid x_{1:t}, z_{1:t})$$

Factorization first introduced by Murphy in 1999

Mapping using Landmarks



Knowledge of the robot's true path renders landmark positions conditionally independent

Factored Posterior

$$
\begin{aligned}
& p(x_{1:t}, l_{1:m} \mid z_{1:t}, u_{0:t-1}) \\
& \quad = p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot p(l_{1:m} \mid x_{1:t}, z_{1:t}) \\
& \quad = p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot \prod_{i=1}^{M} p(l_i \mid x_{1:t}, z_{1:t})
\end{aligned}
$$

Robot path posterior (localization problem)

Conditionally independent landmark positions

Rao-Blackwellization

$$p(x_{1:t}, l_{1:m} \mid z_{1:t}, u_{0:t-1}) =$$

$$p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot \prod_{i=1}^{M} p(l_i \mid x_{1:t}, z_{1:t})$$

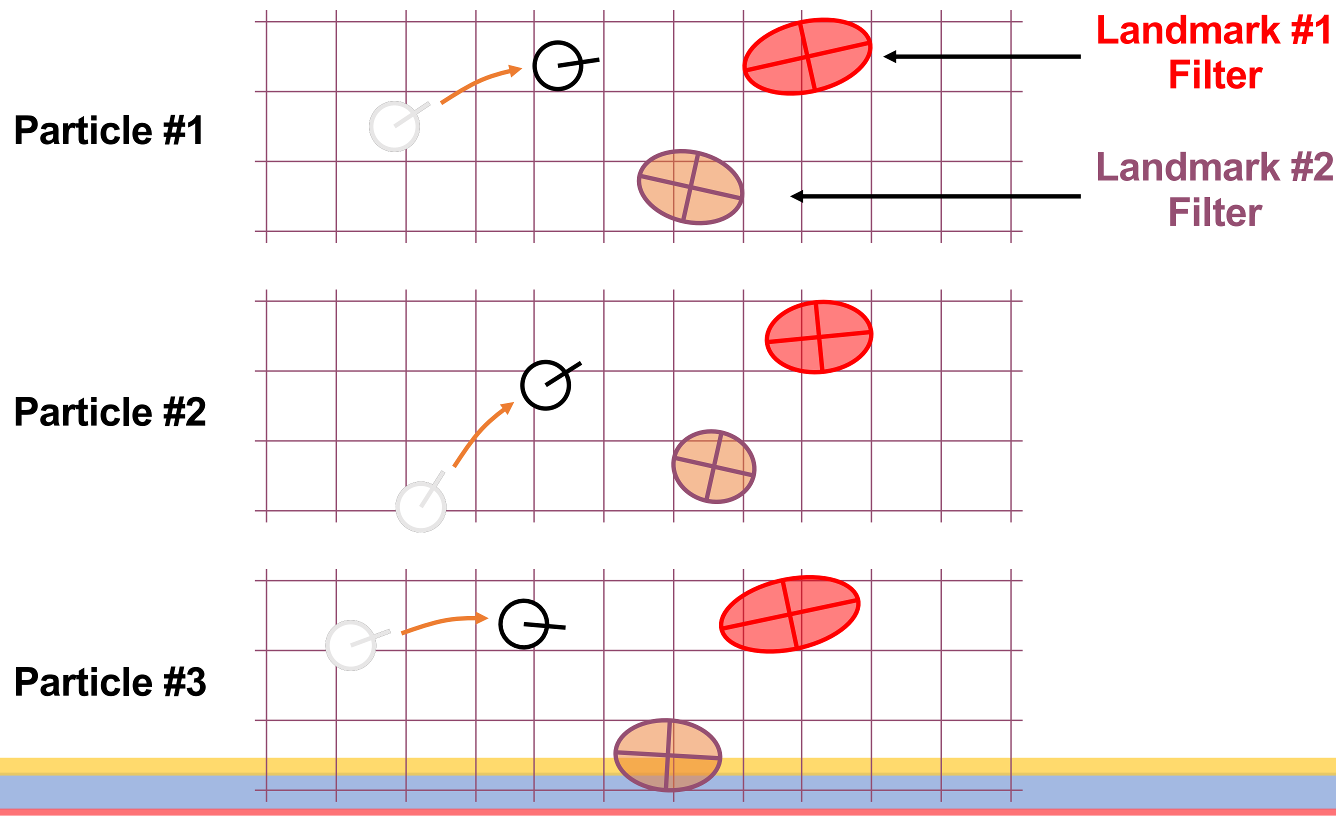
- This factorization is also called Rao-Blackwellization
- Given that the second term can be computed efficiently, particle filtering becomes possible!

FastSLAM

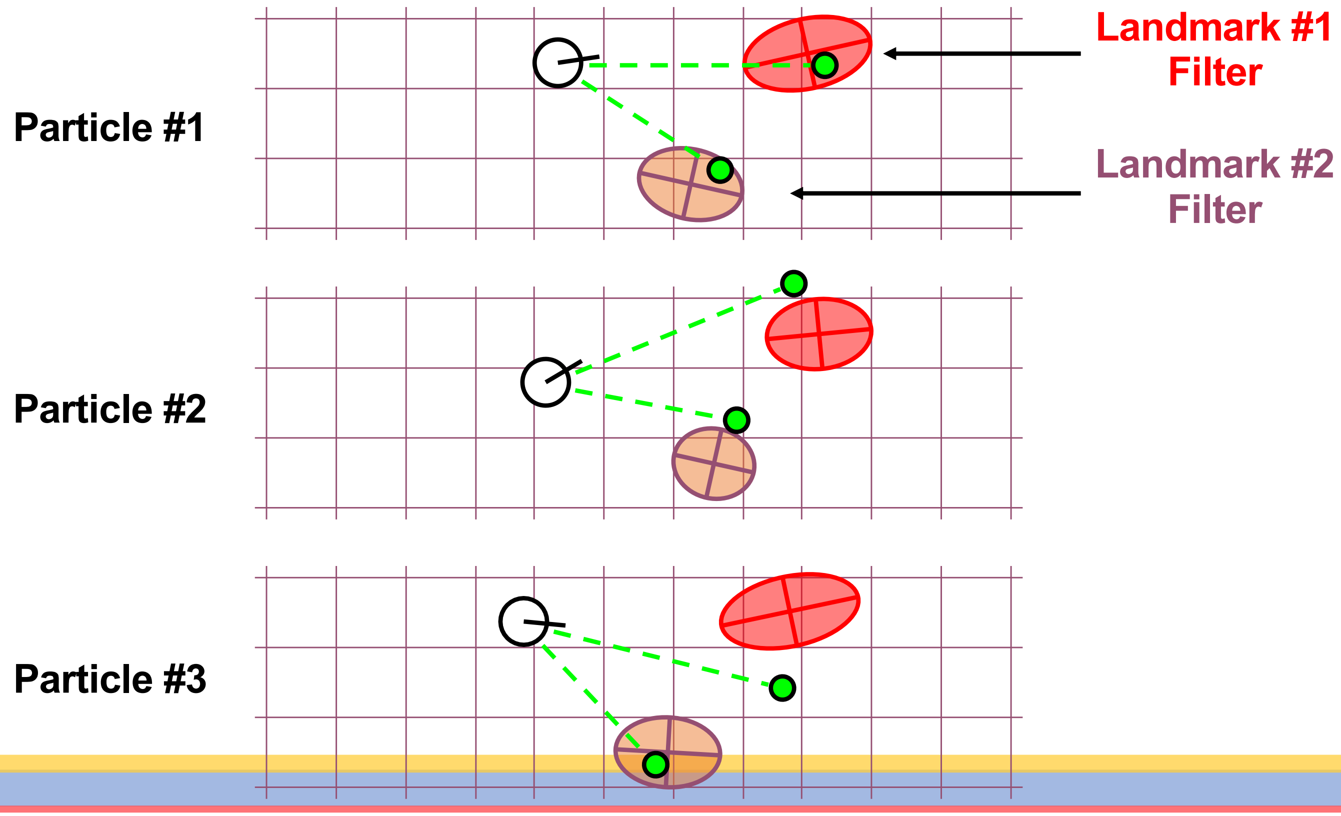
- Rao-Blackwellized particle filtering based on landmarks [Montemerlo et al., 2002]
- Each landmark is represented by a 2x2 Extended Kalman Filter (EKF)
- Each particle therefore has to maintain *M* EKFs

Particle #1	x, y, θ	**Landmark 1**	**Landmark 2**	**…**	**Landmark M**
Particle #2	x, y, θ	**Landmark 1**	**Landmark 2**	**…**	**Landmark M**
⋮					
Particle N	x, y, θ	**Landmark 1**	**Landmark 2**	**…**	**Landmark M**

FastSLAM – Action Update



FastSLAM – Sensor Update

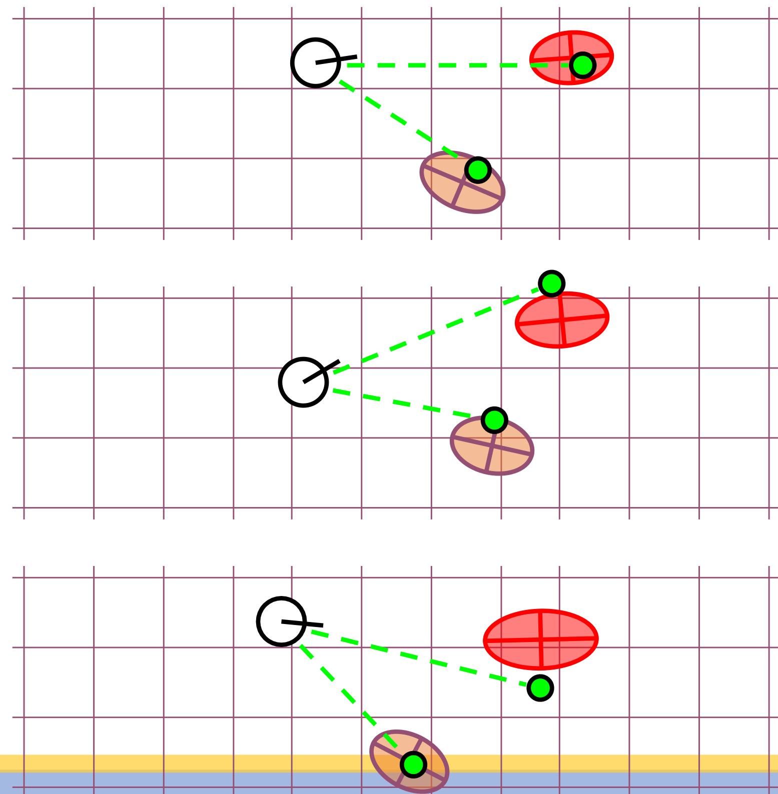


FastSLAM – Sensor Update

Particle #1

Particle #2

Particle #3

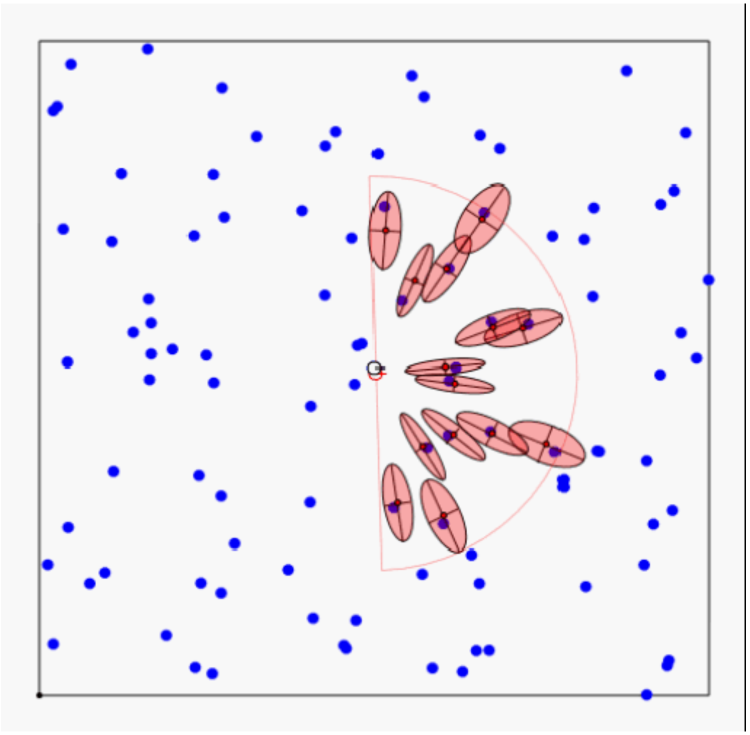


Weight = 0.8

Weight = 0.4

Weight = 0.1

FastSLAM - Video



FastSLAM Complexity

- Update robot particles based on control u_{t-1}

O(N)
Constant time per particle

- Incorporate observation z_t into Kalman filters

O(N•log(M))
Log time per particle

- Resample particle set

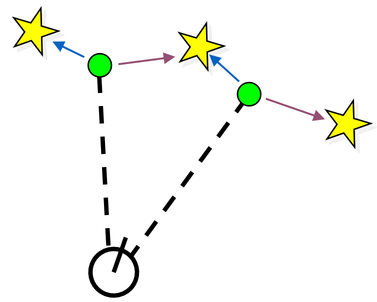
O(N•log(M))
Log time per particle

N = Number of particles
M = Number of map features

O(N•log(M))
Log time per particle

Data Association Problem

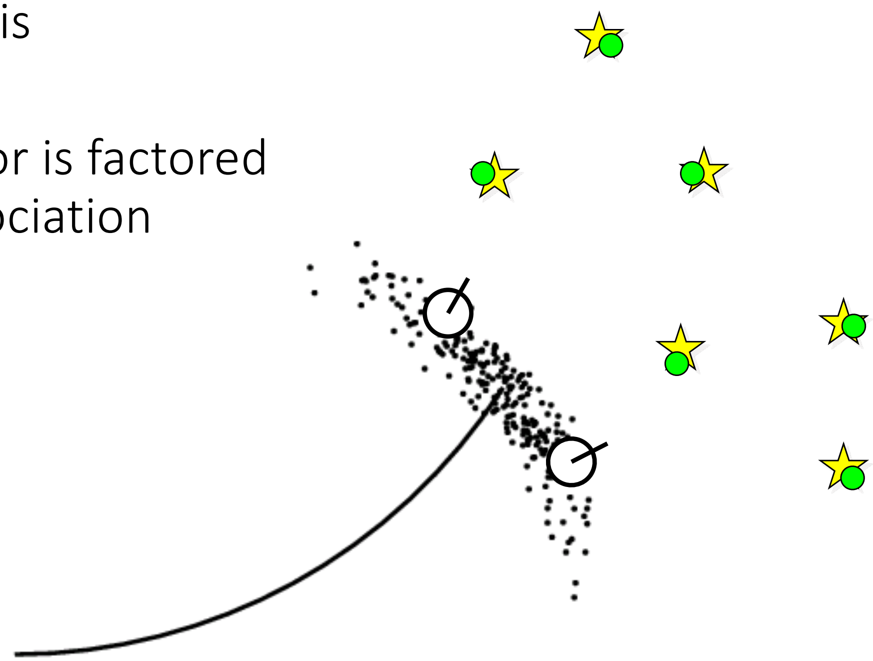
- Which observation belongs to which landmark?



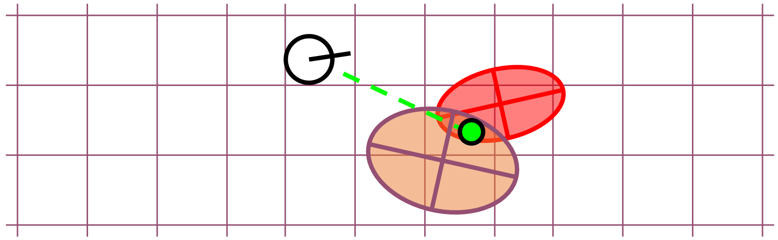
- A robust SLAM must consider possible data associations
- Potential data associations depend also on the pose of the robot

Multi-Hypothesis Data Association

- Data association is done on a per-particle basis
- Robot pose error is factored out of data association decisions



Per-Particle Data Association



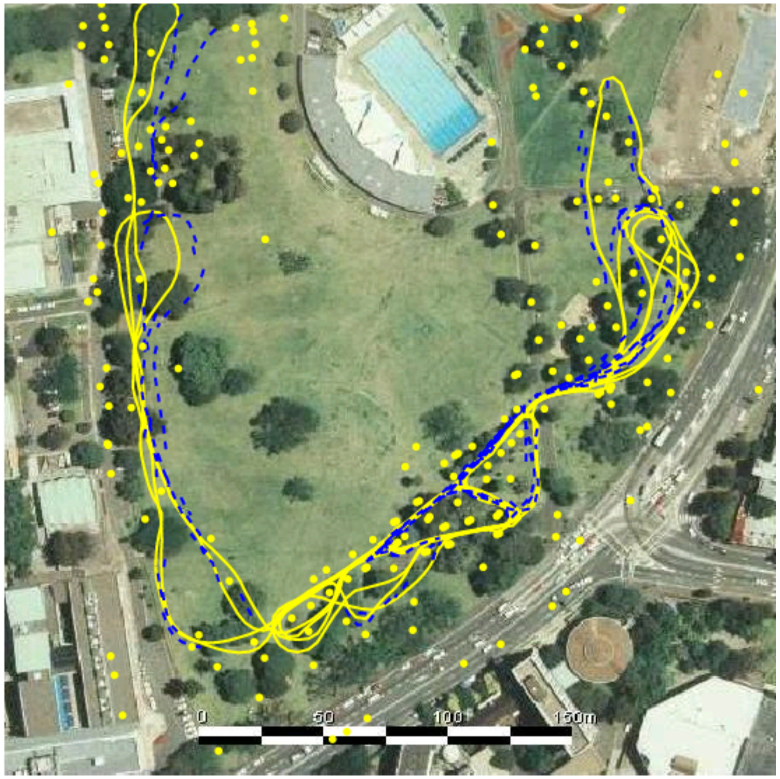
Was the observation generated by the red or the blue landmark?

P(observation|red) = 0.3   P(observation|blue) = 0.7

- Two options for per-particle data association
  - Pick the most probable match
  - Pick an random association weighted by the observation likelihoods
- If the probability is too low, generate a new landmark

Results – Victoria Park

- 4 km traverse
- < 5 m RMS position error
- 100 particles



Dataset courtesy of University of Sydney

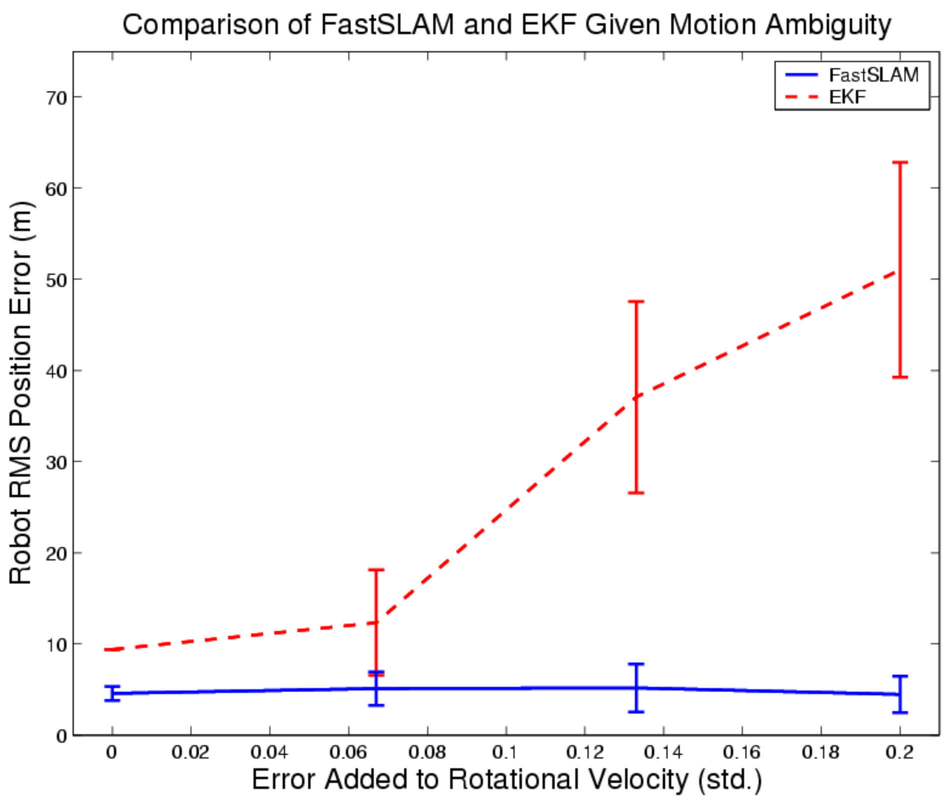
Blue = GPS
Yellow = FastSLAM

Results – Victoria Park

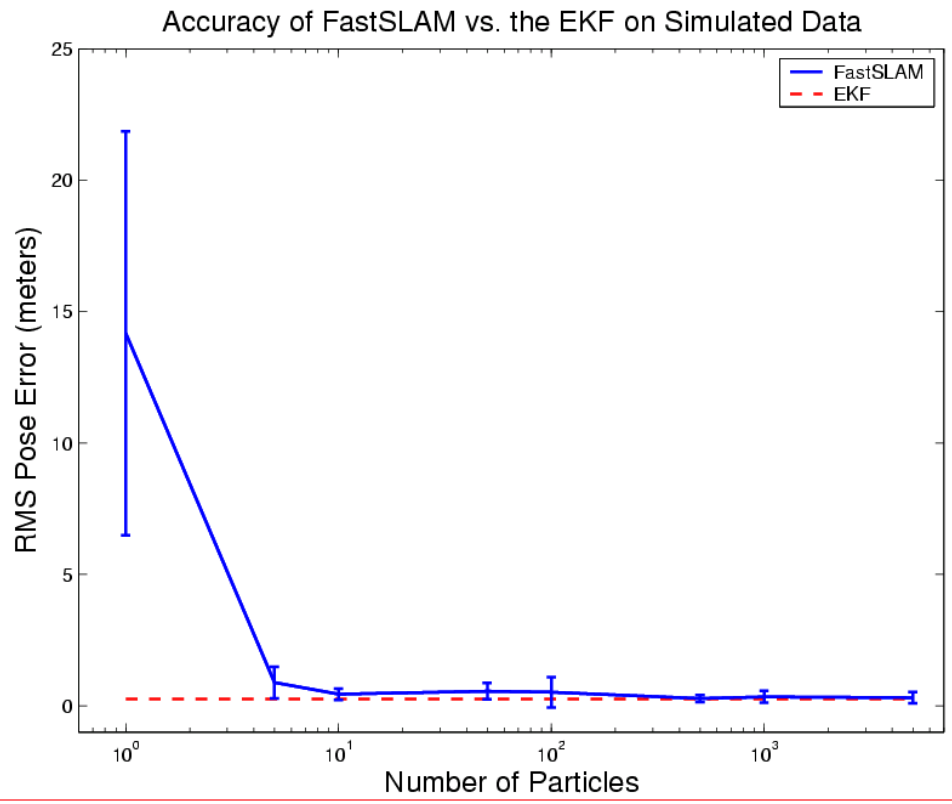
https://www.youtube.com/watch?v=BIOJSNHYSbc

Dataset courtesy of University of Sydney

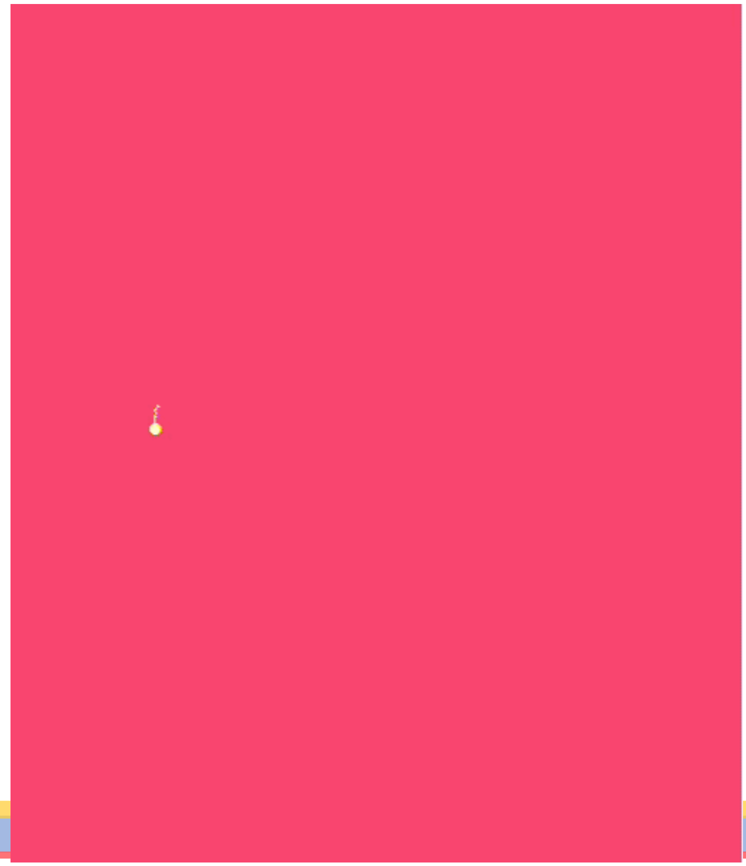
Results – Data Association



Results – Accuracy

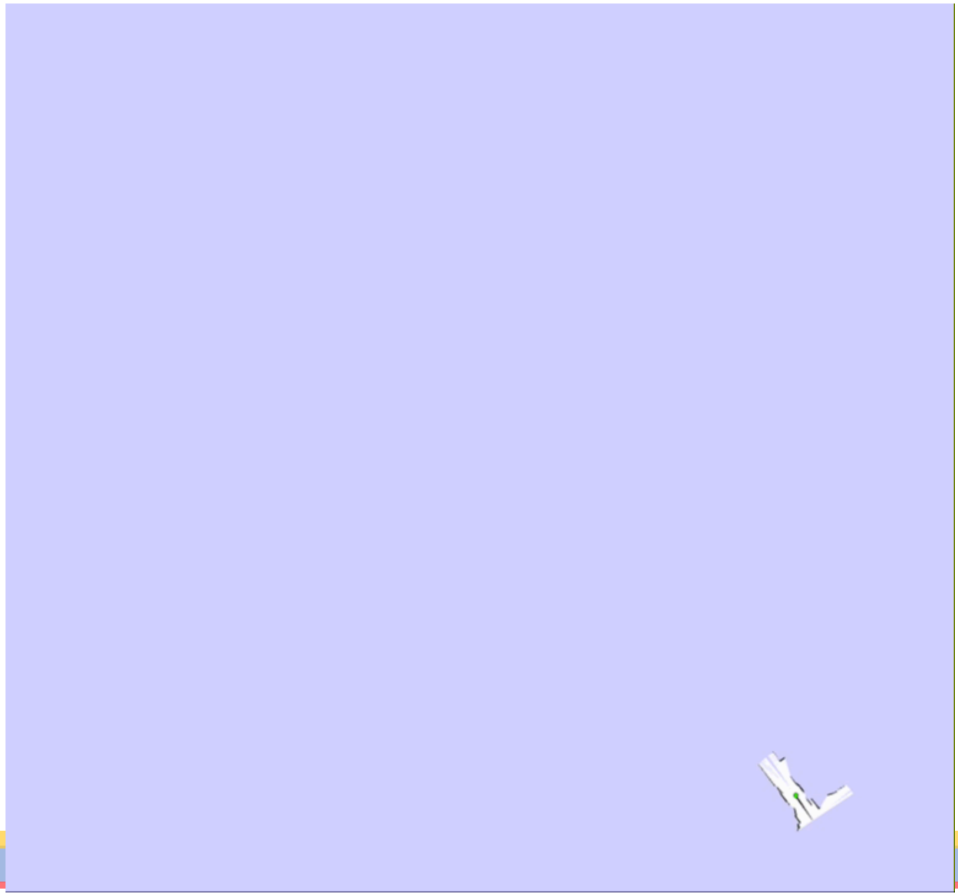


FastSLAM with Scan-Matching



FastSLAM with Scan-Matching

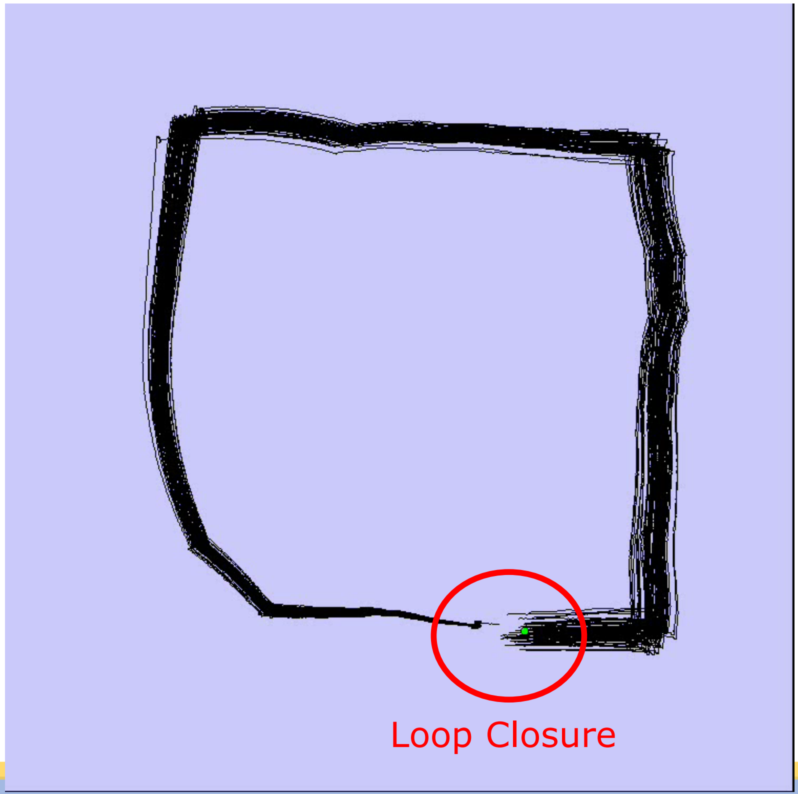
FastSLAM with Scan-Matching



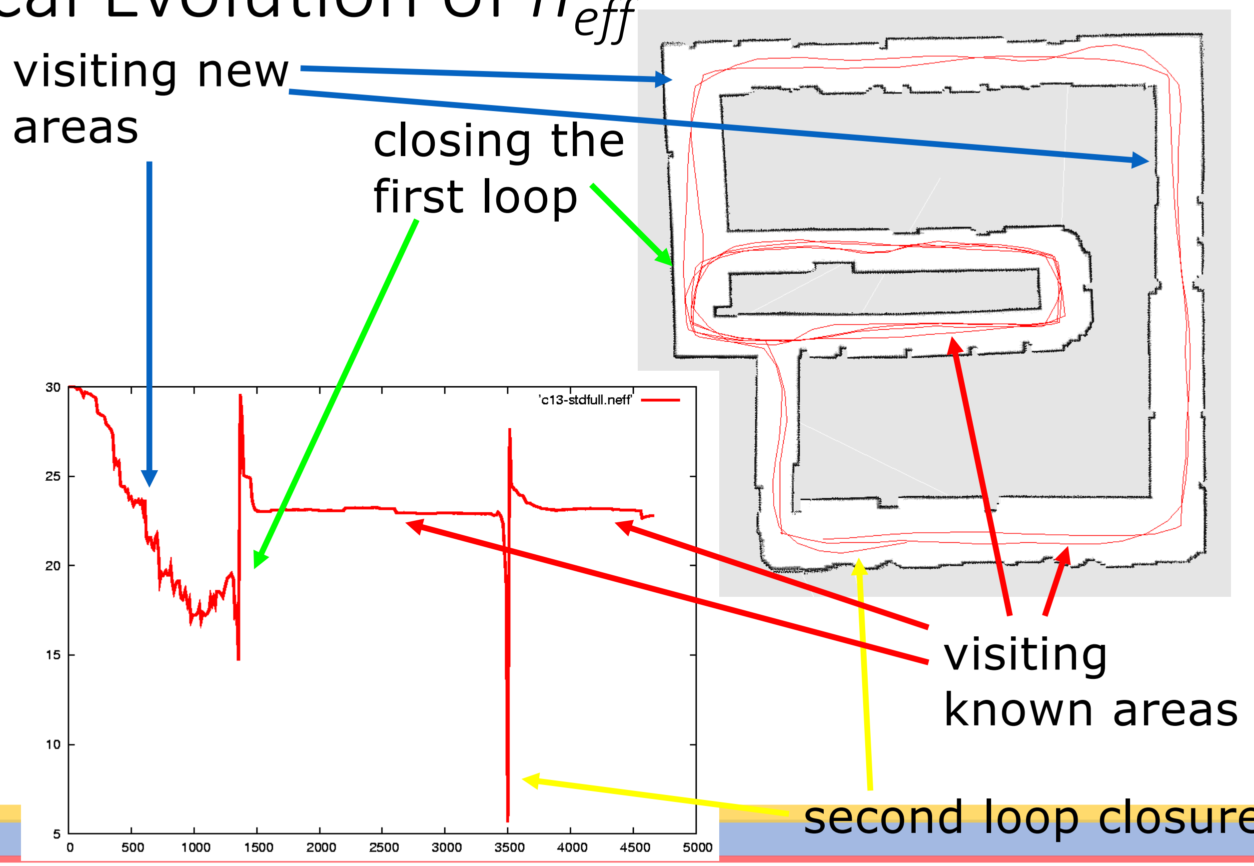
Grid-based SLAM

- Can we solve the SLAM problem if no pre-defined landmarks are available?
- Can we use the ideas of FastSLAM to build grid maps?
- As with landmarks, the map depends on the poses of the robot during data acquisition
- If the poses are known, grid-based mapping is easy ("mapping with known poses")

FastSLAM with Scan-Matching



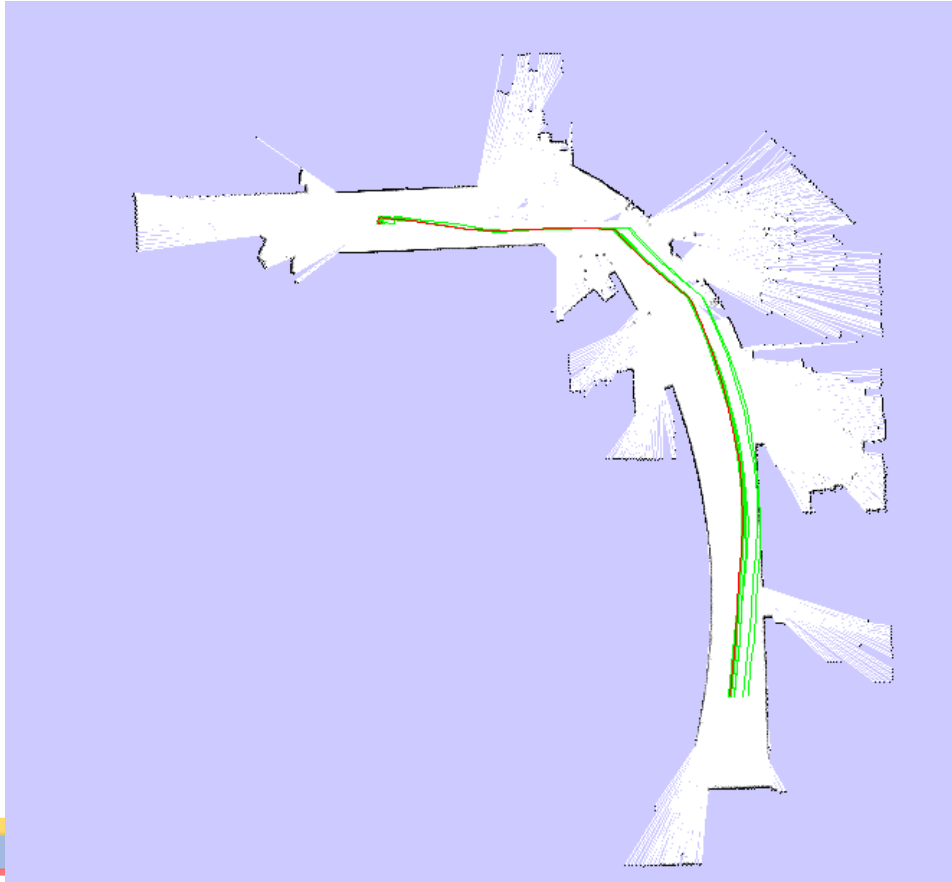
Typical Evolution of n_{eff}



Intel Lab

- **15 particles**
- four times faster than real-time P4, 2.8GHz
- 5cm resolution during scan matching
- 1cm resolution in final map

Intel Lab



- **15 particles**
- Compared to FastSLAM with Scan-Matching, the particles are propagated closer to the true distribution

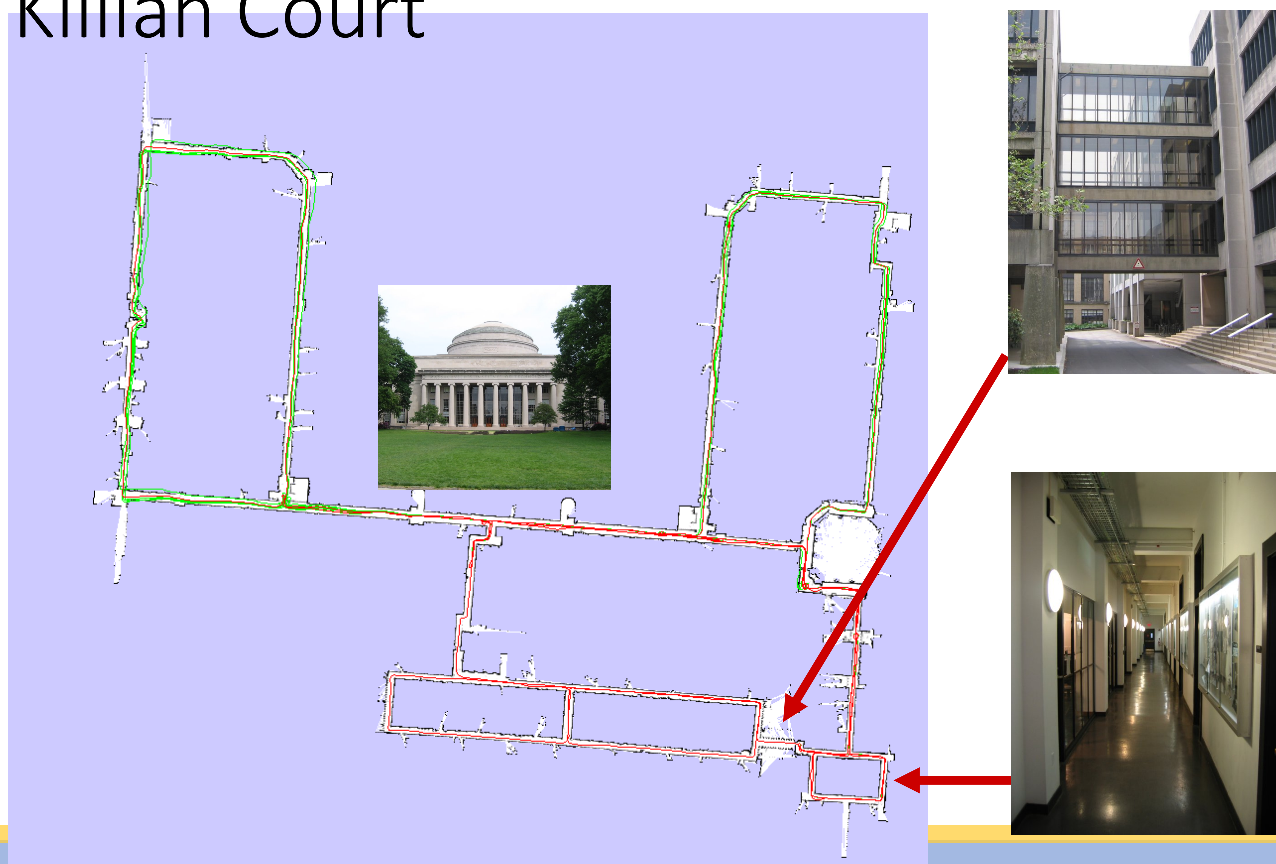
Outdoor Campus Map

- **30 particles**
- $250x250m^2$
- 1.088 miles (odometry)
- 20cm resolution during scan matching
- 30cm resolution in final map

MIT Killian Court

- The **“infinite-corridor-dataset”** at MIT

MIT Killian Court



More Details on FastSLAM

- M. Montemerlo, S. Thrun, D. Koller, and B. Wegbreit. FastSLAM: A factored solution to simultaneous localization and mapping, *AAAI02*

- D. Haehnel, W. Burgard, D. Fox, and S. Thrun. An efficient FastSLAM algorithm for generating maps of large-scale cyclic environments from raw laser range measurements, IROS03

- M. Montemerlo, S. Thrun, D. Koller, B. Wegbreit. FastSLAM 2.0: An Improved particle filtering algorithm for simultaneous localization and mapping that provably converges. IJCAI-2003

- G. Grisetti, C. Stachniss, and W. Burgard. Improving grid-based slam with rao-blackwellized particle filters by adaptive proposals and selective resampling, ICRA05

- A. Eliazar and R. Parr. DP-SLAM: Fast, robust simultanous localization and mapping without predetermined landmarks, IJCAI03

Summary

- Particle filters are an implementation of recursive Bayesian filtering
- They represent the posterior by a set of weighted samples.
- In the context of localization, the particles are propagated according to the motion model.
- They are then weighted according to the likelihood of the observations.
- In a re-sampling step, new particles are drawn with a probability proportional to the likelihood of the observation.